CBSE Class 12

Physics Theory

Previous Year Question Paper 2020

Series: HMJ/1

Code no. 55/1/3

- Please check that this paper contains **11** printed pages.
- Code number given on the right-hand side of the question paper should be written on the title page of the answer- book by the candidate.
- Please check that this question paper contains **10** questions.
- Please write down the Serial Number of the question in the answerbook before attempting it.
- 15-minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

Time Allowed: **3** hours

Maximum Marks: 70

SECTION - A

Note: Select the most appropriate option from those given below each question :

1. Photons of energies 1eV and 2eV are successively incident on a metallic surface of work function 0.5eV. The ratio of kinetic energy of most energetic photoelectrons in the two cases will be 1 Mark

(A)1: 2 (B)1:1 (C)1:3 (D)1:4 **Ans:** For the first photon:

E1 = W + KE1

0.5=1+KE1

KE1=0.5

For the second photon:

E2 = W + KE2

2=0.5+KE2

KE2=1.5

On dividing KE1 and KE2

 $\frac{\mathrm{KE}_{1}}{\mathrm{KE}_{2}} = \frac{0.5}{1.5}$ $\frac{\mathrm{KE}_{1}}{\mathrm{KE}_{2}} = \frac{1}{3}$

So, option C is correct.

2. Which of the following statements is not correct according to Rutherford model ? 1 Mark

(A) Most of the space inside an atom is empty

(B) The electrons revolve around the nucleus under the influence of coulomb force acting on them

(C) Most part of the mass of the atom and its positive charge are concentrated at its center.

(D) The stability of atom was established by the model

Ans: Option D is incorrect according to Rutherford model as he was not able to explain stability of atom

3. The resolving power of a telescope can be increased by increasing: 1 Mark

(A)wavelength of light.

(B)diameter of objective.

(C)length of the tube.

(D)focal length of eyepiece.

Ans: resolving power can be increased by decreasing the wavelength and increasing the diameter of objective.

So, option (B) is correct.

4. The magnetic dipole moment of a current carrying coil does not depend upon 1 Mark

(A) number of turns of the coil.

(B) cross-sectional area of the coil.

(C) current flowing in the coil.

(D) material of the turns of the coil

Ans: The magnetic dipole moment of a current carrying coil depends upon the number of turns, cross sectional area and the current flowing in the coil. So, the correct answer is option D.

5. For glass prism, the angle of minimum deviation will be smallest for the light of 1 Mark

(A) red colour.

(B) blue colour.

(C) yellow colour.

(D) green colour

Ans: Red light is having maximum wavelength so it angle of minimum deviation will be smallest for it.

6. A biconvex lens of glass having refractive index 1.47 is immersed in a liquid. It becomes invisible and behaves as a plane glass refractive index of the liquid is 1 Mark

(A) 1:47

- (B) 1.62 `
- (C) 1.33
- (D) 1.51

Ans: According to lens maker's formula,

$$\frac{1}{f} \left(\frac{\mu_g}{\mu_1} \text{-} 1 \right) \left(\frac{1}{R_1} \text{-} \frac{1}{R_2} \right)$$

As the biconvex lens dipped in a liquid behaves as a plane sheet of glass,

$$f = \infty$$

So, $\frac{1}{f} = 0$
$$\frac{\mu_g}{\mu_1} - 1 = 0$$

$$\mu_g = \mu_1 = 1.47$$

So, the correct answer is option (A).

7. The resistance of a metal wire increases with increasing temperature on account of 1 Mark

- (A) decrease in free electron density
- **(B)** decrease in relaxation time.
- (C) increase in mean free path.
- (D) increase in the mass of electron.

Ans: Relaxation time is defined as the time interval between two successive collisions of electrons in a conductor when current flows through it. The

resistance of a conductor increases with an increase in temperature because the thermal velocity of the free electrons increase as the temperature increases. This results in an increase in the number of collisions between the free electrons and a decrease in the relaxation time.

So, the correct answer is option (B).

8. An electric dipole placed in a non-uniform electric field can experience 1 Mark

(A) a force but not a torque.

(B) a torque but not a force.

(C) always a force and a torque.

(D) neither a force nor a torque

Ans: Given an electric dipole placed in a non-uniform electric field. An electric dipole always experiences a torque when placed in uniform as well as non-uniform electric field. But in a non-uniform electric field, the dipole will also experience net force of attraction. So the electric dipole in a non-uniform electric field experiences both torque and force.

9. If the net electric flux through a closed surface is zero, Then we can infer

(A) no net charge is enclosed by the surface.

(B) uniform electric field exists within the surface.

(C) electric potential varies from point to point inside the surface.

(D) charge is present inside the surface.

1 Mark

Ans: If the net electric flux is zero, then no net charge is enclosed by the closed surface. ... Since electric flux is defined as the rate of flow of electric field in a closed area and if the electric flux is zero, the overall electric charge within the closed boundary will be also zero. So, the correct answer is option A.

10. Kirchhoff's first rule at a junction in an electrical conservation of

(A) energy

(B) charge

(C) momentum

(D) both energy and charge

Ans: The first law of Kirchhoff's is based on charge conservation, as it talks about the summation of current to be zero at any junction, which means that if current is conserved that implies that charge is also conserved.

So, the correct answer is option B.

Note: Fill in the blanks with appropriate answer:

11. A ray of light on passing through an equilateral glass prism, suffers a minimum deviation equal to the angle of the prism. The value of refractive index of the material of the prism is ______ 1 Mark

Ans: The minimum

12. According to Bohr's atomic model, the circumference of the electron orbit is always an _____ multiple of de Broglie wavelength. 1 Mark

Ans: According to Bohr's atomic model, the circumference of the electron orbit is always an **integral** multiple of de Broglie wavelength.

Explanation: Bohr, in his atomic model, considered an electron to be in form of a standing electron wave and if this wave is to be continuous over the circumference of the stationary orbit that the electron lie in, the circumference must be a integral multiple of its wavelength $(n\lambda)$.

Or

In ß-decay, the parent and daughter nuclei have the same number of ______ 1 Mark

Ans: In β -decay, the parent and daughter nuclei have the same number of **protons** and neutrons.

Explanation: In beta decay number, the mass number of the beta particle remains unchanged and we know that the mass number is the number of protons and neutrons.

1 Mark

13. The number of turns of a solenoid are doubled without changing its length and area of cross-section. The self inductance of the solenoid will become _______ times 1 Mark

Ans: The number of turns of a solenoid are doubled without changing its length and area of cross-section. The self inductance of the solenoid will become 4 times.

Explanation: The expression for the self inductance of a solenoid is $L = \frac{\mu_0 N^2 A}{1}$.

So, we can see that $L\alpha N^2$.

So, on doubling the number of terms, the self inductance becomes 4 times.

14. Laminated iron sheets are used to minimize _____ in the core of a transformer.1 Mark

Ans: Laminated iron sheets are used to minimize eddy currents in the core of a transformer.

Explanation: The iron core of a transformer is laminated with the thin sheet; the laminated iron core prevents the formation of eddy currents across the core and thus reduces the loss of energy.

15. The magnetic field lines are _____ by a diamagnetic substance 1 Mark

Ans: The magnetic field lines are feebly repelled by a diamagnetic substance.

Explanation: Diamagnetic substances are those which develop feeble magnetization in the opposite direction of the magnetizing field. Such substances are feebly repelled by magnets and tend to move from stronger to weaker parts of a magnetic field.

16. Why cannot we use Si and Ge fabrication of visible LEDs? 1 Mark

Ans: We cannot use Silicon or germanium in the fabrication of LEDs because they produce energy in the form of heat, and not in the form of Visible light of IR. It is not much sensitive to temperature.

17. The variation of the stopping potential photosensitive surface the frequency (v) of the light incident on two different photosensitive surface M1 and M2 is shown in the figure. Identify the surface which has greater value of the work function. 1 Mark



Ans: As per the figure given above the stopping potential the variation of the stopping potential photosensitive surface the frequency (v) of the light incident on two different photosensitive surface M1 and M2.

So the figure representation is for the same value of V0 but ν differs in both the case and the work function depends on the value of v that means greater the value of greater is the work function hence surface M2 has greater value of work function as, $v_2 > v_1$.

18. How does an increase in doping concentration affect the width of depletion layer of a p-n junction diode ? 1 Mark

Ans: If we increase doping, the number of majority charge carriers (holes on the p-side and electrons on the n-side) will also grow. This would result in an increase in the width of the depletion layer, which is dependent on charge carriers.

19. The nuclear radius of ²⁷₁₃ Al is 4.6 fermi. Find the nuclear radius of ⁶⁴₂₉Cu ? 1 Mark

Ans: We know that the expression for nuclear radius is $R=E_{0}A^{1/3}$

where,

R is the nuclear radius

R_o is a constant

A is the mass number

$$\frac{R_{Al}}{R_{Cu}} = \left(\frac{27}{64}\right)^{1/3}$$
$$\frac{R_{Al}}{R_{Cu}} = \frac{3}{4}$$

This can be written as,

$$R_{Cu} = \frac{4}{3}R_{Al}$$

On putting the value of R_{Al} , we get,

$$R_{Cu} = \frac{4}{3} \times 4.6$$

R_{Cu}=6.1 fermi

So, the nuclear radius of ${}^{64}_{29}$ Cu is R_{Cu}=6.1 fermi.

Or

A proton and an electron have equal speed find the ratio of de Broglie Wavelengths associated with them 1 Mark

Ans: The expression for de Broglie wavelength is,

$$\lambda = \frac{h}{mv}$$

The expression of de Broglie wavelength for an electron is,

$$\lambda_e = \frac{h}{m_e v}$$
(1)

Similarly, the expression of de Broglie wavelength for a proton is,

$$\lambda_{p} = \frac{h}{m_{p}v}$$
(2)

On dividing equation (1) and (2),

$$\frac{\lambda_{e}}{\lambda_{p}} = \frac{m_{p}v}{m_{e}v}$$

$$\frac{\lambda_{e}}{\lambda_{p}} = \frac{m_{p}}{m_{e}}$$
Now, we know that $\frac{m_{p}}{m_{e}} = \frac{1836}{1}$

$$\frac{\lambda_{e}}{\lambda_{p}} = \frac{1836}{1}$$

So, the ratio of de Broglie wavelength associated to an electron and a proton is 1836:1.

20. How is displacement current produced between the plates of a parallel plate capacitor during charging? 1 Mark

Ans: In between the plates of the capacitor due to the time-varying electric field, there is a change in electric flux which constitute a current. This current is known as displacement current.

SECTION-B

21. Two long straight parallel wires A and B separated by a distance d, carry equal current I flowing in same direction as shown in the figure



(a) Find the magnetic field at a point P situated between them at a distance x from one wire.

Ans: The magnetic field due to wire A at the point P is,

² Marks

$$B_1 = \frac{\mu_o I}{2\pi x}$$

The magnetic field due to the wire B at the point P is,

$$B_2 = \frac{\mu_o I}{2\pi (d-x)}$$

So, the net magnetic field is,

B=B1-B2

$$B = \frac{\mu_{o}I}{2\pi} \left(\frac{1}{x} - \frac{1}{d-x} \right)$$
$$B = \frac{\mu_{o}I}{2\pi} \left[\frac{d-2x}{x(d-x)} \right]$$

This is the desired expression.

(b) Show graphically the variation of the magnetic field with distance x for 0 < x < d

Ans: The graphical representation of the variation of the magnetic field with distance x for $0 \le x \le d$ is:



22. Using Bohr's atomic model, derive the expression for the radius of nth orbit of the revolving electron in a hydrogen atom. 2 Marks

Ans: In accordance to the Bohr's postulates,

$$L_n = mv_n r_n = \frac{nh}{2\pi}$$

For a dynamically stable orbit present in the hydrogen atom,

$$F_{e} = F_{c}$$

$$\frac{mv_{n}^{2}}{r_{n}} = \frac{1}{4\pi\varepsilon_{o}} \frac{e^{2}}{r_{n}^{2}}$$

$$mv_{n}^{2} = \frac{1}{4\pi\varepsilon_{o}} \frac{e^{2}}{r_{n}}$$

$$v_{n}^{2} = \frac{e^{2}}{4\pi\varepsilon_{o}mr_{n}}$$

On taking square root on both the sides,

$$v_n = \frac{e}{\sqrt{4\pi\epsilon_o mr_n}}$$
 ...(1)

We also know that,

$$v_n = \frac{nh}{2\pi mr_n} ...(2)$$

On equating equation (1) and equation (3),

$$\frac{e}{\sqrt{4\pi\epsilon_{o}mr_{n}}} = \frac{nh}{2\pi mr_{n}}$$

On squaring both sides, we get,

$$\frac{e^2}{4\pi\varepsilon_o mr_n} = \frac{n^2 h^2}{4\pi^2 m^2 r_n^2}$$
$$\frac{e^2}{\varepsilon_o} = \frac{n^2 h^2}{\pi mr_n}$$
$$r_n = \frac{n^2 h^2 \varepsilon_o}{\pi me^2}$$

This is the desired expression.

Or

2 Marks

(a) Write two main observations of photoelectric effect experiment which could only be explained by Einstein's photoelectric equation.

Ans: The two main observations of photoelectric effect experiment which could only be explained by Einstein's photoelectric equation are:

- (1) There is a particular frequency below which the emission of electrons does not take place. This frequency is known as threshold frequency.
- (2) The kinetic energy of the electron linearly depends on the frequency and does not depend on the intensity of radiation.

(b) Draw graph variation of photocurrent with the anode potential of a photocell

The graph showing the variation of photocurrent with the anode potential of a photocell is:



23. Define the wave front of a travelling wave. Using Huygens principle, obtain the law of refraction at a plane interface when light passes from a rarer to a denser medium. 2 Marks

Ans: Wave front is an imaginary surface over which an optical wave has a constant phase or in same phase and the shape of a wave front is generally determined by the geometry of the source.

Derivation of law of refraction:



Huygens principle states that:

Every point on a primary wave front act as a source for the secondary wavelets.

These secondary wavelets are connected tangential in the forward direction give secondary wave front.

Here AB acts as incident wave front or we can say as primary wave front.

DC act as refracted wave front or we can say secondary wave front.

Consider the light incise on the denser medium having reflective index μ_1 and get refracted through the are medium having refractive index μ_2 .

Now from the figure,

sini sinr	$ \left(\begin{array}{c} \underline{BC}\\ \underline{AC}\\ \underline{AC}\\$		BC AD
sini_	$v_1 \tau_1$		
sinr	$v_2 \tau$	\mathbf{V}_2	
Now we know that,			
$v\alpha \frac{1}{\mu}$			

Hence the above equation become,

 $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \text{constant}$

Or

Using lens marker's formula, derive the lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ for a biconvex lens. 2 Marks

Ans: consider the diagram which will show the geometry of the image formation by a biconvex lens.



Now applying the equation for refraction at the spherical surface ABC of the biconvex lens we will get,



From the figure,

 $\frac{\mathbf{n}_1}{\mathbf{OB}} + \frac{\mathbf{n}_2}{\mathbf{BI}} = \frac{\mathbf{n}_2 - \mathbf{n}_1}{\mathbf{BC}_1}$

Similarly now applying the same procedure on the second surface ADC we will get,



Now from the figure,

 $-\frac{n_2}{DI_1}+\frac{n_1}{DI}=\frac{n_2-n_1}{DC_2}$

From this lens we get,

BI=DI₁

Now adding above two equation we will get,

$$\frac{\mathbf{n}_1}{\mathbf{OB}} + \frac{\mathbf{n}_1}{\mathbf{DI}} = (\mathbf{n}_2 - \mathbf{n}_1) \left(\frac{1}{\mathbf{BC}_1} + \frac{1}{\mathbf{DC}_2} \right) \dots \dots (1)$$

Let us assume that the object is at infinity then,

$$OB \rightarrow \infty$$

and the image will be at focus, DI=f

So we will get,

$$\frac{\mathbf{n}_1}{\mathbf{f}} = (\mathbf{n}_2 - \mathbf{n}_1) \left(\frac{1}{\mathbf{BC}_1} + \frac{1}{\mathbf{DC}_2} \right) \dots \dots (2)$$

Hence from equation (1) and (2) we will get,

$$\frac{\mathbf{n}_1}{\mathbf{OB}} + \frac{\mathbf{n}_1}{\mathbf{DI}} = \frac{\mathbf{n}_1}{\mathbf{f}}$$

Considering their respective lens and applying sigh conversion we will get,

BO = -u and DI = +v

So we will get,

$$\frac{1}{-u} + \frac{1}{+v} = \frac{1}{f}$$
$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Above equation is known as the lens makers formula for a biconvex lens.

24. Explain the principle of working of a meter bridge. Draw the circuit diagram for determination of an unknown resistance using it. 2 Marks

Ans: Meter bridge also known as slide Wire Bridge which is a practical form of wheat-stone bridge, which is used to measure the unknown resistances. The working principle of Meter Bridge is the ratio of the resistance of the two lengths of the wire across the position of jockey, where the galvanometer shows zero deflection which is equal to the ratio of the known resistance R and an unknown resistance S.

Let us assume resistance per cm length of the wire be r.

Now mathematically representing the principle of Meter Bridge,

$$\frac{Lr}{(100-L)r} = \frac{R}{S}$$

$$\Rightarrow S = \frac{100-L}{L} \times R$$

$$\int_{R} \int_{R} \int_{R}$$

25. Explain the terms 'depletion layer, and 'potential barrier, in a p-n junction diode. How are the (a) width of depletion layer, and (b) value of potential barrier affected when the p-n junction is forward biased? 2 Marks

Ans:



a) Width of the depletion layer:

The depletion region is the layer which is created around the p-n junction which is devoid of free change carriers and also has immobile ions. It is created around the p-n junction due to diffusion of majority carriers across the junction.

When the p-n junction diode is biased with forward biasing, the negative terminal of the battery (potential) repels the electron toward the junction and provides the required energy to cross the junction and recombine with the holes which is also being repelled by the positive terminal. This will lead to the decrease in the width of the depletion layer.

b) Potential Barrier:

It is a potential difference or we can say junction voltage that is developed across the junction due to migration of the majority charge carriers across it when the pn junction is formed.

It opposes the further migration of the majority charge carriers across the p-n junction and it appears as if a fictitious battery is connected across the p-n junction. The batter acts in such a way that the positive terminal is to the n-region and the negative terminal is to the p-region of the p-n junction.

The value of the potential barrier is 0.3V for Ge and 0.7V for Si semiconductor diodes. Hence the forward bias voltage opposes the potential barrier and due to the reduction in potential barrier thus the width of depletion layer also decreases.

26. N small conducting liquid droplets, each of radius r, are charged to a potential V each, these droplets coalesce to form a single large drop without any charge leakage find the potential of the large drop. 2 Marks

Ans: Given:

N small conducting droplets are present each of radius r and potential of each droplets is V.

Potential of each liquid droplet,

$$V = \frac{kq}{r}$$
$$\Rightarrow q = \frac{Vr}{k}$$

Where, k is a constant term, r is the radius of the droplet and q is the charge of the conducting droplet.

Now for N such liquid droplets the charge will be,

$$Q=Nq=\frac{NVr}{k}$$

Now the radius of the larger drop will be,

$$R=N^{\frac{1}{3}}r$$

As the volume will remain the same in both the case.

The potential of the new droplet will be,

V'=
$$\frac{kQ}{R'}$$

⇒ V'= $\frac{kNVr}{kN^{\frac{1}{3}}r}$
⇒ V'= $VN^{\frac{2}{3}}$

27. Define activity of a sample of a radioactive substance. The value of the disintegration constant of a radioactive substance is 0.0693 h⁻¹. Find the time after which the activity of a sample of this substance reduces to one-half that of its present value. 2 Marks

Ans: Activity of a sample of a radioactive substance is defined as the number of disintegration that has taken place in a given sample per second. In other word we can say decaying of a radioactive substance.

Given: The value of disintegration constant of the radioactive substance is $0.0693h^{-1}$ or we can say rate constant k= $0.0693h^{-1}$

Here we have to find the time after which the activity of a sample reduces to onehalf, in short we have to calculate the half life time of the radioactive substances.

Half life,
$$t_{\frac{1}{2}} = \frac{0.693}{k}$$

Now putting the k value we will get,

$$t_{\frac{1}{2}} = \frac{0.693}{0.0693 \text{h}^{-1}}$$
$$\Rightarrow t_{\frac{1}{2}} = 10 \text{hours}$$

SECTION - C

28. In a single slit diffraction experiment, light of wavelength λ Illuminates the slit of width 'a' and the diffraction pattern observed on a screen. 3 Marks

Ans: In single slit diffraction experiment, let the wavelength of light be λ and the slit width be 'a'.

(a) show the intensity distribution in the pattern with the angular position θ

Ans: The intensity distribution in the patter with the angular position θ can be shown as,



(b) how are the intensity and angular width of central maxima affected when

Ans: We know,

Angular width is inversely propositional to the width of the slit, 'a' which is represented as,

Angular Width=
$$\frac{2\lambda}{a}$$

And the intensity is directly proportional to the area or we can say width of the slip as well as the separation between the slit and screen while angular width has no relation with separation between the slit and screen.

(i) width of slit is increased, and

Ans: When the width of the sit 'a' increases then the angular width decreases and the intensity of the central maxima increases.

(ii) separation between slit and screen is decreased

Ans: When the separation between the slit and screen decreases then the intensity of the central maxima also decreases while there is no change in the angular width.

29. With the help of a simple diagram, explain the working of silicon solar cells, giving all three basic processes involved. Draw its I-V characteristics. 3 Marks

Ans: Diagram:



The construction of a silicon solar cell is usually made up of thick layer of n-type semiconductor which is layered by a thin layer of p-type semiconductor. Then the electrodes are placed on the top of the p-type semiconductor and then another electrode for collecting current is attached to the bottom of the n-type semiconductor.

Working principle:

When light strikes on the source of the cell, it get penetrated to the p-n junction which is crested by the fusion of p-type semiconductor and n-type semiconductor. Then the photons are able to create electron and hole pairs. These free electrons in the depletion region will migrate to the p-type and two charges are built up on the opposite side of the junction which crest a potential difference across the junction. Hence when load is connected current will flow through it.

The three basic process involves are:

Generation of electron and hole pair are due to the light close to the junction.

Separation of electros to n side and hole to p side is due to the electric field in the depletion region.

And the electrons reaching n side is collected by front contact and the holes reaching p side are collected by back contact.

I-V Characteristics:



30. A resistance R and an inductor L are connected in series to a source $V=V_0sin\omega t$

Find the

Which of them is ahead?

3 Marks

Ans: Diagram:



 $V = V_0 sin\omega t$

The given circuit show that the resistor and the inductor are connected in series. Hence the peak value of current through the circuit will be,

$$I_0 = \frac{V_0}{Z}$$

Where the resultant impedance of the circuit will be,

$$Z = \sqrt{R^2 + X_L^2}$$

Now the current will become,

$$I_0 = \frac{V_0}{\sqrt{R^2 + X_L^2}}$$

(a) peak value of the voltage drops across R and L,

Ans: The peak value of the voltage across the resistor R will be,

$$V_{R} = I_{0}R = \frac{V_{0}R}{\sqrt{R^{2} + X_{L}^{2}}}$$

The peak value of the voltage across the inductor L will be,

$$V_{L} = I_{0}X_{L} = \frac{V_{0}X_{L}}{\sqrt{R^{2} + X_{L}^{2}}}$$

(b) phase difference between the applied voltage and current.

Ans: The phase difference is the angle between the resistor and inductor,

Hence,

$$\tan \phi = \frac{X_{L}}{R}$$
$$\Rightarrow \phi = \tan^{-1} \frac{X_{L}}{R}$$

As in the circuit inductor is present then the voltage will leads the current by an angle of ϕ as the voltage and current in resistor are in same phase.

31.

3 Marks

(a) Write the expression for the speed of light in a material medium of relative permittivity ϵ_r and relative magnetic permeability μ_r .

Ans: The speed of electromagnetic waves is represented as,

$$c=\frac{1}{\sqrt{\mu\epsilon}}$$

Where, ε is the electric permittivity and μ is the magnetic permeability.

Given:

Relative permittivity ε_r

Relative permeability μ_r

We know,

 $\mu = \mu_r \mu_0$

And $\varepsilon = \varepsilon_r \varepsilon_0$

Now using this speed of the light in the material medium,

$$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}}$$

(b) Write the wavelength range and name of the electromagnetic waves are used in

(i) radar systems for aircraft navigation and

Ans: The electromagnetic wave use in radar system for aircraft id the microwave whose wavelength range is in between 1mm to 0.1m.

(ii) Earth satellites to observe the growth of the crops.

Ans: The electromagnetic wave used in earth satellites to observe the growth of crops is the infrared wave or we can say IR ray whose wavelength lies in the range 1mm to 700nm.

32.

3 Marks

(a) Two cells of emf E_1 and E_2 have their internal resistances r_1 and r_2 respectively. Deduce an expression for the equivalent emf and internal resistance of their parallel combination when connected across an external resistance R. Assume that the two cells are supporting each other.

Ans:



Given,

Emf are E_1 and E_2 .

Internal resistances r_1 and r_2 .

Here the total current I is,

$$I = I_1 + I_2 \dots (1)$$

Let V is the potential difference between point A and B.

Then $V=E_1-I_1r_1$

$$\Rightarrow I_1 = \frac{E_1 - V}{r_1} \dots (2)$$

And, $V=E_2-I_2r_2$

$$\Rightarrow I_2 = \frac{E_2 - V}{r_2} \dots (3)$$

Now putting equation (2) and (3) in equation (1) we will get,

$$I = \frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2}$$
$$I = \left(\frac{E_1}{r_1} + \frac{E_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Or we can say,

$$\mathbf{V} = \left(\frac{\mathbf{E}_1 \mathbf{r}_2 + \mathbf{E}_2 \mathbf{r}_1}{\mathbf{r}_1 + \mathbf{r}_2}\right) - \mathbf{I}\left(\frac{\mathbf{r}_1 \mathbf{r}_2}{\mathbf{r}_1 + \mathbf{r}_2}\right)$$

If we replace this with a single cell it can be written as,

$$V \!\!=\!\! E_{\text{equivalenet}} \!\!-\!\! Ir_{\text{equivalent}}$$

Now on comparing we will get,

$$E_{\text{equivalenct}} = \left(\frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}\right)$$
$$r_{\text{equivalent}} = \left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

(b) In case the cell are identical, each of E=5V and internal resistance $r=2\Omega$ calculate the voltage across the external resistance R=10 Ω .

Ans: Given:

 $r_{1} = r_{2} = 2\Omega$ $r_{equivalent} = \left(\frac{2 \times 2}{2 + 2}\right) = 1\Omega$ $E_{1} = E_{2} = 5V$ $E_{equivalenet} = \left(\frac{5 \times 2 + 5 \times 2}{2 + 2}\right) = 5V$

Now the external voltage will be,

 E_{ext} =IR

$$I = \frac{E_{equivalent}}{R+r}$$

Now,

$$E_{ext} = \frac{E_{equivalent}}{R+r} \times R$$
$$\implies E_{ext} = \frac{5V}{10+1} \times 10 = 4.54 \text{ Volts}$$

33.

3 Marks

(a)write an expression of magnetic moment associated with a current (I) carrying circular coil of radius r having N turns

Ans: Magnetic moment of a current carrying circular coil of radius r and having N turns is given as,

M=NIA where A is the area bounded by the circular loop.

Now we can write the area as,

$$A=\pi r^2$$

On putting the magnetic moment will be,

M= NIA= NI πr^2

(b) consider the above-mention places in YZ planes with its magnetic Field due it at point (x,0,0).

Ans:



Now according to the above figure due to the current carrying element dl which at A the magnetic field at P is given as,

$$dB = \frac{\mu_0 I}{4\pi} \frac{\left| \vec{dl} \times \vec{r} \right|}{r^3}$$

Since vector dl and r perpendicular to each other, dl×r=dlr

Now it become,

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl}{r^2}$$

From the figure, $r^2 = X^2 + R^2$

Hence,

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl}{X^2 + R^2}$$

Consider a current elements opposite to that of A that is on B, then we can see that the Y component of the magnetic field the current element get cancelled and that of X component is present.

Now,

 $dB_x = dB\cos\theta$

The net magnetic field at P will be,



Or

3 Marks

(a) Define current sensitivity of a galvanometer. Write its Expression

Ans: Current sensitivity of a galvanometer is defined as the ratio of deflection produced in a galvanometer to the current flowing through it.

We can also say it as the deflection per unit current.

Expression for the current sensitivity as,

$$S_i = \frac{\theta}{I}$$

Where, θ is the deflection and the I is the current.

So its SI unit is radian per ampere.

(b) A galvanometer has resistance G and shows full scale deflection for current Ig.

Ans: We have a galvanometer having resistance G and shows full scale deflection that is Ig.

(i) How can it be converted into an ammeter to measure current up to IO (IO>Ig) ?

Ans: A galvanometer can be converted into an ammeter by connecting a shut parallel to it so as to measure the current up to IO.

Expression for such type of connection is,

$$(I_o - I_g)R_s = I_gG$$
.

Where, RS is the shunt resistance.

(ii) What is the effective resistance of this ammeter?

Ans: As the shut is connected in parallel with the galvanometer the effective resistance will be,

$$R_{eff} = \frac{R_sG}{R_s+G}$$

Where, $R_s || G$.

34. The nucleus ${}^{235}_{92}$ Y, initially at rest, decays into ${}^{231}_{90}$ X by emitting 234 231 4 an α particle ${}_{92}$ Y $\rightarrow {}_{90}$ X+ $_{2}$ He+energy.

