ENGINEERING MECHANICS

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Chapter-1

MECHANICAL PROPERTIES OF MATERIALS

1.1 LOAD

All the external forces acting on a body are called load. On the basis of load the machine or structure is designed. The various possible forces are:

(*i*) Dead weight

(*ii*) Inertia force

(*iii*) Frictional force

(*iv*) Centrifugal force, etc.

1.2 EFFECT OF A LOAD ON A MEMBER

When load is applied on a body, one or more than one effect from following can be produced on body.

(*i*) Tension

(*ii*) Compression

(*iii*) Shearing

(*iv*) Twisting

(*v*) Bending

(*vi*) Cracking

1.3 STRESS

When any external force is applied on a body, an internal resistance force developed within the body. This internal resistance force per unit area is called stress. The intensity of this internal resistance is equal to force applied.

$$\sigma = \frac{P}{A} \qquad ...(1.1)$$

where, P = Force applied in N

$$A = \text{Area of cross seaction in m}^2$$
$$\sigma = \text{Stress in N/m}^2 \text{ or Pa or bar}$$
$$1 \text{ bar} = 10^5 \text{ Pa}$$
$$1 \text{ Pa} = 1 \text{ N/m}^2$$

The stresses can be of two types.

(*i*) **Normal stress:** It can be sub divide into two categories:

(*a*) **Tensile stress:** When two equal and opposite pulls are applied axially on a body such that body tends to elongate as shown in Fig.1.1, the stress produced is known as tensile stress.

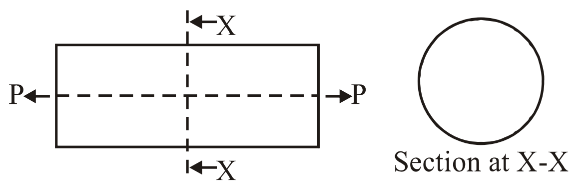

Fig. 1.1: Tensile Stress

Tensile stress is represented by σ_t.

$$\therefore \qquad \sigma_t = \frac{P}{A} \qquad \text{...(1.2)}$$

(*b*) **Compressive stress:** When two equal and opposite pushes are applied axially on a body such that body tends to Shorten in length as shown in Fig. 1.2, the stress produced is known as compressive stress. It is represented by σ_c.

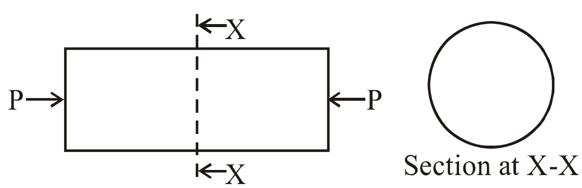

Fig. 1.2: Compressive Stress

$$\sigma_c = \frac{P}{A} \qquad \text{...(1.3)}$$

(*ii*) **Shear stress:** When equal and opposite forces act tangentially on a body, tending to slide its one part over the other at that plane, the stress developed is known as shear stress. *e.g.*, in Riveted Joint, as shown in Fig. 1.3.

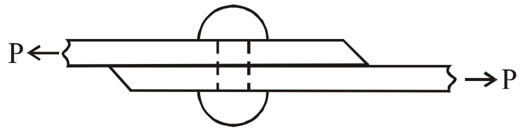

Fig. 1.3: Failure of Rivet in Shear

Consider a Fig. 1.4 in which a body LMNO is fixed with base. When a force P is applied tangentially on face LM, it will distort by angle ϕ and face LM will become L_1M_1.

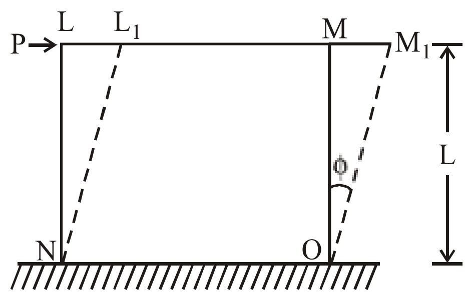

Fig. 1.4

So, shear stress $$\tau = \frac{P}{A}$$

$$= \frac{P}{LM} \quad \text{...(1.4) (Area} = LM \times 1)$$

1.4 STRAINS

Strain is defined as deformation in shape of a body due to load. It is a dimension less quantity. It can also be defined as the ratio of change in dimension per unit original dimension.

i.e., $$\text{strain } \varepsilon = \frac{\text{Change in dimensions}}{\text{Original Dimensions}} \quad \text{...(1.5)}$$

Strain can be classified in to two categories.

