

Exercise – 16A

1. Find the distance between the points

- (i) $A(9,3)$ and $B(15,11)$
- (ii) $A(7,-4)$ and $B(-5,1)$
- (iii) $A(-6,-4)$ and $B(9,-12)$
- (iv) $A(1,-3)$ and $B(4,-6)$
- (v) $P(a+b, a-b)$ and $Q(a-b, a+b)$
- (vi) $P(a \sin \alpha, a \cos \alpha)$ and $Q(a \cos \alpha, -a \sin \alpha)$

Sol:

- (i) $A(9,3)$ and $B(15,11)$

The given points are $A(9,3)$ and $B(15,11)$.

Then $(x_1 = 9, y_1 = 3)$ and $(x_2 = 15, y_2 = 11)$

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(15 - 9)^2 + (11 - 3)^2} \\ &= \sqrt{(15 - 9)^2 + (11 - 3)^2} \\ &= \sqrt{(6)^2 + (8)^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} \\ &= 10 \text{ units} \end{aligned}$$

- (ii) $A(7,-4)$ and $B(-5,1)$

The given points are $A(7,-4)$ and $B(-5,1)$.

Then, $(x_1 = 7, y_1 = -4)$ and $(x_2 = -5, y_2 = 1)$

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-5 - 7)^2 + \{1 - (-4)\}^2} \\ &= \sqrt{(-5 - 7)^2 + (1 + 4)^2} \\ &= \sqrt{(-12)^2 + (5)^2} \\ &= \sqrt{144 + 25} \\ &= \sqrt{169} \end{aligned}$$

$$= 13 \text{ units}$$

$$= \sqrt{144 + 25}$$

$$= \sqrt{169}$$

$$= 13 \text{ units}$$

- (iii) $A(-6, -4)$ and $B(9, -12)$

The given points are $A(-6, -4)$ and $B(9, -12)$

Then $(x_1 = -6, y_1 = -4)$ and $(x_2 = 9, y_2 = -12)$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(9 - (-6))^2 + \{-12 - (-4)\}^2}$$

$$= \sqrt{(9 + 6)^2 + (-12 + 4)^2}$$

$$= \sqrt{(15)^2 + (-8)^2}$$

$$= \sqrt{225 + 64}$$

$$= \sqrt{289}$$

$$= 17 \text{ units}$$

- (iv) $A(1, -3)$ and $B(4, -6)$

The given points are $A(1, -3)$ and $B(4, -6)$

Then $(x_1 = 1, y_1 = -3)$ and $(x_2 = 4, y_2 = -6)$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4 - 1)^2 + \{-6 - (-3)\}^2}$$

$$= \sqrt{(4 - 1)^2 + (-6 + 3)^2}$$

$$= \sqrt{(3)^2 + (-3)^2}$$

$$= \sqrt{9 + 9}$$

$$= \sqrt{18}$$

$$= \sqrt{9 \times 2}$$

$$= 3\sqrt{2} \text{ units}$$

- (v) $P(a+b, a-b)$ and $Q(a-b, a+b)$

The given points are $P(a+b, a-b)$ and $Q(a-b, a+b)$

Then $(x_1 = a+b, y_1 = a-b)$ and $(x_2 = a-b, y_2 = a+b)$

$$\begin{aligned}
 PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{\{(a-b) - (a+b)\}^2 + \{(a+b) - (a-b)\}^2} \\
 &= \sqrt{(a-b-a-b)^2 + (a+b-a+b)^2} \\
 &= \sqrt{(-2b)^2 + (2b)^2} \\
 &= \sqrt{4b^2 + 4b^2} \\
 &= \sqrt{8b^2} \\
 &= \sqrt{4 \times 2b^2} \\
 &= 2\sqrt{2}b \text{ units}
 \end{aligned}$$

(vi) $P(a \sin \alpha, a \cos \alpha)$ and $Q(a \cos \alpha, -a \sin \alpha)$

The given points are $P(a \sin \alpha, a \cos \alpha)$ and $Q(a \cos \alpha, -a \sin \alpha)$