The binding energies per nucleon, the daughter nucleus and α particle are 7.8MeV, 7.835MeV and 7.07MeV respectively. Assuming the daughter nucleus to be formed in the unexcited state and neglecting its share in the energy of the reaction, find the speed of the emitted α particle.

(Mass of α particle = 6.68×10⁻²⁷)

3 Marks

Ans: Given: The binding energies per nucleon is 7.8MeV, the daughter nucleus is 7.835MeV and the α particle is 7.07MeV.

We know the energy released is,

 $Q = \left[M\left(^{231}X\right) + M\left(^{4}He\right) - M\left(^{235}Y\right)\right]c^{2}$

$$= [(7.835 \times 231) + (7.07 \times 4) - (7.8 \times 235)] MeV$$
$$= [1809.9 + 28.28 - 1833] MeV$$
$$= 5.18 \times 1.6 \times 10^{-13} J$$

Now this entire kinetic energy is taken by the α particle as,

$$\frac{1}{2}$$
mv²=5.18×1.6×10⁻¹³J

Mass of the α particle = 6.68×10^{-27}

Now,

$$\frac{1}{2}6.68 \times 10^{-27} v^{2} = 5.18 \times 1.6 \times 10^{-13} J$$
$$\Rightarrow v^{2} = \frac{2 \times 5.18 \times 1.6 \times 10^{-13}}{6.68 \times 10^{-27}}$$
$$\Rightarrow v = \sqrt{\frac{2 \times 5.18 \times 1.6 \times 10^{-13}}{6.68 \times 10^{-27}}}$$

Hence the speed of the α particle is,

$$v=1.57\times10^{7}ms^{-1}$$
.

SECTION - D

35.

5 Marks

(a) Derive the expression for the torque acting on the rectangular current carrying coil of a galvanometer. Why is the magnetic field made radial.

Ans:



From the figure let us consider a loop ABCD in a uniform magnetic field strength donated as B and a current through the path is I.

The magnetic forces of AB and Cd are equal and opposite to each other but have a different kind of action.

Hence the force produce in the rectangular coil ABCD be,

 $\tau = F \times PD$

Where PD is the perpendicular distance between two force arm i.e $b\sin\theta$ and the force is also represented as, F = IIB where, l is the length of the rectangular coil, I is the current flowing through is and B is the magnetic field strength.

Now torque,

τ=IlBbsinθ

Where lb = A,

 τ =IABsin θ

The magnetic field is made radial because I is not directly proportional to ϕ . We can ensure this proportionality by having $\theta = 90^{\circ}$. This is possible only when the magnetic field. In such filed the plane of rotating coil is always parallel to B.

(b) An α particle is accelerated through a potential difference of 10kV and move alone x-axis. It enters in a region of uniform magnetic field B=2×10⁻³T acting along y-axis. Find the radius of its path. (Take mass of the α particle = 6.68×10⁻²⁷kg)

Ans: Given:

Mass of α particle = 6.68×10^{-27} kg

$$B=2\times10^{-3}T$$

V=10kV
Q = 2×1.6×10⁻¹⁹C

We know the radius of circular path is,

$$r = \frac{1}{B} \sqrt{\frac{2mV}{Q}}$$

$$\Rightarrow r = \frac{1}{2 \times 10^{-3}} \sqrt{\frac{2 \times 6.68 \times 10^{-27} \times 10 \times 10^{3}}{2 \times 1.6 \times 10^{-19}}}$$

$$\Rightarrow r = \frac{1}{2 \times 10^{-3}} \frac{1}{50} = \frac{1}{10^{2-3}} = \frac{1}{10^{-1}} = 10m$$

5 Marks

(a) With the help of a labelled diagram, explain the working of a Step-up transformer. Give reasons to explain the following:

Or



Ans: We can explain this with help of the diagram



The transformer work on the principle of the mutual induction that is whenever a current is associated with the primary coil charges then an emf is induced in the secondary coil. Hence when a transformer in which the output that is the secondary voltage is greater than its induce or primary voltage it is known as step up transformer.

Now the induced emf across the primary coil is,

$$E_{p} = -N_{p} \frac{d\phi}{dt}$$

Where, NP is the number of turns in a primary coil, ϕ is the flux associate in the coil.

Similarly induced emf in the secondary coil is,

$$E_s = -N_s \frac{d\phi}{dt}$$

Where, NS is the number of turns in a secondary $coil, \phi$ is the flux associate in the coil.

Taking the ration of both the induced emf,

$$\frac{E_{s}}{E_{p}} = \frac{N_{s}}{N_{p}}$$

(i) the core of the transformer is laminated

Ans: The core of the transformer is laminated so as to reduce the eddy current produces due to the flow of current

(ii) Thick copper wire is used in windings.

Ans: Thick copper wire is used in winding so as to reduce the heat loss because large amount of heat is produce during this process.

(b) A conducting rod PQ of length 20 cm and resistance 0.1 Ω rests on two smooth parallel rails of negligible resistance AA' and CC'. It can slide on the rails and the arrangement is positioned between the poles of a permanent magnet producing uniform magnetic field B = 0.4 T. The rails, the rod and the magnetic field are in three mutually perpendicular directions as shown in the figure. If the ends A and C of the rails are short circuited, find the

Ans: Given:

Length of PQ = 20cm = 0.2m

Resistance = 0.1Ω

B = 0.4T

(i) external force required to move the rod with uniform velocity v = 10 cm/s, and

Ans: v = 10 cm/s 0.1 m/s

Now the external force require to remove the rode will be,

$$F = \frac{B^2 v l^2}{R}$$
$$\Rightarrow F = \frac{0.4^2 \times 0.1 \times 0.2^2}{0.1} = 6.4 \times 10^{-3} N$$

(ii) power required to do so

Ans: We know power is the product of force and velocity.

P=Fv

$$\Rightarrow$$
 P=6.4×10⁻³×0.1=0.64×10⁻³Watt

36.

5 Marks

(a) Draw the ray diagram of an astronomical telescope when the final image is formed at infinity. Write the expression for the resolving power of the telescope.

Ans: The ray diagram of an astronomical telescope when the final image is formed at infinity is as given below:



Here, fO is the focal length of the objective lens and fe is the focal length of the eyepiece lens.

The expression for resolving power of the telescope is gien as,

Resolving power =
$$\frac{D}{1.22\lambda}$$

Where, D is the diameter of aperture objective lens and the λ is the wavelength.

(b) An astronomical telescope has an objective lens of focal Length 20 m and eyepiece of focal length 1 cm .

Ans: Given:

Focal length of the objective lens = 20cm (fO)

Focal length of eyepiece lens = 1 cm = 0.01 m (fe)

(i)Find the angular magnification of the telescope.

Ans: Angular magnification of astronomical telescope is given as,

Angular Magnification = $\frac{f_o}{f_e} = \frac{20m}{0.01m} = 200$

(ii) If this telescope is used to view of the Moon, find the diameter of the image formed by the objective lens.

Given diameter of the Moon is 3.5×10^6 m and radius of lunar orbit 3.5×10^6 is 3.8×10^8 m.

Ans: Given:
Diameter of the Moon is $D=3.5\times10^6 m$.

Radius of lunar orbit is $x=3.8 \times 10^8 m$.

Diameter of the image = ??

We know,

$$\frac{D}{d} = \frac{x}{f_0}$$

$$d = \frac{Df_0}{x} = \frac{3.5 \times 10^6 \times 20}{3.8 \times 10^8} = 18.4 \times 10^{-2} \text{m} = 18.4 \text{cm}$$
Or

5 Marks

(a)An object is placed in front of a concave mirror it is observed that a virtual image is formed. Draw the ray diagram to show the image formation and hence derive the mirror equation.

Ans:



From the diagram,

From $\Delta A'B'F$ and ΔMPF using similarity criteria we get,

 $\frac{A'B'}{MP} = \frac{B'F}{FP}$

We have,

PM=AB
Now,
$\frac{A'B'}{AB} = \frac{B'F}{FP}$
Now from $\Delta A'B'P$ and ΔABP using similarity criteria we get,
$\frac{AB'}{AB} = \frac{B'P}{BP}$
Now equation (1) and (2),
$\frac{B'F}{FP} = \frac{B'P}{BP}$
Where,
B'F = v+f
BP = u
FP = f
B'P = v
Therefore,
$\frac{B'F}{FP} = \frac{B'P}{BP}$
$\frac{v+f}{f} = \frac{v}{u}$
Dividing both side by v and applying sign convention we will get,

 $\frac{1}{v} \cdot \frac{1}{f} = \frac{-1}{u}$ $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

Hence proved.

(b) An object is placed 30 cm in front of a Plano-convex lens with its spherical surface of radius of curvature 20 cm. If the refractive index of the material of the lens is 1.5, find the position and nature of the image formed.

Ans: Given: R=20cm

Object distance, u=30cm

By lens maker formula,

$$\frac{1}{\mathrm{f}} = \left(\mu - 1\right) \left[\frac{1}{\mathrm{R}_1} - \frac{1}{\mathrm{R}_2}\right]$$

For Plano convex lens,

$$R_1 = R$$

$$R_2 = \infty$$

Therefore,

$$\frac{1}{f} = (\mu - 1) \left[\frac{1}{R} \right]$$
$$\Rightarrow \frac{1}{f} = (1.5 - 1) \left[\frac{1}{20 \text{ cm}} \right]$$

f=40cm

Now using mirror formula we will get,

$$\frac{1}{40 \text{ cm}} = \frac{1}{\text{v}} + \frac{1}{30 \text{ cm}}$$
$$\Rightarrow \text{v} = -12 \text{ cm}$$

Therefore the image is virtual.

37.

5 Marks

(a) Using Gauss law, derive expression for electric field due to a spherical shell of uniform charge distribution a and radius R at a point lying at a distance x from the center of shell, such that

Ans: Let us assume that R be the radius of the spherical shell and Q be the charge that is uniformly distributed on the source



(i) 0 < x < R, and (ii) x > R

Ans: 0<x<R

For a point inside the shell is,

By using Gauss's Law we can write,

$$E \times 4\pi x^2 = \frac{Q_{in}}{\varepsilon_0}$$

Here x be the distance from the center of the shell and the charge Qin inside the shell is zero.

Hence,

E=0

(ii) x>R

For a point outside the shell is,

By using Gauss's law we can write,

$$E \times 4\pi x^2 = \frac{Q_{out}}{\varepsilon_0}$$

Where x is the distance from center of shell and the change Qout is on the surface of the shell.

$$E = \frac{Q_{out}}{4\pi x^2 \varepsilon_0}$$

(b) An electric field is uniform and acts along + x direction in the region of positive x. It is also uniform with the same magnitude but acts in - x direction in the region of negative x. The value of the field is E=200 N/C for x > 0 and E = - 200 N/C for x < 0. A right circular cylinder of length 20 cm and radius 5 cm has its center at the origin and its axis along the x-axis so that one flat face is at x = +10 cm and the other is at x = -10 cm. Find :

Ans: Given: E=200 N/C for x > 0 and E = -200 N/C for x < 0.

Right circular cylinder of length 20 cm and radius 5 cm has its center at the origin.

(i) The net outward flux through the cylinder.

Ans: The net outward flux = 2EA

 $\phi = 2EA = 2 \times 200 \times 3.14 \times (0.05)^2 = 3.14 \text{Nm}^2 \text{C}^{-1}$

(ii)The net charge present inside the cylinder.

Ans: The net change present inside the cylinder, Q is

 $Q = \varepsilon_0 \times \phi = 8.854 \times 10^{-12} \times 3.14 = 27.8 \times 10^{-12} C$

Or

5 Marks

(a)Find the expression for the potential energy of a system of two point charges q1 and q2 located at \vec{r}_1 and \vec{r}_2 and respectively in an external electric field \vec{E} .

Ans: Given: two point charges q1 and q2 located at \vec{r}_1 and \vec{r}_2 respectively in an external electric field \vec{E} .

Now work done in bringing q1 from the infinity against the electric field is represented as,

$$W_1 = q_1 V \left| \vec{r_1} \right|$$

Similarly work done in bringing q2 from the infinity against the electric field is represented as,

$$W_1 = q_1 V \left| \vec{r_2} \right|$$

Now work done q2 against the filed due to q1

$$W = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

Hence the potential energy of the system = Total work done in assembling the system.

$$V_{system} = W_1 + W_2 + W = q_1 V |\vec{r}_1| + q_2 V |\vec{r}_2| + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

(b) Draw equipotential surfaces due to an isolated point charge (-q) and depict the electric field lines.

Ans: Equipotential surfaces are always perepmdilar to the electric field. This the diagram of an equipotential surface due to an isolated point charge of -q charge. The electric field lines terminate on negative change or we can say direction of electric field lines are inward.



(c) Three point charges, 1μ C- 1μ C, and 2μ C are initially infinite distance apart. Calculate the work done in assembling these charges at the vertices of an equilateral triangle of side 10 cm.

Ans: Three point charges:

 $q_1{=}1\mu C$, $q_2{=}{-}1\mu C$ and , $q_3{=}2\mu C$ are present in an equilateral triangle of side r=10cm



We know,

Work done = Charge in potential energy

$$W = \left[k \frac{q_1 q_2}{r_{12}} + k \frac{q_1 q_3}{r_{13}} + k \frac{q_2 q_3}{r_{23}} \right]$$

Where,

$$r_{12} = r_{13} = r_{23} = r = 10 \text{ cm} = 0.1 \text{ m}$$
$$W = \frac{k}{r} [q_1 q_2 + q_1 q_3 + q_2 q_3]$$
$$W = \frac{9 \times 10^9}{0.1} [-1 + 2 \cdot 2] = -9 \times 10^{10} \text{ J}$$

CBSE Question Paper 2019 Class 12 Mathematics

Time allowed: 3 hours Maximum Marks: 100

General Instructions:

- (i) **All** questions are compulsory.
- (ii) This question paper contains 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of one mark each, Section B comprises of 8 questions of two marks each, Section C comprises of 11 questions of four marks each and Section D comprises of 6 questions of six marks each.
- (iii) All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the question.
- (iv) There is no overall choice. However, internal choice has been provided in 1 question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculators is not permitted. You may ask logarithmic tables, if required.

SECTION-A

1. If A is a square matrix of order **3** with |A| = 4 then write the value of |-2A|

Solution. Since, order of the matrix, n = 3

$$|\mathbf{A}| = 4$$
$$|-2\mathbf{A}| = (-2)^n |\mathbf{A}|$$

$$\left|-2A\right| = \left(-2\right)^3 \times 4$$

$$|-2A| = -32$$

Therefore, the value of |-2A| is -32

2. If
$$y = \sin^{-1}x + \cos^{-1}x$$
, find $\frac{dy}{dx}$

$$y = \sin^{-1} x + \cos^{-1} x$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \left(\sin^{-1} x + \cos^{-1} x \right)$$

$$= \frac{d}{dx} \left(\sin^{-1} x \right) + \frac{d}{dx} \left(\cos^{-1} x \right)$$

$$= \frac{1}{\sqrt{1 - x^2}} - \frac{1}{\sqrt{1 - x^2}}$$

$$= 0$$

Therefore, $\frac{dy}{dx} = 0$

3. Write the order and degree of the differential equation $\left(\frac{d^4y}{dx^4}\right)^2 = \left[x + \left(\frac{dy}{dx}\right)^2\right]^3$

Solution. Since,

$$\left(\frac{d^4 y}{dx^4}\right)^2 = \left[x + \left(\frac{dy}{dx}\right)^2\right]^3$$
$$\left(\frac{d^4 y}{dx^4}\right)^2 = x^3 + \left(\frac{dy}{dx}\right)^6 + 3x^2\left(\frac{dy}{dx}\right)^2 + 3x\left(\frac{dy}{dx}\right)^4$$

The highest power raised to $\frac{d^4y}{dx^4}$ is 2 and degree of the differential equation is 2

4. If the line has the direction ratios -18,12,-4, then what are its direction cosines?

OR

Find the Cartesian equation of the line which passes through the point (-2,4,-5) is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$

The direction ratios of the lines are -18,12,-4Direction cosines of the lines are $-\frac{18}{\sqrt{18^2 + 12^2 + 4^2}}, \frac{12}{\sqrt{18^2 + 12^2 + 4^2}}, -\frac{4}{\sqrt{18^2 + 12^2 + 4^2}}$ Hence, direction cosine of line are $-\frac{9}{11}, \frac{6}{11}, -\frac{2}{11}$

OR

The cartesion equation of the line which passes through the point (-2, 4, -5) and is parallel to the line $\frac{x+3}{3} = \frac{y-4}{-5} = \frac{z+8}{6}$ is $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$

SECTION - B

5. If * is defined on the set R of all real number by * : $a * b = \sqrt{a^2 + b^2}$ find the identity element if exist in R with respect to *

Solution. As per the question Let b be the identity element then $a^*b = b^*a = a$

$$a * b = b^{2} a - a$$

$$a * b = \sqrt{(a)^{2} + (b)^{2}} = a$$

$$\Rightarrow (a)^{2} + (b)^{2} = (a)^{2}$$

$$\Rightarrow b = 0$$
Similarly,
$$b * a = \sqrt{(b)^{2} + (a)^{2}} = a$$

$$\Rightarrow (b)^{2} + (a)^{2} = (a)^{2}$$

 $\Rightarrow b = 0$

Therefore, 0 is the identity element

6. If
$$A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$$
 and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$ then find the value of k,a and b Solution. Given,

$$kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix} (i)$$

$$A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}, \text{ implies } kA = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} (ii)$$

$$\begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$$

$$-4k = 24 \Longrightarrow k = -6$$

$$3a = 2k \Longrightarrow a = -4$$

$$2b = 3k \Longrightarrow b = -9$$

7. Find
$$\int \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}} dx, 0 < x < \pi/2$$

Solution. According to question, $\lim_{x \to \infty} x - \cos x + 0$

$$let I = \int \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}} dx, 0 < x < \frac{\pi}{2}$$

$$I = \int \frac{\sin x - \cos x}{\sqrt{\sin^2 x + \cos^2 x + 2\sin x \cdot \cos x}} dx$$

$$= \int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2}} dx$$

$$= \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

$$let \sin x + \cos x = t$$

$$\Rightarrow (\cos x - \sin x) dx = dt$$

$$I = \int \frac{-1}{t} dt$$

$$= -\ln t + C$$

$$= \ln\left(\frac{1}{t}\right) + C$$

$$\Rightarrow I = \ln\left(\frac{1}{\sin x + \cos x}\right) + C$$
8. Find $\int \frac{\sin(x - a)}{\sin(x + a)} dx$
OR

Find $\int (\log x)^2 dx$

$$Let I = \int \frac{\sin(x-a)}{\sin(x+a)} dx$$

$$\Rightarrow I = \int \frac{\sin[(x+a)-2a]}{\sin(x+a)} dx$$

$$= \int \frac{\sin(x+a)\cdot\cos(2a) - \cos(x+a)\cdot\sin(2a)}{\sin(x+a)} dx$$

$$= \int \cos(2a) dx - \int \cot(x+a)\cdot\sin(2a) dx$$

$$= x \cdot \cos(2a) - \log|\sin(x+a)| \cdot \sin(2a) + C$$

OR

$$Let I = \int (\log x)^2 dx$$

$$\Rightarrow I = \int 1 (\log x)^2 dx$$

$$\Rightarrow I = x (\log x)^2 - \int \frac{2x \log x}{x} dx$$

$$\Rightarrow I = x (\log x)^2 - I_1 + c_1 \qquad \dots (i)$$

$$I_1 = \int 2 \cdot \log x dx$$

$$\Rightarrow I_1 = 2x \cdot \log x - 2 \int \frac{x}{x} dx$$

$$\Rightarrow I_1 = 2x \cdot \log x - 2x + c_2 \qquad \dots (ii)$$

$$I = x (\log x)^2 - 2x \cdot \log x + 2x + c_1 - c_2$$

$$I = x (\log x)^2 - 2x \cdot \log x + 2x + C \qquad (where C = c_1 - c_2)$$

9. From the differential equation representing the family of curves $y^2 = m(a^2 - x^2)$ by eliminating the arbitrary constant m and a

Solution

The equation $y^2 = m(a^2 - x^2)$ where m and a are arbitrary constants

$$y^{2} = m(a^{2} - x^{2}) \qquad \dots \dots (i)$$

$$2y \frac{dy}{dx} = -2mx \qquad \dots \dots (ii)$$

$$\Rightarrow -2m = 2\frac{y}{x}\frac{dy}{dx}$$

$$2\left[y\frac{d^{2}y}{dx^{2}} + \left(\frac{dy}{dx}\right)^{2}\right] = -2m \qquad \dots \dots (iii)$$

$$2\left[y\frac{d^{2}y}{dx^{2}} + \left(\frac{dy}{dx}\right)^{2}\right] = 2\frac{y}{x}\frac{dy}{dx}$$

$$y\frac{d^{2}y}{dx^{2}} + \left(\frac{dy}{dx}\right)^{2} - \left(\frac{y}{x}\right)\frac{dy}{dx} = 0$$

therefore the required differential equation is $y \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - \left(\frac{y}{x}\right)\frac{dy}{dx} = 0$

10. Find the unit vector perpendicular to both the vectors \vec{a} and \vec{b} , where $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ OR

Show that the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$, $-2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\hat{i} - 3\hat{j} + 5\hat{k}$ are coplanner

Solution

$$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k} \text{ and } \vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$$

let \vec{n} be the vector perpendicular to \vec{a} and \vec{b}
 $\vec{n} = \vec{a} \times \vec{b}$
 $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \end{vmatrix} = 19\hat{j} + 19\hat{k}$

$$\begin{vmatrix} 3 & -2 & 2 \\ \hat{n} = \frac{19\,\hat{j} + 19\hat{k}}{\sqrt{19^2 + 19^2}} = \frac{1}{\sqrt{2}} \left(\hat{j} + \hat{k} \right)$$

OR

let
$$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$$

 $\vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$
 $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$
 $\begin{bmatrix} \vec{a} \, \vec{b} \, \vec{c} \end{bmatrix} = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$
 $= 1(15 - 12) + 2(-10 + 4) + 3(6 - 3)$
 $= 3 - 12 + 9$
 $= 0$

therefore, \vec{a} , \vec{b} , \vec{c} are coplanar

11. Mother, father and son line up at random for a family photo. If A and B are two events given by A = Son on one end, B = Father in the middle, find P(B/A).

Solution

If mother (M), father (F), and son (S) line up for the family picture, then the sample space will be

S = {MFS, MSF, FMS, FSM, SMF, SFM} = A= {MFS, FMS, SMF, SFM} $P(A \cap B) = \frac{2}{6} = \frac{1}{3}$ $P(B) = \frac{2}{6} = \frac{1}{3}$ $P(A) = \frac{4}{6} = \frac{2}{3}$ $P(B | A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

12. Let X be a random variable which assumes values x1, x2, x3, x4 such that 2P(X = x1) = 3P(X = x2) = P(X = x3) = 5P(X = x4). Find the probability distribution of X.

OR

A coin is tossed 5 times. Find the probability of getting (i) at least 4 heads, and (ii) at most 4 heads.

Let
$$P(X = x_3) = x$$

 $P(X = x_1) = \frac{x}{2}$
 $P(X = x_2) = \frac{x}{3}$
 $P(X = x_4) = \frac{x}{5}$
 $\sum_{i=1}^{4} P(x_i) = 1$
 $P(x_1) + P(x_2) + P(x_3) + P(x_4) = 1$
 $\frac{x}{2} + \frac{x}{3} + x + \frac{x}{5} = 1$
 $x = \frac{30}{61}$
 $P(X = x_1) = \frac{15}{61}; P(X = x_2) = \frac{10}{61}; P(X = x_3) = \frac{30}{61}; P(X = x_4) = \frac{6}{61}$
So, the probability distribution function will be
 $X = 1 - 2 - 3 - 4$
 $P(X = x_i) = \frac{15}{61}; \frac{10}{61}; \frac{30}{61}; \frac{6}{61}$

OR

Total number of probability of tossing a coin 5 times is 32(i) Probability of getting atleast 4 heads

$$P(X = 4) + P(X = 5)$$

$${}^{5}C_{4}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{4} + {}^{5}C_{5}\left(\frac{1}{2}\right)^{0}\left(\frac{1}{2}\right)^{5}$$

$$= {}^{5}C_{4}\left(\frac{1}{2}\right)^{5} + {}^{5}C_{5}\left(\frac{1}{2}\right)^{5}$$

$$= \frac{6}{32} = \frac{3}{16}$$
(*ii*) probability of getting at most 4 head

$$P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$$

$${}^{5}C_{1}\left(\frac{1}{2}\right)^{5} + {}^{5}C_{2}\left(\frac{1}{2}\right)^{5} + {}^{5}C_{3}\left(\frac{1}{2}\right)^{5} + {}^{5}C_{4}\left(\frac{1}{2}\right)^{5}$$

$$= \left(\frac{1}{2}\right)^{5} [5 + 10 + 10 + 5]$$

$$= \frac{15}{16}$$

SECTION - C

14. If $\tan^{-1} x - \cot^{-1} x = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right), x > 0$ then find the value of x and hence find the value of $\sec^{-1} \left(\frac{2}{x} \right)$

$$\tan^{-1} x - \cot^{-1} x = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right), x > 0$$

$$\Rightarrow \tan^{-1} x - \tan^{-1} \left(\frac{1}{x} \right) = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$$\Rightarrow \tan^{-1} \left(\frac{x - \frac{1}{x}}{1 + x \cdot \frac{1}{x}} \right) = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$$\Rightarrow \frac{x^2 - 1}{2x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}x^2 - 2x - \sqrt{3} = 0$$

$$\Rightarrow \sqrt{3}x^2 - 3x + x - \sqrt{3} = 0$$

$$\Rightarrow \sqrt{3}x \left(x - \sqrt{3} \right) + 1 \left(x - \sqrt{3} \right) = 0$$

$$\Rightarrow \left(x - \sqrt{3} \right) \left(\sqrt{3}x + 1 \right) = 0$$

$$\Rightarrow x = -\frac{1}{\sqrt{3}}, \sqrt{3}$$

$$\because x > 0, x = \sqrt{3}$$

$$\Rightarrow \sec^{-1} \left(\frac{2}{x} \right) = \sec^{-1} \left(\sec \frac{\pi}{6} \right)$$

$$\Rightarrow \sec^{-1} \left(\frac{2}{x} \right) = \frac{\pi}{6}$$

$$\therefore \operatorname{cot}^{-1} x = \tan^{-1}\left(\frac{1}{x}\right), \ x > 0$$

15. Using properties of determinant prove that

 $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

$$Let \Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$
$$R_1 \rightarrow R_1 - R_2 - R_3$$
$$\Delta = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Expending R₁

$$\Delta = 0 \begin{vmatrix} c+a & b \\ c & a+b \end{vmatrix} - (-2c) \begin{vmatrix} b & b \\ c & a+b \end{vmatrix} + (-2b) \begin{vmatrix} b & c+a \\ c & c \end{vmatrix}$$

= $2c(ab+b^2-bc) - 2b(bc-c^2-ac)$
= $2abc+2cb^2 - 2bc^2 - 2b^2c + 2bc^2 + 2abc$
= $4abc$

16. If
$$(\sin x)^y = x + y$$
, find $\frac{dy}{dx}$

Solution

$$(\sin x)^{y} = x + y$$

$$\log(\sin x)^{y} = \log(x + y) \qquad \dots (i)$$

$$\log(\sin x) \cdot \frac{dy}{dx} + y \cdot \frac{d}{dx} [\log(\sin x)] = \frac{d}{dx} [\log(x + y)]$$

$$\Rightarrow \log(\sin x) \cdot \frac{dy}{dx} + y \cdot \frac{\cos x}{\sin x} = \frac{1}{(x + y)} \cdot (1 + \frac{dy}{dx})$$

$$\Rightarrow \frac{dy}{dx} [\log(\sin x) - \frac{1}{(x + y)}] = \frac{1}{(x + y)} - y \cdot \cot x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1 - (xy + y^{2}) \cdot \cot x}{(x + y) \cdot \log(\sin x) - 1}$$

17. If
$$y = (\sec^{-1} x)^2$$
, $x > 0$ show that $x^2 (x^2 - 1) \frac{d^2 y}{dx^2} + (2x^3 - x) \frac{dy}{dx} - 2 = 0$

$$y = (\sec^{-1} x)^{2}, x > 0$$

$$\Rightarrow \frac{dy}{dx} = 2 \sec^{-1} x \cdot \frac{d(\sec^{-1} x)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 2 \sec^{-1} x \cdot \frac{1}{x\sqrt{x^{2}-1}} \qquad \dots (i)$$

$$\Rightarrow \frac{d^{2} y}{dx^{2}} = 2 \left[\frac{1}{x^{2} (x^{2}-1)} \right] + 2 \sec^{-1} x \left[\frac{-\sqrt{x^{2}-1} - x \left(\frac{2x}{2\sqrt{x^{2}-1}}\right)}{x^{2} (x^{2}-1)} \right]$$

$$\Rightarrow \frac{d^{2} y}{dx^{2}} = 2 \left[\frac{1}{x^{2} (x^{2}-1)} \right] + 2 \sec^{-1} x \cdot \frac{1}{x\sqrt{x^{2}-1}} \left[\frac{x (1-2x^{2})}{x^{2} (x^{2}-1)} \right] \qquad \dots (ii)$$

$$\frac{d^{2} y}{dx^{2}} = 2 \left[\frac{1}{x^{2} (x^{2}-1)} \right] + \frac{dy}{dx} \left[\frac{x (1-2x^{2})}{x^{2} (x^{2}-1)} \right]$$

$$\Rightarrow x^{2} (x^{2}-1) \frac{d^{2} y}{dx^{2}} + (2x^{3}-x) \cdot \frac{dy}{dx} - 2 = 0$$