(i) **Normal strain:** It can be divided into two sub categories, one is tensile strain and other is compressive strain. The ratio of extension in length per unit original length is called tensile strain and the ratio of shorten in length per unit original length is called compressive strain.

Tensile strain $$\varepsilon_T = \frac{\text{Increase in length}}{\text{Oringinal length}} = \frac{\Delta L}{L} \quad \text{...(1.6)}$$

Compressive strain $$\varepsilon_C = \frac{\text{Shorten in length}}{\text{Original length}} = \frac{\Delta L}{L} \quad \text{...(1.7)}$$

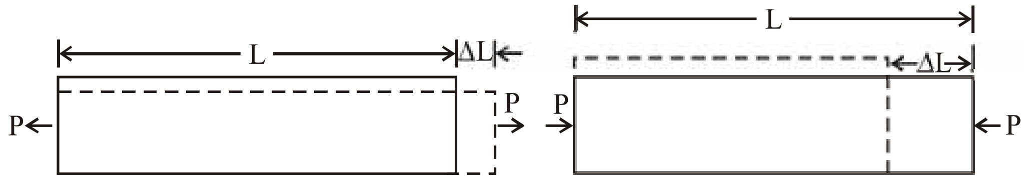

Fig. 1.5

(*ii*) Shear strain: It is a measurement of angle through which a body deformed by applied forces as shown in Fig. 1.4.

$$\text{Shear strain} = \frac{MM_1}{OM} = \tan\phi = \phi \qquad \text{...(1.8) (when } \phi \text{ is very small)}$$

1.5 VOLUMETRIC STRAIN

When a uniform force is applied on all three axis of a body, then all the three dimensions will change or there is a change in volume. The ratio of change in volume per unit original volume is known as volumetric strain.

$$\therefore \text{ Volumetric strain } \varepsilon_V = \frac{\text{Change in volume}}{\text{Original volume}}$$

Let us consider a rectangular block of length *l*, width *b* and height *h* as shown in Fig. 1.6. This block is subjected to tensile force on X-X', Y-Y' and Z-Z' axis.

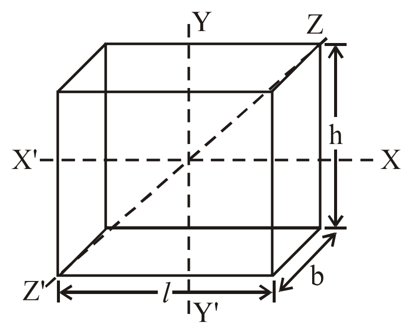

Fig. 1.6: Rectangular Block

Strain in X-X' direction

$$\varepsilon_X = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta l}{l}$$

$$\therefore \qquad \text{Increase in length} = \Delta l = \varepsilon_X l$$

$$\text{Total increased length} = l + \varepsilon_X l = l(1 + \varepsilon_X)$$

$$\text{Similarly increased width} = b(1 + \varepsilon_Z)$$

$$\text{Similarly increased height} = h(1 + \varepsilon_Y)$$

$$\text{Total increased volume} = l \times b \times h(1 + \varepsilon_X)(1 + \varepsilon_Z)(1 + \varepsilon_Y)$$

$$= lbh(1 + \varepsilon_X + \varepsilon_Y + \varepsilon_Z)$$

(Since ε_X, ε_Y and ε_Z are very small, so product of their will also be very small and can be neglected)

$$\text{Change in volume} = \text{Increased volume} - \text{Original volume}$$
$$= lbh(1+\varepsilon_X+\varepsilon_Y+\varepsilon_Z) - lbh$$
$$= lbh(\varepsilon_X+\varepsilon_Y+\varepsilon_Z)$$

$$\text{Volumetric strain} = \varepsilon_V = \frac{\text{Change in volume}}{\text{Original volume}}$$

$$\text{Volumetric strain} = \frac{lbh(\varepsilon_X+\varepsilon_Y+\varepsilon_Z)}{lbh}$$

$$\varepsilon_V = \varepsilon_X+\varepsilon_Y+\varepsilon_Z \quad \text{...(1.9)}$$

Equation 1.9: Shows that volumetric strain is the algebric sum of all the axial strains. It should be kept in mind that tensile strain considered with positive sign and compressive strain considered with negative sign.

