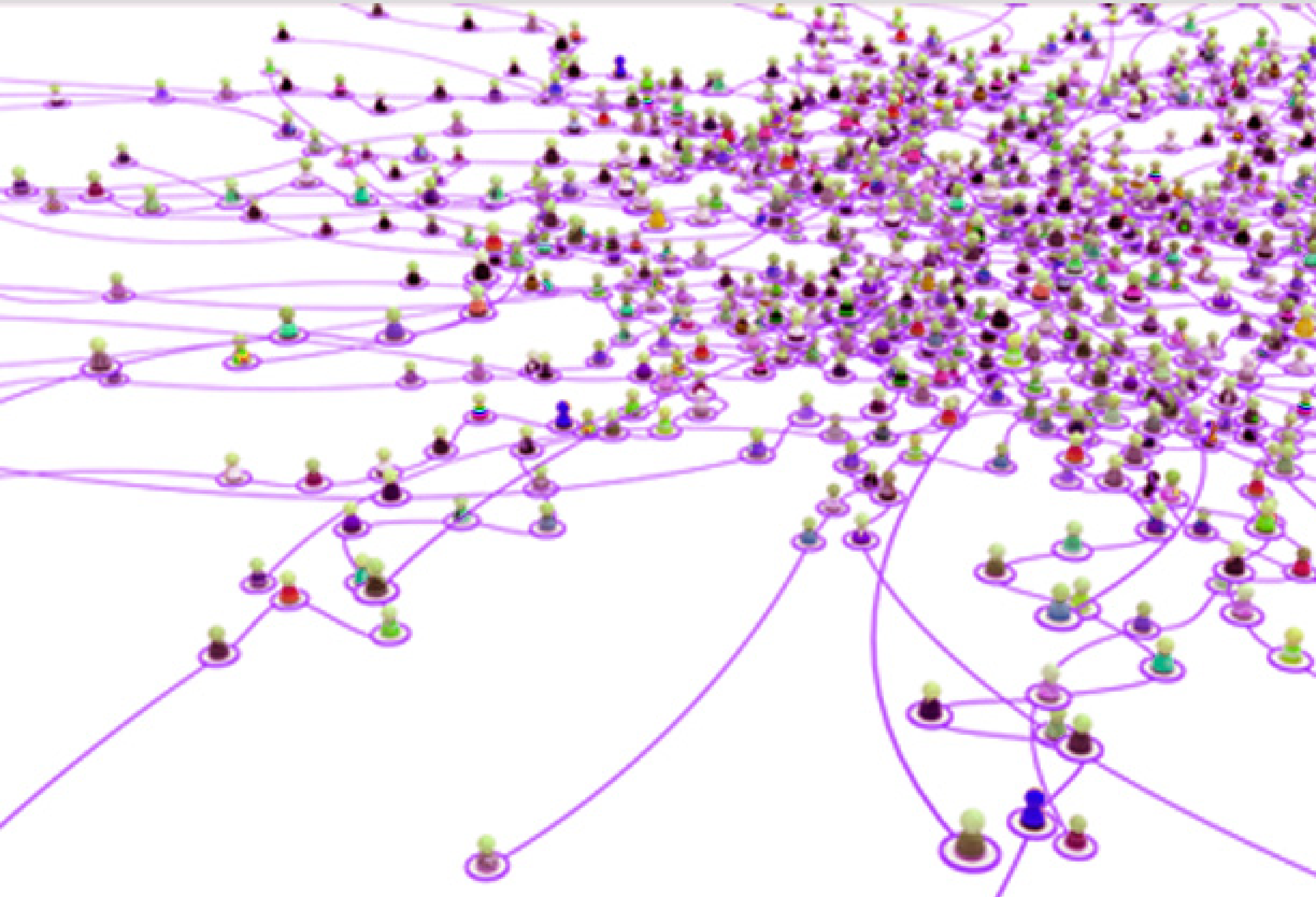
# VTU eNotes On Network Analysis



## Electrical And Electronics Engineering

# Chapter 1 Circuit Concepts and Network Simplification Techniques

## 1.1 Introduction

Today we live in a predominantly electrical world. Electrical technology is a driving force in the changes that are occurring in every engineering discipline. For example, surveying is now done using lasers and electronic range finders.

Circuit analysis is the foundation for electrical technology. An indepth knowledge of circuit analysis provides an understanding of such things as cause and effect, feedback and control and, stability and oscillations. Moreover, the critical importance is the fact that the concepts of electrical circuit can also be applied to economic and social systems. Thus, the applications and ramifications of circuit analysis are immense.