18. Find the equation of a tangent and the normal to the curve $y = \frac{(x-7)}{(x-2)(x-3)}$ at the point where it cuts the x-axis

Equation of the curve is

$$y = \frac{(x-7)}{(x-2)(x-3)}$$

put y=0 in the above equation we get x=7

$$\frac{dy}{dx} = \frac{(x-2)(x-3) - (x-7)(2x-5)}{(x-2)^2 (x-3)^2}$$

The slope of the tangent at point (7,0) is

$$m_{t} = \frac{dy}{dx}\Big|_{(7,0)} = \frac{20}{400} = \frac{1}{20}$$
$$(y-0) = \frac{1}{20}(x-7) \Longrightarrow x - 20y - 7 = 0$$
$$m_{t} \cdot m_{n} = -1$$
$$\Longrightarrow m_{n} = \frac{-1}{\frac{1}{20}} = -20$$

Equation of the normal is

 $(y-0) = -20(x-7) \Longrightarrow 20x + y - 140 = 0$

19. Find
$$\int \frac{\sin 2x}{\left(\sin^2 x + 1\right)\left(\sin^2 x + 3\right)} dx$$

$$\int \frac{\sin 2x}{(\sin^2 x + 1)(\sin^2 x + 3)} dx$$
$$\Rightarrow I = \int \frac{2\sin x \cdot \cos x}{(\sin^2 x + 1)(\sin^2 x + 3)} dx$$

let $\sin^2 x + 3 = t \Rightarrow 2\sin x \cdot \cos x dx = dt$ Therefore,

$$I = \int \frac{dt}{(t-2)t}$$

$$\Rightarrow I = \frac{1}{2} \int \left(\frac{1}{t-2} - \frac{1}{t}\right) dt$$

$$\Rightarrow I = \frac{1}{2} \left[\ln(t-2) - \ln t \right] + c$$

$$\Rightarrow I = \frac{1}{2} \ln\left(\frac{t-2}{t}\right) + c$$

$$\Rightarrow I = \ln\sqrt{\frac{t-2}{t}} + c$$

$$\Rightarrow I = \ln\sqrt{\frac{\sin^2 x + 1}{\sin^2 x + 3}} + c$$

20. Prove that
$$\int_{a}^{b} f(x) dx = \int_{a}^{b} f(a+b-x) dx$$
 and hence evaluate
$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$$

$$let a + b - x = t$$

$$\Rightarrow dx = -dt$$

$$when x = a, t = b and x = b, t = a$$

$$\int_{a}^{b} f(x) dx = -\int_{b}^{a} f(a + b - t) dt$$

$$= \int_{a}^{b} f(a + b - t) dt$$

$$= \int_{a}^{b} f(a + b - x) dx$$

$$\left[\because \int_{a}^{b} f(x) dx = -\int_{b}^{a} f(x) dx \right]$$

$$let I = \int_{a}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan x}} = \int_{\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x} dx}{\sqrt{\cos x + \sqrt{\sin x}}}$$

$$I = \int_{\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{\sqrt{\cos \left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)} dx}{\sqrt{\cos \left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)} + \sqrt{\sin \left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)}}$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x} dx}{\sqrt{\sin x} + \sqrt{\cos x}} \qquad \dots (iii)$$

$$2I = \int_{\frac{\pi}{3}}^{\frac{\pi}{3}} dx = [x] \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

$$\Rightarrow I = \frac{\pi}{12}$$

21. Show that $(1+x^2)dy + 2xydx = \cot xdx$

 $(1+x^{2})dy + 2xydx = \cot xdx$ $\frac{dy}{dx} + \frac{2xy}{1+x^{2}} = \frac{\cot x}{1+x^{2}}$ The Linear differential equation is IF=e^{$\int pdx$} = e^{$\int \frac{2x}{1+x^{2}} = 1+x^{2}$ the general solution is $y(1+x^{2}) = \int \left[\frac{\cot x}{1+x^{2}}(1+x^{2})\right]dx + C$}

$$y(1+x^{2}) = \int \left\lfloor \frac{\cot x}{1+x^{2}}(1+x^{2}) \right\rfloor dx + 0$$
$$= y(1+x^{2}) = \log[\sin x] + C$$

22. let $\vec{a}, \vec{b}, \vec{c}$ be the three vectors such that $|\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 3$. If the projection of \vec{a} and \vec{b} is equal to the projection of \vec{c} along \vec{a} and \vec{b}, \vec{c} are perpendicular to each other then find $|3\vec{a} - 2\vec{b} + 2\vec{c}|$

Solution

$$\begin{aligned} |\vec{a}| &= 1, |\vec{b}| = 2, |\vec{c}| = 3 \\ \text{the projection of } \vec{b} \text{ along } \vec{a} &= \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|} \\ \text{the projection of } \vec{c} \text{ along } \vec{a} &= \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|} \\ \Rightarrow \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|} &= \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|} \\ \Rightarrow \vec{b} \cdot \vec{a} &= \vec{c} \cdot \vec{a} \qquad \dots(i) \\ (3\vec{a} - 2\vec{b} + 2\vec{c})((3\vec{a} - 2\vec{b} + 2\vec{c})) = 9|\vec{a}|^2 - 6\vec{a} \cdot \vec{b} + 6\vec{a} \cdot \vec{c} - 6\vec{b} \cdot \vec{a} + 4|\vec{b}|^2 - 4\vec{b} \cdot \vec{c} + 6\vec{c} \cdot \vec{a} - 4\vec{c} \cdot \vec{b} + 4|\vec{c}|^2 \\ |3\vec{a} - 2\vec{b} + 2\vec{c}|^2 &= 9|\vec{a}|^2 + 4|\vec{b}|^2 + 4|\vec{c}|^2 - 12\vec{a} \cdot \vec{b} + 12\vec{a} \cdot \vec{c} - 8\vec{b} \cdot \vec{c} \\ |3\vec{a} - 2\vec{b} + 2\vec{c}|^2 &= 9|\vec{a}|^2 + 4|\vec{b}|^2 + 4|\vec{c}|^2 \\ \Rightarrow |3\vec{a} - 2\vec{b} + 2\vec{c}|^2 &= 9\times 1 + 4 \times 4 + 4 \times 9 = 61 \\ \Rightarrow |3\vec{a} - 2\vec{b} + 2\vec{c}| &= \sqrt{61} \end{aligned}$$

SECTION - D

23. Find the value of λ for which the following lines are perpendicular to each other

$$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}; \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$$

hence, find whether the lines intersect or not

Solution

$$\frac{x-5}{5\lambda+2} = \frac{y-2}{-5} = \frac{z-1}{1} \qquad \dots(1)$$

and
$$\frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3} \qquad \dots(2)$$

$$a_1 = 5\lambda+2, \ b_1 = -5, \ c_1 = 1 \ and$$

$$a_2 = 1, \ b_2 = 2\lambda, \ c_2 = 3$$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$(5\lambda+2) - 5(2\lambda) + 1(3) = 0$$

$$-5\lambda+5 = 0$$

$$\Rightarrow \lambda = -1$$

24.If
$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$$
, find \mathbf{A}^{-1}

hence, solve the following system of equations

x + y + z = 6y + 3z = 11x - 2y + z = 0

 $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$ Cofactors $A_{11} = 7, A_{12} = 3, A_{13} = -1$ $A_{21} = -3, A_{22} = 0, A_{23} = 3$ $A_{31} = 2, A_{32} = -3, A_{33} = 1$ $A^{-1} = \frac{Adj(A)}{|A|}$ $Adj(A) = \begin{bmatrix} 7 & 3 & -1 \\ -3 & 0 & 3 \\ 2 & -3 & 1 \end{bmatrix}^{T} = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$ |A| = 9 $A^{-1} = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$ For system of equations AX = B $X = A^{-1}B$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix}$

25. Show that the height of a cylinder, which is open at the top, having a given surface area and greatest volume, is equal to the radius of its base.

Solution

x = 1, y = 2, z = 3

Let R be the radius *H* be the height V be the volume S be the total surface area $V = \pi R^2 H$ $S = \pi R^2 + 2\pi R H$ $\Rightarrow H = \frac{S - \pi R^2}{2 \pi R}$ Substituting value of H in V $V = \frac{1}{2} \left(SR - \pi R^3 \right)$ $\frac{dV}{dR} = \frac{1}{2} \left(S - 3\pi R^2 \right)$ $\frac{dV}{dR} = 0$ $\Rightarrow \frac{1}{2} \left(S - 3\pi R^2 \right) = 0$ $R = \sqrt{\frac{S}{3\pi}}$ $\frac{d^2V}{dR^2} = \frac{1}{2} \left(0 - 6\pi R \right)$ $=-3\pi R$ *V* is greatest when $R = \sqrt{\frac{S}{3\pi}}$ $H = \frac{S - \pi \times \frac{S}{3\pi}}{2\pi \sqrt{\frac{S}{3\pi}}}$ $H = \frac{2S}{\frac{3}{2\sqrt{\frac{\pi S}{3}}}}$ $H = \sqrt{\frac{S}{3\pi}}$

26. Find the area of the triangle whose vertices are (-1, 1), (0, 5) and (3, 2), using integration.



Let A(-1,1), B(0,5) and C(3,2)The equation of line AB is $y-1=\frac{5-1}{0+1}(x+1)$ y = 4x+5The equation of line BC is $y-5=\frac{2-5}{3-0}(x-0)$ y = -x+5The equation of line CA is $y-2=\frac{1-2}{-1-3}(x-3)$ $y = \frac{x}{4} + \frac{5}{4}$ Required area = Area of $\triangle ABC$ The equation of line CA is

$$y - 2 = \frac{1 - 2}{-1 - 3} (x - 3)$$
$$y = \frac{x}{4} + \frac{5}{4}$$

27. Find the equation of the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{2i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x-axis. Hence, find the distance of the plane from x-axis.

Solution

$$\vec{a} = 2\hat{i} + 5\hat{j} - 3\hat{k}, \vec{b} = -2\hat{i} - 3\hat{j} + 5\hat{k}, \vec{c} = 5\hat{i} + 3\hat{j} - 3\hat{k}$$

$$(\vec{r} - \vec{a}) \cdot \left[(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a}) \right] = 0$$

$$\Rightarrow \left[\vec{r} - \left(2\hat{i} + 5\hat{j} - 3\hat{k} \right) \right] \cdot \left[\left(-4\hat{i} - 8\hat{j} + 8\hat{k} \right) \times \left(3\hat{i} - 2\hat{j} \right) \right] = 0$$

$$\Rightarrow \left[\vec{r} - \left(2\hat{i} + 5\hat{j} - 3\hat{k} \right) \right] \cdot \left[2\hat{i} + 3\hat{j} + 4\hat{k} \right] = 0$$

$$\begin{vmatrix} x - 2 & y - 5 & z + 3 \\ -2 - 2 & -3 - 5 & 5 + 3 \\ 5 - 2 & 3 - 5 & -3 + 3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x - 2 & y - 5 & z + 3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (x - 2)(16) - (y - 5)(-24) + (z + 3)(32) = 0$$

$$\Rightarrow 2x + 3y + 4z = 7$$

$$2(2\lambda + 3) + 3(2\lambda + 1) + 4(3\lambda + 5) = 7$$

$$\Rightarrow 22\lambda = -22$$

$$\Rightarrow \lambda = -1$$

Therefore, point of intersection is (1, -1, 2)

28. There are two boxes I and II. Box I contains 3 red and 6 Black balls. Box II contains 5 red and black balls. One of the two boxes, box I and box II is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box II is ' a find the value of n

$$E_{1} = selecting box I$$

$$E_{2} = selecting box II$$

$$A = getting a red ball from selected box$$

$$P(E_{1}) = \frac{1}{2}, P(E_{1}) = \frac{1}{2}$$

$$P\left(\frac{A}{E_{1}}\right) = \frac{3}{9} = \frac{1}{3}$$

$$P\left(\frac{A}{E_{2}}\right) = \frac{5}{n+5}$$

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$
$$\frac{3}{5} = \frac{\frac{1}{2} \times \frac{5}{n+5}}{\frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{5}{n+5}}$$
$$\frac{3}{5} = \frac{15}{n+20}$$
$$(n+20)3 = 75$$
$$3n = 15$$

n = 5

XII CBSE - BOARD - MARCH - 2018

CODE (65/2) Mathematics - Solutions

Date: 21.03.2018

Section-A

1. If a * b denotes the larger of 'a' and 'b' and if $a \circ b = (a * b) + 3$, then write the value of $(5) \circ (10)$, where * and \circ are binary operations.

Sol: $(5) \circ (10) = (5*10) + 3 = 10 + 3 = 13$

- 2. Find the magnitude of each of the two vectors \vec{a} and \vec{b} , having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$
- Sol: Given:

$$|\overline{\mathbf{a}}| = |\overline{\mathbf{b}}| \text{ and } \theta = 60^{\circ} \text{ and } \overline{\mathbf{a}} \cdot \overline{\mathbf{b}} = \frac{9}{2}$$

$$\therefore \ \overline{\mathbf{a}} \cdot \overline{\mathbf{b}} = |\overline{\mathbf{a}}| |\overline{\mathbf{b}}| \cos \theta$$

$$\frac{9}{2} = |\overline{\mathbf{a}}| |\overline{\mathbf{a}}| \cos 60^{\circ}$$

$$\frac{9}{2} = |\overline{\mathbf{a}}|^{2} \times \frac{1}{2}$$

$$|\overline{\mathbf{a}}|^{2} = 9$$

$$|\overline{\mathbf{a}}| = 3 = |\overline{\mathbf{b}}|$$

3. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b'.

Sol: ... A is skew symmetric matrix

$$a_{12} = -a_{21} \Longrightarrow a = -2$$

and $a_{31} = -a_{13} \Longrightarrow b = 3$

4. Find the value of
$$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$$

Sol: $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3}) = k$ (say)
as $\cot^{-1}(-x) = \pi - \cot^{-1} x$
 $\therefore k = \tan^{-1}(\sqrt{3}) - (\pi - \cot^{-1}(\sqrt{3}))$
 $= \frac{\pi}{3} - (\pi - \frac{\pi}{6})$
 $= \frac{\pi}{3} - \pi + \frac{\pi}{6}$
 $= \frac{\pi}{2} - \pi$
 $= -\frac{\pi}{2}$

Section-B

5. The total cost C(x) associated with the production of x units of an item is given by

 $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of total cost at any level of output.

Sol: $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$

Marginal cost
$$(C_M) = \frac{d}{dx} (C(x)) = 0.005 \times 3x^2 - 0.02 \times 2x + 30$$

 $\therefore x = 3$
 $C_M = 0.005 \times 3 \times 9 - 0.02 \times 2 \times 3 + 30$
 $= 0.135 - 0.12 + 30$
 $= 30.135 - 0.12$
 $= 30.015$

6. Differentiate
$$\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$
 with respect to x.
Sol: Let $y = \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

$$\therefore y = \tan^{-1} \left(\frac{2 \cos^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} \right)$$
$$= \tan^{-1} \left(\cot \frac{x}{2} \right)$$
$$= \tan^{-1} \left(\tan \left(\frac{\pi}{2} - \frac{x}{2} \right) \right)$$
$$\therefore y = \frac{\pi}{2} - \frac{x}{2}$$
$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{\pi}{2} - \frac{x}{2} \right) = -\frac{1}{2}$$

7. Given
$$A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$
 compute A^{-1} and show that $2A^{-} = 9I - A$.

Sol: $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$$\begin{bmatrix} -4 & 7 \end{bmatrix}$$

$$|A| = 14 - 12 = 2$$

$$\therefore A_{11} = 7 \qquad A_{12} = 4 \qquad A_{31} = 3 \qquad A_{22} = 2$$

$$adj(A) = \begin{bmatrix} A_{11} & A_{22} \\ A_{21} & A_{22} \end{bmatrix}^{T} = \begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}^{T} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adj(A) = \frac{1}{2} \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$LHS. = 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$R.H.S. = 9I - A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$L.H.S. = R.H.S.$$

8. Prove that :
$$3\sin^{-1} x = \sin^{-1} (3x - 4x^3), X \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

Sol: When
$$-\frac{1}{2} \le x \le \frac{1}{2}$$

We have,

$$-\frac{1}{2} \le x \le \frac{1}{2} \Longrightarrow -\frac{\pi}{6} \le \theta \le \frac{\pi}{6} \Longrightarrow -\frac{\pi}{2} \le 3\theta \le \frac{\pi}{2}$$

Also,
$$-\frac{1}{2} \le x \le \frac{1}{2} \Longrightarrow -1 \le 3x - 4x^3 \le 1$$

 $\therefore \sin 3\theta = 3x - 4x^3$
 $\Rightarrow 3\theta = \sin^{-1}(3x - 4x^3)$
 $\Rightarrow 3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$

9. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Sol:
$$S = \{(1,1), (1,2), ..., (6,6)\}$$

 $\therefore n(s) = 36$
 $A = \text{Red die resulted in a number less than 4.}$
 $= \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (4,1), (4,2), (4,3),$
 $(5,1), (5,2), (5,3), (6,1), (6,2), (6,3)\}$
 $n(A) = {}^{18}C_1 = 18$
 $B = \text{sum of number is 8}$
 $B = \{(4,4), (6,2), (2,6), (5,3), (3,5)\}$
 $n(B) = {}^{5}C_1 = 5$
 $A \cap B = \{(5,3), (6,2)\}$
 $n(A \cap B) = {}^{2}C_1 = 2$
 $\therefore P\left(\frac{B}{A}\right) = \text{Probability of sum of number 8 when Red die resulted in a number less than}$
 $4 = \frac{P(A \cap B)}{P(A)} = \frac{n(A \cap B)}{n(A)} = \frac{2}{18} = \frac{1}{9}$

10. If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$ find $\sin \theta$.

Sol:

$$\overline{a} = \hat{i} - 2j + 3\hat{k}, \ \overline{b} = 3\hat{i} - 2\hat{j} + \hat{k}$$

 $\overline{a} \times \overline{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 3 & -2 & 1 \end{vmatrix} = (4)\hat{i} + 8\hat{j} + 4\hat{k}$
 $\left| \overline{a} \times \overline{b} \right| = \sqrt{(4)^2 + (8)^2 + (4)^2} = \sqrt{16 + 64 + 16} = \sqrt{96}$

$$= 4\sqrt{6}$$
$$|\overline{a}| = \sqrt{(1) + (4) + 9} = \sqrt{14}$$
$$|\overline{b}| = \sqrt{9 + 4 + 1} = \sqrt{14}$$
$$\sin \theta = \left|\frac{\sqrt{96}}{\sqrt{14} \times \sqrt{14}}\right| = \frac{4\sqrt{16}}{14} = \frac{2\sqrt{6}}{7}$$

11. Find the differential equation representing the family of curves $y = ae^{bx+5}$, where *a* and *b* are arbitrary constants.

Sol:
$$y = ae^{bx} \times e^{5}$$

 $y = ae^{bx} \times e^{5}$
 $y = \alpha e^{bx}$ where $e^{5}a = \alpha$
Differentiate w.r.t. 'x'
 $\frac{dy}{dx} = \alpha be^{bx}$
 $\therefore \frac{dy}{dx} = by$
 $\frac{dy}{dx} = b$
Again differentiate w.r.t. 'x'
 $\frac{y\frac{d^{2}y}{dx^{2}} - \frac{dy}{dx} \times \frac{dy}{dx}}{y^{2}} = 0$
 $y\frac{d^{2}y}{dx^{2}} - \left(\frac{dy}{dx}\right)^{2} = 0$

12. Evaluate:
$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$$

Sol:
$$I = \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$$
$$I = \int \frac{1 - 2\sin^2 x + 2\sin^2 x}{\cos^2 x} dx$$
$$I = \int \sec^2 x \, dx$$
$$I = \tan x + C$$

Section-C

13. If
$$y = \sin(\sin x)$$
, prove that $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$

Sol: $y = sin(sin x) \dots (1)$

$$\frac{dy}{dx} = \cos(\sin x) \times \cos x \Rightarrow \frac{\frac{dy}{dx}}{\cos x} = \cos(\sin x)....(2)$$
$$\frac{d^2y}{dx^2} = -\cos(\sin x) \times \sin x - \cos x \sin(\sin x) \cos x \qquad(3)$$
Put (1) and (2) in (3)
$$\frac{d^2y}{dx^2} = -\left(\frac{\frac{dy}{dx}}{\cos x}\right) \times \sin x - y \cos^2 x$$

$$\frac{d^2y}{dx^2} = -\frac{dy}{dx}\tan x - y\cos^2 x$$
$$\frac{d^2y}{dx^2} + \frac{dy}{dx}\tan x + y\cos^2 x = 0$$

14. Find the particular solution of the differential equation $e^x \tan y dx + (2 - e^x) \sec^2 y dy = 0$, given that $y = \frac{\pi}{4}$ when x = 0

Sol: $e^x \tan y \, dx + (2 - e^x) \sec^2 y \, dy = 0$

 $e^x \tan y \, dx = (e^x - 2) \sec^2 y \, dy$

 $\int \frac{e^{x} dx}{e^{x} - 2} = \int \frac{\sec^{2} y \, dy}{\tan y}$ $\ln |e^{x} - 2| = \ln |\tan y| + \ln C$ $\ln |e^{x} - 2| = \ln (C \tan y)$ $e^{x} - 2 = C \tan y$ Given: $x = 0, \ y = \frac{\pi}{4}$ $e^{o} - 2 = C \tan \left(\frac{\pi}{4}\right)$

$$e^{o} - 2 = C \tan\left(\frac{\pi}{4}\right)$$
$$1 - 2 = C \times 1 \implies C = -1$$
$$\therefore e^{x} - 2 = -\tan y$$
$$e^{x} - 2 + \tan y = 0$$

(OR)

Find the particular solution of the differential equation $\frac{dy}{dx} + 2y \tan x = \sin x$, given that y = 0 when $x = \frac{\pi}{3}$

Sol:

$$\frac{dy}{dx} + (2 \tan x)y = \sin x$$

$$\frac{dy}{dx} + py = Q$$

$$P = 2 \tan x \text{ and } Q = \sin x$$

$$I.F = e^{\int Pdx} = e^{2\int \tan x \, dx} = e^{2\ln \sec x}$$

$$= e^{\ln \sec^2 x} = \sec^2 x$$
Soln. $y(I.F) = \int Q(I.F) \, dx$

$$y \cdot \sec^2 x = \int \sin x \cdot \sec^2 x \, dx$$

$$y \sec^2 x = \int \tan x \sec x \, dx$$

$$y \sec^2 x = \sec x + C$$
Given $y = 0$

$$x = \frac{\pi}{3}$$

$$\sec \frac{\pi}{3} + C = 0$$

$$C = -\sec \frac{\pi}{3} = -2$$

$$\therefore y \sec^2 x = \sec x - 2$$

$$y \sec^2 x - \sec x + 2 = 0$$
15. Find the shortest distance between the lines.

$$\vec{r} = (4\hat{i} - \hat{j}) + \lambda (\hat{i} + 2\hat{j} - 3\hat{k}) \text{ and } \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu (2\hat{i} + 4\hat{j} - 5\hat{k})$$
Sol:

$$\vec{r} = (4\hat{i} - \hat{j}) + \lambda (\hat{i} + 2\hat{j} - 3\hat{k}) = \vec{a} + \lambda \vec{b} \quad (\text{say})$$

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu (2\hat{i} + 4\hat{j} - 5\hat{k}) = \vec{c} + \mu \vec{d} \quad (\text{say})$$

$$\therefore \vec{c} - \vec{a} = (\hat{i} - \hat{j} + 2\hat{k}) - (4\hat{i} - \hat{j}) = -3\hat{i} + 0\hat{j} + 2\hat{k}$$

$$\therefore \vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i} - \hat{j} + 0\hat{k}$$

$$|\vec{b} \times \vec{d}| = \sqrt{4 + 1} = \sqrt{5}$$
Shortest distance = $\left| \frac{(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d})}{|\vec{b} \times \vec{d}|} \right|$

$$= \left| \frac{-6}{\sqrt{5}} \right| = \frac{6}{\sqrt{5}} \text{ units}$$

16. Two numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean and variance of X.

Sol: X can take values as 2,3,4,5 such that

P(X = 2) = probability that the larger of two number 2.

= prob. of getting 1 in first selection and 2 in second selection getting 2 in first selection and 1 in second selection.

$$\therefore P(X=2) = \frac{1}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{4} = \frac{2}{20}$$

similarly,

$$\therefore P(X=3) = \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4} = \frac{4}{20}$$
$$\therefore P(X=4) = \frac{3}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} = \frac{6}{20}$$
$$\therefore P(X=5) = \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{4}{4} = \frac{8}{20}$$

Х	2	3	4	5
P(X)	2	4	6	8
	20	20	20	20

$$E(X) = 2 \times \frac{2}{20} + 3 \times \frac{4}{20} + 4 \times \frac{6}{20} + 5 \times \frac{8}{20}$$

= $\frac{80}{20} = 4$
$$E(X^{2}) = 4 \times \frac{2}{20} + 9 \times \frac{4}{20} + 16 \times \frac{6}{20} + 25 \times \frac{8}{20}$$

= $\frac{340}{20} = 17$
$$V(X) = E(X^{2}) - (E(X))^{2}$$

= $17 - 16$
= 1

17. Using propeties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} = 9(3xyz + xy + yz + zx)$$

Sol:

$$L.H.S. = \begin{vmatrix} 1 & 1 & 1 + 3x \\ 1 + 3y & 1 & 1 \\ 1 & 1 + 3z & 1 \end{vmatrix}$$
$$C_1 \to C_1 - C_2 ; C_3 \to C_3 - C_2$$
$$= \begin{vmatrix} 0 & 1 & 3x \\ 3y & 1 & 0 \\ -3z & 1 + 3z & -3z \end{vmatrix}$$
$$= (3 \times 3) \begin{vmatrix} 0 & 1 & x \\ y & 1 & 0 \\ -z & 1 + 3z & -z \end{vmatrix}$$
$$= 9 \begin{bmatrix} -1(-yz - 0) + x(y + 3zy + z) \end{bmatrix}$$
$$= 9 (yz + xy + 3xyz + xz)$$

=9(3xyz + xy + yz + zx) = R.H.S.Hence proved.

18. Find the equations of the tangent and the normal, to the curve $16x^2 + 9y^2 = 145$ at the point (x_1, y_1) where $x_1 = 2$ and $y_1 > 0$

: $P(x_1, y_1) \equiv (2, y_1)$ lies on $16x^2 + 9y^2 = 145$ Sol: $16(2)^2 + 9y_1^2 = 145$ $9y_1^2 = 145 - 64$ $9y_1^2 = 81$ $v_1^2 = 9$ $y_1 = \pm 3$ But $y_1 > 0$ $\therefore y_1 = 3$ $\therefore P \equiv (2,3)$ $16x^2 + 9y^2 = 145$...(i) $32x + 18y \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{-32x}{18y} = \frac{-16x}{9y}$ Slope of tangent = $m_{(2,3)} = \frac{-16 \times 2}{9 \times 3} = \frac{-32}{27}$ Slope of normal $= m'_{(2,3)} = \frac{27}{32}$ Equation of tangent is, $(y-3) = \frac{-32}{27}(x-2)$ 27y - 81 = -32x + 6432x + 27y - 145 = 0Equation of normal is, $(y-3) = \frac{27}{32}(x-2)$ 32v - 96 = 27x - 5427x - 32y - 54 + 96 = 027x - 32y + 42 = 0

Find the intervals in which the function $f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$ is

(a) strictly increasing, (b) strictly decreasing.

19. Find:
$$\int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)} dx$$

Sol: Let
$$I = \int \frac{2\cos x}{(1 - \sin x)(1 + \sin^2 x)} dx$$

Let $\sin x = t$

 $\cos dx = dt$

$$\therefore I = \int \frac{2}{(1-t)(1+t^2)} dt$$

Consider

$$\frac{2}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{t^2+1}$$
$$= \frac{A(t^2+1) + (Bt+C)(1-t)}{(1-t)(t^2+1)}$$
$$\therefore 2 = At^2 + A + Bt + C - Bt^2 - Ct$$
$$= (A-B)t^2 + (B-C)t + (A+C)$$
$$\therefore A - B = 0, B - C = 0 \quad A + C = 2$$

$$A=1, B=1, C=1$$

$$\therefore I = \int \left(\frac{1}{1-t} + \frac{2t}{2(t^2+1)} + \frac{1}{t^2+1} \right) dt$$

$$= -\log|1-t| + \frac{1}{2}\log|t^2+1| + \tan^{-1}(t) + C$$

$$= \frac{1}{2}\log\left|\frac{t^2+1}{(1-t)^2}\right| + \tan^{-1}(t) + C$$

$$= \frac{1}{2}\log\left|\frac{\sin^2 x + 1}{(1-\sin x)^2}\right| + \tan^{-1}(\sin x) + C$$

- 20. Suppose a girl throws a die. If she gets 1 or 2 she tosses a coin three times and notes the number of tails. If she gets 3,4,5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3,4,5 or 6 with the ride ?
- Sol: Let *A* be the event that girl will get 1 or 2

$$P(A) = \frac{2}{6} = \frac{1}{3}$$

Let *B* be the event that girl will get 3, 4, 5 or 6
$$P(B) = \frac{4}{6} = \frac{2}{3}$$

$$P(T/A) = \text{Probability of exactly one till given she will get 1 or 2 = \frac{3}{8}$$

$$P(T/B) = \text{Probability of exactly one till given she will get 3, 4, 5 or 6 = \frac{1}{2}$$

$$P\left(\frac{B}{T}\right) = \frac{P(B) \times P\left(\frac{T}{B}\right)}{P(A) \times P\left(\frac{T}{A}\right) + P(B) \times P\left(\frac{T}{B}\right)}$$
$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}}$$
$$= \frac{\frac{1}{3}}{\frac{1}{8} + \frac{1}{3}}$$

$$=\frac{\frac{1}{3}}{\frac{11}{8\times3}}$$
$$=\frac{8}{11}$$

- 21. Let $\vec{a} = 4\hat{i} + 5\hat{j} \hat{k}$, $\vec{b} = \hat{i} 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} \hat{k}$. Find a vector \vec{d} which is perpendicular to both \vec{c} and \vec{b} and $\vec{d} \cdot \vec{a} = 21$
- Sol: Since \overline{d} is perpendicular to both \overline{c} and \overline{b} , therefore, if is parallel to $\overline{c} \times \overline{b}$

$$\therefore \overline{d} = \lambda (\overline{c} \times \overline{b})$$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 1 & -4 & 5 \end{vmatrix}$$

$$= \lambda \left\{ (5-4)\hat{i} - (15+1)\hat{j} + (-12-1)\hat{k} \right\}$$

$$= \lambda \left\{ \hat{i} - 16\hat{j} - 13\hat{k} \right\}$$

Given that

_ _

$$d \cdot a = 21$$

$$\lambda \left\{ \hat{i} - 16\hat{j} - 13\hat{k} \right\} \cdot \left(4\hat{i} + 5\hat{j} - \hat{k} \right) = 21$$

$$\lambda \left(4 - 80 + 13 \right) = 21$$

$$\lambda = \frac{21}{-63} = -\frac{1}{3}$$

$$\therefore \ \overline{d} = -\frac{1}{3} \left(\hat{i} - 16\hat{j} - 13\hat{k} \right)$$

$$= \left(-\frac{1}{3} \right) \hat{i} + \left(\frac{16}{3} \right) \hat{j} + \left(\frac{13}{3} \right) \hat{k}$$

22. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width. If the cost is to be least when depth of the tank is half of its width. If the cost is to be borne by nearby settled lower income families, for whom water will be provided, what kind of value is hidden in this question ?