Then $(x_1 = a \sin \alpha, y_1 = a \cos \alpha)$ and $(x_2 = a \cos \alpha, y_2 = -a \sin \alpha)$

$$\begin{aligned}
 PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(a \cos \alpha - a \sin \alpha)^2 + (-a \sin \alpha - a \cos \alpha)^2} \\
 &= \sqrt{(a^2 \cos^2 \alpha + a^2 \sin^2 \alpha - 2a^2 \cos \alpha \times \sin \alpha) + (a^2 \sin^2 \alpha + a^2 \cos^2 \alpha + 2a^2 \cos \alpha \times \sin \alpha)} \\
 &= \sqrt{2a^2 \cos^2 \alpha + 2a^2 \sin^2 \alpha} \\
 &= \sqrt{2a^2 (\cos^2 \alpha + \sin^2 \alpha)} \\
 &= \sqrt{2a^2 (1)} \quad (\text{From the identity } \cos^2 \alpha + \sin^2 \alpha = 1) \\
 &= \sqrt{2a^2} \\
 &= \sqrt{2}a \text{ units}
 \end{aligned}$$

2. Find the distance of each of the following points from the origin:

(i) $A(5, -12)$ (ii) $B(-5, 5)$ (iii) $C(-4, -6)$

Sol:

(i) $A(5, -12)$

Let $O(0, 0)$ be the origin

$$OA = \sqrt{(5-0)^2 + (-12-0)^2}$$

$$= \sqrt{(5)^2 + (-12)^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13 \text{ units}$$

(ii) $B(-5, 5)$

Let $O(0, 0)$ be the origin.

$$OB = \sqrt{(-5 - 0)^2 + (5 - 0)^2}$$

$$= \sqrt{(-5)^2 + (5)^2}$$

$$= \sqrt{25 + 25}$$

$$= \sqrt{50}$$

$$= \sqrt{25 \times 2}$$

$$= 5\sqrt{2} \text{ units}$$

(iii) $C(-4, -6)$

Let $O(0, 0)$ be the origin

$$OC = \sqrt{(-4 - 0)^2 + (-6 - 0)^2}$$

$$= \sqrt{(-4)^2 + (-6)^2}$$

$$= \sqrt{16 + 36}$$

$$= \sqrt{52}$$

$$= \sqrt{4 \times 13}$$

$$= 2\sqrt{13} \text{ units}$$

3. Find all possible values of x for which the distance between the points $A(x, -1)$ and $B(5, 3)$ is 5 units.

Sol:

Given $AB = 5 \text{ units}$

Therefore, $(AB)^2 = 25 \text{ units}$

$$\Rightarrow (5 - a)^2 + \{3 - (-1)\}^2 = 25$$

$$\Rightarrow (5 - a)^2 + (3 + 1)^2 = 25$$

$$\Rightarrow (5 - a)^2 + (4)^2 = 25$$

$$\Rightarrow (5-a)^2 + 16 = 25$$

$$\Rightarrow (5-a)^2 = 25 - 16$$

$$\Rightarrow (5-a)^2 = 9$$

$$\Rightarrow (5-a) = \pm\sqrt{9}$$

$$\Rightarrow 5-a = \pm 3$$

$$\Rightarrow 5-a = 3 \text{ or } 5-a = -3$$

$$\Rightarrow a = 2 \text{ or } 8$$

Therefore, $a = 2$ or 8 .

4. Find all possible values of y for which distance between the points $A(2, -3)$ and $B(10, y)$ is 10 units.

Sol:

The given points are $A(2, -3)$ and $B(10, y)$

$$\therefore AB = \sqrt{(2-10)^2 + (-3-y)^2}$$

$$= \sqrt{(-8)^2 + (-3-y)^2}$$

$$= \sqrt{64 + 9 + y^2 + 6y}$$

$$\because AB = 10$$

$$\therefore \sqrt{64 + 9 + y^2 + 6y} = 10$$

$$\Rightarrow 73 + y^2 + 6y = 100 \quad (\text{Squaring both sides})$$

$$\Rightarrow y^2 + 6y - 27 = 0$$

$$\Rightarrow y^2 + 9y - 3y - 27 = 0$$

$$\Rightarrow y(y+9) - 3(y+9) = 0$$

$$\Rightarrow (y+9)(y-3) = 0$$

$$\Rightarrow y+9 = 0 \text{ or } y-3 = 0$$

$$\Rightarrow y = -9 \text{ or } y = 3$$

Hence, the possible values of y are -9 and 3 .

