ENGINEERING MATHEMATICS

(FOR COCHIN UNIVERSITY OF SCIENCE AND TECHNOLOGY, KERALA)
SEMESTER I & II

N.P. Bali
Dr. Remadevi.S

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SYLLABUS

For B. Tech. Semester I & II
Cochin University of Science and Technology, Kerala

CE/CS/EB/EC/EE/EI/IT/ME/SE 1101 ENGINEERING MATHEMATICS–I

Module I: Ordinary Differential Equations

First order differential equations-exact differential equations, Bernoulli's equation—Methods of solution and Simple applications.

Linear differential equations of higher orders with constant co-efficient-Methods of solution of these equations. Cauchy's Linear differential equations. Simultaneous linear differential equations—Simple applications of linear differential equations in engineering problems— Electrical Circuits, Mechanical Systems.

Module II: Infinite Series

Integral test, comparison test, ratio test, Cauchy's root test, Raabe's test, series of positive and negative terms, concept of absolute convergence, alternating series, Leibnitz test (No proofs for any of the above tests).

Power Series

Taylor's and Maclaurin's series of functions, Leibnitz formula for the nth derivative of the product of two functions (No proof), use of Leibnitz formula for the determination of co-efficients of the power series.

Module III: Partial Differentiation

Partial differentiation—Concept of partial derivative—Chain rule—Total derivative—Euler's theorem for homogeneous functions, Differentials and their applications in errors and approximations, Jacobians—Maxima minima of functions of two variables (Proof of the result not required)—Simple applications.

Co-ordinate Systems

Rectangular co-ordinates—Polar co-ordinates—In plane and in Space—Cylindrical polar co-ordinates—Spherical polar co-ordinates.

Module IV: Integral Calculus

Application of definite integrals: Area, Volume, Arc length, Surface area.

Multiple integrals: Evaluation of double integrals—Change of order of integration. Evaluation of triple integrals—Change of Variables in integrals.

Applications of multiple integrals Plane Area, Surface area and Volumes of solids.

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PREFACE TO THE SECOND EDITION

With the grace of the Almighty and the over whelming response given by the teachers and the students, the book is gaining increasing popularity. We express our sincerest thanks and sense of gratitude for their appreciation of the book.

The new edition has been revised with lot of care, dedication and patience. The book has been divided into four modules containing 13 chapters and covers the entire portion in the B.Tech. First Year Cochin University Engineering Mathematics–I revised new syllabus. Many new types of questions have been added, in examples as well as in exercises, to enhance the utility of the book. The book presents the subject matter in a very systematic, simple and lucid style, so that students can solve the questions on their own. To make the students faimiliar with the university pattern, all the questions set in university papers have been included.

The most outstanding and distinguishing feature of the book is the large number of typical solved examples followed by well graded problems.

All suggestions for further improvement of the book will be highly appreciated and gratefully acknowledged.

—Authors

PREFACE TO THE FIRST EDITION

This book is a part of the original book **"A Textbook of Engineering Mathematics"** (*with 28 chapters and covering the syllabi of engineering courses of all semesters of all the Indian Universities*) running its seventh edition and very well received by the students and teachers of all Indian Universities. The rapid sale of the seventh edition bears testimony to the overwhelming response. We thank them all for the appreciation. Some topics included in this book were taken from the book **"Differential Calculus"** and **"Integral Calculus"** by N.P. Bali.

The present form of the book is divided into four modules containing 14 chapters and covers the entire portion in the B.Tech. First Year Cochin University Engineering Mathematics-I syllabus. Problems taken from Cochin University Question Papers are included with solutions in each unit.

There is no dearth of books on Engineering Mathematics but the students find it difficult to solve most of the problems in the exercise in the absence of adequate number of solved examples. An outstanding and distinguishing feature of the book is the large number of typical solved examples followed by well-graded problems.

We have endeavoured to present the fundamental concepts in a comprehensive and lucid manner. We are indebted to all authors, Indian and Foreign, whose works we have frequently consulted.

All efforts have been made to keep the book free from errors. Answers to all exercises have been re-checked. All suggestions for improvement will be highly appreciated and gratefully acknowledged.

—Authors

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MODULE–I

DIFFERENTIAL EQUATIONS AND APPLICATIONS

1

Differential Equations of First Order

1.1. DEFINITIONS

(*i*) A **differential equation** is an equation involving differentials or differential coefficients.