Sol: Let the length, width and height of the open tank be x, x and y units respectively. Then, its volume is x^2y and the total surface area is $x^2 + 4xy$.

It is given that the tank can hold a given quantity of water. This means that its volume is constant. Let it be V. Then,

 $V = x^2 y$

The cost of the material will be least if the total surface area is least. Let S denote the total surface area. Then,

$$S = x^2 + 4xy$$

We have to minimize S object to the condition that the volume V is constant. Now,

 $S = x^{2} + 4xy$ $\Rightarrow S = x^{2} + \frac{4V}{x}$ $\Rightarrow \frac{dS}{dx} = 2x - \frac{4V}{x^{2}} \text{ and } \frac{d^{2}S}{dx^{2}} = 2 + \frac{8V}{x^{3}}$ The critical numbers of S are given by $\frac{dS}{dx} = 0$.
Now, $\frac{dS}{dx} = 0$ $\Rightarrow 2x - \frac{4V}{x^{2}} = 0$ $\Rightarrow 2x^{3} - 4V = 0$ $\Rightarrow 2x^{3} = 4x^{2}y$ $\Rightarrow x = 2y$



Hence, S is minimum when x = 2y i.e. the depth (height) of the tank is half of its width. Comment : Base is directly proportional to height.

23. If
$$(x^2 + y^2)^2 = xy$$
, find $\frac{dy}{dx}$.

Sol: Given:

(x² + y²)² = xyx⁴ + y⁴ + 2x²y² = xydiff. w.r.t. x.



$$4x^{3} + 4y^{3} \frac{dy}{dx} + 2\left(2x^{2}y\frac{dy}{dx} + 2xy^{2}\right) = \left(x\frac{dy}{dx} + y\right)$$

$$4y^{3} \frac{dy}{dx} + 4x^{2}y\frac{dy}{dx} - x\frac{dy}{dx} = y - 4x^{3} - 4xy^{2}$$

$$\frac{dy}{dx}\left(4y^{3} + 4x^{2}y - x\right) = y - 4x^{3} - 4xy^{2}$$

$$\frac{dy}{dx} = \frac{y - 4x^{3} - 4xy^{2}}{4y^{3} + 4x^{2}y - x}$$

(OR)

If
$$x = a(2\theta - \sin 2\theta)$$
 and $y = a(1 - \cos 2\theta)$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$.
Sol: $y = a[1 - \cos 2\theta]$, $\frac{dy}{d\theta} = a(0 - 2\sin 2\theta)$
 $\therefore \frac{dy}{d\theta} = -2a\sin 2\theta$
 $x = a(2\theta - \sin 2\theta)$, $\frac{dx}{d\theta} = a(2 - 2\cos 2\theta)$
 $\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2a\sin 2\theta}{2a[1 - \cos 2\theta]}$ $\left(\because \frac{dx}{d\theta} \neq 0\right)$
 $\Rightarrow \frac{-2\sin\theta\cos\theta}{2\sin^2\theta} = -\cot\theta$
 $\therefore \frac{dy}{dx} = -\cot\left(\frac{\pi}{3}\right) = -\frac{1}{\sqrt{3}}$
Section- D

24. Evaluate:
$$\int_{0}^{\pi/4} \frac{\sin x + \cos x}{16 + 9\sin 2x} dx$$

Sol: Let I =
$$\int_{0}^{\pi/4} \frac{\sin x + \cos x}{16 + 9\sin 2x} dx$$

Here, we express the denominator in terms $\sin x - \cos x$ which is integration of numerator.

Clearly,
$$(\sin x - \cos x)^2 = \sin^2 x + \cos^2 x - 2\sin x \cos x = 1 - \sin 2x$$

$$\Rightarrow \sin 2x = 1 - (\sin x - \cos x)^2$$

$$\therefore I = \int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9\left\{1 - (\sin x - \cos x)^2\right\}} dx$$

$$\Rightarrow I = \int_0^{\pi/4} \frac{\sin x + \cos x}{25 - 9\left(\sin x - \cos x\right)^2} dx$$

Let $\sin x - \cos x = t$. Then, $d(\sin x - \cos x) = dt \Rightarrow (\cos x + \sin x) dx = dt$.

Also,
$$x = 0 \Longrightarrow t = \sin 0 - \cos 0 = -1$$
 and $x = \frac{\pi}{4} \Longrightarrow t = \sin \frac{\pi}{4} - \cos \frac{\pi}{4} = 0$

$$\therefore I = \int_{-1}^{0} \frac{dt}{25 - 9t^2} = \frac{1}{9} \int_{-1}^{0} \frac{dt}{\frac{25}{9} - t^2} = \frac{1}{9} \int_{-1}^{0} \frac{dt}{\left(\frac{5}{3}\right)^2 - t^2}$$

$$\Rightarrow I = \frac{1}{9} \times \frac{1}{2(5/3)} \left[\log \left| \frac{5/3 + t}{5/3 - t} \right| \right]_{-1}^{0}$$

$$\Rightarrow I = \frac{1}{30} \left[\log 1 - \log \left(\frac{2/3}{8/3} \right) \right] = \frac{1}{30} \left[\log 1 - \log \left(\frac{1}{4} \right) \right] = \frac{1}{30} \left[\log 1 + \log 4 \right] = \frac{1}{30} \log 4 = \frac{1}{15} \log 2$$

Evaluate:
$$\int_{1}^{3} (x^2 + 3x + e^x) dx$$

 $I = \int_{1}^{3} (x^{2} + 3x + e^{x}) dx = \int_{a}^{b} f(x) dx \text{ (say)}$ when $f(\mathbf{x}) = \mathbf{x}^2$

when
$$f(x) = x^2 + 3x + e^x$$
; $a = 1, b = 3$

$$h = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$$

$$f(a+rh) = f(1+rh) = (1+rh)^2 + 3(1+rh) + e^{1+rh}$$

$$= 4 + 5rh + r^2h^2 + e \times e^{rh}$$

$$= r^2h^2 + 5rh + 4 + e \times e^{rh}$$

$$\int_{a}^{b} f(x)dx = \lim_{n \to \infty} \sum_{r=1}^{n} h f(a+rh)$$

$$= \lim_{n \to \infty} \sum_{r=1}^{n} h(r^{2}h^{2} + 5rh + 4 + e \times e^{rh})$$

$$= \lim_{n \to \infty} \left(\sum_{r=1}^{n} r^{2}h^{3} + 5\sum_{r=1}^{n} rh^{2} + \sum_{r=1}^{n} 4h + e\sum_{r=1}^{n} e^{rh} \times h \right)$$

$$= \lim_{n \to \infty} \left(\frac{8}{n^{3}} \times \frac{n(n+1)(2n+1)}{6} + 5 \times \frac{4}{n^{2}} \times \frac{n(n+1)}{2} + 4 \times \frac{2}{n} \times n + e\left(e^{h}\left(\frac{e^{nh} - 1}{e^{h} - 1}\right)\right) \cdot h\right)$$

$$= \lim_{n \to \infty} \left[\left(\frac{8}{6} \times \frac{n}{n} \times \left(\frac{n}{n} + \frac{1}{n}\right) \times \left(\frac{2n}{n} + \frac{1}{n}\right) + \frac{20}{2} \times \frac{n}{n} \times \left(\frac{n}{n} + \frac{1}{n}\right) + 8 + \frac{e^{n+1}(e^{2} - 1)}{\left(\frac{e^{h} - 1}{h}\right)} \right) \right]$$

as $n \rightarrow \infty$: $h \rightarrow 0$

$$\int_{a}^{b} f(x)dx = \frac{4}{3} \times 1 \times \lim_{n \to \infty} \left(1 + \frac{1}{n}\right) \times \lim_{n \to \infty} \left(2 + \frac{1}{n}\right) + \lim_{n \to \infty} 10 \times 1 \times \left(1 + \frac{1}{n}\right) + 8 + \lim_{h \to 0} \frac{e^{h+1}(e^{2} - 1)}{\left(\frac{e^{h} - 1}{h}\right)}$$

$$= \frac{4}{3} \times 1 \times 2 + 10 \times 1 \times 1 + 8 + \frac{e(e^{2} - 1)}{1}$$
$$= \frac{8}{3} + 10 + 8 + e^{3} - e$$
$$= \frac{8}{3} + 18 + e^{3} - e$$
$$= \frac{62}{3} + e^{3} - e$$

- 25. A factory manufactures two types of screws A and B, each type requiring the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on the hand operated machines to manufacture a packet of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screws 'A' at a profit of 70 paise and screws 'B' at a profit of Rs. 1. Assuming that he can sell all the screws he manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit ? Formulate the above LPP and solve it graphically and find the maximum profit.
- Sol: Let the factory manufactures x screws of type A and y screws of type B on each day.

 $\therefore x \ge 0, y \ge 0$ Given that

	Screw A	Screw B	Availibility
Automatic machine	4	6	$4 \times 60 = 240$ minutes
Hand operate machine	6	3	$4 \times 60 = 240$ minutes
Profit	70 paise	1 rupee	

The constraints are

 $4x + 6y \le 240$

 $6x + 3y \le 240$

Total profit

z = 0.70x + 1y

 \therefore L.P.P. is

maximise z = 0.7x + y

subject to,

 $2x + 3y \le 120$

 $2x + y \le 80$



∴ common feasible region is *OCBAO*

Correct point	Z = 0.7x + y
A(40,0)	Z(A) = 28
B(30, 20)	Z(B) = 41 maximum
C(0, 40)	Z(C) = 40
<i>O</i> (0,0)	Z(O) = 0

The maximum value of 'Z' is 41 at (30, 20). Thus the factory showed produce 30 packages at screw A and 20 packages of screw B to get the maximum profit of Rs.41

26. Let $A = \{x \in Z : 0 \le x \le 12\}$ show that $R = \{(a,b) : a, b \in A, |a-b|\}$ is divisible by 4} is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

Sol: We have,

 $R = \{(a,b) : |a-b| \text{ is a multiple of } 4\}, \text{ where } a, b \in A = \{x \in Z : 0 \le x \le 12\} = \{0,1,2,...,12\}.$ We observe the following properties of relation R.

Reflexivity : For any $a \in A$, we have

|a-a|=0, which is a multiple of 4.

$$\Rightarrow$$
 $(a,a) \in \mathbb{R}$

Thus, $(a,a) \in \mathbb{R}$ for all $a \in \mathbb{A}$.

So, R is reflexive.

```
Symmetry : Let (a, b) \in \mathbb{R}. Then,
```

 $(a,b) \in \mathbb{R}$

```
\Rightarrow |a-b| is a multiple of 4
```

 $\Rightarrow |a-b| = 4\lambda \text{ for some } \lambda \in \mathbb{N}$

$$\Rightarrow |b-a| = 4\lambda \text{ for some } \lambda \in \mathbb{N} \qquad \qquad \left[\because |a-b| = |b-a|\right]$$

```
\Rightarrow (b,a) \in R
```

So, R is symmetric.

Transitivity : Let $(a,b) \in R$ and $(b,c) \in R$. Then,

$$(a,b) \in R$$
 and $(b,c) \in R$

 \Rightarrow |a-b| is a multiple of 4 and |b-c| is a multiple of 4

$$\Rightarrow$$
 $|a-b| = 4\lambda$ and $|b-c| = 4\mu$ for some $\lambda, \mu \in \mathbb{N}$

$$\Rightarrow$$
 $a-b=\pm 4\lambda$ and $b-c=\pm 4\mu$

$$\Rightarrow \qquad a-c=\pm 4\lambda\pm 4\mu$$

 \Rightarrow a-c is a multiple of 4

 \Rightarrow |a-c| is a multiple of 4

$$\Rightarrow$$
 $(a,c) \in \mathbb{R}$

Thus, $(a,b) \in R$ and $(b,c) \in R \Longrightarrow (a,c) \in R$

So, R is transitive.

Hence, R is an equivalence relation.

Let x be an element of A such that $(x,1) \in \mathbb{R}$. Then,

 $\begin{aligned} |x-1| &\text{ is a multiple of } 4 \\ \Rightarrow & |x-1| = 0, 4, 8, 12 \\ \Rightarrow & x-1 = 0, 4, 8, 12 \\ \Rightarrow & x = 1, 5, 9 \\ \text{Hence, the set of all elements of A which are related to 1 is } \{1, 5, 9\} \text{ i.e. } [1] = [1, 5, 9]. \end{aligned}$

& [2] = [2, 6, 10]

(OR)

Show that the function $f: R \to R$ defined by $f(x) = \frac{x}{x^2 + 1}$, $\forall x \in R$ is neither one – one nor onto. Also, if $g: R \to R$ is defined as g(x) = 2x - 1 find fog(x)

Sol: $f: R \to R, f(x) = \frac{x}{x^2 + 1}, \forall x \in R$ $f(x_1) = \frac{x_1}{x_2^2 + 1}$ $f(x_1) = f(x_2)$ $\frac{x_1}{x_1^2 + 1} = \frac{x_2}{x_2^2 + 1}$

$$x_1 x_2^2 + x_1 = x_2 x_1^2 + x_2$$

$$x_1 x_2^2 - x_2 x_1^2 + x_1 - x_2 = 0$$

$$x_1 x_2 (x_2 - x_1) - 1(x_2 - x_1) = 0$$

 $(x_1x_2-1)(x_2-x_1)=0$

 $x_1 x_2 = 1$ or $x_1 = x_2$

 $\therefore f(x)$ is not one-one

also
$$y = \frac{x}{x^2 + 1}$$
$$x^2 y - x + y = 0$$
$$\Delta \ge 0 \quad \text{if } x \text{ is real}$$
$$\therefore B^2 - 4AC \ge 0$$
$$(-1)^2 - 4 \times y \times y \ge 0$$
$$1 - 4y^2 \ge 0$$
$$(1 - 2y)(1 + 2y) \ge 0$$
$$(2y - 1)(2y + 1) \le 0$$
$$\therefore -\frac{1}{2} \le y \le \frac{1}{2}$$

Codomain
$$\in R$$

But range $\in \left[-\frac{1}{2}, \frac{1}{2}\right]$

 \therefore Function is not onto

$$f(x) = \frac{x}{x^2 + 1} \text{ as } f: R \to R$$

$$g(x) = 2x - 1 \text{ as } g: R \to R$$

$$(fog)(x) = f(g(x)) = \frac{g(x)}{(g(x)^2 + 1)}$$

$$= \frac{2x - 1}{(2x - 1)^2 + 1}$$

$$= \frac{2x - 1}{4x^2 - 4x + 1 + 1}$$

$$= \frac{2x - 1}{4x^2 - 4x + 2}$$

- 27. Using integration, find the area of the region in the first quadrant enclosed by the x axis, the line y = xand the circle $x^2 + y^2 = 32$
- Sol: Put y = x in $x^2 + y^2 = 32$

$$\therefore x^2 + x^2 = 32$$

$$2x^{2} = 32$$

$$x^{2} = 16$$

$$x = 4$$

$$A = \int_{0}^{4} y_{\text{line}} dx + \int_{4}^{\sqrt{52}} y_{\text{circle}} dx$$

$$A = \int_{0}^{4} x dx + \int_{4}^{\sqrt{52}} (\sqrt{32 - x^{2}}) dx$$

$$= \left(\frac{x^{2}}{2}\right)_{0}^{4} + \int_{4}^{\sqrt{52}} \sqrt{(\sqrt{32})^{2} - x^{2}} dx$$

$$= (8) + \left(\frac{x}{2}\sqrt{32 - x^{2}} + \frac{32}{2}\sin^{-1}\left(\frac{x}{\sqrt{32}}\right)\right)^{\sqrt{52}}$$

$$= (8) + \left(0 + 16 \times \frac{\pi}{2} - \left(2\sqrt{16} + 16\sin^{-1}\left(\frac{4}{\sqrt{32}}\right)\right)\right)$$

$$= 8 + 8\pi - 8 - 16\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$=8\pi - 16 \times \frac{\pi}{4} = 8\pi - 4\pi = 4\pi \text{ sq units}$$

28. If
$$A = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$$
, find A^{-1} . Use it solve the system of equations.
 $2x - 3y + 5z = 11$
 $3x + 2y - 4z = -5$

$$x + y - 2z = -3$$

Sol:

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\therefore |A| = 2(-4+4) + 3(-6+4) + 5(3-2) = 0 - 6 + 5 = -1 \neq 0$$

Now,
$$A_{11} = 0$$
, $A_{12} = 2$, $A_{12} = 1$
 $A_{21} = -1$, $A_{22} = -9$, $A_{23} = -5$
 $A_{31} = 2$, $A_{32} = 23$, $A_{33} = 13$
 $\therefore A^{-1} = \frac{1}{|A|} (adjA) = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ (1)

Now, the given system of equations can be written in the form of AX = B, where

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ x \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

The solution of the system f equations is given by $X = A^{-1}B$,

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \qquad [Using(1)]$$

$$= \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 39 \\ -11 - 25 + 39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, x = 1, y = 2, and z = 3

(OR)

Using elementary row transformations, find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

Sol:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$
$$|A| = 1(-25 + 28) - 2(-10 + 14) + 3(-8 + 10)$$
$$= 3 - 2(4) + 3(2) = 9 - 8 = 1 \neq 0$$
$$A^{-1} \text{ exists.}$$
$$A \cdot A^{-1} = I$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$R_{2} \Rightarrow R_{2} - 2R_{1} ; R_{3} \Rightarrow R_{3} + 2R_{1}$$
$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$
$$R_{1} \Rightarrow R_{1} - 2R_{2}$$
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$
$$R_{1} \Rightarrow R_{1} - R_{3}; R_{2} \Rightarrow R_{2} - R_{3}$$
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$
$$I \cdot A^{-1} = A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

- 29. Find the distance of the point (-1, -5, -10) from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$
- Sol: Cartesian equation of line and plane,

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$$
 :(Line)
$$x - y + z - 5 = 0$$
 : (Plane)

Let Q (α, β, γ) be point of intersection of line and plane which will satisfy both equation.

$$\frac{\alpha - 2}{3} = \frac{\beta + 1}{4} = \frac{\gamma - 2}{2} = \lambda \text{ (say)}$$

$$\alpha = 3\lambda + 2, \beta = 4\lambda - 1, \gamma = 2\lambda + 2$$

also $\alpha - \beta + \gamma - 5 = 0$
$$3\lambda + 2 - 4\lambda + 1 + 2\lambda + 2 - 5 = 0$$

$$\lambda = 0$$

$$\therefore \alpha = 2, \beta = -1, \gamma = 2 \Longrightarrow Q \equiv (2, -1, 2)$$

$$\ell(PQ) = \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2}$$

$$= \sqrt{9+16+144}$$

$$= \sqrt{169}$$

$$= 13 \text{ units}$$

SET-1

MATHEMATICS

Paper & Solution

Code: 65/1 Max. Marks: 100

Series GBM Time: 3 Hrs.

General Instruction:

(i) All questions are compulsory

(ii) The question paper consists of **29** questions divided into four section A, B, C and D. Sections A comprises of questions of **one mark** each, Section B comprises of **8** questions of **two marks** each, Section C comprises of **11** questions of **four marks** each and Section D comprises of **6** questions of **six marks** each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 3 questions of four marks each and 3 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is **not** permitted. You may ask for logarithmic tables, if required.

SECTION – A

Question numbers 1 to 4 carry 1 mark each

1. If for any 2 × 2 square matrix A, A (adj A) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of |A|.

Solution:

A (adj A) =
$$\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$

by using property

A (adj A) = |A| I_n

$$\Rightarrow |A| I_n = \begin{bmatrix} 8 & 0\\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow |A| I_n = 8 \begin{bmatrix} 1 & 0\\ 0 & 1 \end{bmatrix} \Rightarrow |A| = 8$$

2. Determine the value of 'k' for which the following function is continuous at x = 3:

= 3

$$f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x-3} & , x \neq 3 \\ k & , x = 3 \end{cases}$$

Solution:

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{(x+3)^2 - 36}{x-3}$$
$$= \lim_{x \to 3} \frac{(x+3-6)(x+3+6)}{(x-3)}$$
$$= 12$$
given that f(x) is continuous at x
$$\therefore \lim_{x \to 3} f(x) = f(3)$$

 $\Rightarrow k = 12$

3. Find:
$$\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$$

Solution:
$$\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$$
$$= 2\int \frac{-\cos 2x}{\sin x} dx$$
$$= -2\int \cot 2x dx$$
$$= \frac{-2\log|\sin 2x|}{2} + C$$
$$= -\log|\sin 2x| + C$$

4. Find the distance between the planes 2x - y + 2z = 5 and $5x - 2 \cdot 5y + 5z = 20$.

Solution:

- $2x y + 2z = 5 \dots (1)$
- $5x 2 \cdot 5 y + 5z = 20$

or $2x - y + 2z = 8 \dots (2)$

Distance between plane (1) & (2)

$$= \left| \frac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}} \right| = \left| \frac{3}{\sqrt{9}} \right| = 1$$

SECTION - B

Question numbers 5 to 12 carry 2 marks each

5. If A is a skew-symmetric matrix of order 3, then prove that det A = 0. **Solution:**

Let $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ be a skew symmetric matrix of order 3 $\therefore |A| = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ |A| = -a(0+bc) + b(ac-0)= -abc + abc = 0 Proved 6. Find the value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in $[-\sqrt{3}, 0]$. Solution: $f(x) = x^3 - 3x$ (i) f(x) being a polynomial is continuous on $[-\sqrt{3}, 0]$ (ii) $f(-\sqrt{3}) = f(0) = 0$ (iii) $f(x) = 3x^2 - 3$ and this exist uniquely on $[-\sqrt{3}, 0]$ \therefore f(x) is derivable on $(-\sqrt{3}, 0)$ \therefore f(x) satisfies all condition of Rolle's theorem \therefore There exist at least one $c \in (-\sqrt{3}, 0)$ where f'(c) = 0 $\Rightarrow 3c^2 - 3 = 0$ $\Rightarrow c = \pm 1 \Rightarrow c = -1$

7. The volume of a cub is increasing at the rate of 9 cm $\frac{3}{s}$. How fast is it surface area increasing when the

length of an edge is 10 cm? Solution: Assumed volume of cube = V Given that, $\frac{dV}{dt} = 9cm^3/\sec$ $\frac{dA}{dt} = ?$ l = 10 cm $\frac{dV}{dt} = \frac{d}{dt}(l)^3 = 9 \Rightarrow 3l^2 \frac{dl}{dt} = 9$ $\frac{dl}{dt} = \frac{3}{l^2}$(1) Now $\frac{dA}{dt} = \frac{d}{dt}(6l^2) = 12l \frac{dl}{dt} = 12l \times \frac{3}{l^2}$ (form (1)) $= \frac{36}{l} = \frac{36}{10} = 3.6 \text{ cm}^2/\sec$

8. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.

Solution: $f(x) = x^2 - 3x^2 + 6x - 100$ $f'(x) = 3x^2 - 6x + 6$ $f'(x) = 3 (x^2 - 2x + 2)$ $f'(x) = 3 [(x - 1)^2 + 1]$ f'(x) > 0 for all $x \in \mathbb{R}$ So, f(x) is increasing on \mathbb{R} . **9.** The x- coordinate of a point on the joining the points P(2, 2, 1) and Q(5, 1, -2) is 4. Find its z-coordinate. **Solution:**

$$\frac{k}{P} + \frac{1}{R} = Q$$
(2, 2, 1)
(5, 1, -2)
Let **R** divides **PQ** in the ratio **k**: 1
$$R\left(\frac{5k+2}{k+1}, \frac{k+2}{k+1}, \frac{-2k+1}{k+1}\right)$$
Given x co-ordinate of R = 4
$$\therefore \frac{5k+2}{k+1} = 4$$

$$\Rightarrow k = 2$$

$$\therefore \text{ z co- ordinate } = \frac{-(2)+1}{2+1} = -1$$

10. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.

Solution:

A = {2, 4, 6} $P(A) = \frac{3}{6} = \frac{1}{2}$ B = {1, 2, 3} A \cap B = {2} $P(B) = \frac{3}{6} = \frac{1}{2}$ P(A \cap B) = $\frac{1}{6}$ Here, P(A) P(B) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ Since, P(A \cap B) \neq P(A) P(B), so events A and B are not independent events.

11. Tow tailors, A and B, earn ₹ 300 and ₹ 400 per day respectively A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labor cost, formulate this as an LPP.

Solution:

	Tailor A	Tailor B	Minimum Total No.
No. of shirts	6	10	60
No. of trousers	4	4	32
Wage	Rs 300/day	Rs 400/day	

Let tailor A and B works for X days and Y days respectively

 $\therefore x \ge 0, y \ge 0$

Minimum number of shirts = 60

 $\therefore 6x + 10y \ge 60$ $3x + 5y \ge 8$ Minimum no of trouser = 32 $\therefore 4x + 4y \ge 32$ $\Rightarrow x + y \ge 8$ Let z be the total labor cost $\therefore z = 300 x + 400 y$ \therefore The given L. P. problem reducers to: z = 300x + 400y $x \ge 0, y \ge 0, 3x + 5y \ge 30$ and $x + y \ge 8$

12. Find:
$$\int \frac{dx}{5-8x-x^2}$$

Solution:

$$= -\int \frac{dx}{\{(x+4)^2 - 21\}}$$
$$= \int \frac{dx}{(\sqrt{21})^2 - (x+4)^2}$$
$$= \frac{1}{2\sqrt{21}} \log \left| \frac{\sqrt{21} + (x+4)}{\sqrt{21} - (x+4)} \right| + C$$

SECTION - C

Question numbers 13 to 23 carry 4 marks each

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13. If $\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}$, then find the value of x.

Solution:

$$\tan^{-1} \left[\frac{\frac{x-3}{x-4} + \frac{x+3}{x+4}}{1 - \left(\frac{x^2 - 9}{x^2 - 16}\right)} \right] = \frac{\pi}{4}$$
$$\frac{(x+4)(x-3) + (x+3)(x-4)}{(x^2 - 16) - (x^2 - 9)} = 1$$
$$2x^2 - 24 = -7$$
$$2x^2 = -7 + 24$$
$$x^2 = \frac{17}{2}$$
$$x = \pm \sqrt{\frac{17}{2}}$$

14 Using properties of determinants prove that

 $\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a - 1)^3$

OR

Find matrix A such that

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$$

Solution:

Use R₁ = R₁ - R₂; R₂ = R₂ - R₃; R₃ = R₃ L. H. S. $\begin{vmatrix}
a^{2}-1 & a-1 & 0 \\
2a-2 & a-1 & 0 \\
3 & 3 & 1
\end{vmatrix}$ $= \begin{vmatrix}
(a-1)(a+1) & (a-1) & 0 \\
2(a-1) & (a-1) & 0 \\
3 & 3 & 1
\end{vmatrix}$ Taking common $(a-1)^{2}$ $= (a-1)^{2} \begin{vmatrix}
(a+1) & 1 & 0 \\
2 & 1 & 0 \\
3 & 3 & 1
\end{vmatrix}$ $= (a-1)^{2} [(a+1)(1-0) - 1(2-0)]$ $= (a-1)^{2} [(a+1)-2]$ $= (a-1)^{3}$ = R. H. S.