5. Find value of x for which the distance between the points $P(x, 4)$ and $Q(9, 10)$ is 10 units.

Sol:

The given points are $P(x, 4)$ and $Q(9, 10)$.

$$\therefore PQ = \sqrt{(x-9)^2 + (4-10)^2}$$

$$\begin{aligned} &= \sqrt{(x-9)^2 + (-6)^2} \\ &= \sqrt{x^2 - 18x + 81 + 36} \\ &= \sqrt{x^2 - 18x + 117} \\ \therefore PQ &= 10 \\ \therefore \sqrt{x^2 - 18x + 117} &= 10 \\ \Rightarrow x^2 - 18x + 117 &= 100 \quad (\text{Squaring both sides}) \\ \Rightarrow x^2 - 18x + 17 &= 0 \\ \Rightarrow x^2 - 17x - x + 17 &= 0 \\ \Rightarrow x(x-17) - 1(x-17) &= 0 \\ \Rightarrow (x-17)(x-1) &= 0 \\ \Rightarrow x-17 = 0 \text{ or } x-1 &= 0 \\ \Rightarrow x = 17 \text{ or } x = 1 \end{aligned}$$

Hence, the values of x are 1 and 17.

6. If the point $A(x, 2)$ is equidistant from the points $B(8, -2)$ and $C(2, -2)$, find the value of x . Also, find the value of x . Also, find the length of AB .

Sol:

As per the question

$$AB = AC$$

$$\Rightarrow \sqrt{(x-8)^2 + (2+2)^2} = \sqrt{(x-2)^2 + (2+2)^2}$$

Squaring both sides, we get

$$(x-8)^2 + 4^2 = (x-2)^2 + 4^2$$

$$\Rightarrow x^2 - 16x + 64 + 16 = x^2 + 4 - 4x + 16$$

$$\Rightarrow 16x - 4x = 64 - 4$$

$$\Rightarrow x = \frac{60}{12} = 5$$

Now,

$$AB = \sqrt{(x-8)^2 + (2+2)^2}$$

$$= \sqrt{(5-8)^2 + (2+2)^2} \quad (\because x = 5)$$

$$= \sqrt{(-3)^2 + (4)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5$$

Hence, $x = 5$ and $AB = 5$ units.

7. If the point $A(0, 2)$ is equidistant from the points $B(3, p)$ and $C(p, 5)$ find the value of p . Also, find the length of AB .

Sol:

As per the question

$$AB = AC$$

$$\Rightarrow \sqrt{(0-3)^2 + (2-p)^2} = \sqrt{(0-p)^2 + (2-5)^2}$$

$$\Rightarrow \sqrt{(-3)^2 + (2-p)^2} = \sqrt{(-p)^2 + (-3)^2}$$

Squaring both sides, we get

$$(-3)^2 + (2-p)^2 = (-p)^2 + (-3)^2$$

$$\Rightarrow 9 + 4 + p^2 - 4p = p^2 + 9$$

$$\Rightarrow 4p = 4$$

$$\Rightarrow p = 1$$

Now,

$$AB = \sqrt{(0-3)^2 + (2-p)^2}$$

$$= \sqrt{(-3)^2 + (2-1)^2} \quad (\because p=1)$$

$$= \sqrt{9+1}$$

$$= \sqrt{10} \text{ units}$$

Hence, $p = 1$ and $AB = \sqrt{10}$ units

8. Find the point on the x -axis which is equidistant from the points $(2, -5)$ and $(-2, 9)$.

Sol:

Let $(x, 0)$ be the point on the x axis. Then as per the question, we have

$$\sqrt{(x-2)^2 + (0+5)^2} = \sqrt{(x+2)^2 + (0-9)^2}$$

$$\Rightarrow \sqrt{(x-2)^2 + (5)^2} = \sqrt{(x+2)^2 + (-9)^2}$$

$$\Rightarrow (x-2)^2 + (5)^2 = (x+2)^2 + (-9)^2 \quad (\text{Squaring both sides})$$

$$\Rightarrow x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$$

$$\Rightarrow 8x = 25 - 81$$

$$\Rightarrow x = -\frac{56}{8} = -7$$

Hence, the point on the x -axis is $(-7, 0)$.