1.6 POISSON'S RATIO

When a force is applied on a body it experienced longitudinal change and also lateral change. The ratio of lateral strain to the longitudinal strain is a constant and is known as Poisson's Ratio. It is denoted by v or $\frac{1}{m}$.

$$\therefore \quad \text{Poisson's ratio} = v = \frac{1}{m} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} \quad \text{...(1.10)}$$

For most of materials, its value lies between $\frac{1}{3}$ to $\frac{1}{4}$. For cork its value is zero. It may be noted that in any case the value of v cannot be greater than 0.5. This can be prooved as follows:

Consider a cube of unit size subjected to an axial pure P in X-direction.

the strain in X direction = ε

The strain in Y and Z direction = $-\varepsilon v$ in each direction

Final length in X-direction = $1+\varepsilon$

Final length in Y-direction = $1-\varepsilon \cdot v$

Final length in Z-direction = $1-\varepsilon v$

Now, New Volume of cube = $(1+\varepsilon)(1-\varepsilon v)(1-\varepsilon v)$

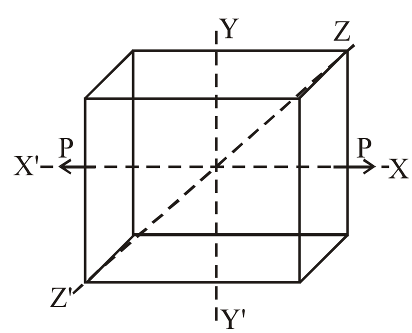

Fig. 1.7: Cube of Unit Size

$$= 1+\varepsilon-2\varepsilon v+......$$

as ε is very small so ε^2 and higher power will be very small and can be neglected.

$\therefore$ Change in volume = New volume – Original volume

$$= (1+\varepsilon-2\varepsilon v)-1$$

$$= \varepsilon-2\varepsilon v$$

Which is always greater than zero

So, $\varepsilon-2\varepsilon v > 0$

or $\varepsilon > 2\varepsilon v$

or $v < 0.5$ ...(1.11)

1.7 ELASTICITY AND ELASTIC LIMIT

Elasticity is the property of material. A material is said to be be perfectly elastic only when strain or deformation produced by any external force, disappears completely after removal of force.

As the material is loaded again and again with slight increase in load every time, There is a limiting value of load within in which the deformation disappears totally. As that the load increase from that limiting value the material never regain its original shape. There is a permanent deformation. This limiting value is known as elastic limit.

1.8 HOOK'S LAW

Hook's law states that strain is directly proportional to stress within elastic limit.

$$\text{Stain } \alpha \text{ Stress}$$

$$\frac{\text{Stress}}{\text{Strain}} = \text{constant}, E \qquad ...(1.12)$$

where E = Young's modulus of elasticity

1.9 MODULUS

There are three modulus

(*a*) Young's modulus of elasticity: It is defined as the ratio of normal stress to normal strain.

$$E = \frac{\text{Stress}}{\text{Strain}} \qquad ...(1.13)$$

The unit of this modulus is N/m^2 or Pa.

(*b*) Modulus of Rigidity: It is defined as the ratio of shear stress to shear strain. It is denoted by *G* or *C*.

$$G = \frac{\text{Shear Stress}}{\text{Shear Strain}} \quad \text{...(1.14)}$$

The unit of this modulus is also N/m^2 or Pa.

(*c*) Bulk Modulus: It is defined as the ratio normal stress to volumetric strain. It is denoted by *K*.

$$K = \frac{\text{Normal Stress}}{\text{Volumetric Strain}} \quad \text{...(1.15)}$$

The unit of this modulus is also N/m^2 or Pa.

1.10 STRESS AND STRAIN IN SIMPLE AND COMPOUND BAR

(*a*) Simple Bar

Let a bar of uniform cross section Area A. An axial force *P* is applied as shown in Fig. 1.8.

$$\text{Stress} = \frac{P}{A}$$

$$\text{Strain} = \frac{\Delta L}{L}$$

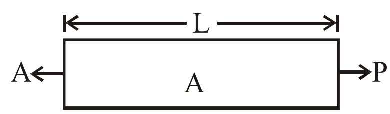

Fig. 1.8: Simple Bar

(*b*) Stepped Bar

Let us consider a stepped bar of different Areas as shown in Fig. 1.9.