OR

Let matrix A is

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$$

$$\begin{bmatrix} 2a - c & 2b - d \\ a & b \\ -3a + 4c & -3b + 4d \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$$

Comparing both the sides

2a - c = -1,

After solving we get

$$C = 3, d = -4$$
So, $A = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$
15. If $x^{y} + y^{x} = d^{b}$, then find $\frac{dy}{dx}$.
OR
If $e^{y}(x + 1) = 1$, then show that $\frac{d^{2}y}{dx^{2}} = \left(\frac{dy}{dx}\right)^{2}$.
Solution:
We have $x^{y} + y^{x} = d^{b}$
Differentiating W. r. t. x, we get $\frac{d}{dx}(x^{y}) + \frac{d}{dx}(y^{x}) = 0$(1)
Let $u = x^{y}$: log $u = y \log x$
 $\Rightarrow \frac{1}{u}\frac{du}{dx} = y.\frac{1}{x} + \log x.\frac{dy}{dx}; \Rightarrow \frac{du}{dx} = u\left(\frac{y}{x} + \log x.\frac{dy}{dx}\right)$
or $\frac{d}{dx}(x^{y}) = x^{y}\left(\frac{y}{x} + \log x.\frac{dy}{dx}\right)$ (2)
Let $v = y^{x}$: log $v = x \log v$
 $\Rightarrow \frac{1}{v}\frac{dv}{dx} = x.\frac{1}{y}\frac{dy}{dx} + \log y.1; \Rightarrow \frac{dy}{dx} = v\left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$
or $\frac{d}{dx}(y^{x}) = y^{x}\left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$ (3)
Using (2) and (3) in (1),
We get. $x^{y}\left(\frac{y}{x} + \log x.\frac{dy}{dx}\right) + y^{x}\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$(4)
 $\Rightarrow (x^{y}\log x + xy^{x-1})\frac{dy}{dx} = -(y^{x}\log y + yx^{y-1})$ or $\frac{dy}{dx} = -\frac{y^{x}\log y + yx^{y-1}}{x^{y}\log x + xy^{y-1}}$
OR

$$e^{y}(1) + (x+1)e^{y} \frac{dy}{dx} = 0$$
$$\Rightarrow (x+1)\frac{dy}{dx} + 1 = 0 \dots \dots (1)$$

Again differentiating W. r. t . x,

$$\therefore (x+1)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right) \cdot 1 = 0$$
$$\frac{d^2y}{dx^2} = -\frac{\frac{dy}{dx}}{(x+1)}$$
$$\frac{d^2y}{dx^2} = -\frac{dy}{dx} \cdot \frac{dy}{dx} \text{ [equation (1)]}$$
$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

16. Find:
$$\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)} d\theta$$

Solution:

$$\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)} d\theta$$
$$= \int \frac{\cos\theta}{(4+\sin^2\theta)(5-4(1-\sin^2\theta))}$$
$$\int \frac{\cos\theta}{(\sin^2\theta+4)(4\sin^2\theta+1)}$$

Put sin $\theta = t$

 $\cos\theta \ d\theta = dt$

:.
$$I = \int \frac{1}{(4+t^2)(1+4t^2)} dt$$

Consider

$$\frac{1}{(4+t^{2})(1+4t^{2})} = \frac{At+B}{4+t^{2}} + \frac{Ct+D}{1+4t^{2}}$$

$$1 = (At+B)(1+4t^{2}) + (Ct+D)(4+t^{2})$$

$$= At+B+4At^{3}+4Bt^{2}+4Ct+Ct^{3}+4D+Dt^{2}$$

$$= (4A+C)t^{3} + (4B+D)t^{2} + (A+4C)t + (B+4D)$$

$$4A+C = 0 \Longrightarrow C = -4A$$

$$4B+D = 0 \Longrightarrow D = -4B$$

$$A+4C = 0 \Longrightarrow A = -4C$$

$$B+4D = 1$$

By solving we get A = 0, B =
$$-\frac{1}{15}$$
, $C = \frac{4}{15}$

$$\therefore \frac{1}{(4+t^2)(1+4t^2)} = \frac{\frac{-1}{15}}{4+t^2} + \frac{\frac{4}{15}}{1+4t^2}$$

$$\therefore I = -\frac{1}{15} \int \frac{1}{4+t^2} dt + \frac{4}{15} \times \frac{1}{4} \int \frac{1}{\frac{1}{4}+t^2} dt$$

$$= -\frac{1}{15} \times \frac{1}{2} \tan^{-1} \left(\frac{t}{2}\right) + \frac{1}{15} \times \frac{1}{\frac{1}{2}} \tan^{-1} \left(\frac{t}{\frac{1}{2}}\right) + C$$

$$= -\frac{1}{30} \tan^{-1} \left(\frac{t}{2}\right) + \frac{2}{15} \tan^{-1} (2t) + C$$

$$= \frac{2}{15} \tan^{-1} (2\sin\theta) - \frac{1}{30} \tan^{-1} \left(\frac{\sin\theta}{2}\right) + C$$

17. Evaluate:
$$\int_{0}^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$$

OR

Evaluate:
$$\int_{1}^{4} \{ |x-1| + |x-2| + |x-4| \} dx$$

Solution:

$$I = \int_{0}^{\pi} \frac{x \tan x}{\sec x + \tan x} dx \qquad \dots (1)$$
$$I = \int_{0}^{\pi} \frac{x(\pi - x)(-\tan x)}{-\sec x - \tan x} dx$$
$$I = \int_{0}^{\pi} \frac{(\pi - x) \tan x}{\sec x + \tan x} dx \qquad \dots (2)$$

Adding (1) & (2)

$$\begin{aligned} 2I &= \int_0^{\pi} \frac{\pi \tan x}{\sec x + \tan x} dx \\ \Rightarrow &2I = 2\pi \int_0^{\pi/2} \frac{\tan x}{\sec x + \tan x} dx \\ \left\{ \because \int_0^{2a} f(x) dx &= 2 \int_0^a f(x) dx \text{ whenever } f(2a - x) = f(x) \right\} \\ I &= \pi \int_0^{\pi/2} \frac{\tan x}{\sec x + \tan x} dx \\ I &= \pi \int_0^{\pi/2} \frac{\tan x(\sec x - \tan x)}{\sec^2 x - \tan^2 x} dx \\ I &= \pi \int_0^{\pi/2} (\sec x \tan x - \tan^2 x) dx \\ = \pi \int_0^{\pi/2} (\sec x \tan x - \sec^2 x + 1) dx \\ I &= \pi \left[\sec x - \tan x + x \right]_0^{\pi/2} \\ = \pi \left[\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) + \frac{\pi}{2} - \sec 0 \right] \\ = \pi \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} + \frac{\pi^2}{2} - \pi \\ = \pi \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin^2 x}{\cos x(1 + \sin x)} + \frac{\pi^2}{2} - \pi \\ = \frac{\pi^2}{2} - \pi \end{aligned}$$
OR
Let $f(x) = |x - 1| + |x - 2| + |x - 4|$
We have three critical points $x = 1, 2, 4$
(i) when $x < 1$
(ii) when $x < 2$
(iii) when $x < 2$
(iii) when $x \geq 4$
F(x) = $-(x - 1) - (x - 2) - (x - 4)$ if $x < 1$
 $= (x - 1) + (x - 2) + (x - 4)$ if $1 \le x < 2$
 $= (x - 1) + (x - 2) + (x - 4)$ if $1 \le x < 2$
 $= (x - 1) + (x - 2) + (x - 4)$ if $x < 4$
 $\therefore f(x) = -3x + 7$ if $x < 1$
 $= -x + 5$ if $1 \le x < 2$
 $x > 4$

$$\therefore I = \int_{1}^{4} f(x)dx$$

$$\therefore I = \int_{1}^{2} f(x)dx + \int_{2}^{4} f(x+1)dx$$

$$\therefore I = \int_{1}^{2} (-x+5)dx + \int_{2}^{4} (x+1)dx$$

$$= \left[-\frac{x^{2}}{2} + 5x\right]_{1}^{2} + \left[\frac{x^{2}}{2} + x\right]_{2}^{4}$$

$$= \left(-\frac{4}{2} + 10\right) - \left(-\frac{1}{2} + 5\right) + \left(\frac{16}{2} + 4\right) - \left(\frac{4}{2} + 2\right)$$

$$= 8 - \frac{9}{2} + 12 - 4 = \frac{23}{2}$$

18. Solve the differential equation $(\tan^{-1} x - y)dx = (1 + x^2) dy$.

Solution:

We have

 $\frac{dy}{dx} = \frac{\tan^{-1} x - y}{1 + x^2}$ $\frac{dy}{dx} + \frac{y}{1 + x^2} = \frac{\tan^{-1} x}{1 + x^2}$ $I.F. = e^{\int \frac{1}{1 + x^2} dx} = e^{\tan^{-1} x}$ $y.e^{\tan^{-1}} = \int \frac{\tan^{-1} x}{1 + x^2} \times e^{\tan^{-1}} dx$ Put t = tan⁻¹ $dt = \frac{1.dx}{1 + x^2}$ $= t.e^t - \int 1.e^t dt$ $y.e^{\tan^{-1} x} = te^t - e^t + c$ $y.e^{\tan^{-1} x} = (\tan^{-1} x - 1)e^{\tan^{-1} x} + c$ $y = \tan^{-1} x - 1 + ce^{\tan^{-1} x}$

19. Show that the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively, are the vertices of a right-angled triangle, Hence find the area of the triangle.

Solution:

 $\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}$ $\overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}$ $\overrightarrow{CA} = \hat{i} - 3\hat{j} - 5\hat{k}$ $\overrightarrow{BC}.\overrightarrow{CA} = 0$ $\overrightarrow{BC} \perp \overrightarrow{CA}$ А в C $\therefore \Delta ABC$ is a right angled triangle $\Delta = \frac{1}{2} |\overrightarrow{BC}| |\overrightarrow{AC}|$ $\Delta = \frac{1}{2}\sqrt{4+1+1}\sqrt{1+9+25}$ $=\frac{1}{2}\sqrt{6}\sqrt{35}$ $=\frac{1}{2}\sqrt{210}$

20. Find the value of λ , if four points with position vectors $3\hat{i} + 6\hat{j} + 9\hat{k}$, $\hat{i} + 2\hat{j} + 3\hat{k}$, $2\hat{i} + 3\hat{j} + \hat{k}$ and $4\hat{i} + 6\hat{j} + \lambda\hat{k}$, are coplanar.

Solution:

We have

P.V. of A = $3\hat{i} + 6\hat{j} + 9\hat{k}$ P.V. of B = $\hat{i} + 2\hat{j} + 3\hat{k}$ $\overline{AB} = -2\hat{i} - 4\hat{j} - 6\hat{k}$ $\overline{AD} = -\hat{i} - 3\hat{j} - 8\hat{k}$ $\overline{AD} = \hat{i} + (\lambda - 9)\hat{k}$ Now,

$$\overrightarrow{AB}.(\overrightarrow{AC} \times \overrightarrow{AD}) \Rightarrow \begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & (\lambda - 9) \end{vmatrix} = 0$$
$$\Rightarrow -2(-3\lambda + 27) + 4(-\lambda + 9 + 8) - 6(0 + 3) = 0$$
$$\Rightarrow 6\lambda - 54 - 4\lambda + 68 - 18 = 0$$
$$2\lambda - 4 = 0$$
$$\lambda = 2$$

 $\therefore \overrightarrow{AB}, \overrightarrow{AC}, \overleftarrow{AD}$ are coplanar and so the points A, B, C and D are coplanar.

21. There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean and variance of X.

Solution:

Х	denote	sum o	of the	numl	bers s	50, X	can	be 4,	, 6, 8	8, 10	0, 1	2

X	Number on card	P(x)	X P(x)	$X^2 P(x)$
4	(1, 3)	$\frac{1}{4} \times \frac{1}{3} \times 2 = \frac{1}{6}$	2/3	8/3
6	(1, 6)	$\frac{1}{4} \times \frac{1}{3} \times 2 = \frac{1}{6}$	1	6
8	(3, 5) or (1, 7)	$\frac{1}{4} \times \frac{1}{3} \times 2 + \frac{1}{4} \times \frac{1}{3} \times 2 = \frac{1}{3}$	8/3	64/3
10	(3, 7)	$\frac{1}{4} \times \frac{1}{3} \times 2 = \frac{1}{6}$	5/3	50/3
12	(5, 7)	$\frac{1}{4} \times \frac{1}{3} \times 2 = \frac{1}{6}$	2	24

Mean = $\Sigma XP(x) = 8$

Variance
$$= \Sigma X^2 P(x) - (\Sigma X P(x))^2 = \frac{212}{3} - 64 = \frac{20}{3}$$

22. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at

random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance? Is regularity required only in school? Justify your answer.

Solution:

Let E_1 be students having 100% attendance E_2 be students having irregular attendance E be students having A grade

$$P(E_1) = \frac{30}{100} \qquad P(E_2) = \frac{70}{100}$$
$$P\left(\frac{E}{E_1}\right) = \frac{70}{100} \times \frac{30}{100} = 21\%$$
$$P\left(\frac{E}{E_2}\right) = \frac{10}{100} \times \frac{70}{100} = 7\%$$

By Baye's theorem,

So,
$$P\left(\frac{E_1}{E}\right) = \frac{P(E_1)P\left(\frac{E}{E_1}\right)}{P(E_1)P\left(\frac{E}{E_2}\right) + P(E_2)P\left(\frac{E}{E_2}\right)} = \frac{\frac{30}{100} \times \frac{21}{100}}{\frac{30}{100} \times \frac{21}{100} + \frac{70}{100} \times \frac{7}{100}} = \frac{63}{63 + 49} = \frac{63}{112}$$

23. Maximize Z = x + 2ySubject to the constraints

 $x + 2y \ge 100$ $2x - y \le 0$ $2x + y \le 200$ $x, y \ge 0$

Solve the above LPP graphically.

Solution:

 $\begin{aligned} x + 2y &= 100 \\ 2x - y &= 0 \dots(1) \\ 2x + y &= 200 \dots(2) \\ x &= 0, \ y &= 0 \dots(3) \end{aligned}$



Corner points are A (100, 0), B(50, 100), C(20, 40)

Corner points	$\mathbf{Z} = \mathbf{x} + 2\mathbf{y}$	[
A(100, 0)	100 🗲	— minimum
B(50, 100)	250 🗲	- maximum
C(20, 40)	100 ←	— minimum

Maximum at point B and maximum value 250

SECTION – D

Question numbers 24 to 29 carry 6 marks each

24. Determine the product
$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$
 and use it to solve the system of equations $x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1.$
Solution:
Product of the matrices
$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} -4 + 4 + 8 & 4 - 8 + 4 & -4 - 8 + 12 \\ -7 + 1 + 6 & 7 - 2 + 3 & -7 - 2 + 9 \\ 5 - 3 - 2 & -5 + 6 - 1 & 5 + 6 - 3 \end{bmatrix}$$
$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I_3$$
$$Hence \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

 $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8} \begin{bmatrix} -16 & +36 & +4 \\ -28 & +9 & +3 \\ 20 & -27 & -1 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}$ $x = \frac{24}{8}, y = \frac{-16}{8}, z = \frac{-8}{8}$ x = 3, y = -2, z = -1

25. Consider $f: R - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f

and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.

OR

Let $A = Q \times Q$ and let * be a binary operation on A defined by (a, b) * (c, d) = (ac, b + ad) for (a, b), (c, d) \in A. Determine, whether * is commutative and associative. Then, with respect to * on A

(i) Find the identity element in A.

(ii) Find the invertible elements of A.

Solution:

$$f(x) = \frac{4x+3}{3x+4}, x \in R - \left\{-\frac{4}{3}\right\}$$

F is one - one \rightarrow
Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ and $f(x_1) = f(x_2)$
 $\Rightarrow \frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4}$
 $\Rightarrow 12x_1x_2 + 16x_1 + 9x_2 + 12 = 12x_1x_2 + 9x_1 + 16x_2 + 12$
 $\Rightarrow 7x_1 = 7x_2 \Rightarrow x_1 = x_2$
 \therefore f is one - one
F is ort $x_1 \to 0$

Let $k \in R - \left\{\frac{4}{3}\right\}$ be any number $f(x) = k \Longrightarrow \frac{4x+3}{3x+4}$ $\Rightarrow 4x + 3 = 3kx + 4k$ $\Rightarrow x = \frac{4k - 3}{4 - 3k}$ Also $\frac{4k-3}{4-3k} = -\frac{4}{3}$ implies -9 = -16 (which is impossible) $\therefore f\left(\frac{4k-3}{4-3k}\right) = k$ i.e. f is onto \therefore The function f is invertible i.e. f⁻¹ exist inverse of f Let $f^{-1}(x) = k$ f(k) = x $\Rightarrow \frac{4k+3}{3k+4} = x$ $\Rightarrow k = \frac{4x-3}{4-3x}$ $\therefore f^{-1}(x) = \frac{4x+3}{4-3x}, x \in R - \left\{-\frac{4}{3}\right\}$ $f^{-1}(0) = -\frac{3}{4}$ and when $f^{-1}(x) = 2$ $\Rightarrow \frac{4x-3}{4-3x} = 2$ $\Rightarrow 4x - 3 = 8 - 6x$ $\Rightarrow 10x = 11$ $\Rightarrow x = \frac{11}{10}$ (i) Let (e, f) be the identify element for * \therefore for (a, b) $\in Q \times Q$, we have (a, b) * (e, f) = (a, b) = (e, f) * (a, b) \Rightarrow (ae, af + b) = (a, b) = (ea, eb + f) \Rightarrow ae = a, af + b = b, a = ea, b = eb + f \Rightarrow e = 1, af = 0, e = 1, b = (1) b + f

OR

(:: a need not be '0') \Rightarrow e = 1, f = 0, e = 1, f = 0 \therefore (e, f) = (1, 0) $\in \mathbf{Q} \times \mathbf{Q}$ 1 0) 1 1 1 ...

(ii) Let
$$(a, b) \in Q \times Q$$

Let $(c, d) \in Q \times Q$
such that
 $(a, b) * (c, d) = (1, 0) = (c, d) * (a, b)$
 $\Rightarrow (ac, ad + b) = (1, 0) = (ca, cb + d)$
 $\Rightarrow ac = 1, ad + b = 0, ca = 1, cb + d = 0$
 $\Rightarrow c = \frac{1}{a}, d = -\frac{b}{a}, (\frac{1}{a})b + d = 0(a \neq 0)$
 $\therefore (c, d) = (\frac{1}{a}, \frac{-b}{a})(a \neq 0)$
 $\therefore \text{ for } a \neq 0, (a, b)^{-1} = (\frac{1}{a}, \frac{-b}{a})$

26. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

Solution: If each side of square base is x and height is h then volume

$$V = x^{2}h \Rightarrow h = \frac{V}{x^{2}}$$

S is surface area then
$$S = 4hx + 2x^{2} = 4\left(\frac{V}{x^{2}}\right)x + 2x^{2}$$
$$\Rightarrow S = \frac{4V}{x} + 2x^{2}$$
Diff. w. r. to x
$$\frac{dS}{dx} = -\frac{4V}{x^{2}} + 4x \text{ and } \frac{d^{2}S}{dx^{2}} = +\frac{8V}{x^{3}} + 4$$
Now $\frac{dS}{dx} = 0 \Rightarrow 4x = \frac{4V}{x^{2}}$
$$\Rightarrow x^{3} = V \Rightarrow x = V^{1/3}$$
at $x = V^{1/3}, \frac{d2S}{dx^{2}} > 0$
$$\Rightarrow S \text{ is minimum when } x = V^{1/3}$$
and $h = \frac{V}{x^{2}} = \frac{V}{V^{2/3}} = V^{1/3} \Rightarrow x = h$
$$\Rightarrow x = h \text{ means it is a cube}$$

27. Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are A (4, 1), B (6, 6) and C (8, 4).

OR

Find the area enclosed between the parabola $4y = 3x^2$ and the straight line 3x - 2y + 12 = 0. Solution:
B (6, 6)

$$y = \frac{1}{4}$$
 (4, 1)
 $y = \frac{1}{4}$ (5, 2)
 $y = \frac{1}{4}$ (5, 2)
Equation of AB is $y - 1 = \frac{6-1}{6-4}(x-4)$
 $y = \frac{4-6}{8-6}(x-6)$
 $y = -x + 12$
Equation of AC is
 $y = -x + 12$
Equation of AC is
 $y = -x + 12$
Equation of AC is
 $y = \frac{3x}{4} - 2$
Area of $\triangle ABC$ = area ABED + area BEFC – area ADFC
 $= \int_{4}^{6} (\frac{5x}{2} - 9) dx + \int_{6}^{8} (-x+12) dx - \int_{4}^{6} (\frac{3x}{4} - 2) dx$
 $= \left[(\frac{5x^{2}}{4} - 9x)_{4}^{6} \right] + \left[(\frac{-x^{2}}{2} + 12x)_{6}^{8} \right] - \left[(\frac{3x^{2}}{8} - 2x)_{4}^{6} \right] = 7$ sq units
OR
Parabola 4y = 3x^{2} ...(1)
line 3x - 2y + 12 = 0 ...(2)
from (2) $y = \frac{3x + 12}{2}$
putting this value of y is (1) we get
 $6x + 24 = 3x^{2}$
 $\Rightarrow x = 4, -2$
when $x = 4$ then $y = 12$
 $x = -2$ then $y = 3$
Required area
 $= \int_{-2}^{4} (y \text{ of line}) dx - \int_{-2}^{4} (y \text{ of parabola}) dx$

OR



28. Find the particular solution of the differential equation $(x - y) \frac{dy}{dx} = (x + 2y)$, given that y = 0 when x = 0

1.

Solution:

$$(x - y)\frac{dy}{dx} = (x + 2y)$$

$$\frac{dy}{dx} = \frac{x + 2y}{x - y}$$
Let $y = Vx$

$$\frac{dy}{dx} = V + x\frac{dV}{dx}$$

$$\Rightarrow V + x\frac{dV}{dx} = \frac{x + 2(Vx)}{x - Vx}$$

$$\Rightarrow V + x\frac{dV}{dx} = \frac{1 + 2V}{1 - V}$$

$$\Rightarrow x\frac{dV}{dx} = \frac{1 + 2V - V + V^2}{1 - V}$$

$$\Rightarrow \int \frac{1 - V}{1 + V + V^2} dV = \int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2} \int \left\{ \frac{(2V + 1) - 3}{1 + V + V^2} \right\} dV = \int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2} \left[\int \frac{2V + 1}{1 + V + V^2} dV - 3 \int \frac{dV}{1 + V + V^2} \right] = \int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2}\log|1+V+V^{2}| + \frac{3}{2}\int \frac{dV}{\left(V+\frac{1}{2}\right)^{2} + \left(\frac{\sqrt{3}}{2}\right)^{2}} = \log|x| + C$$
$$\Rightarrow -\frac{1}{2}\log|1+V+V^{2}| + \frac{3}{2}\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{V+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = \log|x| + C$$
$$\Rightarrow -\frac{1}{2}\log\left|1+\frac{y}{x}+\frac{y^{2}}{x^{2}}\right| + \sqrt{3}\tan^{-1}\left(\frac{2\frac{y}{x}+1}{\sqrt{3}}\right) = \log|x| + C$$

we have y = 0 when x = 1

$$\Rightarrow 0 + \sqrt{3} \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = 0 + C$$
$$\Rightarrow C = \sqrt{3} \tan^{-1} \frac{1}{\sqrt{3}}$$

 \therefore Solution

$$\Rightarrow -\frac{1}{2}\log\left|1 + \frac{y}{x} + \frac{y^2}{x^2}\right| + \sqrt{3}\tan^{-1}\left(\frac{2\frac{y}{x} + 1}{\sqrt{3}}\right) = \log|x| + \sqrt{3}\tan^{-1}\frac{1}{\sqrt{3}}$$

29. Find the coordinates of the point where the line through the points (3, -4, -5) and (2, -3, 1), crosses the plane determined by the points (1, 2, 3), (4, 2, -3) and (0, 4, 3).

OR

A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$.

Solution:

Equation of line passing through (3, -4, -5) and (2, -3, 1) $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$...(1) Equation of plane passing through (1, 2, 3) (4, 2, -3) and (0, 4, 3) $\begin{vmatrix} x-1 & y-2 & z-3 \\ 3 & 0 & -6 \\ -1 & 2 & 0 \end{vmatrix} = 0$ $\Rightarrow (x-1) (12) - (y-2) (-6) + (z-3) (6) = 0$ $\Rightarrow 2x + y + z - 7 = 0 ...(2)$ Let any point on line (1)



OR

Let the equation of plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad \dots(1)$$

T

It cut the co-ordinate axes at A, B and C \therefore A (a, 0, 0), B (0, b, 0), C (0, 0, c) Let the centroid of \triangle ABC be (x, y, z)

$$\therefore \left(x = \frac{a}{3}, y = \frac{b}{3}, z = \frac{c}{3} \right) \dots (2)$$

Т

given that distance of plane (1) from origin is 3p

$$\therefore \left| \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} \right| = 3p$$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{9p^2}$$

from (2)

$$\Rightarrow \frac{1}{9x^2} + \frac{1}{9y^2} + \frac{1}{9z^2} = \frac{1}{9p^2}$$

$$\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$$
 Proved

CBSE-XII-2015 EXAMINATION

MATHEMATICS

Series SSO/1

Time: 3 Hrs.

Paper & Solution

Code: 65/1/1/D Max. Marks: 100

General Instructions:

(i) All questions are compulsory.

(ii) Please check that this Question Paper contains 26 Questions.

(iii) Marks for each question are indicated against it.

(iv) Questions 1 to 6 in Section-A are Very Short Answer Type Questions carrying one mark each.

(v) Questions 7 to 19 in Section-B are Long Answer I Type Questions carrying 4 marks each.

(vi) Questions 20 to 26 in Section-C are Long Answer II Type Questions carrying 6 marks each

(vii) Please write down the serial number of the Question before attempting it.

SECTION – A

Question numbers 1 to 6 carry 1 mark each.

1. If $\vec{a} = 7\hat{i} + \hat{j} - 4k$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3k$, then find the projection of \vec{a} on \vec{b} . Solution:

 $p = \frac{a.b}{|\vec{b}|} = \frac{8}{7}$

2. Find λ_i , if the vectors $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{j} + 3\hat{k}$ are coplanar.

Solution:

1 3 $\begin{vmatrix} 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix} = 0 \Longrightarrow \lambda = 7$

3. If a line makes angles 90°,60° and θ with x, y and z- axis respectively, where θ is Acute, then find θ . Solution:

 $\cos^2 \frac{\pi}{2} + \cos^2 \frac{\pi}{3} + \cos^2 \theta = 1 \Longrightarrow \theta = \frac{\pi}{6}$

4. Write the element a_{23} of a 3 × 3 matrix A = (a_{ij}) whose elements a_{ij} are given $a_{ij} = \frac{|i-j|}{2}$.

Solution:

 $a_{23} = \frac{|2-3|}{2} = \frac{1}{2}$

5. Find the differential equation representing the family of curves $v = \frac{A}{r} + B$, where A and B are arbitrary constants.

Solution:

$$\frac{dv}{dr} = -\frac{A}{r^2}, \Longrightarrow r^2 \frac{d^2v}{dr^2} + 2r \frac{dv}{dr} = 0$$

SET-1

CBSE-XII-2015 EXAMINATION

6. Find the integrating factor of the differential equation

$$\left(\frac{e^{-2}\sqrt{x}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1.$$

Solution:

$$I.F = \int_{e} \frac{1}{\sqrt{x}} dx = e^{2\sqrt{x}}$$

SECTION – B

Question numbers 7 to 19 carry 4 marks each.

7. If
$$A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$$
 find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = O$

OR

If
$$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$$
, find $(A')^{-1}$.
Solution:
Getting $A^2 = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$
 $\begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$
 $\begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$

$$A^{2} - 5A + 4I = \begin{bmatrix} 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

$$\therefore X = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{pmatrix}$$

 $A' = \begin{pmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{pmatrix}$ $|A'| = 1(-9) - 2(-5) = -9 + 10 = 1 \neq 0$ Adj $A' = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix}$

OR

$$(A')^{-1} = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix}$$
8. if $f(x) = \begin{bmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{bmatrix}$, using properties of determinants find the value of $F(2x) - f(x)$.
Solution:

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$$

$$R_2 \to R_2 - xR_1 and R_3 \to R_3 - x^2R^4$$

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ 0 & a+x & -1 \\ 0 & a+x^2 & a \end{vmatrix}$$
 (For bringing 2 zeroes in any row/column

$$f(x) = a(a^2 + 2ax + x^2) = a(x + a)^2$$

$$\therefore f(2x) - f(x) = a[2x + a]^2 - a(x + a)^3$$

$$= ax (3x + 2a)$$
9. Find : $\int \frac{dx}{\sin x + \sin 2x}$
OR
Integrate the following w. r. t. x

$$\frac{x^2 - 3x + 1}{\sqrt{1 - x^2}}$$
Solution:

$$\int \frac{dx}{\sin x + \sin 2x} = \int \frac{dx}{\sin x (1 + 2\cos x)} = \int \frac{\sin x \cdot dx}{(1 - \cos x)(1 + \cos x)(1 + 2\cos x)}$$

$$= -\int \frac{dt}{(1 - t)(1 + t)(1 + 2t)}$$
where $\cos x = t$

$$= \int \left(\frac{-\frac{1}{2}}{1 - t} + \frac{1}{1 + t} - \frac{4}{3} \\ 1 - t + \frac{1}{2} \log|1 + t| - \frac{2}{3} \log|1 + 2\cos x| + c$$
OR

$$\int \frac{x^2 - 3x + 1}{\sqrt{1 - x^2}} dx = \int \frac{2 - 3x - (1 - x^2)}{\sqrt{1 - x^2}} dx$$

$$= 2\int \frac{1}{\sqrt{1-x^2}} dx - 3\int \frac{x}{\sqrt{1-x^2}} dx - \int \sqrt{1-x^2} dx$$
$$= 2\sin^{-1}x + 3\sqrt{1-x^2} - \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\sin^{-1}x + c$$
or
$$= \frac{3}{2}\sin^{-1}x + \frac{1}{2}(6-x)\sqrt{1-x^2} + c$$

10. Evaluate :
$$\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 dx$$

Solution:

π

$$I = \int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 dx = \int_{-\pi}^{\pi} (\cos^2 ax + \sin^2 bx) dx - \int_{-\pi}^{\pi} 2\cos ax \sin bx dx$$
$$= I_1 - I_2$$

$$I_{1} = 2 \int_{0}^{\pi} (\cos^{2} ax + \sin^{2} bx) dx \text{ (being an even fun.)}$$

$$I_{2} = 0 \text{ (being an odd fun.)}$$

$$\therefore I = I_{1} = \int_{0}^{\pi} (1 + \cos 2ax + 1 - \cos 2bx) dx$$

$$= \left[2x + \frac{\sin 2ax}{2a} - \frac{\sin 2bx}{2b} \right]_{0}^{\pi}$$

$$= \left[2\pi + \frac{1}{2a} \cdot \sin 2a\pi - \frac{\sin 2b\pi}{2b} \right] \text{ or } 2\pi$$

11. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag b. If two balls are drawn at random (without replacement) from the select bag, find the probability of one of them being red and another black.