9. Find the points on the x-axis, each of which is at a distance of 10 units from the point A(11, -8).

Sol:

Let $P(x, 0)$ be the point on the x-axis. Then as per the question we have

$$AP = 10$$

$$\Rightarrow \sqrt{(x-11)^2 + (0+8)^2} = 10$$

$$\Rightarrow (x-11)^2 + 8^2 = 100 \quad (\text{Squaring both sides})$$

$$\Rightarrow (x-11)^2 = 100 - 64 = 36$$

$$\Rightarrow x - 11 = \pm 6$$

$$\Rightarrow x = 11 \pm 6$$

$$\Rightarrow x = 11 - 6, 11 + 6$$

$$\Rightarrow x = 5, 17$$

Hence, the points on the x-axis are $(5, 0)$ and $(17, 0)$.

10. Find the points on the y-axis which is equidistant from the points $A(6, 5)$ and $B(-4, 3)$

Sol:

Let $P(0, y)$ be a point on the y-axis. Then as per the question, we have

$$AP = BP$$

$$\Rightarrow \sqrt{(0-6)^2 + (y-5)^2} = \sqrt{(0+4)^2 + (y-3)^2}$$

$$\Rightarrow \sqrt{(6)^2 + (y-5)^2} = \sqrt{(4)^2 + (y-3)^2}$$

$$\Rightarrow (6)^2 + (y-5)^2 = (4)^2 + (y-3)^2 \quad (\text{Squaring both sides})$$

$$\Rightarrow 36 + y^2 - 10y + 25 = 16 + y^2 - 6y + 9$$

$$\Rightarrow 4y = 36$$

$$\Rightarrow y = 9$$

Hence, the point on the y-axis is $(0, 9)$.

11. If the points $P(x, y)$ is point equidistant from the points $A(5, 1)$ and $B(-1, 5)$, Prove that

$3x = 2y$. **Sol:**

As per the question, we have

$$AP = BP$$

$$\Rightarrow \sqrt{(x-5)^2 + (y-1)^2} = \sqrt{(x+1)^2 + (y-5)^2}$$

$$\Rightarrow (x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2 \quad (\text{Squaring both sides})$$

$$\Rightarrow x^2 - 10x + 25 + y^2 - 2y + 1 = x^2 + 2x + 1 + y^2 - 10y + 25$$

$$\Rightarrow -10x - 2y = 2x - 10y$$

$$\Rightarrow 8y = 12x$$

$$\Rightarrow 3x = 2y$$

$$\text{Hence, } 3x = 2y$$

12. If $p(x, y)$ is point equidistant from the points $A(6, -1)$ and $B(2, 3)$, show that $x - y = 3$

Sol:

The given points are $A(6, -1)$ and $B(2, 3)$. The point $P(x, y)$ equidistant from the points A and B So, $PA = PB$

$$\text{Also, } (PA)^2 = (PB)^2$$

$$\Rightarrow (6-x)^2 + (-1-y)^2 = (2-x)^2 + (3-y)^2$$

$$\Rightarrow x^2 - 12x + 36 + y^2 + 2y + 1 = x^2 - 4x + 4 + y^2 - 6y + 9$$

$$\Rightarrow x^2 + y^2 - 12x + 2y + 37 = x^2 + y^2 - 4x - 6y + 13$$

$$\Rightarrow x^2 + y^2 - 12x + 2y - x^2 - y^2 + 4x + 6y = 13 - 37$$

$$\Rightarrow -8x + 8y = -24$$

$$\Rightarrow -8(x - y) = -24$$

$$\Rightarrow x - y = \frac{-24}{-8}$$

$$\Rightarrow x - y = 3$$

Hence proved.