Thus, $$\frac{dy}{dx} = x^2 - 1 \qquad ...(1)$$

$$\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + y = 0 \qquad ...(2)$$

$$(x^2 + y^2 - 3y)\,dx = (x^2 + 3x + y)\,dy \qquad ...(3)$$

$$y = x\frac{dy}{dx} + \frac{c}{\frac{dy}{dx}} \qquad ...(4)$$

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2}\cdot\frac{dy}{dx} + x^2\left(\frac{dy}{dx}\right)^3 = 0 \qquad ...(5)$$

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = k\frac{d^2y}{dx^2} \qquad ...(6)$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu \qquad ...(7)$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = x + y \qquad ...(8)$$

are all differential equations.

(*ii*) Differential equations which involve only one independent variable and the differential co-efficients with respect to it are called **ordinary differential equations.**

Thus equations (1) to (6) are all ordinary differential equations.

(*iii*) Differential equations which involve two or more independent variables and partial derivatives with respect to them are called **partial differential equations.**

Thus equations (7) and (8) are partial differential equations.

(*iv*) The **order** of a differential equation is the order of the highest ordered derivative occurring in the differential equation.

Thus equations (1), (3) and (4) are of first order ; equations (2) and (6) are of the second order while equation (5) is of the third order.

(*v*) The **degree** of a differential equation is the degree of the highest ordered derivative which occurs in it provided the equation has been made free of the radical signs and fractional powers as far as the derivatives are concerned.

Thus, equations (1), (2), (3) and (5) are of the first degree.

Equation (4) is $$y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2 + c$$

It is of the second degree.

Equation (6) is $$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = k^2\left(\frac{d^2y}{dx^2}\right)^2$$

It is of the second degree.

(*vi*) **Solution of a Differential Equation.** A solution (or integral) of a differential equation is a relation, free from derivatives, between the variables which satisfies the given equation.

Thus if $y = f(x)$ be the solution, then by replacing y and its derivatives with respect to x, the given differential equation will reduce to an identity.

For example, $$y = c_1 \cos x + c_2 \sin x$$

is the solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$

Since $$\frac{dy}{dx} = -c_1 \sin x + c_2 \cos x$$

$$\frac{d^2y}{dx^2} = -c_1 \cos x - c_2 \sin x = -y.$$

The **general** (or **complete**) **solution** of a differential equation is that in which the number of independent arbitrary constants is equal to the order of the differential equation.

Thus, $y = c_1 \cos x + c_2 \sin x$ (involving two arbitrary constants c_1, c_2) is the general solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$ of second order.

A **particular solution** of a differential equation is that which is obtained from its general solution by giving particular values to the arbitrary constants.

For example, $y = c_1e^x + c_2e^{-x}$ is the general solution of the differential equation $\frac{d^2y}{dx^2} - y = 0$, whereas $y = e^x - e^{-x}$ or $y = e^x$ are its particular solutions.

The solution of a differential equation of nth order is its particular solution if it contains less than n arbitrary constants.

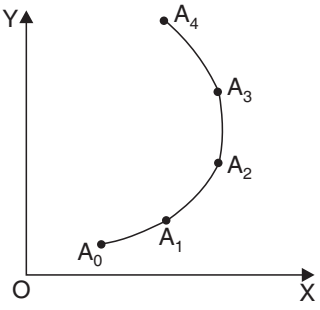
1.2. GEOMETRICAL MEANING OF A DIFFERENTIAL EQUATION OF THE FIRST ORDER AND FIRST DEGREE

Let $$f\left(x, y, \frac{dy}{dx}\right) = 0 \quad \ldots(1)$$

be a differential equation of the first order and first degree.

We know that the direction of a curve at a particular point is determined by drawing a tangent line at that point, *i.e.*, its slope is given by $\frac{dy}{dx}$ at that particular point.

Let $A_0\ (x_0, y_0)$ be any point in the plane. Let $m_0 = \frac{dy_0}{dx_0}$ be the slope of the curve at A_0 derived from (1). Take a neighbouring point $A_1\ (x_1, y_1)$ such that the slope of A_0A_1 is m_0. Let $m_1 = \frac{dy_1}{dx_1}$ be the slope of the curve at A_1 derived from (1). Take a neighbouring point $A_2\ (x_2, y_2)$ such that the slope of A_1A_2 is m_1. Continuing like this, we get a succession of points. If the points are taken sufficiently close to each other, they approximate a smooth curve $C : y = \phi(x)$ which is a solution of (1) corresponding to the initial point $A_0\ (x_0, y_0)$. Any point on C and the slope of the tangent at that point satisfy (1). If the moving point starts at any other point, not on C and moves as before, it will describe another curve. The equation of each such curve is a *particular solution* of the differential equation (1). The equation of the system of all such curves is the general solution of (1).