OR

An unbiased coin is tossed 4 times. Find the mean and variance of the number of heads obtained. **Solution:**

. Let E_1 : selecting bag A, and E_2 : selecting bag B.

$$\therefore P(E_1) = \frac{1}{3}, P(E_2) = \frac{2}{3}$$

Let A : Getting one Red and one balck ball

$$\therefore P(A/E_1) = \frac{{}^4C_1 \cdot {}^6C_1}{{}^{10}C_2} = \frac{8}{15}, P(A/E_2) = \frac{{}^7C_1 \cdot {}^3C_1}{{}^{10}C_2} = \frac{7}{15}$$

$$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)$$

$$= \frac{1}{3} \cdot \frac{8}{15} + \frac{2}{3} \cdot \frac{7}{15} = \frac{22}{45}$$

Х	:	0	1	2	3	4
P (x)	:	${}^{4}\mathrm{C}_{0}\left(\frac{1}{2}\right)^{4}$	${}^{4}C_{1}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)$	${}^{4}\mathrm{C}_{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{2}$	${}^{4}\mathrm{C}_{3}\left(\frac{1}{2}\right)\!\left(\frac{1}{2}\right)^{\!\!3}$	${}^4\mathrm{C_4}\left(\frac{1}{2}\right)^{\!\!4}$
	:	$=\frac{1}{16}$	$=\frac{4}{16}$	$=\frac{6}{16}$	$=\frac{4}{16}$	$=\frac{1}{16}$
x P (x	:) :	0	$\frac{4}{16}$	$\frac{12}{16}$	$\frac{12}{16}$	$\frac{4}{16}$
x²P (x	x) :	0	$\frac{4}{16}$	$\frac{24}{16}$	$\frac{36}{16}$	$\frac{16}{16}$
		32				

Mean =
$$\sum x P(x) = \frac{1}{16} = 2$$

Variance = $\sum x^2 P(x) - \left(\sum x P(x)\right)^2 = \frac{80}{16} - \left(2\right)^2 = 1$

12. If
$$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\mathbf{k}$$
, find $(\vec{\mathbf{r}} \times \hat{\mathbf{i}}) \cdot (\vec{\mathbf{r}} \times \mathbf{j}) + xy$
Solution:
 $\vec{r} \times \vec{i} = (x\hat{i} + y\hat{j} + z\hat{k})x\hat{i} = -y\hat{k} + z\hat{j}$
 $\vec{r} \times \vec{j} = (x\hat{i} + y\hat{j} + z\hat{k})\hat{j} = x\hat{k} - z\hat{i}$
 $(\vec{r} \times \hat{i}), (\vec{r} \times \vec{j}) = (o\hat{i} + z\hat{j} - y\hat{k}) \cdot (-z\hat{i} + o\hat{j} + x\hat{k}) = -xy$
 $(\vec{r} \times \hat{i}) \cdot (\vec{r} \times \vec{j}) + xy = -xy + xy + 0$

13. Find the distance between the point (-1, -5, -10) and the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane x - y + z = 5. **Solution:** . Any point on the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ is $(3\lambda + 2, 4\lambda - 1, 12\lambda + 2)$ If this is the point of intersection with plane x - y + z = 5

Then
$$3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 - 5 = 0 \Longrightarrow \lambda = 0$$

 \therefore Point of intersection is (2, -1, 2)

Required distance = $\sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = 13$

14. If $\sin\left[\cot^{-1}(x+1)\right] = \cos(\tan^{-1}x)$, then find x. **OR**

If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then find x.

Solution:

CBSE-XII-2015 EXAMINATION

Writing
$$\cot^{-1}(x+1) = \sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}}$$

and $\tan^{-1}x = \cos^{-1} \frac{1}{\sqrt{1+x^2}}$
 $\therefore \sin\left(\sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}}\right) = \cos\left(\cos^{-1} \frac{1}{\sqrt{1+x^2}}\right)$
 $1+x^2+2x+1=1+x^2 \Rightarrow x=-\frac{1}{2}$
($\tan^{-1}x$)² + $\left(\cot^{-1}x\right)^2 = \frac{5\pi^2}{8} \Rightarrow \left(\tan^{-1}x\right)^2 + \left(\frac{\pi}{2} - \tan^{-1}x\right)^2 = \frac{5\pi^2}{8}$
 $\therefore 2\left(\tan^{-1}x\right)^2 - \pi \tan^{-1}x - \frac{3\pi^2}{8} = 0$
 $\tan^{-1}x = \frac{\pi \pm \sqrt{\pi^2 + 3\pi^2}}{4} = 3\frac{\pi}{4}, -\frac{\pi}{4}$
 $\Rightarrow x=-1$
15. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{4}\right), x^2 \le 1$, then find $\frac{dy}{4}$.

15. If
$$y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right), x^2 \le 1$$
, then find $\frac{dy}{dx}$

Solution:

Putting
$$x^2 = \cos\theta$$
, we get

$$y = \tan^{-1} \left(\frac{\sqrt{1 + \cos\theta} + \sqrt{1 - \cos\theta}}{\sqrt{1 + \cos\theta} - \sqrt{1 - \cos\theta}} \right)$$

$$= \tan^{-1} \left(\frac{\cos\theta/2 + \sin\theta/2}{\cos\theta/2 - \sin\theta/2} \right) = \tan^{-1} \left(\frac{1 + \tan\theta/2}{1 - \tan\theta/2} \right)$$

$$y = \frac{\pi}{4} + \frac{\theta}{2} = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$$

$$\frac{dy}{dx} = -\frac{1}{2}\frac{1}{\sqrt{1 - x^4}} \cdot 2x = -\frac{x}{\sqrt{1 - x^4}}$$

16. If $x = a \cos \theta + b \sin \theta$, $y = a \sin \theta - b \cos \theta$, show that $y^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 0$.

Solution:

 $\frac{dx}{d\theta} = -a\sin\theta + b\cos\theta$ $\frac{dy}{d\theta} = a\cos\theta + b\sin\theta$

$$\therefore \frac{dy}{dx} = \frac{a\cos\theta + b\sin\theta}{a\sin\theta + b\cos\theta} = -\frac{x}{y}$$

Or $y\frac{dy}{dx} + x = 0$
$$\therefore y\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{dy}{dx} + 1 = 0$$

17. The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing when the side of the triangle is cm ?

Solution:

Let x be the side of an equilateral triangle

$$\therefore \frac{dx}{dt} = 2cm/s.$$
Area (A) $= \frac{\sqrt{3x^2}}{4}$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2}x\frac{dx}{dt}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} \cdot (20) \cdot (2) = 20\sqrt{3} cm^2/s$$

18. Find :
$$\int (x+3)\sqrt{3-4x-x^2} dx$$
. Solution:

Writing
$$x+3 = -\frac{1}{2}(-4-2x)+1$$

$$\therefore \int (x+3)\sqrt{3-4x-x^2} \, dx = -\frac{1}{2} \int (-4-2x)\sqrt{3-4x-x^2} \, dx + \int \sqrt{7-(x+2)^2} \, dx$$

$$= -\frac{1}{3}(3-4x-x^2)^{\frac{3}{2}} + \frac{x+2}{2}\sqrt{3-4x-x^2} + \frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + c$$

19. Three schools A, B and C organized a mela for collecting found for helping the rehabilitation of flood victims. They sold hand made fans, mats and plates from recycled material at a cost of Rs. 25, Rs. 100, Rs. 50, each. The number of articles sold are given below :

School Article	А	В	С
Hand-fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Find the found collected by each school separately by selling the above articles, Also Find the total founds collected for the purpose. Write one value generated by the above situation. **Solution:**

HF. M P

A	(40	50	20)	$\left(25\right)$		(7000)	١
B	25	40	30	100	=	6125	
С	35	50	40)	50		(7875)	

Funds collected by school A : Rs. 7000, School B : Rs. 6125, School C : Rs. 7875 Total collected : Rs. 21000 For writing one value

SECTION - C

Question numbers 20 to 26 carry 6 marks each.

20. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by (a, b) R (c, d) if ad (b + c) = bc (a + d). show that R is an equivalence relation.

Solution:

 $\forall a, b \in N, (a, b)R(a, b) as ab (b+a) = ba(a+b)$ $\therefore R \text{ is reflexive(i)}$ Let (a, b) R (c, d) for (a, b), (c, d) $\in N \times N$ $\therefore ad (b+c) = bc (a+d) \dots (ii)$ Also (c, d) R (a, b) \therefore cb (d + a) = da (c + b) (using ii) $\therefore R \text{ is symmetric(iii)}$ Let (a, b) R (c, d) and (c, d) R (e, f), for a, b, c, d, e, f $\in N$ $\therefore ad (b+c) = bc (a+d) and cf (d+e) = de (c+f)$ $\therefore \frac{b+c}{bc} = \frac{a+d}{ad} and \frac{d+e}{de} = \frac{c+f}{cf}$

bc ad de cf i.e $\frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ and $\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$ adding we get $\frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$ \Rightarrow af (b + e) = be (a + f) Hence (a, b) R (e, f) \therefore R is transitive(iv)

Form (i), (iii) and (iv) R is an equivalence relation

21. Using integration find the area of the triangle formed by positive x- zxis and tangent and normal to the circle $x^2 + y^2 = 4$ at $(1,\sqrt{3})$.

OR

Evaluate $\int_{1}^{3} (e^{2-3x} + x^2 + 1)$ as a limit of a sum. Solution:

CBSE-XII-2015 EXAMINATION



Eqn. of normal (OP) : $y = \sqrt{3}x$ Eqn. of tangent (PQ) is

$$y - \sqrt{3} = -\frac{1}{\sqrt{3}}(x-1)$$
 i.e $y = \frac{1}{\sqrt{3}}(4-x)$

Coordinates of Q(4, 0)

$$\therefore \text{ Req. area} = \int_{0}^{1} \sqrt{3x} \, dx + \int_{1}^{4} \frac{1}{\sqrt{3}} (4-x) \, dx$$
$$= \sqrt{3} \frac{x^2}{2} \int_{0}^{1} + \frac{1}{\sqrt{3}} \left[4x - \frac{x^2}{2} \right]_{1}^{4}$$
$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}} \left[16 - 8 - 4 + \frac{1}{2} \right] = 2\sqrt{3} \text{ sq. units}$$

$$\int_{1}^{3} \left(e^{2-3x} + x^{2} + 1\right) dx here h = \frac{2}{n}$$

$$= \lim_{h \to 0} h\left[f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)\right]$$

$$= \lim_{h \to 0} h\left[\left(e^{-1} + 2\right) + \left(e^{-1-3h} + 2 + 2h + h^{2}\right) + \left(e^{-1-6h} + 2 + 4h + 4h^{2}\right) + \dots + \left(e^{-1-3(n-1)h} + 2 + 2(n-1)h + (n-1)^{2}h^{2}\right)\right]$$

$$= \lim_{h \to 0} h\left[e^{-1}\left(1 + e^{-3h} + e^{-6h} + \dots + e^{-3(n-1)h}\right) + 2n + 2h\left(1 + 2 + \dots + (n-1)^{2}\right)\right]$$

$$= \lim_{h \to 0} h\left[e^{-1} \cdot \frac{e^{-3nh} - 1}{e^{-3n} - 1} \cdot h + 2nh + 2\frac{nh(nh-h)}{2} + \frac{nh(nh-h)(2nh-h)}{6}\right]$$

$$= e^{-1}\frac{\left(e^{-6} - 1\right)}{-3} + 4 + 4 + \frac{8}{3} = -e^{-1}\frac{e^{-6} - 1}{3} + \frac{32}{3}$$

22. Solve the differential equation : $(\tan^{-1}y - x)dy = (1 + y^2)dx$. OR

Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$ given that y = 1, When x = 0.

Solution:

Given differential equation can be written as

$$\frac{dx}{dy} + \frac{1}{1+y^2} \cdot x = \frac{\tan^{-1} y}{1+y^2}$$

CBSE-XII-2015 EXAMINATION

 $\therefore \text{ Integrating factor is } e^{\tan^{-1}y}$ $\therefore \text{ Solution is : } x \cdot e^{\tan^{-1}y} = \int \frac{\tan^{-1}y \cdot e^{\tan^{-1}y}}{1+y^2} \, dy$ $\Rightarrow x \cdot e^{\tan^{-1}y} = \int t \, e^t \, dt \text{ where } \tan^{-1}y = t$ $= t \, e^t - e^t + c = e^{\tan^{-1}y} \left(\tan^{-1}y - 1\right) + c$ $or \ x = \tan^{-1}y - 1 + c \, e^{-\tan^{-1}y}$

OR

Given differential equation is
$$\frac{dy}{dx} = \frac{\frac{y}{x}}{1 + \left(\frac{y}{x}\right)^2}$$

Putting
$$\frac{y}{x} = v$$
 to get $v + x \frac{dv}{dx} = \frac{v}{1 + v^2}$
 $\therefore x \frac{dv}{dx} = \frac{v}{1 + v^2} - v = \frac{\left|-v^3\right|}{1 + v^2}$
 $\Rightarrow \int \frac{v^2 + 1}{v^3} dv = -\int \frac{dx}{x}$
 $\Rightarrow \log |v| - \frac{1}{2v^2} = -\log |x| + c$
 $\therefore \log y - \frac{x^2}{2y^2} = c$
 $x = 0, y = 1 \Rightarrow c = 0 \therefore \log y - \frac{x^2}{2y^2} = 0$

23. If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y+k}{2} = \frac{z}{1}$ intersect, then find the value of K and hence find the equation of the plane containing these lines.

Solution:

Any point on line
$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$$
 is $(2\lambda+1, 3\lambda-1, 4\lambda+1)$

$$\therefore \frac{2\lambda+1-3}{1} = \frac{3\lambda-1-k}{2} = \frac{4\lambda+1}{1} \Longrightarrow \lambda = -\frac{3}{2}$$
, hence $k = \frac{9}{2}$

Eqn. of plane containing three lines is

 $\begin{vmatrix} x - 1 & y + 1 & z - 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$ $\Rightarrow -5(x - 1) + 2(y + 1) + 1(z - 1) = 0$ i.e. 5x - 2y - z - 6 = 0 **24.** If A and B are two independent events such that $P(\overline{A} \cap B) = \frac{2}{15}$ and $P(A \cap \overline{B}) = \frac{1}{6}$, then find P (A) and P (B).

Solution:

$$P(\overline{A} \cap B) = \frac{2}{15} \Rightarrow P(\overline{A}) \cdot P(B) = \frac{2}{15}$$

$$P(A \cap \overline{B}) = \frac{1}{6} \Rightarrow P(\overline{A}) \cdot P(\overline{B}) = \frac{1}{6}$$

$$\therefore (1 - P(A))P(B) = \frac{2}{15} \text{ or } P(B) - P(A) \cdot P(B) = \frac{2}{15} \dots (i)$$

$$P(A)(1 - P(B)) = \frac{1}{6} \text{ or } P(A) - P(A) \cdot P(B) = \frac{1}{6} \dots (ii)$$
From (i) and (ii) $P(A) - P(B) = \frac{1}{6} - \frac{2}{15} = \frac{1}{30}$
Let P (A) = x, P (B) = y $\therefore x = \left(\frac{1}{30} + y\right)$

$$(i) \Rightarrow y| - \left(\frac{1}{30} + y\right)y = \frac{2}{15} \therefore 30y^2 - 29y + 4 = 0$$
Solving to get $y = \frac{1}{6}$ or $y = \frac{4}{5}$

$$\therefore x = \frac{1}{5}$$
 or $x = \frac{5}{6}$
Hence $P(A) = \frac{1}{5}$, $P(B = \frac{1}{6})$ OR $P(A) = \frac{5}{6}$, $P(B = \frac{4}{5})$

25. Find the local maxima and local minima, of the function $f(x) = \sin x$, $0 < x < 2\pi$. Also find the local maximum and local minimum values.

Solution:

$$f(x) = \sin x - \cos x, 0 < x < 2\pi$$

$$f'(x) = 0 \Rightarrow \cos x + \sin x = 0 \text{ or } \tan x = -1,$$

$$\therefore x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$f''\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \text{ i.e ve so, } x = \frac{3\pi}{4} \text{ is LocalMaxima}$$

and
$$f''\left(\frac{7\pi}{4}\right) = -\frac{1}{\sqrt{2}} + \frac{1}{2} \text{ i.e ve so, } x = \frac{7\pi}{4} \text{ is LocalMinima}$$

Local Maximum value
$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

Local Minimum value
$$= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \sqrt{2}$$

26. Find graphically, the maximum value of z = 2x + 5y, subject to constraints given below :

CBSE-XII-2015 EXAMINATION



Correctly shading feasible region Vertices are A (0, 2), B (1.6, 1.2), C (2, .0) Z = 2x + 5y is maximum at A (0, 2) and maximum value = 10

CBSE-XII-2014 EXAMINATION

MATHEMATICS

Paper & Solution

Code: 65/1 Max. Marks: 100

Time: 3 Hrs.

General Instructions:

(i) All question are compulsory.

(ii) The question paper consists of **29** questions divided into three sections A, B and C. Section A comprises of **10** questions of **one mark** each, Section B comprises of **12** questions of **four marks** each and Section C comprises of **7** questions of **six marks** each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is **not** permitted. You may ask for logarithmic tables, if required.

SECTION A

Question numbers 1 to 10 carry 1 mark each.

1. If $R = \{(x, y) : x + 2y = 8\}$ is a relation on N, write the range of R.

Solution:

 $R = \{(x, y): x + 2y = 8\}$ is a relation on N

Then we can say 2y = 8 - x

$$y = 4 - \frac{x}{2}$$

so we can put the value of x, x = 2, 4, 6 only

we get y = 3 at x = 2we get y = 2 at x = 4we get y = 1 at x = 6so range = {1, 2, 3} Ans.

2. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, xy < 1, then write the value of x + y + xy.

Solution:

 $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$ $\Rightarrow \tan^{-1} \frac{x+y}{1-xy} = \frac{\pi}{4}$ $\Rightarrow \frac{x+y}{1-xy} = \tan\frac{\pi}{4}$ $\Rightarrow \frac{x+y}{1-xy} = 1 \text{ or, } x+y = 1-xy$

or, x + y + xy = 1 Ans.

3. If A is a square matrix such that $A^2 = A$, then write the value of $7A - (I + A)^3$, where I is an identity matrix.

Solution:

$$\begin{array}{l} A^2 = A \\ 7A - (I + A)^3 \\ 7A - [(I + A)^2(I + A)] = 7A - [(II + AA + 2AI) (I + A)] \\ = 7A - [I + A^2 + 2AI] [I + A] \\ = 7A - [I + A + 2A] [I + A] \\ = 7A - [I + 3A] [I + A] \\ = 7A - [I + 3A] [I + A] \\ = 7A - [I + IA + 3AI + 3A^2] \\ = 7A - [I + A + 3A + 3A] \\ = 7A - [I + 7A] \\ = -I \text{ Ans.} \end{array}$$

4. If
$$\begin{bmatrix} x - y & z \\ 2x - y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$$
, find the value of x + y.

Solution:

$$If\begin{bmatrix} x-y & z\\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4\\ 0 & 5 \end{bmatrix} \text{ then } x+y=?$$

we can compare the element of 2 matrices. so $x - y = -1 \dots (1)$ $2x - y = 0 \dots (2)$ On solving both eqⁿ we get $\rightarrow x = 1, y = 2$ so x + y = 3 Ans.

5. If
$$\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$$
, find the value of x.

Solution:

$$\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$$

on expanding both determinants we get 12x + 14 = 32 - 42 12x + 14 = -10 12x = -24x = -2 Ans.

6. If
$$f(x) = \int_{0}^{x} t \sin t \, dt$$
, then write the value of $f'(x)$.

Solution:

$$f(x) = \int_{0}^{\infty} t \sin t \, dt$$

$$\Rightarrow f'(x) = 1 \cdot x \sin x - 0$$

 $= x \sin x Ans.$

7. Evaluate :

$$\frac{x}{x^2+1}dx$$

Solution:

$$I = \int_{2}^{4} \frac{x}{x^{2} + 1} dx$$

$$\Rightarrow 2xdx = dt$$

Put x²+1=t

$$xdx = \frac{1}{2}dt$$

$$dt x = 2$$

$$t = 5$$

$$at x = 4$$

$$t = 17$$

$$\therefore I = \int_{4}^{17} \frac{1/2}{t} dt$$
$$= \frac{1}{2} \left[\log |t| \right]_{4}^{17}$$
$$= \frac{1}{2} \left[\log 17 - \log 4 \right]$$
$$= \frac{1}{2} \log (17/4) Ans.$$

8. Find the value of 'p' for which the vectors $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are parallel. **Solution:**

Let $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$, $\vec{b} = \hat{i} - 2p\hat{j} + 3\hat{k}$ If \vec{a} , \vec{b} are parallel vector then their exist a, λ such that $\vec{a} = \lambda \vec{b}$ So $(3\hat{i} + 2\hat{j} + 9\hat{k}) = \lambda (\hat{i} - 2p\hat{j} + 3\hat{k})$ compare $3 = \lambda$ $2 = -2p\lambda$ $p = -\frac{1}{3}Ans.$ 9.Find $\vec{a} . (\vec{b} \times \vec{c})$, if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$.

Solution:

If
$$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}, \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$$

Then
$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

expand along R₁ = 2[4 - 1] - 1 [-2 - 3] + 3 [-1 - 6]
= 6 + 5 - 21 = -10

10. If the Cartesian equations of a line are $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, write the vector equation for the line.

Solution:

Cartesian eqⁿ of line is $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, we can write it as $\frac{x-3}{-5} = \frac{y+4}{7} = \frac{z-3}{2}$ so vector eqⁿ is $\vec{r} = (3i-4j+3k) + \lambda (-5\hat{i}+7\hat{j}+2\hat{k})$ where λ is a constant

SECTION B

Question numbers 11 to 22 carry 4 marks each.

11. If the function $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^2 + 2$ and $g : \mathbb{R} \to \mathbb{R}$ be given by $g(x) = \frac{x}{x-1}$, $x \neq 1$, find fog and gof and hence find fog (2) and gof (-3).

Solution: $f: R \rightarrow R; f(x) = x^2 + 2$

$$g: R \to R; g(x) = \frac{x}{x-1}, x \neq 1$$

$$fog = f(g(x))$$

$$= f\left(\frac{x}{x-1}\right) = \left(\frac{x}{x-1}\right)^2 + 2$$

$$= \frac{x^2}{(x-1)^2} + 2$$

$$= \frac{x^2 + 2(x-1)^2}{(x-1)^2}$$

$$= \frac{x^2 + 2x^2 - 4x + 2}{(x-1)^2}$$

$$= \frac{3x^2 - 4x + 2}{(x-1)^2}$$

$$gof = g(f(x)) = g(x^{2} + 2) = \frac{(x^{2} + 2)}{(x^{2} + 2) - 1} = \frac{x^{2} + 2}{x^{2} + 1} = 1 + \frac{1}{x^{2}}$$

$$\therefore fog(2) = \frac{3(2)^{2} - 4(2) + 2}{(2 - 1)^{2}} = 6$$

$$gof'(-3) = 1 + \frac{1}{(-3)^{2} + 1} = \frac{11}{10} = 1\frac{1}{10}$$

12. Prove that $\tan^{-1}\left[\frac{\sqrt{1 + x} - \sqrt{1 - x}}{\sqrt{1 + x} + \sqrt{1 - x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, \frac{-1}{\sqrt{2}} \le x \le 1$
OR
If $\tan^{-1}\left(\frac{x - 2}{x - 4}\right) + \tan^{-1}\left(\frac{x + 2}{x + 4}\right) = \frac{\pi}{4}$, find the value of x.
Solution:

$$\tan^{-1}\left[\frac{\sqrt{1 + x} - \sqrt{1 - x}}{\sqrt{1 + x} + \sqrt{1 - x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, \frac{1}{\sqrt{2}} \le x \le 1$$

In LHS
put x = cos 2 θ
 $\tan^{-1}\left[\frac{\sqrt{1 + cos 2\theta} - \sqrt{1 - cos 2\theta}}{\sqrt{1 + cos 2\theta} - \sqrt{1 - cos 2\theta}}\right]$
 $= \tan^{-1}\left[\frac{\sqrt{1 + cos^{2}\theta - 1} - \sqrt{1 - 1 + 2sin^{2}\theta}}{\sqrt{1 + cos^{2}\theta - 1} + \sqrt{1 - cos 2\theta}}\right]$
 $= \tan^{-1}\left[\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta}\right]$
 $= \tan^{-1}\left[\frac{\tan(\pi/4) - \tan \theta}{1 + \tan(\pi/4) \cdot \tan \theta}\right]$
 $= \tan^{-1}\left[\frac{\tan(\pi/4) - \theta}{1 + \tan(\pi/4) - \theta}\right]$
 $= \frac{\pi}{4} - \theta \quad as \begin{cases} x = cos 2\theta \\ so, \theta \frac{\cos^{-1} x}{2} \end{cases}$

$$\tan^{-1}\left(\frac{x-2}{x-4}\right) + \tan^{-1}\left(\frac{x+2}{x+4}\right) = \frac{\pi}{4} \quad (1)$$

Use formula, $\tan^{-1}\left[\frac{\frac{x-2}{x-4} + \frac{x+2}{x+4}}{1-\left(\frac{x-2}{x-4}\right)\cdot\left(\frac{x+2}{x+4}\right)}\right] = \frac{\pi}{4}$
$$\Rightarrow \tan^{-1}\left[\frac{(x-2)(x+4) + (x+2).(x-4)}{(x-4).(x+4) - (x-2).(x+2)}\right] = \frac{\pi}{4}$$
$$\Rightarrow \frac{(x-2)(x+4) + (x+2).(x-4)}{(x-4).(x+4) - (x-2).(x+2)} = 1$$
$$\Rightarrow \frac{x^2 - 8 + 2x + x^2 - 8 - 2x}{x^2 - 16 - x^2 + 4} = 1$$
$$\Rightarrow \frac{2x^2 - 16}{-12} = 1$$
$$\Rightarrow 2x^2 = -12 + 16 = 4$$
$$\Rightarrow x^2 = 2 \qquad \Rightarrow x = \pm\sqrt{2}$$

13. Using properties of determinants, prove that

 $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^{3}$

Solution:

To prove,
$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^{3}$$

LHS =
$$\begin{vmatrix} x & x & x \\ 5x & 4x & 2x \\ 10x & 8x & 3x \end{vmatrix} + \begin{vmatrix} y & x & x \\ 4y & 4x & 2x \\ 8y & 8x & 3x \end{vmatrix}$$
$$= x^{3} \begin{vmatrix} 1 & 1 & 1 \\ 5 & 4 & 2 \\ 10 & 8 & 3 \end{vmatrix} + yx^{2} \begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 2 \\ 8 & 8 & 3 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 - C_2$, $C_2 \rightarrow C_2 - C_3$ in the first determinant

$$= x^{3} \begin{vmatrix} 0 & 0 & 1 \\ 1 & 2 & 2 \\ 2 & 5 & 3 \end{vmatrix} + yx^{2} \times 0$$

As the first two columns of the 2^{nd} determinant are same. Expanding the first determinant through R_1

$$= x^{3} \cdot 1 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = x^{3} (5 - 4)$$

 $= x^3 = RHS$ thus proved

14. Find the value of
$$\frac{dy}{dx}$$
 at $\theta = \frac{\pi}{4}$, if $x = ae^{\theta}(\sin \theta - \cos \theta)$ and $y = ae^{\theta}(\sin \theta + \cos \theta)$.

Solution:

$$y = ae^{\theta}(\sin\theta + \cos\theta)$$

$$x = ae^{\theta}(\sin\theta - \cos\theta)$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} \text{ (Applying parametric differentiation) ... (1)}$$
Now, $\frac{dy}{d\theta} = ae^{\theta}(\cos\theta - \sin\theta) + ae^{\theta}(\sin\theta + \cos\theta)$

$$= 2ae^{\theta}(\cos\theta) \text{ (Applying product Rule)}$$

$$\frac{dx}{d\theta} = ae^{\theta}(\cos\theta + \sin\theta) + ae^{\theta}(\sin\theta - \cos\theta)$$

$$= 2ae^{\theta}(\sin\theta)$$
Substituting the values of $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ in (1)
$$\frac{dy}{dx} = \frac{2ae^{\theta}\cos\theta}{2ae^{\theta}\sin\theta} = \cot\theta$$
Now $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$

$$[\cot\theta]_{\theta=\pi/4} = \cot\frac{\pi}{4} = 1.$$

15. If $y = Pe^{ax} + Qe^{bx}$, show that

$$\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = 0$$

Solution:

If
$$y = Pe^{ax} + Qe^{bx}$$
 ...(1)

$$\frac{dy}{dx} = aPe^{ax} + bQe^{bx}$$
 ...(2)

$$\frac{d2y}{dx^2} = a^2Pe^{ax} + b^2Qe^{bx}$$
 ...(3)
multiplying ... (1) by ab
we get, aby = abPe^{ax} + abQe^{bx} ... (4)
multiplying (2) by (a + b)
we get,, $(a+b)\frac{dy}{dx} = (a+b)(aPe^{ax} + bQe^{bx}) = (a^2Pe^{ax} + b^2Pe^{bx}) + (abPe^{ax} + abQe^{bx})$

or,
$$(a^2bPe^{ax}+b^2Qe^{bx})-(a+b)\frac{dy}{dx}+(abPe^{ax}+abQe^{bx})$$

or,
$$\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = 0$$

16. Find the value(s) of x for which $y = [x (x - 2)]^2$ is an increasing function. **OR**

Find the equations of the tangent and normal to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(\sqrt{2a}, b)$.

