Exercise – 16D

1. Points A(-1, y) and B(5,7) lie on the circle with centre O(2, -3y).Find the value of y.

Sol:

The given points are $A(-1, y)$, $8(5,7)$ and $O(2,-3y)$.

Here, AO and BO are the radii of the circle. So

$AO = BO \Rightarrow AO^2 = BO^2$

$\Rightarrow (2+1)^2 + (-3y-y)^2 = (2-5)^2 + (-3y-7)^2$

$\Rightarrow 9 + (4y)^2 = (-3)^2 + (3y+7)^2$

$\Rightarrow 9 + 16y^2 = 9 + 9y^2 + 49 + 42y$

$\Rightarrow 7y^2 - 42y - 49 = 0$

$\Rightarrow y^2 - 6y - 7 = 0$

$\Rightarrow y^2 - 7y + y - 7 = 0$

$\Rightarrow y(y-7) + 1(y-7) = 0$

$\Rightarrow (y-7)(y+1) = 0$

$\Rightarrow y = -1 \ or \ y = 7$

Hence, $y = 7 \ or \ y = -1$.

2. If the point A(0,2) is equidistant from the points B(3,p) and C(p, 5), find p.

Sol:

The given ports are $A(0,2), B(3, p)$ and $C(p,5)$.

$AB = AC \Rightarrow AB^2 = AC^2$

$\Rightarrow (3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$

$\Rightarrow 9 + p^2 - 4p + 4 = p^2 + 9$

$\Rightarrow 4p = 4 \Rightarrow p = 1$

Hence, $p = 1$.

3. ABCD is a rectangle whose three vertices are A(4,0), C(4,3) and D(0,3). Find the length of one its diagonal.

Sol:

The given vertices are *B*(4, 0), *C*(4, 3) and *D*(0, 3) Here, *BD* one of the diagonals So

$BD = \sqrt{(4-0)^2 + (0-3)^2}$

$= \sqrt{(4)^2 + (-3)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$

Hence, the length of the diagonal is 5 units.

4. If the point P(k-1, 2) is equidistant from the points A(3,k) and B(k,5), find the value of k.

Sol:

The given points are $P(k-1, 2), A(3, k)$ *and* $B(k, 5)$.

$\because AP = BP$

$\therefore AP^2 = BP^2$

$\Rightarrow (k-1-3)^2 + (2-k)^2 = (k-1-k)^2 + (2-5)^2$

$\Rightarrow (k-4)^2 + (2-k)^2 = (-1)^2 + (-3)^2$

$\Rightarrow k^2 - 8y + 16 + 4 + k^2 - 4k = 1 + 9$

$\Rightarrow k^2 - 6y + 5 = 0$

$\Rightarrow (k-1)(k-5) = 0$

$\Rightarrow k = 1 \text{ or } k = 5$

Hence, $k = 1 \text{ or } k = 5$

5. Find the ratio in which the point P(x,2) divides the join of A(12, 5) and B(4, -3).

Sol:

Let *k* be the ratio in which the point $P(x, 2)$ divides the line joining the points

$A(x_1 = 12,\ y_1 = 5)$ *and* $B(x_2 = 4, y_2 = -3)$. Then

$x=\frac{k\times 4+12}{k+1}$ *and* $2=\frac{k\times(-3)+5}{k+1}$

Now,

$2=\frac{k\times(-3)+5}{k+1}\Rightarrow 2k+2=-3k+5\Rightarrow k=\frac{3}{5}$

Hence, the required ratio is $3:5$.

6. Prove that the diagonals of a rectangle ABCD with vertices A(2,-1), B(5,-1) C(5,6) and D(2,6) are equal and bisect each other.