Stress in part 1 $\quad \sigma_1 = \dfrac{P}{A_1}$

Stress in part 2 $\quad \sigma_2 = \dfrac{P}{A_2}$

Stress in part 3 $\quad \sigma_3 = \dfrac{P}{A_3}$

Strain in part 1 $\quad \varepsilon_1 = \dfrac{\Delta L_1}{L_1}$

Strain in part 2 $\quad \varepsilon_2 = \dfrac{\Delta L_2}{L_2}$

Strain in part 3 $\quad \varepsilon_3 = \dfrac{\Delta L_3}{L_3}$

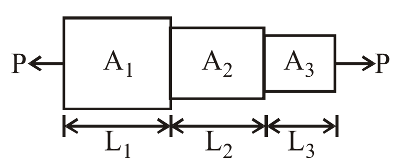

Fig. 1.9: Stepped bar

Total stress in bar $= \sigma_1 + \sigma_2 + \sigma_3$ ...(1.16)

Total strain in bar $= \varepsilon_1 + \varepsilon_2 + \varepsilon_3$ ...(1.17)

(*c*) Extension of Taper Bar

Let us consider a taper bar of length L and larger diameter D_1 and smaller diameter D_2 as shown in Fig. 1.10

Diameter at a distance x from larger diameter

$$= D_1 - \left(\frac{D_1 - D_2}{L}\right)x$$

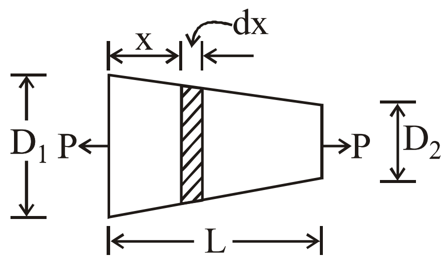

Fig. 1.10: Taper bar

Change in length for $dx = \dfrac{4Pdx}{\pi\left[D_1 - \left(\frac{D_1 - D_2}{L}\right)x\right]^2 E}$

Total extension $$= \int_0^L \frac{4Pdx}{\pi\left[D_1 - \left(\frac{D_1 - D_2}{L}\right)x\right]^2 E}$$

$$= \frac{4PL}{\pi D_1 D_2 E} \quad ...(1.18)$$

(*d*) Compound Bar or Composite Section

A composite section means a section made of different metals connected rigidly together from both ends as shown in Fig. 1.11. There is a compound section in which central rod is of copper and surrounding by tube is made up of brass. They connected firmly and a force P is applied on this compound section.

Let L = Length of composite section

A_b = Area of brass tube

A_c = Area of copper rod

E_b = Young's modulus of elasticity of brass

E_c = Young's modulus of elasticity of copper

σ_b = Stress in brass tube

σ_c = Stress in copper rod

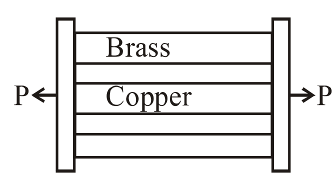

Fig. 1.11: Compound Section

As Brass tube and copper rod are held together and a common force is there so, there elongation or strain will be same.

or $$\varepsilon_c = \varepsilon_b$$

$$\frac{P_c L}{A_c E_c} = \frac{P_b L}{A_b E_b}$$

or $$P_c = \frac{P_b A_c E_c}{A_b E_b} \quad \text{...(1.19)}$$

or $$P_b = \frac{P_c A_b E_b}{A_c E_c} \quad \text{...(1.20)}$$

and $$P = P_b + P_c$$

1.11 PRINCIPLE OF SUPER POSITION

According to this principle when a number of axial forces acting on a body, the total resulting strain is equals to algebric sum of strains due to individual force.

i.e., $$\Delta L = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \frac{P_3 L_3}{A_3 E_3} + \text{........} = \sum_{J=1}^{n} \left(\frac{PL}{AE} \right)_J \quad \text{...(1.21)}$$

1.12 TEMPERATURE STRESS AND STRAIN

When the temperature of a body increase or decrease, the dimensions are also increase or decrease. When this change in dimension is prevented, the stresses are set up in the body, these stresses are known as temperature stress or thermal stress.