Solution:

 $y = [x (x - 2)]^2$

we know, for increasing function we have $f'(x) \ge 0$

$$\therefore f'(x) = 2[x(x-2)] \left[\frac{d}{dx} x(x-2) \right]$$

Or, $f'(x) = 2[x(x-2)] \frac{d}{dx} (x^2 - 2x)$
$$= 2x(x-2) (2x-2)$$

$$= 4x(x-2) (x-1)$$

For $f'(x) \ge 0$
i.e., $4x(x-1) (x-2) \ge 0$

the values of x are :

CBSE-XII-2014 EXAMINATION



 $x\!\in\![0,\!1]\!\cup\![2,\!\infty]$

OR

The slope of the tangent at $(\sqrt{2} a, b)$ to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{2x}{a^2} - \frac{2yy'}{b^2} = 0$$
$$\Rightarrow y' = \frac{b^2 x}{a^2 y} \Big]_{(\sqrt{2}a,b)} = \frac{b^2 \sqrt{2}a}{a^2 b} = \frac{b\sqrt{2}}{a}$$

The equation of the tangent :

 $y-b = \frac{b\sqrt{2}}{a}(x-\sqrt{2}a) \text{ {using point-slope form : } y-y_1 = m(x-x_1) }$ $ay-ab = b\sqrt{2}x-2ab$ or $b\sqrt{2}x-ay-ab = 0$ Normal : The slope of the normal $= \frac{-1}{dy/dx}$ $= \frac{-1}{\frac{b\sqrt{2}}{a}} = -\frac{a}{b\sqrt{2}}$ Equation of Normal :

$$y-b = \frac{a}{b\sqrt{2}}(x-\sqrt{2}a)$$
$$yb\sqrt{2}-b^2\sqrt{2} = -ax+\sqrt{2}a^2$$
or
$$ax+b\sqrt{2}y-\sqrt{2}(a^2+b^2) = 0$$

-a

17. Evaluate :

 $\int_{0}^{\pi} \frac{4x \sin x}{1 + \cos^2 x} dx$

Evaluate :

$$\int \frac{x+2}{\sqrt{x^2+5x+6}} dx$$

Solution:

$$I = \int_{0}^{\pi} \frac{4x \sin x}{1 + \cos^{2} x} dx$$
$$I = \int_{0}^{\pi} \frac{4(\pi - x) \sin(\pi - x)}{1 + \cos^{2}(\pi - x)} dx \quad \left\{ Applying \int f(a - x) = \int f(x) \right\}$$
$$I = \int_{0}^{\pi} \frac{4\pi \sin x}{1 + \cos^{2} x} dx - \int_{0}^{\pi} \frac{4x \sin x}{1 + \cos^{2} x} dx$$

Or,

$$I = \int_{0}^{\pi} \frac{4\pi \sin x}{1 + \cos^{2} x} dx - I$$

$$2I = 4\pi \int_{0}^{\pi} \frac{\sin x}{1 + \cos^{2} x} dx$$

$$2I = 4\pi . 2 \times \int_{0}^{\pi/2} \frac{\sin x}{1 + \cos^{2} x} dx \quad \left\{ \text{Applying } \int_{0}^{2a} f(x) dx = 2 \int_{0}^{2} f(x) dx \text{ if } f(2a - x) = f(x) \right\}$$

$$I = 4\pi \int_{0}^{\pi/2} \frac{\sin x}{1 + \cos^{2} x} dx$$

put cos x = t $\Rightarrow -\sin x dx = dt$
as well for x = 0, x = $\pi/2$
t = 1 t = 0
 $\therefore I = 4\pi \int_{0}^{0} \frac{-dt}{1 + t^{2}}$

$$I = 4\pi \int_{0}^{0} \frac{-dt}{1 + t^{2}} \quad \left\{ \int_{a}^{b} f(x) dx = -\int_{b}^{a} f(x) dx \right\}$$

$$I = 4\pi \left[\tan^{-1} 1 - \tan^{-1} 0 \right]$$

$$= 4\pi \times \frac{\pi}{4} = \pi^{2}.$$

CBSE-XII-2014 EXAMINATION

$$\int \frac{x+2}{\sqrt{x^2+5x+6}} dx$$
put, $x + 2 = \lambda \left(\frac{d}{dx} (x^2+5x+6) \right) + \mu$
 $x + 2 = 2\lambda x + 5\lambda + \mu$
comparing coefficients of x both sides
 $1 = 2\lambda \Rightarrow \lambda = 1/2$
comparing constant terms both sides,
 $2 = 5\lambda + \mu$
or, $2 = 5\left(\frac{1}{5}\right) + \mu$
or, $2 = 5\left(\frac{1}{5}\right) + \mu$
or, $\mu = 2 - \frac{5}{2} = \frac{-1}{2}$
 $\therefore \int \frac{x+2}{\sqrt{x^2+5x+6}} dx = \int \frac{1}{2} \frac{(2x+5) - \frac{1}{2}}{\sqrt{x^2+5x+6}} dx$ {as $x + 2 = \lambda(2x+5) + \mu$ }
 $\therefore I = \int \frac{1}{\sqrt{x^2+5x+6}} dx - \frac{1}{2} \int \frac{dx}{\sqrt{x^2+5x+6}}$
 $I_1 = \frac{1}{2} \int \frac{(2x+5)}{\sqrt{x^2+5x+6}} dx$, put $x^2 + 5x + 6 = t$
 $\therefore (2x+5) dx = dt$
 $= \frac{1}{2} \int \frac{dt}{\sqrt{t}} = \frac{1}{2} \left(\frac{t^{-1/241}}{-\frac{1}{2}+1} \right) + C = t^{1/2} + C = \sqrt{x^2+5x+6} + C$
 $I_2 = \frac{1}{2} \int \frac{dx}{\sqrt{x^2+5x+6}}$

$$\frac{1}{2} \cdot \log\left[\left(x+\frac{5}{2}\right)+\sqrt{\left(x+\frac{5}{2}\right)^2-\left(\frac{1}{2}\right)^2}\right]+C$$
$$\frac{1}{2} \cdot \log\left[\left(x+\frac{5}{2}\right)+\sqrt{x^2+5x+6}\right]+C$$

Substituting the values of I_1 and I_2 in (1) we get,

$$I = \sqrt{x^2 + 5x + 6} + \frac{1}{2} \log \left[\left(x + \frac{5}{2} \right) + \sqrt{x^2 + 5x + 6} \right] + c$$

18. Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that y = 0 when x = 1.

Solution:

$$\frac{dy}{dx} = (1+x) + y(1+x)$$
Or, $\frac{dy}{dx} = (1+y)(1+x)$
Or, $\frac{dy}{1+y} = (1+x)dx$

$$\int \frac{dy}{1+y} = \int (1+x)dx$$

$$\log |1+y| = x + \frac{x^2}{2} + C$$
given $y = 0$ when $x = 1$
i.e., $\log |1+0| = 1 + \frac{1}{2} + C$

$$\Rightarrow C = -\frac{3}{2}$$
 \therefore The particular solution

$$\log|1+y| = \frac{x^2}{2} + x - \frac{3}{2}.$$

or the answer can expressed as

is

$$\log |1+y| = \frac{x^2 + 2x - 3}{2}$$

or $1 + y = e^{(x^2 + 2x - 3)/2}$
or, $y = e^{(x^2 + 2x - 3)} - 1$.

19. Solve the differential equation $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$.

Solution:

$$(1+x^{2})\frac{dy}{dx} + y = e^{\tan^{-1}x}$$
$$\frac{dy}{dx} + \frac{y}{1+x^{2}} = \frac{e\tan^{-1}}{1+x^{2}}$$

It is a linear differential equation of 1st order. comparing with standard LDE

$$\frac{dy}{dx} + P(x)y = Q(x)$$
$$P(x) = \frac{1}{1 + x^2}; Q(x) = \frac{e \tan^{-1x}}{1 + x^2}$$

Integrating factor
$$IF = e^{\int Pdx} = e^{\int \frac{1}{1+x^2}dx} = e^{\tan(1x)}$$

Solution of LDE

$$y.IF = \int IF \ Q(x)dx + C$$

$$\therefore y.e^{\tan^{-1}x} = \int e^{\tan^{-1}x} \cdot \frac{e^{\tan^{-1}x}}{1 + x^2} dx + C$$

$$y.e^{\tan^{-1}x} = \int \frac{(e^{\tan^{-1}x})^2}{1 + x^2} dx + C \quad \dots (1) \text{ y}$$

To solving $\int \frac{(e^{\tan^{-1}x})^2}{1 + x^2} dx$
Put $e^{\tan^{-1}x} = t$
or $e^{\tan^{-1}x} \cdot \frac{1}{1 + x^2} = dt$

$$\therefore \int \frac{e^{\tan^{-1}x} \cdot e^{\tan^{-1}x}}{1+x^2} dx = \int t dt$$
$$= \frac{t^2}{2} + C = \frac{(e^{\tan^{-1}x})^2}{2} + C$$

Substituting in (1)

$$y.e^{\tan^{-1}x} = \frac{(e^{\tan^{-1}x})^2}{2} + C$$

20. Show that the four points A, B, C and D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $4(-\hat{i} + \hat{j} + \hat{k})$ respectively are coplanar.

OR

The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

Solution:

If P.V of $\vec{A} = 4\hat{i} + 5\hat{j} + \hat{k}$ $\vec{B} = -\hat{j} - \hat{k}$ $\vec{C} = 3\hat{i} + 9\hat{j} + 4\hat{k}$ $\vec{D} = 4(-\hat{i} + \hat{j} + \hat{k})$ Points $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ all Coplanar if $\left[\overrightarrow{AB} \overrightarrow{AC} \overrightarrow{AD}\right] = 0 \Rightarrow (1)$ So, $\overrightarrow{AB} = P.V.$ of $\vec{B} - P.V.$ of $\vec{A} = -4\hat{i} - 6\hat{j} - 2\hat{k}$ $\overrightarrow{AC} = P.V.$ of $\vec{C} - P.V.$ of $\vec{A} = -\hat{i} - 4\hat{j} + 3\hat{k}$ $\overrightarrow{AD} = P.V.$ of $\vec{D} - P.V.$ of $\vec{A} = -8\hat{i} - \hat{j} + 3\hat{k}$ So, so for $\left[\overrightarrow{AB} \overrightarrow{AC} \overrightarrow{AD}\right]$ $= \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$ expand along R₁ \rightarrow -4[12 + 3] + 6[-3 + 24] - 2[1 + 32]= -60 + 126 - 66 OR

= 0 So, we can say that point A, B, C, D are Coplanar proved

Given
$$\rightarrow \vec{a} = \hat{i} + \hat{j} + \hat{k}$$

 $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$
 $\vec{c} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}$
So, $\vec{b} + \vec{c} = (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}$
Unit vector along $(\vec{b} + \vec{c}) = \frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2 + \lambda)^2 + 36 + 4}}$
 $= \frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2 + \lambda)^2 + 40}}$

given that dot product of \vec{a} with the unit vector of $\vec{b} + \vec{c}$ is equal to 1 So, apply given condition

$$\frac{(2+\lambda)+6-2}{\sqrt{(2+\lambda)^2+40}} = 1$$

$$\Rightarrow 2+\lambda+4 = \sqrt{(2+\lambda)^2+40}$$

Squaring $36+\lambda^2+12\lambda = 4+\lambda^2+4\lambda+40$

$$\Rightarrow 8\lambda = 8$$

$$\Rightarrow \lambda = 1.$$

21. A line passes through (2, -1, 3) and is perpendicular to the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and

 $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$. Obtain its equation in vector and Cartesian form.

Solution:

Line L is passing through point $=(2\hat{i}-\hat{j}+3\hat{k})$

If
$$L_1 \Rightarrow \vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$$

 $L_2 \Rightarrow \vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

Let dr of line $L = a_1, a_2, a_3$ The eqⁿ of L in vector form \Rightarrow

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + k(a_1\hat{i} + a_2\hat{j} + a_3\hat{k})$$
k is any constant.
so by condition that L1 is perpendicular to L $a_1a_2 + b_1b_2 + c_1c_2 = 0$
 $2a_1 - 2a_2 + a_3 = 0 \dots (1)$
and also
 $L \perp L_2$
so, $a_1 + 2a_2 + 2a_3 = 0 \dots (2)$
Solve (1), (2)
 $3a_1 + 3a_3 = 0$
 $\Rightarrow a_3 = -a_1$
put it in (2)
 $a_1 + 2a_2 - 2a_1 = 0$
 $a_2 = \frac{a_1}{2}$ let
so dr of L = $\left(a_1, \frac{a_1}{2}, -a_1\right)$
so we can say dr of L = $\left(1, \frac{1}{2}, -1\right)$
so eqⁿ of L in vector form
 $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + k\left(\hat{i} + \frac{\hat{j}}{2} - \hat{k}\right)$
3-D form $\rightarrow \frac{x-2}{1} = \frac{y+1}{1/2} = \frac{z-3}{-1}$

22. An experiment succeeds thrice as often as it fails. Find the probability that in the next five trials, there will be at least 3 successes.

Solution:

In Binomial distribution

 $(p+q)^{n} = {}^{n} C_{0} \cdot p^{n} + {}^{n} C_{1} \cdot p^{n-1} \cdot q^{1} + {}^{n} C_{2} \cdot p^{n-2} \cdot q^{2} + \dots + {}^{n} C_{n} \cdot q^{n}$ if p = probability of success q = prob. of fail given that p = 3q ...(1) we know that p + q = 1

so, 3q + q = 1

$$q = \frac{1}{4}$$

So, $p = \frac{3}{4}$

Now given \Rightarrow n = 5 we required minimum 3 success $(p+q)^5 = {}^5C_0.p^5 + {}^5C_1.p^4.q^1 + {}^5C_2.p^3.q^2$ $= {}^5C_0.\left(\frac{3}{4}\right)^5 + {}^5C_1.\left(\frac{3}{4}\right)^4.\left(\frac{1}{4}\right) + {}^5C_2.\left(\frac{3}{4}\right)^3.\left(\frac{1}{4}\right)^2$ $3^5 + 5.3^4 + 10.3^3$

$$=\frac{3^{5}+5.3^{4}+10.3^{3}}{4^{5}}=\frac{3^{3}[9+15+10]}{4^{5}}=\frac{34\times27}{16\times64}=$$

SECTION C

459

512

Question numbers 23 to 29 carry 6 marks each.

23. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award \gtrless x each, \gtrless y each and \gtrless z each for the three respective values to 3, 2 and 1 students respectively with a total award money of \gtrless 1,600. School B wants to spend \gtrless 2,300 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is \gtrless 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award.

Solution:

Let Matrix D represents number of students receiving prize for the three categories :

D =					
Number of students	SINCERITY	TRUTHFULNESS	HELPFULNESS		
of school					
А	3	2	1		
В	4	1	3		
One student for each	1	1	1		
value					

 $X = \begin{vmatrix} y \\ z \end{vmatrix}$ where x, y and z are rupees mentioned as it is the question, for sincerity, truthfulness and

helpfulness respectively.

[1600]

х

 $E = \begin{vmatrix} 2300 \\ 900 \end{vmatrix}$ is a matrix representing total award money for school A, B and for one prize for each value.

We can represent the given question in matrix multiplication as : DX = Eor $\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$ Solution of the matrix equation exist if $|\mathbf{D}| \neq 0$ 3 2 1 i.e., $\begin{vmatrix} -2 & -2 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 3[1-3] - 2[4-3] + 1[4-1]$ = -6 - 2 + 3= -5therefore, the solution of the matrix equation is $X = D^{-1} E$ To find D⁻¹; D⁻¹ = $\frac{1}{|D|} adj(D)$ Cofactor Matrix of D -2 -1 3 $= \begin{bmatrix} -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}$ Adjoint of D = adj (D) $= \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$ {transpose of Cofactor Matrix} $\therefore D^{-1} = \frac{1}{-5} \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$ Now, $X = D^{-1}E$ $=\frac{1}{-5}\begin{bmatrix} -2 & -1 & 5\\ -1 & 2 & -5\\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 1600\\ 2300\\ 900 \end{bmatrix}$ [200] y = 300| *z*. | 400 \therefore x = 200, y = 300, z = 400. Ans. Award can also be given for Punctuality.

24. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also show that the maximum volume of the cone is $\frac{8}{27}$ of the volume of the sphere.

Solution:

Let R and h be the radius and height of the cone. r be the radius of sphere.

To show
$$h = \frac{4r}{3}$$

and Maximum Volume of Sphere
 $= \frac{8}{27}$ Volume of Sphere
In $\triangle ABX$, $AC = h - r$
 $\therefore (h - r)^2 + R^2 = r^2$ {Pythagorus Theorem}
 $\Rightarrow R^2 = r^2 - (h - r)^2$
Volume of cone : $V = \frac{1}{3}\pi R^2 h$
or, $V = \frac{1}{3}\pi (r^2 - (h - r)^2)h$
 $V = \frac{1}{3}\pi [r^2 - h^2 - r^2 + 2hr]h$
 $V = \frac{1}{3}\pi [2h^2r - h^3]$
For maxima or minima, $\frac{dV}{dh} = 0$
Now, $\frac{dV}{dh} = \frac{1}{3}\pi [4hr - 3h^2]$
Putting, $\frac{dV}{dh} = 0$
We get $4hr = 3h^2$
 $\Rightarrow h = \frac{4r}{3}$
 $\frac{d^2V}{dh^2} = \frac{1}{3}\pi [4r - 6h]$
Putting $h = \frac{4r}{3}$
 $\frac{d^2V}{dh^2} = \frac{1}{3}\pi (4r - \frac{6.4r}{3})$
 $= -\frac{1}{3}\pi [4r]$

Which is less than zero, therefore

$$h = \frac{4r}{3}$$
 is a Maxima

and the Volume of the cone at $h = \frac{4r}{3}$

will be maximum,

$$V = \frac{1}{3}\pi R^{2}h$$

= $\frac{1}{3}\pi [r^{2} - (h - r)^{2}]h$
= $\frac{1}{3}\pi \left[r^{2} - \left(\frac{4r}{3} - r\right)^{2}\right] \left[\frac{4r}{3}\right]$
= $\frac{1}{3}\pi \left[\frac{8r^{2}}{9}\right] \left[\frac{4r}{3}\right]$
= $\frac{8}{27} \left(\frac{4\pi r^{3}}{3}\right)$
= $\frac{8}{27}$ (Volume of the sphere)

25. Evaluate :

 $\int \frac{1}{\cos^4 x + \sin^4 x} dx$
Solution:

$$\int \frac{1}{\cos^4 x + \sin^4 x} dx$$

$$= \int \frac{\frac{1}{\cos^4 x}}{1 + \tan^4 x}$$

$$= \int \frac{\sec^2 x \sec^2 x dx}{1 + \tan^4 x}$$

$$= \int \frac{(1 + \tan^2 x) \sec^2 x dx}{1 + \tan^4 x}$$
put tan x = t $\Rightarrow \sec^2 x dx = dt$

$$= \int \frac{(1 + t^2) dt}{1 + t^4}$$

$$= \int \frac{(\frac{1}{t^2} + 1) dt}{\frac{1}{t^2} + t^2} \{\text{dividing each by } t^2\}$$

$$= \int \frac{\left(1 + \frac{1}{t^2}\right) dt}{\left(t - \frac{1}{t}\right)^2 + 2}$$
Put
$$t - \frac{1}{t} = z \implies \left(1 + \frac{1}{t^2}\right) dt = dz$$

$$= \int \frac{dz}{z^2 + 2} = \frac{1}{\sqrt{2}} \tan^{-1} z + C$$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \left(\tan x - \frac{1}{\tan x}\right) + C$$

$$= \frac{1}{\sqrt{2}} \tan^{-1} (\tan x - \cot x) + C$$

26. Using integration, find the area of the region bounded by the triangle whose vertices are (-1, 2), (1, 5) and (3, 4).

Solution:

Let A = (-1, 2) B = (1, 5) C = (3, 4)A (-1, 2) L M N

We have to find the area of $\triangle ABC$

C (3, 4)

Find eqn of Line AB
$$\rightarrow y-5 = \left(\frac{2-5}{-1-1}\right) \cdot (x-1)$$

 $y-5 = \frac{3}{2}(x-1)$
 $2y-10 = 3x-3$
 $3x-2y+7=0...(1)$
 $y = \frac{3x+7}{2}$
Eqn of BC $\rightarrow y-4 = \left(\frac{5-4}{1-3}\right) \cdot (x-3)$
 $y-4 = \frac{1}{-2}(x-3)$
 $2y-8 = -x+3$
 $x+2y-11=0$ (2)
 $y = \frac{11-x}{2}$
Eqn of AC $\rightarrow y-4 = \left(\frac{2-4}{-1-3}\right) \cdot (x-3)$
 $y-4 = \frac{1}{2}(x-3) \Rightarrow 2y-8 = x-3$
 $x-2y+5 = 0...(3)$

$$\Rightarrow y = \frac{x+5}{2}$$

So, required area $= \int_{-1}^{1} \left(\frac{3x+7}{2}\right) dx + \int_{1}^{3} \left(\frac{11-x}{2}\right) dx - \int_{-1}^{3} \left(\frac{x+5}{2}\right) dx$
 $= \frac{1}{2} \left[\frac{3x^{2}}{2} + 7x\right]_{-1}^{1} + \frac{1}{2} \left[11x - \frac{x^{2}}{2}\right]_{1}^{3} - \frac{1}{2} \left[\frac{x^{2}}{2} + 5x\right]_{-1}^{3}$
 $= \frac{1}{2} \left[\left(\frac{3}{2} + 7\right) - \left(\frac{3}{2} - 7\right)\right] + \frac{1}{2} \left[\left(33 - \frac{9}{2}\right) - \left(11 - \frac{1}{2}\right)\right] - \frac{1}{2} \left[\left(\frac{9}{2} + 15\right) - \left(\frac{1}{2} - 5\right)\right]$
 $= \frac{1}{2} [14 + 22 - 4 - 24] = \frac{1}{2} [36 - 28] = 4$ square unit

27. Find the equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 which is perpendicular to the plane x - y + z = 0. Also find the distance of the plane obtained above, from the

origin.

OR

Find the distance of the point (2, 12, 5) from the point of intersection of the line $\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - 2\hat{j} + \hat{k}) = 0$.

Solution:

Eqⁿ of given planes are $P_1 \Longrightarrow x + y + z - 1 = 0$ $P_2 \Longrightarrow 2x + 3y + 4z - 5 = 0$ Eq^n of plane through the line of intersection of planes P_1 , P_2 is $P_1+\;\lambda\,P_2=0$ $(x + y + z - 1) + \lambda (2x + 3y + 4z - 5) = 0$ $(1 + 2\lambda) x + (1 + 3\lambda) y + (1 + 4\lambda) z + (-1 - 5\lambda) = 0 \dots (1)$ given that plane represented by $eq^{n}(1)$ is perpendicular to plane x - y + z = 0so we use formula $a_1a_2 + b_1b_2 + c_1c_2 = 0$ so $(1 + 2\lambda).1 + (1 + 3\lambda).(-1) + (1 + 4\lambda).1 = 0$ $1+2\lambda-1-3\lambda+1+4\lambda=0$ $3\lambda + 1 = 0$ $\lambda = \frac{-1}{3}$ Put $\lambda = -\frac{1}{3}$ in eqⁿ (1) so we get $\left(1-\frac{2}{3}\right)x+(1-1)y+\left(1-\frac{4}{3}\right)z+\frac{2}{3}=0$ $\frac{x}{3} - \frac{z}{3} + \frac{2}{3} = 0$ x - z + 2 = 0 Ans.

General points on the line:

 $\begin{aligned} \mathbf{x} &= 2 + 3\lambda, \, \mathbf{y} = -4 + 4\lambda, \, \mathbf{z} = 2 + 2\lambda \\ \text{The equation of the plane :} \\ \vec{r}.(\hat{i} - 2\hat{j} + \hat{k}) = 0 \\ \text{The point of intersection of the line and the plane :} \\ \text{Substituting general point of the line in the equation of plane and finding the particular value of } \lambda. \\ &[(2 + 3\lambda)\hat{i} + (-4 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}].(\hat{i} - 2\hat{j} + \hat{k}) = 0 \\ &(2 + 3\lambda).1 + (-4 + 4\lambda)(-2) + (2 + 2\lambda).1 = 0 \\ &12 - 3\lambda = 0 \text{ or, } \lambda = 4 \\ \therefore \text{ the point of intersection is :} \\ &(2 + 3 (4), -4 + 4(4), 2 + 2(4)) = (14, 12, 10) \\ &\text{Distance of this point from } (2, 12, 5) \text{ is} \\ &= \sqrt{(14 - 2)^2 + (12 - 12)^2 + (10 - 5)^2} \quad \text{{Applying distance formula}} \\ &= \sqrt{12^2 + 5^2} \\ &= 13 \text{ Ans.} \end{aligned}$

28. A manufacturing company makes two types of teaching aids A and B of Mathematics for class XII. Each type of A requires 9 labour hours of fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ₹ 80 on each piece of type A and ₹120 on each piece of type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit? Make it as an LPP and solve graphically. What is the maximum profit per week?

Solution:

Let pieces of type A manufactured per week = x

Let pieces of type B manufactured per week = y

Companies profit function which is to be maximized : Z = 80x + 120y

	Fabricating hours	Finishing hours
А	9	1
В	12	3

Constraints : Maximum number of fabricating hours = 180

 $\therefore 9x + 12y \le 180 \implies 3x + 4y \le 60$ K

Where 9x is the fabricating hours spent by type A teaching aids, and 12y hours spent on type B. and Maximum number of finishing hours = 30

 $\therefore x + 3y \le 30$

where x is the number of hours spent on finishing aid A while 3y on aid B.

So, the LPP becomes :

Z (MAXIMISE) = 80x + 120 y Subject to $3x + 4y \le 60$ $x + 3y \le 30$

 $x \ge 0$

 $y \ge 0$

Solving it Graphically :



29. There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the two-headed coin?

OR

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of the random variable X, and hence find the mean of the distribution.

Solution:

If there are 3 coins.

Let these are A, B, C respectively

For coin A \rightarrow Prob. of getting Head P(H) = 1

For coin B \rightarrow Prob. of getting Head P(H) = $\frac{3}{4}$

For coin C \rightarrow Prob. of getting Head P(H) = 0.6

we have to find P(A/H) = Prob. of getting H by coin A

So, we can use formula

$$P(A/H) = \frac{P(H/A).P(A)}{P(H/A).P(A) + P(H/B).P(B) + P(H/C).P(C)}$$

Here P(A) = P(B) = P(C) = $\frac{1}{3}$ (Prob. of choosing any one coin)

$$P(H_A) = 1, P(H_B) = \frac{3}{4}, P(H_C) = 0.6$$

Put value in formula so

$$P(A/H) = \frac{1 \cdot \frac{1}{3}}{1 \cdot \frac{1}{3} + \frac{3}{4} \cdot \frac{1}{3} + \frac{1}{3}(0.6)} = \frac{1}{1 + 0.75 + 0.6}$$
$$= \frac{100}{235}$$
$$= \frac{20}{47} Ans.$$

OR

First six numbers are 1, 2, 3, 4, 5, 6. X is bigger number among 2 number so

Variable (X)	2	3	4	5	6
Probability					
P(X)					

if X = 2

for P(X) = Prob. of event that bigger of the 2 chosen number is 2 So, Cases = (1, 2)

3), (2, 3)

So,
$$P(X) = \frac{1}{{}^{6}C_{2}} = \frac{1}{15}....(1)$$

if X = 3
So, favourable cases are = (1,

 $P(x) = \frac{2}{6\pi} = \frac{2}{12}$(2)

if $X = 4 \implies$ favourable casec = (1, 4), (2, 4), (3, 4)

$$P(X) = \frac{3}{15}...(3)$$

if $X = 5 \implies$ favourable casec = (1, 5), (2, 5), (3, 5), (4, 5)

$$P(X) = \frac{4}{15}....(4)$$

if $X = 6 \implies$ favourable casec = (1, 6), (2, 6), (3, 6), (4, 6), (5, 6)

$$P(X) = \frac{5}{15}....(5)$$

We can put all value of P(X) in chart, So

Variable (X)	2	3	4	5	6
Probability	1	2	3	4	5
P(X)	15	15	15	15	15
and required mean $= 2 \cdot \left(\frac{1}{15}\right) + 3 \cdot \left(\frac{2}{15}\right) + 4 \left(\frac{3}{15}\right) + 5 \cdot \left(\frac{4}{15}\right) + 6 \cdot \left(\frac{5}{15}\right)$					
$=\frac{70}{15}=\frac{14}{3}$ Ans.					

MATHEMATICS

Paper & Solution

Code: 65/1 Max. Marks: 70

Time: 3 Hrs.

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section Comprises of 7 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the questions.

