3E1704

B. Tech. (Sem. III) (Main) Examination, January - 2013
Petroleum Engg.
3PE1 Mathematics - III (Common with 3EC1, EIC, BM, AI, CR)

Time : **3** Hours]

[Total Marks : **80**
[Min. Passing Marks : **24**

*Attempt any **five** questions, selecting **one question** from each unit. All questions carry equal marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.
Units of quantities used/calculated must be stated clearly.
Figures on the right hand side indicate full marks.*

Use of following supporting material is permitted during examination.

(Mentioned in form No. 205)

1. __Non-programmable Scientific Calculator__ 2. _____Nil_____

UNIT - I

1 (a) The error function is defined as

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} \, du$$

Find the Laplace transform of the error function and hence prove that

$$L\left[erfc \sqrt{t}\right] = \frac{1}{\sqrt{s+1}\left[\sqrt{s+1}+1\right]}$$

8

(b) Solve the ordinary linear differential equation

$$\frac{d^2y}{dt^2} + m^2 y = a \cos nt, t > 0$$

give that $y(0) = y_0$ and $y'(0) = y_1$. State the condition when the solution so obtained does not exist.

8

OR

1   **(a)**   Find the inverse Laplace transform of the following :

      (i) $\dfrac{1}{(8s-27)^{\frac{1}{3}}}$ (ii) $\dfrac{se^{-as}}{s^2-\omega^2}$

<div align="right">3+3</div>

   (b)   Give the statement of convolution theorem on inverse Laplace transform. Justify it with a suitable example.

<div align="right">2</div>

   (c)   The equation

$$Y(x,s)=c_1e^{xs}+c_2e^{-xs}-\frac{x}{s^4}$$ denotes the general solution

of a PDE obtained by Laplace transform method. If the solution satisfies the initial conditions $y(x,0)=y_t(x,0)=0$ and y remains finite for $x\to\infty, t\to\infty$ followed by $y(0,t)=0$, then find the ultimate solution of the PDE.

<div align="right">8</div>

<div align="center">

UNIT-II

</div>

2   **(a)**   Find the Fourier series for the function $f(x)$ defined by

$$f(x)=\begin{cases} \text{sgn}(x), & x\neq 0 \ (\text{lies between} -\pi \text{ and } \pi) \\ 0 & , x=0 \end{cases}$$

     where $\text{sgn}(x)$ denotes the signum function.   8

   (b)   Find the z-transform of the Dirac's delta function given by

$$\delta_n = \begin{bmatrix} 1 & , & n=0 \\ 0 & , & n\neq 0 \end{bmatrix}$$

<div align="right">4</div>

   (c)   Find the inverse z-transform of $\log\left(\dfrac{z}{z+1}\right)$ using power series method.

<div align="right">4</div>

<div align="center">

OR

</div>

2   **(a)**   Compute approximately, the Fourier coefficients a_0, a_1, a_2 and b_1, b_2 in the Fourier series expansion of function tabulated as follows :

$x:$	0	1	2	3	4	5
$y:$	9	18	24	28	26	20

Find the amplitude of the first harmonic.

<div align="right">10</div>

(b) Find the inverse z-transform of $\dfrac{3z+2z}{z^2-3z+2}, 1<|z|<2$.

6

UNIT-III

3 (a) Find the Fourier transform of the function $f(x)=|x|e^{-|x|}$.

6

(b) Use the method of Fourier transform to obtain the displacement $u(x,t)$ of an infinite string, given that the string is initially at rest and the initial displacement is $f(x), -\infty<x<\infty$. Show that the solution can be represented by $u(x,y)=\dfrac{1}{2}\left[f(x+ct)+f(x-ct)\right]$.

10

OR

3 (a) Use inverse cosine transform to find $f(x)$ in

$$\int_0^\infty f(x)\cos\lambda x\, dx = e^{-\lambda}$$

6

(b) Find the temperature distribution in a bar of length L, with its both ends and lateral surface insulated when the initial temperature in the bar is $f(x)$. Deduce the expression for temperature if $f(x)=x^2$ and $L=10$.

10

UNIT-IV

4 (a) Determine where the function $f(z)=\begin{cases}\dfrac{z^2+3iz-2}{z+i} &, z\neq -i \\ 5 &, z=i\end{cases}$ is continuous ? Can the function be refined to make it continuous at $z=-i$?

8

(b) Evaluate $\displaystyle\oint_C \dfrac{e^{3z}\,dz}{(z-\ln 2)^4}$ where C is the square with vertices at $\pm 1, \pm i$.

4

(c) Find the analytic function $w = u + iv$, if the real part is given by $u = e^x \cos y$. **4**

OR

4 (a) Prove that the cross-ratio of four points is invariant under a bilinear transformation. rtuonline.com **6**

(b) The ML-inequality of an analytic function $f(z)$ is

$$\left| \int_C f(z)\, dz \right| \le ML$$

where $|f(z)| \le M$ everywhere on C and L is the length of the curve. Use this inequality to find an upper bound for the absolute value of the integral $\int_c \left(e^z - \bar{z} \right) dz$ where c is the boundary of the triangle with vertices at $Z = 0, -4, 3i$. **10**

UNIT - V

5 (a) Expand $f(z) = \sin z$ in a Taylor's series about $z = \dfrac{\pi}{4}$. **6**

(b) Evaluate $I = \displaystyle\int_0^{\pi} \dfrac{a\, d\theta}{a^2 + \sin^2 \theta}, \ a > 0$. **10**

OR

5 (a) Show, by contour integration, that

$$\int_0^{\infty} \dfrac{\sin x}{x\left(a^2 + x^2\right)}\, dx = \dfrac{\pi}{2a^2}\left(1 - e^{-a}\right), \ a > 0.$$ **8**

rtuonline.com

(b) Find the Laurent series of $f(z) = \dfrac{7z^2 + 9z - 18}{z^3 - 9z}$ in the regions :

(i) $0 < |z + 3| < 3$

(ii) $3 < |z + 3| < 6$

(iii) $|z + 3| > 6$. **8**