In this chapter, we shall introduce some of the basic quantities that will be used throughout the text. *An electric circuit or electric network is an interconnection of electrical elements linked together in a closed path so that an electric current may continuously flow.* Alternatively, an electric circuit is essentially a pipe-line that facilitates the transfer of charge from one point to another.

## 1.2 Current, voltage, power and energy

The most elementary quantity in the analysis of electric circuits is the electric charge. Our interest in electric charge is centered around its motion results in an energy transfer. Charge is the intrinsic property of matter responsible for electrical phenomena. The quantity of charge $q$ can be expressed in terms of the charge on one electron. which is $-1.602 \times 10^{-19}$ coulombs. Thus, $-1$ coulomb is the charge on $6.24 \times 10^{18}$ electrons. The current flows through a specified area $A$ and is defined by the electric charge passing through that area per unit time. Thus we define $q$ as the charge expressed in coulombs.

***Charge is the quantity of electricity responsible for electric phenomena.***

The time rate of change constitutes an electric current. Mathemetically, this relation is expressed as

$$i(t) = \frac{dq(t)}{dt} \tag{1.1}$$

or

$$q(t) = \int_{-\infty}^{t} i(x)dx \tag{1.2}$$

**The unit of current is ampere(A)**; an ampere is 1 coulomb per second.

***Current is the time rate of flow of electric charge past a given point.***

The basic variables in electric circuits are current and voltage. If a current flows into terminal $a$ of the element shown in Fig. 1.1, then a voltage or potential difference exists between the two terminals $a$ and $b$. Normally, we say that a voltage exists across the element.

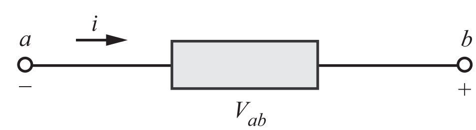


Figure 1.1 Voltage across an element

***The voltage across an element is the work done in moving a positive charge of 1 coulomb from first terminal through the element to second terminal. The unit of voltage is volt, V or Joules per coulomb.***

We have defined voltage in Joules per coulomb as the energy required to move a positive charge of 1 coulomb through an element. If we assume that we are dealing with a differential amount of charge and energy,

then

$$v = \frac{dw}{dq} \tag{1.3}$$

Multiplying both the sides of equation (1.3) by the current in the element gives

$$vi = \frac{dw}{dq}\left(\frac{dq}{dt}\right) \quad \Rightarrow \quad \frac{dw}{dt} = p \tag{1.4}$$

which is the time rate of change of energy or power measured in Joules per second or watts ($W$).

$p$ could be either positive or negative. Hence it is imperative to give sign convention for power. If we use the signs as shown in Fig. 1.2., the current flows out of the terminal indicated by $x$, which shows the positive sign for the voltage. In this case, the element is said to provide energy to the charge as it moves through. Power is then provided by the element.

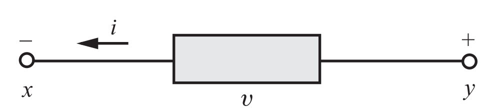


Figure 1.2 An element with the current leaving from the terminal with a positive voltage sign

Conversely, power absorbed by an element is $p = vi$, when $i$ is entering through the positive voltage terminal.

***Energy is the capacity to perform work.*** Energy and power are related to each other by the following equation:

$$Energy = w = \int_{-\infty}^{t} p\ dt$$

EXAMPLE 1.1

Consider the circuit shown in Fig. 1.3 with $v = 8e^{-}t$ V and $i = 20e^{-t}$ A for $t \geq 0$. Find the power absorbed and the energy supplied by the element over the first second of operation. we assume that $v$ and $i$ are zero for $t < 0$.

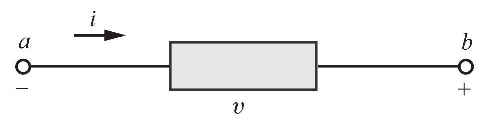



Figure 1.3

SOLUTION

The power supplied is

$$\begin{aligned} p &= vi = (8e^{-t})(20e^{-t}) \\ &= 160e^{-2t}\ \text{W} \end{aligned}$$

The element is providing energy to the charge flowing through it.

The energy supplied during the first seond is

$$\begin{aligned} w &= \int_{0}^{1} p\ dt = \int_{0}^{1} 160e^{-2t} dt \\ &= 80(1 - e^{-2}) = \mathbf{69.17\ Joules} \end{aligned}$$

## 1.3 Linear, active and passive elements

A linear element is one that satisfies the principle of superposition and homogeneity.
In order to understand the concept of superposition and homogeneity, let us consider the element shown in Fig. 1.4.