13. Find the co-ordinates of the point equidistant from three given points $A(5, 3)$, $B(5, -5)$ and $C(1, -5)$

Sol:

Let the required point be $P(x, y)$. Then $AP = BP = CP$

$$\text{That is, } (AP)^2 = (BP)^2 = (CP)^2$$

$$\text{This means } (AP)^2 = (BP)^2$$

$$\Rightarrow (x-5)^2 + (y-3)^2 = (x-5)^2 + (y+5)^2$$

$$\Rightarrow x^2 - 10x + 25 + y^2 - 6y + 9 = x^2 - 10x + 25 + y^2 + 10y + 25$$

$$\Rightarrow x^2 - 10x + y^2 - 6y + 34 = x^2 - 10x + y^2 + 10y + 50$$

$$\Rightarrow x^2 - 10x + y^2 - 6y - x^2 + 10x - y^2 - 10y = 50 - 34$$

$$\Rightarrow -16y = 16$$

$$\Rightarrow y = -\frac{16}{16} = -1$$

$$\text{And } (BP)^2 = (CP)^2$$

$$\Rightarrow (x-5)^2 + (y+5)^2 = (x-1)^2 + (y+5)^2$$

$$\Rightarrow x^2 - 10x + 25 + y^2 + 10y + 25 = x^2 - 2x + 1 + y^2 + 10y + 25$$

$$\Rightarrow x^2 - 10x + y^2 + 10y + 50 = x^2 - 2x + y^2 + 10y + 26$$

$$\Rightarrow x^2 - 10x + y^2 + 10y - x^2 + 2x - y^2 - 10y = 26 - 50$$

$$\Rightarrow -8x = -24$$

$$\Rightarrow x = \frac{-24}{-8} = 3$$

Hence, the required point is $(3, -1)$.

14. If the points $A(4,3)$ and $B(x,5)$ lie on a circle with the centre $O(2,3)$. Find the value of x .

Sol:

Given, the points $A(4,3)$ and $B(x,5)$ lie on a circle with center $O(2,3)$.

Then $OA = OB$

$$\text{Also } (OA)^2 = (OB)^2$$

$$\Rightarrow (4-2)^2 + (3-3)^2 = (x-2)^2 + (5-3)^2$$

$$\Rightarrow (2)^2 + (0)^2 = (x-2)^2 + (2)^2$$

$$\Rightarrow 4 = (x-2)^2 + 4$$

$$\Rightarrow (x-2)^2 = 0$$

$$\Rightarrow x-2 = 0$$

$$\Rightarrow x = 2$$

Therefore, $x = 2$

15. If the point $C(-2,3)$ is equidistant from the points $A(3,-1)$ and $B(x,8)$, find the value of x .

Also, find the distance between BC

Sol:

As per the question, we have

$$AC = BC$$

$$\Rightarrow \sqrt{(-2-3)^2 + (3+1)^2} = \sqrt{(-2-x)^2 + (3-8)^2}$$

$$\Rightarrow \sqrt{(5)^2 + (4)^2} = \sqrt{(x+2)^2 + (-5)^2}$$

$$\Rightarrow 25 + 16 = (x+2)^2 + 25 \quad (\text{Squaring both sides})$$

$$\Rightarrow 25 + 16 = (x+2)^2 + 25$$

$$\Rightarrow (x+2)^2 = 16$$

$$\Rightarrow x+2 = \pm 4$$

$$\Rightarrow x = -2 \pm 4 = -2-4, -2+4 = -6, 2$$

Now

$$BC = \sqrt{(-2-x)^2 + (3-8)^2}$$

$$= \sqrt{(-2-2)^2 + (-5)^2}$$

$$= \sqrt{16+25} = \sqrt{41} \text{ units}$$

Hence, $x = 2$ or -6 and $BC = \sqrt{41}$ units

16. If the point $P(2,2)$ is equidistant from the points $A(-2,k)$ and $B(-2k,-3)$, find k . Also, find the length of AP .

Sol:

As per the question, we have

$$AP = BP$$

$$\Rightarrow \sqrt{(2+2)^2 + (2+k)^2} = \sqrt{(2+2k)^2 + (2+3)^2}$$

$$\Rightarrow \sqrt{(4)^2 + (2-k)^2} = \sqrt{(2+2k)^2 + (5)^2}$$

$$\Rightarrow 16+4+k^2-4k = 4+4k^2+8k+25 \quad (\text{Squaring both sides})$$

$$\Rightarrow k^2+4k+3=0$$

$$\Rightarrow (k+1)(k+3)=0$$

$$\Rightarrow k = -3, -1$$

Now for $k = -1$

$$AP = \sqrt{(2+2)^2 + (2-k)^2}$$

$$= \sqrt{(4)^2 + (2+1)^2}$$

$$= \sqrt{16+9} = 5 \text{ units}$$

For $k = -3$

$$AP = \sqrt{(2+2)^2 + (2-k)^2}$$

$$= \sqrt{(4)^2 + (2+3)^2}$$

$$= \sqrt{16+25} = \sqrt{41} \text{ units}$$

Hence, $k = -1, -3$; $AP = 5$ units for $k = -1$ and $AP = \sqrt{41}$ units for $k = -3$.