Let α = Co-efficient of thermal expansion

L = Length of bar at temperature T_o

ΔT = Change in temperature

$\therefore$ Change in length due to increase in temperature by $\Delta T = \alpha L \Delta T$

$$\text{Temp. Strain} = \frac{\alpha L \Delta T}{L} = \alpha \Delta T \quad \text{...(1.22)}$$

$$\text{Temperature Stress} = E \times \alpha \Delta T \quad \text{...(1.29)}$$

There is a gap in two railway line joints just for avoiding thermal stresses. If the body is prevent from expansion then compressive stress set up and if body is prevent from contraction the tensile stress set up in the body.

1.13 RELATIONS BETWEEN *E*, *G* AND *K*

This expression is in three parts

(*a*) Relation between *E* and *G*.

(*b*) Relation between *E* and *K*.

(*c*) Relation between *E*, *G* and *K*.

(*a*) Relation between *E* and *G*:

Let us consider a cube of unit size. The shear stress τ is acted upon it and distort the cube such that *NO* will become N_1O_1.

From *N*, drop a perpendicular *NP* on diagonal LN_1, such that *LN* = *LP*.

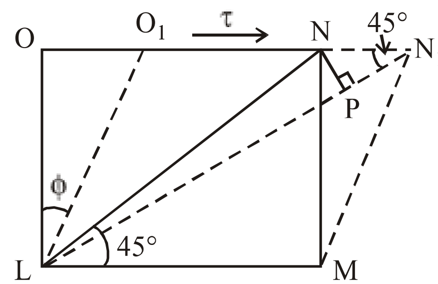

Fig. 1.12

Now, Shear strain $\phi = \dfrac{NN_1}{NM} = \dfrac{\tau}{G}$

Diagonal $LN_1 = LP + PN_1$

or $LN_1 = LN + PN_1$

$PN_1 = NN_1 \cos 45°$

$= \dfrac{NN_1}{\sqrt{2}}$

and $LN = \sqrt{2}\,LM$

(By Pythagorous theorem)

$\therefore$ Strain in diagonal $LN = \dfrac{PN_1}{LN} = \dfrac{NN_1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}\,LM}$

$$= \frac{NN_1}{2LM} = \frac{1}{2}\frac{\tau}{G} \quad \text{...(1)} \quad (\because LM = NM)$$

Thus the strain in diagonal of a cube subjected to simple shear is half the shear strain.

If the cube is subjected to simple shear stress then the normal stress on diagonal is equal to shear stress as shown in Fig. 1.13.

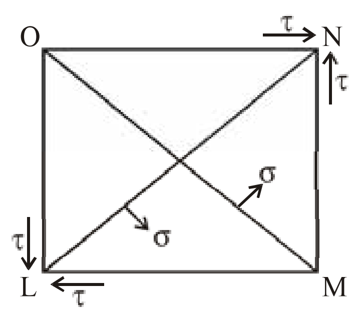

Fig. 1.13

$\therefore$ $\sigma = \tau$

So, Strain in diagonal $LN = \dfrac{1}{2}\dfrac{\sigma}{G}$ ...(*i*)

If σ is acting alone in direction *LN*, the strain will be $\dfrac{\sigma}{E}$ but there is equal and opposite stress in diagonal *OM*. So, the strain in direction *LN* due to stress in diagonal *OM* is

$$= v\frac{\sigma}{E}$$

Total strain in $LN = \dfrac{\sigma}{E} + v\dfrac{\sigma}{E}$

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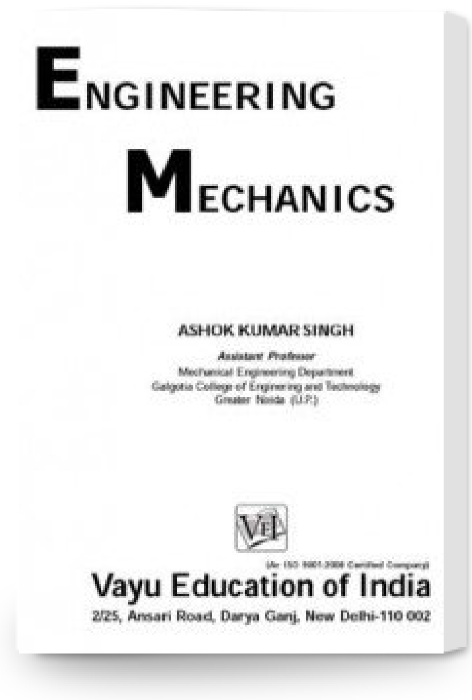

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