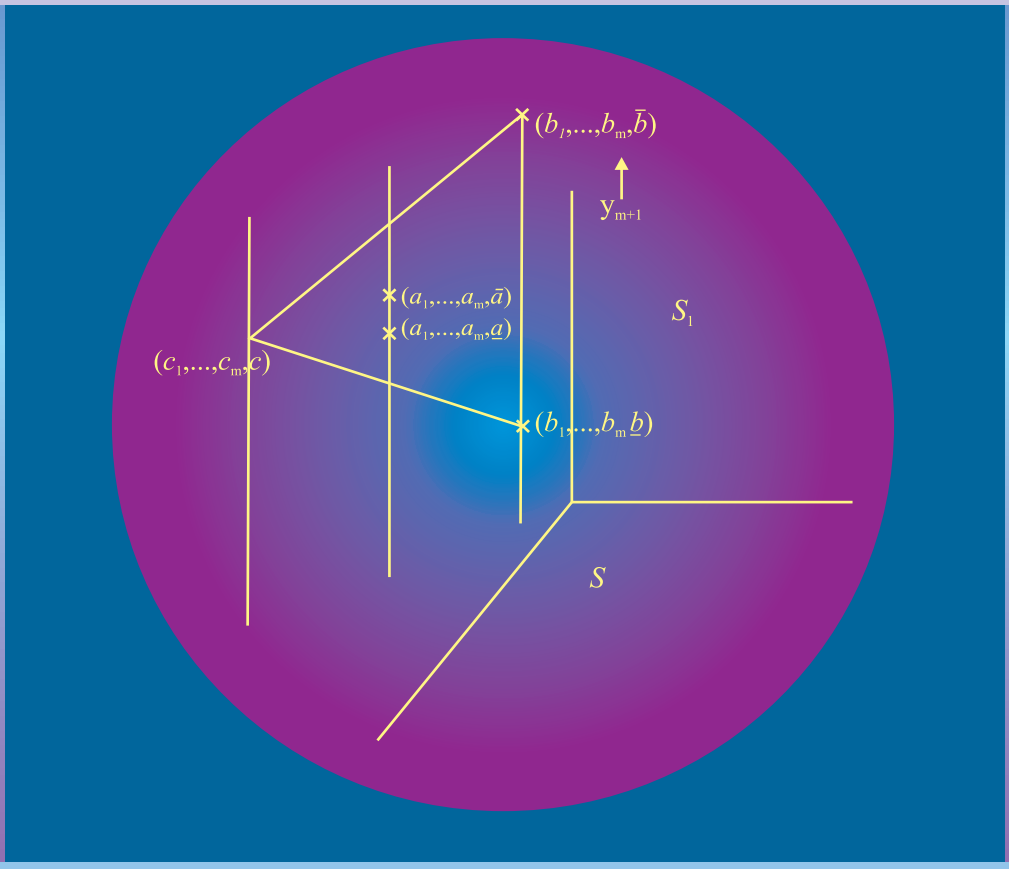
Eastern Economy Edition

STATISTICAL INFERENCE

Testing of Hypotheses



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STATISTICAL INFERENCE
Testing of Hypotheses

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STATISTICAL INFERENCE: TESTING OF HYPOTHESES
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Preface

During our teaching at master's level we realized the need of a book for a core paper on testing of hypotheses, which could cover most of the syllabi of undergraduate and postgraduate courses, and competitive examinations, prescribed by different Universities. We felt that the students require a book which is solely dedicated to testing of hypothesis syllabi, emphasizing on concept building, detailing of proofs to main theorems, their real-life applications, and critical and analytical remarks to explain and develop more insight of the subject. The book has been written to serve these academic requirements of the students.

The text is full semester course on Testing of Hypotheses and non-parametric tests. Chapter 1 of the book assumes knowledge of measure theory and mathematical analysis elaborately.

The questions are framed according to the master's level syllabi of various Indian universities, I.A.S., I.S.S., UGC-NET. The book provides a sufficient number of examples along with the applications of different theorems and results at the end of each chapter. The problems discussed are of varied nature at different levels of difficulty. It is for this reason the book would prove beneficial for those who are preparing for any of the competitive examinations.

Chapter 2 explains the concepts and definitions on decision theoretic setup for hypothesis testing while giving the genesis and proof of Neyman and Pearson lemma. A large number of important application exercises have been given at the end of this chapter.

Chapter 3 deals with treatments given to one-sided problems by Karlin and Rubin (1956) by way of characterizing Polya type-3 family of distributions. Uniformly Most Powerful (UMP) tests, both for Regular and Pitman family, have also been derived with some interesting and critical remarks. A large number of applications are given in support of these theorems.

UMP and UMP unbiased tests were derived for two-sided testing problems in Chapter 4. Further, the class of two-sided tests has been shown

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