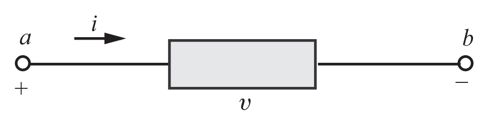



Figure 1.4 An element with excitation $i$ and response $v$

The excitation is the current, $i$ and the response is the voltage, $v$. When the element is subjected to a current $i_1$, it provides a response $v_1$. Furthermore, when the element is subjected to a current $i_2$, it provides a response $v_2$. If the principle of superposition is true, then the excitation $i_1 + i_2$ must produce a response $v_1 + v_2$.

Also, it is necessary that the magnitude scale factor be preserved for a linear element. If the element is subjected to an excitation $\beta i$ where $\beta$ is a constant multiplier, then if principle of homogencity is true, the response of the element must be $\beta v$.

We may classify the elements of a circuir into categories, passive and active, depending upon whether they absorb energy or supply energy.

### 1.3.1 Passive Circuit Elements

An element is said to be passive if the total energy delivered to it from the rest of the circuit is either zero or positive.

Then for a passive element, with the current flowing into the positive (+) terminal as shown in Fig. 1.4 this means that

$$w = \int_{-\infty}^{t} vi\ dt \geq 0$$

Examples of passive elements are resistors, capacitors and inductors.

#### 1.3.1.A Resistors

Resistance is the physical property of an element or device that impedes the flow of current; it is represented by the symbol $R$. Resistance of a wire element is calculated using the relation:

Figure 1.5 Symbol for a resistor R

$$R = \frac{\rho l}{A} \tag{1.5}$$

where $A$ is the cross-sectional area, $\rho$ the resistivity, and $l$ the length of the wire. The practical unit of resistance is ohm and represented by the symbol $\Omega$.

***An element is said to have a resistance of 1 ohm, if it permits 1A of current to flow through it when 1V is impressed across its terminals.***

**Ohm's law**, which is related to voltage and current, was published in 1827 as

$$v = Ri \tag{1.6}$$

or

$$R = \frac{v}{i}$$

where $v$ is the potential across the resistive element, $i$ the current through it, and $R$ the resistance of the element.

The power absorbed by a resistor is given by

$$p = vi = v\left(\frac{v}{R}\right) = \frac{v^2}{R} \tag{1.7}$$

Alternatively,

$$p = vi = (iR)i = i^2R \tag{1.8}$$

Hence, the power is a nonlinear function of current $i$ through the resistor or of the voltage $v$ across it.

The equation for energy absorbed by or delivered to a resistor is

$$w = \int_{-\infty}^{t} p d\tau = \int_{-\infty}^{t} i^2R\ d\tau \tag{1.9}$$

Since $i^2$ is always positive, the energy is always positive and the resistor is a passive element.

#### *1.3.1.B Inductors*

Whenever a time-changing current is passed through a coil or wire, the voltage across it is proportional to the rate of change of current through the coil. This proportional relationship may be expressed by the equation

$$v = L\frac{di}{dt} \tag{1.10}$$

Where $L$ is the constant of proportionality known as inductance and is measured in Henrys (H). Remember $v$ and $i$ are both funtions of time.

Let us assume that the coil shown in Fig. 1.6 has $N$ turns and the core material has a high permeability so that the magnetic fluk $\phi$ is connected within the area $A$. The changing flux creates an induced voltage in each turn equal to the derivative of the flux $\phi$, so the total voltage $v$ across $N$ turns is

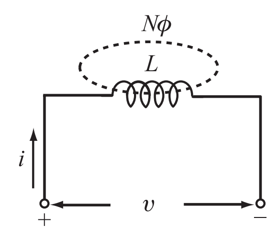


Figure 1.6 Model of the inductor

$$v = N\frac{d\phi}{dt} \tag{1.11}$$

Since the total flux $N\phi$ is proportional to current in the coil, we have

$$N\phi = Li \tag{1.12}$$

Where $L$ is the constant of proportionality. Substituting equation (1.12) into equation(1.11), we get

$$v = L\frac{di}{dt}$$

The power in an inductor is

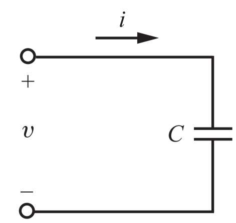
$$p = vi = L\left(\frac{di}{dt}\right)i$$

The energy stored in the inductor is

$$\begin{aligned} w &= \int_{-\infty}^{t} p\ d\tau \\ &= L\int_{i(-\infty)}^{i(t)} i\ di = \frac{1}{2}Li^2 \text{ Joules} \end{aligned} \tag{1.13}$$

Note that when $t = -\infty, i(-\infty) = 0$. Also note that $w(t) \geq 0$ for all $i(t)$, so the inductor is a passive element. The inductor does not generate energy, but only stores energy.