1.3. FORMATION OF A DIFFERENTIAL EQUATION

Differential equations are formed by elimination of arbitrary constants. To eliminate two arbitrary constants, we require two more equations besides the given relation, leading us to second order derivatives and hence a differential equation of the second order. Elimination of n arbitrary constants leads us to n^{th} order derivatives and hence a differential equation of the nth order.

ILLUSTRATIVE EXAMPLES

Example 1. *Eliminate the constants from the equation* $y = e^x (A \cos x + B \sin x)$ $\ldots(1)$ *and obtain the differential equation.*

Sol. There are two arbitrary constants A and B in equation (1).

Differentiating (1) w.r.t. x, we have

$$\frac{dy}{dx} = e^x (A \cos x + B \sin x) + e^x (-A \sin x + B \cos x)$$

$$= y + e^x (-A \sin x + B \cos x) \quad \ldots(2)$$

Differentiating again w.r.t. x, we have

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x)$$

$$= \frac{dy}{dx} + \left(\frac{dy}{dx} - y\right) - y \qquad \text{[Using (1) and (2)]}$$

or $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

which is the required differential equation.

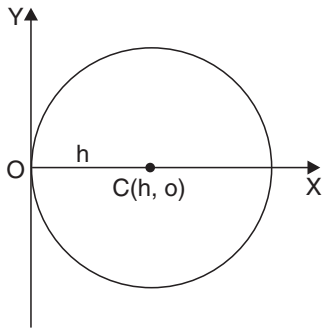
Example 2. *Find the differential equation of all circles touching the axis of y at the origin and centres on the axis of x.*

Sol. The equation of such a circle is

$$(x-h)^2 + y^2 = h^2$$

or $$x^2 + y^2 - 2hx = 0 \qquad \ldots(1)$$

where h is the only arbitrary constant.



Differentiating (1) w.r.t. x, we have

$$2x + 2y\frac{dy}{dx} - 2h = 0$$

or $$h = x + y\frac{dy}{dx}$$

Substituting the value of h in (1), we have

$$x^2 + y^2 - 2x\left(x + y\frac{dy}{dx}\right) = 0$$

or $$2xy\frac{dy}{dx} + x^2 - y^2 = 0$$

which is the required differential equation.

Example 3. *Form the differential equation of all circles of radius a.*

Sol. The equation of any circle of radius a is $(x-h)^2 + (y-k)^2 = a^2$ ...(1)

where (h, k) the coordinates of the centre are arbitrary.

Differentiating (1) w.r.t. x, we have $2(x-h) + 2(y-k)\frac{dy}{dx} = 0$

or $$(x-h) + (y-k)\frac{dy}{dx} = 0 \qquad \ldots(2)$$

Differentiating again, we have $$1 + (y-k)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0 \qquad \ldots(3)$$

From (3), $$y - k = -\frac{1 + \left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$$

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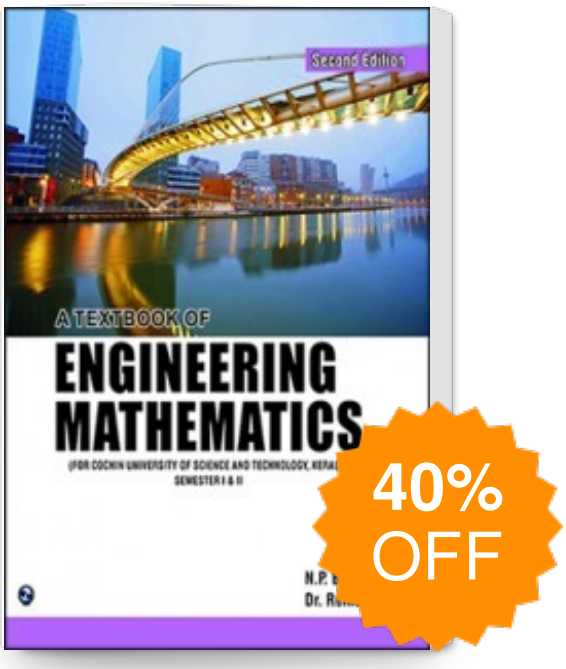

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