Sol:

The vertices of the rectangle *ABCD* are $A(2,-1), B(5,-1), C(5,6)$ and $D(2,6)$. Now

Coordinates of midpoint of $AC=\left(\frac{2+5}{2},\frac{-1+6}{2}\right)=\left(\frac{7}{2},\frac{5}{2}\right)$

Coordinates of midpoint of $BD=\left(\frac{5+2}{2},\frac{-1+6}{2}\right)=\left(\frac{7}{2},\frac{5}{2}\right)$

Since, the midpoints of *AC* and *BD* coincide, therefore the diagonals of rectangle *ABCD* bisect each other

7. Find the lengths of the medians AD and BE of ΔABC whose vertices are A(7,-3), B(5,3) and C(3,-1)

Sol:

The given vertices are $A(7,-3), B(5,3)$ *and* $C(3,-1)$.

Since *D* and *E* are the midpoints of *BC* and *AC* respectively. therefore

Coordinates of $D=\left(\frac{5+3}{2},\frac{3-1}{2}\right)=(4,1)$

Coordinates of $E=\left(\frac{7+3}{2},\frac{-3-1}{2}\right)=(5,-2)$

Now

$AD=\sqrt{(7-4)^2+(-3-1)^2}=\sqrt{9+16}=5$

$BE=\sqrt{(5-5)^2+(3+2)^2}=\sqrt{0+25}=5$

Hence, *AD* = *BE* = 5 units.

8. If the point C(k,4) divides the join of A(2,6) and B(5,1) in the ratio 2:3 then find the value of k.

Sol:

Here, the point $C(k,4)$ divides the join of $A(2,6)$ and $B(5,1)$ in ratio $2:3$. So

$k = \frac{2 \times 5 + 3 \times 2}{2 + 3}$

$= \frac{10 + 6}{5}$

$= \frac{16}{5}$

Hence, $k = \frac{16}{5}$.

9. Find the point on x-axis which is equidistant from points A(-1,0) and B(5,0)

Sol:

Let $P(x, 0)$ be the point on $x - axis$. Then

$AP = BP \Rightarrow AP^2 = BP^2$

$\Rightarrow (x+1)^2 + (0-0)^2 = (x-5)^2 + (0-0)^2$

$\Rightarrow x^2 + 2x + 1 = x^2 - 10x + 25$

$\Rightarrow 12x = 24 \Rightarrow x = 2$

Hence, $x = 2$

10. Find the distance between the points $A\left(\frac{-8}{5}, 2\right)$ and $B\left(\frac{2}{5}, 2\right)$

Sol:

The given points are $A\left(\frac{-8}{5}, 2\right)$ and $B\left(\frac{2}{5}, 2\right)$

Then, $\left(x_1 = \frac{-8}{5}, y_1 = 2\right)$ *and* $\left(x_2 = \frac{2}{5}, y_2 = 2\right)$

Therefore,

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left\{\frac{2}{5} - \left(\frac{-8}{5}\right)\right\}^2 + (2-2)^2}$

$= \sqrt{(2)^2 + (0)^2}$

$= \sqrt{4 + 0}$

$= \sqrt{4}$

$= 2$ *units*.

11. Find the value of a, so that the point $(3,a)$ lies on the line represented by $2x-3y=5$.

Sol:

The points $(3,a)$ lies on the line $2x-3y=5$.

If point $(3,a)$ lies on the line $2x-3y=5$, then $2x-3y=5$

$\Rightarrow (2\times 3)-(3\times a)=5$

$\Rightarrow 6-3a=5$

$\Rightarrow 3a=1$

$\Rightarrow a=\frac{1}{3}$

Hence, the value of a is $\frac{1}{3}$.

12. If the points $A(4,3)$ *and* $B(x,5)$ lie on the circle with center $O(2,3)$, find the value of x.

Sol:

The given points $A(4, 3)$ and $B(x, 5)$ lie on the circle with center $O(2, 3)$.