#### *1.3.1.C Capacitors*




1.7 Circuit symbol for a capacitor

A capacitor is a two-terminal element that is a model of a device consisting of two conducting plates seperated by a dielectric material. Capacitance is a measure of the ability of a deivce to store energy in the form of an electric field. ***Capacitance is defined as the ratio of the charge stored to the voltage difference between the two conducting plates or wires,***

$$C = \frac{q}{v}$$

The current through the capacitor is given by

$$i = \frac{dq}{dt} = C\frac{dv}{dt} \tag{1.14}$$

The energy stored in a capacitor is

$$w = \int_{-\infty}^{t} vi \; d\tau$$

Remember that $v$ and $i$ are both functions of time and could be written as $v(t)$ and $i(t)$.

Since
$$i = C\frac{dv}{dt}$$
we have
$$w = \int_{-\infty}^{t} v \; C\frac{dv}{d\tau} \; d\tau$$
$$= C\int_{v(-\infty)}^{v(t)} v \; dv = \frac{1}{2}Cv^2 \Big|_{v(-\infty)}^{v(t)}$$

Since the capacitor was uncharged at $t = -\infty$, $v(-\infty) = 0$.

Hence
$$w = w(t)$$
$$= \frac{1}{2}Cv^2(t) \text{ Joules} \tag{1.15}$$

Since $q = Cv$, we may write

$$w(t) = \frac{1}{2C}q^2(t) \text{ Joules} \tag{1.16}$$

Note that since $w(t) \geq 0$ for all values of $v(t)$, the element is said to be a passive element.

### 1.3.2 Active Circuit Elements (Energy Sources)

An active two-terminal element that supplies energy to a circuit is a source of energy. An ideal voltage source is a circuit element that maintains a prescribed voltage across the terminals regardless of the current flowing in those terminals. Similarly, an ideal current source is a circuit element that maintains a prescribed current through its terminals regardless of the voltage across those terminals.

These circuit elements do not exist as practical devices, they are only idealized models of actual voltage and current sources.

Ideal voltage and current sources can be further described as either independent sources or dependent sources. An independent source establishes a voltage or current in a circuit without relying on voltages or currents elsewhere in the circuit. The value of the voltage or current supplied is specified by the value of the independent source alone. In contrast, a dependent source establishes a voltage or current whose value depends on the value of the voltage or current elsewhere in the circuit. We cannot specify the value of a dependent source, unless you know the value of the voltage or current on which it depends.

The circuit symbols for ideal independent sources are shown in Fig. 1.8.(a) and (b). Note that a circle is used to represent an independent source. The circuit symbols for dependent sources are shown in Fig. 1.8.(c), (d), (e) and (f). A diamond symbol is used to represent a dependent source.

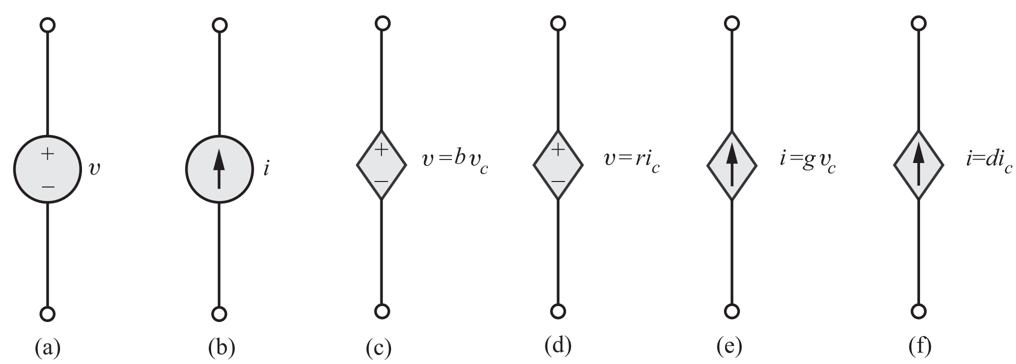



Figure 1.8 (a) An ideal independent voltage source
(b) An ideal independent current source
(c) voltage controlled voltage source
(d) current controlled voltage source
(e) voltage controlled current source
(f) current controlled current source

## 1.4 Unilateral and bilateral networks

A Unilateral network is one whose properties or characteristics change with the direction. An example of unilateral network is the semiconductor diode, which conducts only in one direction.