(iv) There is no overall choice, However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

1. Write the principal value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$

Solution:

$$\tan^{-1}(\sqrt{3}) = \pi / 3$$

$$\cot^{-1}(-\sqrt{3}) = \pi - \pi / 6$$

Hence

$$\pi/3 - (\pi - \pi/6) = -\pi/2$$

2. Write the value of
$$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$$
.

Solution:

$$\therefore \cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6}$$

= $\tan^{-1}(2\sin(2\pi/6))$
= $\tan^{-1}\left(2.\sin\frac{\pi}{3}\right)$
= $\tan^{-1}\left(2.\frac{\sqrt{3}}{2}\right) = \tan^{-1}\sqrt{3} = \pi/3$

3. For what value of x, is the matrix A = $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix ?

Sol. The value of determinant of skew symmetric matrix of odd order is always equal to zero

 $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = 0$ -1(0-3x)-2(3-0) = 0 $\Rightarrow 3x-6=0 \Rightarrow \boxed{x=2}$ 4. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then write the value of k. **Sol.** Given A2 = kA $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \Rightarrow \boxed{k=2}$

5. Write the differential equation representing the family of curves y = mx, where m is an arbitrary constant. Sol. y = mx(1)

differentiating with respect to x, we get

dy/dx = m

 \therefore differential equation of curve

$$y = \frac{xdy}{dx}$$

6. If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$, then write the value of $a_{32} \cdot A_{32}$.

Sol. $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$ $A_{32} = (-1)^{3+2} M_{32} \text{ where } M_{32} \text{ is the min or of } a_{32}.$ $A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix}$ $A_{32} = -\begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} \Rightarrow A_{32} = -(8-30)$ $\boxed{A_{32} = 22}$ $\therefore a_{32}A_{32} = 5(22) = 110$

7. P and Q are two points with position vectors $3\vec{a}-2\vec{b}$ and $\vec{a}+\vec{b}$ respectively. Write the position vector of point R which divides the line segment PQ in the ratio 2 : 1 externally. 1

Sol. P.V. of P is
$$3\vec{a} - 2\vec{b}$$

P.V. of Q is $\vec{a} + \vec{b}$
Point R divides segment PQ in ratio 2 : 1 externally.
P.V.of R = $\frac{(P.V.of p)1 - (P.V.of Q)(2)}{1 - 2}$
P.V.of R = $\frac{(3\vec{a} - 2\vec{b})(1) - (\vec{a} + \vec{b})(2)}{1 - 2} = \frac{\vec{a} - 4\vec{b}}{-1}$
P.V.of R = $4\vec{b} - \vec{a}$
8. Find $|\vec{x}|$, if for a unit vector \vec{a} , $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 15$.
Sol. Given $|\vec{a}| = 1$
 $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 15$
 $|\vec{x}|^2 - |\vec{a}|^2 = 15$
 $|\vec{x}|^2 - 1 = 15$
 $|\vec{x}|^2 = 15 + 1$
 $|\vec{x}|^2 = 16$
 $|\vec{x}| = 4$

9. Find the length of the perpendicular drawn from the origin to the plane 2x - 3y + 6z + 21 = 0.

Sol.
$$p = \left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$

$$p = \left| \frac{0 + 0 + 0 + 21}{\sqrt{2^2 + 3^2 + 6^2}} \right| \Longrightarrow p = \frac{21}{\sqrt{49}} \implies p = \frac{21}{7} \implies p = 3$$

10. The money to be spent for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (marginal revenue). If the total revenue (in rupees) received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$, find the marginal revenue, when x = 5, and write which value does the equations indicate.

Sol. R(x) = $3x^2 + 36x + 5$ MR = $\frac{dR}{dx} = 6x^2 + 36$ when x = 5 MR = 30 + 36 + 66

11. Consider $f : R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by

 $f^{-1}(y) = \sqrt{y-4}$, where R₊ is the set of all non-negative real numbers.

Sol. $f: R^+ \rightarrow [4.\infty)$ $f(x) = x^2 + 4$ $f(x) = x^2 > 4$ \therefore (one-one) As $f(x) = x^2 + 4 \ge 4$ \Rightarrow Rage = $[4.\infty) = co-domain$ \therefore onto Further : $y = x^2 + 4$ so f is invertible. $\Rightarrow y - 4 = x^2 \Rightarrow x = \pm \sqrt{y - 4}$ As x > 0 so $x = \sqrt{y - 4}$ $\therefore y = \sqrt{x - 4} = f^{-1}(x)$ Or $f^{-1}(y) - \sqrt{y - 4}$

12. Show that :

$$\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$$
OR

Solve the following equation :

$$\cos\left(\tan^{-1} x\right) = \sin\left(\cot^{-1} \frac{3}{4}\right)$$

Solution:

Let
$$\frac{1}{2}\sin^{-1}\frac{3}{4} = \theta$$
 then $\frac{3}{4} = \sin 2\theta$
Now $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \tan\theta$

if
$$\sin 2\theta = \frac{3}{4}$$
 then $\frac{2 \tan \theta}{1 + \tan^2 \theta} = \frac{3}{4}$
8 $\tan \theta = 3 + 3 \tan^2 \theta$
 $3 \tan^2 \theta - 8 \tan \theta + 3 = 0$
 $\tan \theta = \frac{8 \pm \sqrt{64 - 4 \times 3 \times 3}|}{6}$
 $\tan \theta = \frac{8 \pm \sqrt{28}}{6} = \frac{4 \pm \sqrt{7}}{3}$
 $\tan \theta = \frac{4 + \sqrt{7}}{3}$ or $\frac{4 - \sqrt{7}}{3}$
 $\tan \left(\frac{1}{2} \sin^{-1} \frac{3}{4}\right) = \frac{4 - \sqrt{7}}{3}$ Hence proved.
OR
 $\cos(\tan^{-1} x)$
LHS. Irt $\tan^{-1} x = \theta \implies x = \tan \theta$

$$\cos \theta = \frac{1}{\sqrt{1 + \tan^2 \theta}} = \frac{1}{\sqrt{1 + x^2}}$$

Hence $\cos (\tan^{-1} x) = \frac{1}{\sqrt{1 + x^2}}$
R.H.S Let $\cot^{-1} \frac{3}{4} = \theta \Rightarrow \frac{3}{4} = \cot \theta$
then $\sin \theta = \frac{1}{\sqrt{1 + \cot^2 \theta}} = \frac{1}{\sqrt{1 + \frac{9}{16}}} = \frac{4}{5}$
Now LHS = RHS

$$\frac{1}{\sqrt{1+x^2}} = \frac{4}{5}$$
$$25 = 16 + 16x^2$$
$$x^2 = \frac{9}{16} \implies x = \frac{3}{4}$$

13. Using properties of determinants, prove the following :

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^{2}(x+y)$$

Solution:

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^{2}(x+y)$$

$$LHS \begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$$
Now, apply $C_{1} \rightarrow C_{1} + C_{2} + C_{3}$

$$\begin{vmatrix} 3x+3y & x+y & x+2y \\ 3x+3y & x & x+y \\ 3x+3y & x & x+y \\ 3x+3y & x & x+y \end{vmatrix}$$

$$3(x+y) \begin{vmatrix} -y & 2y \\ -2y & y \end{vmatrix}$$

$$3(x+y) \begin{vmatrix} -y & 2y \\ -2y & y \end{vmatrix}$$

$$3y^{2}(x+y) \begin{vmatrix} -1 & 2 \\ -2 & 1 \\ 3y^{2}(x+y) \end{vmatrix} = 9y^{2}(x+y).$$
 Hence proved.
$$14. \text{ If } y^{x} = e^{y-x}, \text{ prove that } \frac{dy}{dx} = \frac{(1+\log y)^{2}}{\log y}$$
Sol. $y^{x} = e^{y-x}$

$$\Rightarrow x \log_{e} y = y - x \qquad \dots(1)$$
Differentiating w.r.t.x
$$\Rightarrow \log_{e} y + x \cdot \frac{1}{y} \frac{dy}{dx} = \frac{dy}{dx} - 1$$

$$\Rightarrow \log_{e} y + 1 = \frac{dy}{dx} \left(1 - \frac{1}{1 + \log_{e} y}\right)$$

$$\Rightarrow (\log_{e} y + 1) = \frac{dy}{dx} \left(1 - \frac{1}{1 + \log_{e} y}\right)$$

$$\Rightarrow (\log_{e} y + 1) = \frac{dy}{dx} \left(1 - \frac{\log_{e} y}{1 + \log_{e} y}\right)$$

15. Differentiate the following with respect to x :

$$\sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$$

Solution:

$$y = \sin^{-1} \left[\frac{2^{x+1} \cdot 3^x}{1 + (36)^x} \right]$$
$$y = \sin^{-1} \left[\frac{2^x \cdot 2 \cdot 3^x}{1 + (36)^x} \right]$$
$$y = \sin^{-1} \left[\frac{2 \cdot (6)^x}{1 + (6)^{2x}} \right]$$
$$y = 2 \tan^{-1} (6)^x$$
$$\frac{dy}{dx} = \frac{2}{1 + (6)^{2x}} \cdot 6^x \log 6$$
$$\frac{dy}{dx} = \frac{2 \cdot 6^x \log 6}{1 + (36)^x}$$

16. Find the value of k, for which
$$f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & \text{if } -1 \le x < 0\\ \frac{x}{x-1}, & \text{if } 0 \le x < 1 \end{cases}$$

OR
OR
OR

If $x = a \cos^3 \theta$ and $y = a \sin^3 \theta$, then find the value of $\frac{d^2 y}{dx^2}$ at $\theta = \frac{\pi}{6}$.

Solution:

$$f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & \text{if } -1 \le x < 0\\ \frac{2x+1}{x-1}, & \text{if } 0 \le x < 1 \end{cases}$$

function f(x) is continuous at x = 0

OR

$$\therefore f(0) = \lim_{x \to 0} f(x)$$

$$\Rightarrow \frac{0+1}{0-1} = \lim_{x \to 0} \left(\frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} \right)$$

$$\Rightarrow -1 = \lim_{x \to 0} \left(\frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} \right) \left(\frac{\sqrt{1+kx} + \sqrt{1-kx}}{\sqrt{1+kx} + \sqrt{1-kx}} \right)$$

$$\Rightarrow -1 = \lim_{x \to 0} \frac{(1+2k) - (1-kx)}{x[\sqrt{1+kx} + \sqrt{1-kx}]}$$

$$\Rightarrow -1 = \lim_{x \to 0} \frac{2k}{\sqrt{1+kx} + \sqrt{1-kx}}$$

$$\Rightarrow -1 = \frac{2k}{2} \Rightarrow k = -1$$

$$x = a \cos^{3}\theta \qquad \text{and} \qquad y = a \sin^{3}\theta$$

$$\frac{dx}{d\theta} = -3a \cos^{2\theta} \sin\theta \text{ and} \qquad \frac{dy}{d\theta} = 3a \sin^{2}\theta \cos\theta$$

$$x = a \cos^{3}\theta \qquad \text{and} \qquad y = a \sin^{3}\theta$$

$$\frac{dx}{d\theta} = -3a \cos^{2\theta} \sin\theta \text{ and} \qquad \frac{dy}{d\theta} = 3a \sin^{2}\theta \cos\theta$$

$$\frac{dx}{d\theta} = -3a \cos^{2\theta} \sin\theta \text{ and} \qquad \frac{dy}{d\theta} = 3a \sin^{2}\theta \frac{\cos\theta}{d\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\Rightarrow \frac{dy}{dx} = -\tan\theta$$

$$\Rightarrow \frac{d^{2}y}{dx^{2}} = -\sec^{2}\theta \frac{1}{(-3a\cos^{2}\theta \sin\theta)}$$

$$\Rightarrow \frac{d^{2}y}{dx^{2}} = \frac{1}{3a}\sec^{4}\theta \csc\theta$$

$$\left(\frac{d^{2}y}{dx^{2}}\right)_{\theta\frac{\pi}{6}} = \frac{1}{3a}\left(\frac{2}{\sqrt{3}}\right)^{4} \cdot 2 = \frac{32}{27a}$$

17. Evaluate :

$$\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$$
OR

Evaluate :

$$\int \frac{x+2}{\sqrt{x^2+2x+3}} dx$$

Solution:

$$\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$$
$$= \int \frac{(2\cos^2 x - 1) - (2\cos^2 \alpha - 1)}{\cos x - \cos \alpha} dx$$
$$= \int \frac{2(\cos^2 x - 1)(2\cos^2 \alpha - 1)}{\cos x - \cos \alpha} dx$$
$$= 2\int (\cos x + \cos \alpha) dx$$
$$= 2(\sin x + x \cos \alpha) + c$$

OR

$$\begin{split} I &= \int \frac{x+2}{\sqrt{x^2+2x+3}} \, dx \\ I &= \int \left\{ \frac{(x+1)+1}{\sqrt{x^2+2x+3}} \right\} dx \\ I &= \int \left\{ \frac{(x+1)}{\sqrt{x^2+2x+3}} \right\} dx + \int \left\{ \frac{1}{\sqrt{x^2+2x+3}} \right\} dx \\ I &= I_1 + I_2 \\ In I_1 \text{let } x^2 + 3 = t^2 \\ \therefore \quad (2x+2) dx = 2t \ 2t \\ \Rightarrow (x+1) dx = t dt \\ \therefore \quad I_1 &= \int \frac{t.dt}{t} = t \\ I_1 &= \sqrt{x^2+2x+3} \\ \text{Now in } I_1 &= \int \frac{1}{\sqrt{x^2+2x+3}} \, dx = \int \frac{dx}{\sqrt{x^2+2x+3}} \\ I_2 &= \log[(x+1)^2 + \sqrt{(x+1)^2+2}] \\ \text{Now } I &= I_1 + I_2 \\ \Rightarrow I &= \sqrt{x^2+2x+3} + \log(x+1+\sqrt{x^2+2x+|3}) + c \end{split}$$

18. Evaluate :

 $\int \frac{\mathrm{d}x}{x(x^5+3)}$

Solution:

$$I = \int \frac{dx}{x(x^5 + 3)}$$

$$I = \int \frac{x^4 dx}{x(x^5 + 3)}$$
Let $x^5 = t \Rightarrow 5x^4 dx = dt$

$$I = \frac{1}{5} \int \frac{dt}{t(t + 3)}$$

$$I = \frac{1}{5} \cdot \frac{1}{3} \int \left(\frac{1}{t} - \frac{1}{t + 3}\right) dt$$

$$I = \frac{1}{15} \{\log t - \log(t + 3)\} + c$$

$$I = \frac{1}{15} \log \int \left(\frac{t}{t + 3}\right) + c$$

$$I = \frac{1}{15} \log \int \left(\frac{x^5}{x^5 + 3}\right) + c$$

19. Evaluate

$$\int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx$$

Solution:

$$I = \int_{0}^{2\pi} \frac{1}{1 + e^{\sin x}} dx \qquad \dots(1)$$
$$I = \int_{0}^{2\pi} \frac{1}{1 + e^{\sin(2\pi - x)}} dx$$
$$I = \int_{0}^{2\pi} \frac{1}{1 + e^{-\sin x}} dx$$
$$I = \int_{0}^{2\pi} \frac{e^{\sin x}}{e^{\sin x} + 1} dx$$
Adding (1) & (2) we get
$$\Rightarrow 21 = \int_{0}^{2\pi} \left(\frac{1 + e^{\sin x}}{1 + e^{\sin x}}\right) dx$$
$$\Rightarrow 21 = [x]_{0}^{2\pi}$$
$$\Rightarrow 21 = 2\pi$$
$$I = \pi$$

20. If $\vec{a} = \vec{i} - \vec{j} + 7k$ and $\vec{b} = 5\vec{i} + \vec{j} + \lambda \vec{k}$, then find the value of λ , so that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors. Solution:

$$\vec{a} = \vec{i} - \vec{j} + 7k$$

$$\vec{b} = 5\vec{i} + \vec{j} + \lambda\vec{k}$$

$$\vec{a} + \vec{b} = 6\vec{i} - 2\vec{j} + (7 + \lambda)\vec{k}$$

$$\vec{a} - \vec{b} = 4\vec{i} - 0\vec{j} + (7 - \lambda)\vec{k}$$

given $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular

$$\therefore \quad (\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 0$$

$$\{6\vec{i} - 2\vec{j} + (7 + \lambda)\vec{k}\} \times \{4\vec{i} - 0\vec{j} + (7 - \lambda)\vec{k}\} = 0$$

$$6(-4) + 0(-2) + (7 + \lambda)(7 - \lambda) = 0$$

$$-24 + 49 - \lambda^2 = 0$$

$$\lambda^2 = 25 \implies \boxed{\lambda = \pm 5}$$

21. Show that the lines

 $\vec{r} = 3\vec{i} + 2\vec{j} - 4\vec{k} + \lambda(\vec{i} + 2\vec{j} + 2\vec{k});$ $\vec{r} = 3\vec{i} + 2\vec{j} + \mu(3\vec{i} + 2\vec{j} + 6\vec{k})$

are intersecting. Hence find their point of intersection.

OR

Find the vector equation of the plane through the points (2, 1, -1) and (-1, 3, 4) and perpendicular to the plane x - 2y + 4z = 10.

Sol. If the given lines are intersecting then the shortest distance between the lines is zero and also they have same

common point
$$\vec{r} = 3\vec{i} + 2\vec{j} - 4\vec{k} + \lambda(\vec{i} + 2\vec{j} + 2\vec{k})$$

$$\Rightarrow \frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2} (=\lambda)(\text{Let})$$
Let P is $(\lambda + 3, 2\lambda + 2, 2\lambda - 4)$
Also, $\vec{r} = 5\vec{i} - 2\vec{j} + \mu(3\vec{i} + 2\vec{j} + 6\vec{k})$

$$\Rightarrow \frac{x-5}{3} = \frac{y+2}{2} = \frac{z-0}{6} (=\mu)(\text{Let})$$
Let Q is $(\mu + 5, 2\mu - 2, 6\mu)$
If lines are intersecting then P and Q will be same.
 $\lambda + 3 = 3\mu + 5$ (1)
 $2\lambda + 2 = 2\mu - 2$ (2)

 $2\lambda - 4 = 6\mu$ (3)

Solve(2) & (3)

$$\lambda + 1 = \mu - 1$$

 $2\lambda - 2 = 3\mu$
 $\frac{- + -}{3 = 2\mu - 1}$
 $4 = 2\mu$
 $\mu = -2$ (3)
 $2\lambda - 4 = 6(-2)$
 $2\lambda = -12 + 4$
 $2\lambda = -8$
 $\lambda = -4$
put $\mu & \lambda in (1)$
 $\lambda + 3 = 3\mu + 5$
 $-4 + 3 = 3(-2) + 5$
 $-1 = -1$
 \therefore from $\lambda = -4$ then P is (-1, -6, -12)
from $\mu = -2$ then Q is (-1, -6, -12)
as P and Q are same

 \therefore lines are intersecting lines and their point of intersection is (-1, -6, -12).

OR



 $\therefore \vec{n}_1 = \hat{i} - 2\hat{j} + 4k$

The required plane is perpendicular to given plane.

Therefore n r of required plane will be perpendicular to $\vec{n_1}$ and AB.

$$\therefore \vec{n} \parallel (\vec{n}_{1} \times \vec{AB}) \vec{n}_{1} = \hat{i} - 2\hat{j} + 4k \vec{AB} = -3\hat{i} + 2\hat{j} - 5k \therefore \vec{n}_{1} \times \vec{AB} = -18\hat{i} - 17\hat{j} - 4k \therefore required plane is \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n} \vec{r} \cdot (-18\hat{i} - 17\hat{j} - 4k) = (2\hat{i} + \hat{j} - k) \cdot (-18\hat{i} - 17\hat{j} - 4k) \vec{r} \cdot (-18\hat{i} - 17\hat{j} - 4k) = -36 - 17 + 4 \vec{r} \cdot (-18\hat{i} - 17\hat{j} - 4k) = -49 18x + 17y + 4z = 49$$

22. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively.

Assuming that the events, 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time. Write at least one advantage of coming to school in time. Sol. If P(A come in school time) = 3/7

P (B come in school time) = 5/7

P (A not come in school time) = 4/7

P (B not come in school time) = 2/7

P (only one of them coming school in time)

$$= P(A) \times P(B) + P(A).P(B)$$
$$= \frac{3}{7} \times \frac{2}{7} + \frac{5}{7} \times \frac{4}{7} = \frac{26}{49}$$

23. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

OR

Find the equations of tangents to the curve $3x^2 - y^2 = 8$, which pass through the point $\left(\frac{4}{3}, 0\right)$



OR

Area of reactangle A = 2a = cos θ . 2b sin θ A = 2ab. sin 2 θ \therefore A_{max} = 2ab Let a point (x_1, y_1) $3x^2 - y^2 = 8 \Rightarrow 6x - 2y.y' = 0$ $\Rightarrow y' = \frac{3x}{y}$ \therefore Tangent $y - y_1 = \frac{3x_1}{y_1}(x - x_1)$ It passing throgh $\left(\frac{4}{3}, 0\right)$ $-y_1 = \frac{3x_1}{y_1} \left(\frac{4}{3} - x_1\right)$ $\Rightarrow -y_1^2 = 4x_1 - 3x_1^2 \Rightarrow y_1^2 = 4x_1 - 3x_1^2$ $\Rightarrow 3x_1^2 - 8 = 3x_1^2 - 4x_1$ \therefore $x_1 = 2$ So $12 - y^2 = 8$ $\Rightarrow y^2 = 4 \Rightarrow y_1 = \pm 2$

24. Find the area of the region bounded by the parabola $y = x^2$ and y = |x|. Sol.



Required area = 2[area of ΔOAB – Area of curve OCBA]

$$A = 2 \left[\frac{1}{2} (1)(1) - \int_0^1 x^2 dx \right]$$
$$A = 2 \left[\frac{1}{2} - \frac{1}{3} \right] \implies A = 2 \left[\frac{1}{6} \right] = \frac{1}{3}$$

25. Find the particular solution of the differential equation $(tan-1y - x)dy = (1 + y^2)dx$, given that when x = 0, y=0

Sol.
$$(\tan^{-1}y - x)dy = (1 + y^2)dx$$

$$\Rightarrow \frac{dx}{dy} = \frac{\tan^{-1} y}{1 + y^2} - \frac{x}{1 + y^2}$$
$$\Rightarrow \frac{dx}{dy} + \frac{x}{1 + y^2} = \frac{\tan^{-1} y}{1 + y^2}$$
$$IF = e^{\int \frac{1}{1 + y^2} dy}$$
$$IF = e^{\tan^{-1} y}$$
$$x. IF = \int Q.IF \, dy + c$$
$$\Rightarrow x. e^{\tan^{-1} y} = \int \frac{\tan^{-1} y}{(1 + y^2)} e^{\tan^{-1} y} dy + c$$
$$Put \tan^{-1} y = t$$
$$\Rightarrow x. e^{\tan^{-1} y} = (t.e^t) - (e^t) + c$$
$$\Rightarrow x. e^{\tan^{-1} y} = \tan^{-1} y. e^{\tan^{-1} y} - e^{\tan^{-1} y} + c$$

26. Find the equation of the plane passing through the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j}) - 6 = 0$ and $\vec{r} \cdot (3\hat{j} - \hat{j} - 4k) = 0$, whose perpendicular distance from origin is unity.

OR

Find the vector equation of the line passing through the point (1, 2, 3) and parallel to the planes $\vec{r}.(3\hat{i}-\hat{j}-2k) = 5$ and $\vec{r}.(3\hat{i}+\hat{j}+k) = 6$ P_1 is $\vec{r}.(3\hat{i}+\hat{j})-6=0$

Sol.

 P_1 is x + 3y - 6 = 0 P_2 is $\vec{r} . (3\hat{j} - \hat{j} - 4k) = 0$ $P_2 is 3x - y - 4z = 0$

Equation of plane passing through intersection of P_1 and P_2 is $P_1 + \lambda P_2 = 0$

 $(x+3y-6)+\lambda(3x-y-4z)=0$ $(1+3\lambda)x + (3-\lambda)y + (-4\lambda)z + (-6) = 0$

Its distance from (0, 0, 0) is 1.

$$\begin{vmatrix} 0+0+0-6 \\ \hline \sqrt{(1+3\lambda)^2 + (3-\lambda)^2 + (-4\lambda)^2} \end{vmatrix}$$

36 = (1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2
36 = 1 + 9\lambda^2 + 6\lambda + 9 + \lambda^2 - 6\lambda + 16\lambda^2
36 = 26\lambda^2 + 10 \Rightarrow 26\lambda^2 = 26 \Rightarrow \lambda^2 = 1 \Rightarrow \boxed{\lambda = \pm 1}

Hence required plane is

For $\lambda = 1$, (x + 3y - 6) + 1 (3x - y - 4z) = 04x + 2y - 4z - 6 = 0

For
$$\lambda = -1$$
, $(x + 3y - 6) - 1(3x - y - 4z) = 0$
 $-2x + 4y + 4z - 6 = 0$
OR
P₁is $\vec{r} . (\hat{i} - \hat{j} + 2k) = 5$
 $\therefore \vec{n_1} = \hat{i} - \hat{j} + 2k$
P₂is $\vec{r} . (3\hat{i} - \hat{j} + 2k) = 6$
 $\vec{n_2} = 3\hat{i} - \hat{j} + k$

The line parallel to plane P_1 & P_2 will be perpendicular to \vec{n}_1 & \vec{n}_2

$$\therefore \quad b \parallel (n_1 \times n_2)$$

$$\vec{n}_1 = \hat{i} - \hat{j} + 2k$$

$$\vec{n}_2 = 3\hat{i} - \hat{j} + k$$

$$\vec{n}_1 \times \vec{n}_2 = -3\hat{i} + 5\hat{j} + 4k$$

$$\therefore \quad \vec{b} = -3\hat{i} + 5\hat{j} + 4k$$
Point is (1, 2, 3)
$$\therefore \quad \vec{a} = \hat{i} - \hat{j} + 3k$$

$$\therefore \quad \text{required line is } \vec{r} = \vec{a} + 4\hat{j} \vec{b}$$

$$\vec{r} = (\hat{i} + 2\hat{j} + 3k) + \lambda(-3\hat{i} + 5\hat{j} + 4|k)$$

27. In a hockey match, both teams A and B scored same number of goals up to the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, find the irrespective probabilities of winning the match and state whether the decision of the referee was fair or not.

Sol. P(6 get) = 1/6
P(6 not get) = P^(6get) = 5/6
P.(A win) = P(Aget6) + P(6get).P(6get)P(6get) + P(6get).P(6get).P(6get).P(6get).P(6get) ++∞
P.(A win) =
$$\frac{1}{6} \times \frac{5}{6} \times \frac{1}{6} + ... + ∞$$

 $= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + ... + ∞$
 $\boxed{\because S_{\infty} = \frac{a}{1-r}}$
 $= \frac{\left(\frac{1}{6}\right)}{\left(1-\frac{25}{36}\right)} = \frac{36}{11 \times 6} = \frac{6}{11}$

Similarly winning for B

P(B win) = 1 - P(A win) $=1-\frac{6}{11}=\frac{5}{11}$

28. A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods A and B. To produce one unit of A, 2 workers and 3 units of capital are required while 3workers and 1 unit of capital is required to produce one unit of B. If A and B are priced at j-100 and j-120per unit respectively, how should he use his resources to maximise the total revenue ? Form the above as an LPP and solve graphically. Do you agree with this view of the manufacturer that men and women workers are equally efficient and so should be paid at the same rate?

Sol. if z_{max} . = 100x + 120y				
		typeA	typeB	
work	ter	2	3	30
capit	1	3	1	17
Subject to,				
2x+3y≤30				
3x+y≤17				
x≥0				
Let c	objec	t of type A	A = x	
Obje	ct of	type B =	у	
pts	c00	rdinate	Z ^{max} =100z	x+ 120
0	(0,0))	Z=0	
А	$\left(\frac{17}{2}\right)$	(-,0)	$Z = \frac{1700}{2}$	
	$\sqrt{3}$	J	3	
E	(3.8	3)	Z=300+96	50=126

pts	coordinate	$Z^{max} = 100x + 120y$
0	(0,0)	Z=0
А	$\left(\frac{17}{3},0\right)$	$Z = \frac{1700}{3}$
E	(3.8)	Z=300+960=1260
C	(0.10)	Z=1200
	•	10.00

maximum revenue = 1260.

29. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some other (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation

and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these value, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards.

```
Sol. Given
x + y + z = 12 .....(1)
3(y+z) + 2x = 33 .....(2)
(x + z) = 2y \dots (3)
x + y + z = 12
2x + 3y + 3z = 33
x - 2y + z =
\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}
AX = B
A-1(AX) = A-1(B)
I \cdot X = A - 1 \cdot B
X = A - 1.B
X = \frac{(AdjA) \cdot B}{|A|}
|\mathbf{A}| = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}
|A| = 1(3+6) - 1(2-3) + 1(-4-3)
|A| = 9 + 1 - 7 = 3
|\mathbf{A}| \neq 0
(\text{Adj. A}) = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}
(\text{Adj. A}) \cdot \mathbf{B} = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}_{3\times 3} \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}_{3\times 3}
(Adj. A).B = \begin{bmatrix} 9\\12\\15 \end{bmatrix}
\therefore X = \frac{(Adj.A) \cdot B}{|A|}
```

$$X = \frac{1}{3} \begin{bmatrix} 9\\12\\15 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 3\\4\\5 \end{bmatrix} \Rightarrow \begin{bmatrix} x\\y\\z \end{bmatrix} = \begin{bmatrix} 3\\4\\5 \end{bmatrix}$$
$$x = 3, y = 4, z = 5.$$