Then, $OA = OB$

$\Rightarrow \sqrt{(x-2)^2+(5-3)^2}=\sqrt{(4-2)^2+(3-3)^2}$

$\Rightarrow (x-2)^2+2^2=2^2+0^2$

$\Rightarrow (x-2)^2=(2^2-2^2)$

$\Rightarrow (x-2)^2=0$

$\Rightarrow x-2=0$

$\Rightarrow x=2$

Hence, the value of $x=2$

13. If $P(x,y)$ is equidistant from the points $A(7,1)$ *and* $B(3,5)$, find the relation between x and y.

Sol:

Let the point $P(x, y)$ be equidistant from the points $A(7, 1)$ and $B(3, 5)$

Then,

$PA=PB$

$\Rightarrow PA^2=PB^2$

$\Rightarrow (x-7)^2+(y-1)^2=(x-3)^2+(y-5)^2$

$\Rightarrow x^2+y^2-14x-2y+50=x^2+y^2-6x-10y+34$

$\Rightarrow 8x-8y=16$

$\Rightarrow x-y=2$

14. If the centroid of ΔABC having vertices $A(a,\ b),\ B(b,\ c)$ and $C(c,\ a)$ is the origin, then find the value of $(a+b+c)$.

Sol:

The given points are $A(a,b), B(b,c)$ *and* $C(c,a)$

Here,

$(x_1 = a, y_1 = b), (x_2 = b, y_2 = c)$ *and* $(x_3 = c, y_3 = a)$

Let the centroid be (x, y).

Then,

$$x = \frac{1}{3}(x_1 + x_2 + x_3)$$
$$= \frac{1}{3}(a+b+c)$$
$$= \frac{a+b+c}{3}$$
$$y = \frac{1}{3}(y_1 + y_2 + y_3)$$
$$= \frac{1}{3}(b+c+a)$$
$$= \frac{a+b+c}{3}$$

But it is given that the centroid of the triangle is the origin.

Then, we have

$$\frac{a+b+c}{3} = 0$$
$$\Rightarrow a+b+c = 0$$

15. Find the centroid of ΔABC whose vertices are $A(2,2), B(-4,-4)$ *and* $C(5,-8)$.

Sol:

The given points are $A(2,2), B(-4,-4)$ *and* $C(5,-8)$.

Here, $(x_1 = 2, y_1 = 2), (x_2 = -4, y_2 = -4)$ *and* $(x_3 = 5, y_3 = -8)$

Let $G(x, y)$ be the centroid of ΔABC Then,

$$x = \frac{1}{3}(x_1 + x_2 + x_3)$$
$$= \frac{1}{3}(2-4+5)$$
$$= 1$$

$y = \frac{1}{3}(y_1 + y_2 + y_3)$

$= \frac{1}{3}(2 - 4 - 8)$

$= \frac{-10}{3}$

Hence, the centroid of ΔABC is $G\left(1, \frac{-10}{3}\right)$.

16. In what ratio does the point $C(4,5)$ divides the join of $A(2,3)$ *and* $B(7,8)$?

Sol:

Let the required ratio be *k : 1*

Then, by section formula, the coordinates of C are

$C\left(\frac{7k+2}{k+1}, \frac{8k+3}{k+1}\right)$

Therefore,

$\frac{7k+2}{k+1} = 4 \text{ and } \frac{8k+3}{k+1} = 5 \quad [\because C(4,5) \text{ is given}]$

$\Rightarrow 7k + 2 = 4k + 4$ and $8k + 3 = 5k + 5 \Rightarrow 3k = 2$

$\Rightarrow k = \frac{2}{3}$ in each case

So, the required ratio is $\frac{2}{3}:1$, which is same as $2:3$.

17. If the points $A(2,3)$, $B(4,k)$ *and* $C(6,-3)$ are collinear, find the value of *k*.

Sol:

The given points are $A(2,3)$, $B(4,k)$ *and* $C(6,-3)$

Here, $(x_1 = 2, y_1 = 3), (x_2 = 4, y_2 = k)$ and $(x_3 = 6, y_3 = -3)$

It is given that the points *A, B* and *C* are collinear. Then,

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$\Rightarrow 2(k+3) + 4(-3-3) + 6(3-k) = 0$

$\Rightarrow 2k + 6 - 24 + 18 - 6k = 0$

$\Rightarrow -4k = 0$

$\Rightarrow k = 0$