A bilateral network is one whose properties or characteristics are same in either direction. For example, a transmission line is a bilateral network, because it can be made to perform the function equally well in either direction.

## 1.5 Network simplification techniques

In this section, we shall give the formula for reducing the networks consisting of resistors connected in series or parallel.

### 1.5.1 Resistors in Series

When a number of resistors are connected in series, the equivalent resistance of the combination is given by

$$R = R_1 + R_2 + \cdots + R_n \tag{1.17}$$

Thus the total resistance is the algebraic sum of individual resistances.

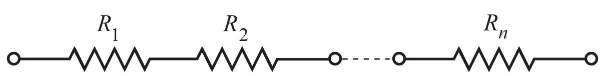



Figure 1.9 Resistors in series

### 1.5.2 Resistors in Parallel

When a number of resistors are connected in parallel as shown in Fig. 1.10, then the equivalent resistance of the combination is computed as follows:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ....... + \frac{1}{R_n} \tag{1.18}$$

Thus, the reciprocal of a equivalent resistance of a parallel combination is the sum of the reciprocal of the individual resistances. Reciprocal of resistance is conductance and denoted by $G$. Consequently the equivalent conductance,

$$G = G_1 + G_2 + \cdots + G_n$$

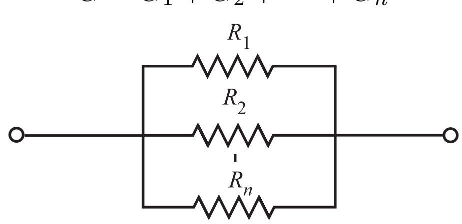



Figure 1.10 Resistors in parallel

### 1.5.3 Division of Current in a Parallel Circuit

Consider a two branch parallel circuit as shown in Fig. 1.11. The branch currents $I_1$ and $I_2$ can be evaluated in terms of total current $I$ as follows:

$$I_1 = \frac{IR_2}{R_1 + R_2} = \frac{IG_1}{G_1 + G_2} \tag{1.19}$$

$$I_2 = \frac{IR_1}{R_1 + R_2} = \frac{IG_2}{G_1 + G_2} \tag{1.20}$$

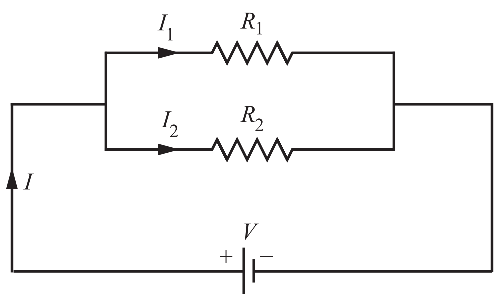



Figure 1.11 Current division in a parallel circuit

That is, current in one branch equals the total current multiplied by the resistance of the other branch and then divided by the sum of the resistances.

**EXAMPLE 1.2**

The current in the $6\Omega$ resistor of the network shown in Fig. 1.12 is 2A. Determine the current in all branches and the applied voltage.

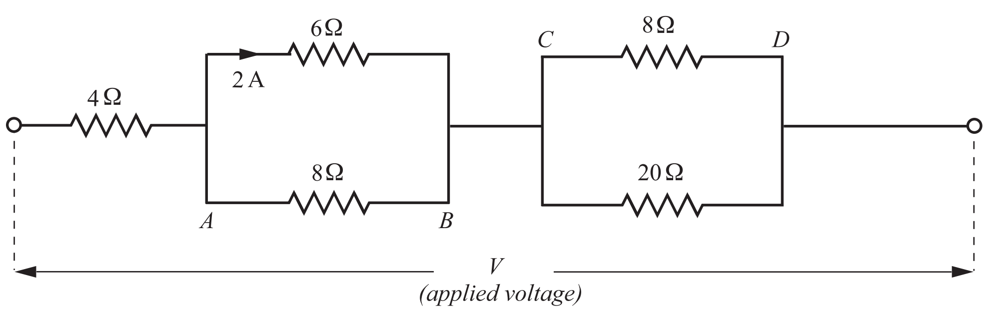



Figure 1.12

**SOLUTION**

Voltage across $6\Omega = 6 \times 2$

$= 12$ volts

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