17. If the point (x, y) is equidistant from the points $(a+b, b-a)$ and $(a-b, a+b)$, prove that $bx = ay$.

Sol:

As per the question, we have

$$\begin{aligned}\sqrt{(x-a-b)^2 + (y-b+a)^2} &= \sqrt{(x-a+b)^2 + (y-a-b)^2} \\ \Rightarrow (x-a-b)^2 + (y-b+a)^2 &= (x-a+b)^2 + (y-a-b)^2 \quad (\text{Squaring both sides}) \\ \Rightarrow x^2 + (a+b)^2 - 2x(a+b) + y^2 + (a-b)^2 - 2y(a-b) &= x^2 + (a-b)^2 - 2x(a-b) + y^2 \\ &+ (a+b)^2 - 2y(a+b) \\ \Rightarrow -x(a+b) - y(a-b) &= -x(a-b) - y(a+b) \\ \Rightarrow -xa - xb - ay + by &= -xa + bx - ya - by \\ \Rightarrow by &= bx\end{aligned}$$

Hence, $bx = ay$.

18. Using the distance formula, show that the given points are collinear:

- (i) $(1, -1)$, $(5, 2)$ and $(9, 5)$ (ii) $(6, 9)$, $(0, 1)$ and $(-6, -7)$
(iii) $(-1, -1)$, $(2, 3)$ and $(8, 11)$ (iv) $(-2, 5)$, $(0, 1)$ and $(2, -3)$

Sol:

- (i) Let $A(1, -1)$, $B(5, 2)$ and $C(9, 5)$ be the given points. Then

$$\begin{aligned}AB &= \sqrt{(5-1)^2 + (2+1)^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5 \text{ units} \\ BC &= \sqrt{(9-5)^2 + (5-2)^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5 \text{ units} \\ AC &= \sqrt{(9-1)^2 + (5+1)^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10 \text{ units} \\ \therefore AB + BC &= (5+5) \text{ units} = 10 \text{ units} = AC\end{aligned}$$

Hence, the given points are collinear

- (ii) Let $A(6, 9)$, $B(0, 1)$ and $C(-6, -7)$ be the given points. Then

$$\begin{aligned}AB &= \sqrt{(0-6)^2 + (1-9)^2} = \sqrt{(-6)^2 + (-8)^2} = \sqrt{100} = 10 \text{ units} \\ BC &= \sqrt{(-6-0)^2 + (-7-1)^2} = \sqrt{(-6)^2 + (-8)^2} = \sqrt{100} = 10 \text{ units} \\ AC &= \sqrt{(-6-6)^2 + (-7-9)^2} = \sqrt{(-12)^2 + (-16)^2} = \sqrt{400} = 20 \text{ units} \\ \therefore AB + BC &= (10+10) \text{ units} = 20 \text{ units} = AC\end{aligned}$$

Hence, the given points are collinear

- (iii) Let $A(-1, -1)$, $B(2, 3)$ and $C(8, 11)$ be the given points. Then

$$AB = \sqrt{(2+1)^2 + (3+1)^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5 \text{ units}$$

$$BC = \sqrt{(8-2)^2 + (11-3)^2} = \sqrt{(6)^2 + (8)^2} = \sqrt{100} = 10 \text{ units}$$

$$AC = \sqrt{(8+1)^2 + (11+1)^2} = \sqrt{(9)^2 + (12)^2} = \sqrt{225} = 15 \text{ units}$$

$$\therefore AB + BC = (5+10) \text{ units} = 15 \text{ units} = AC$$

Hence, the given points are collinear

(iv) Let $A(-2, 5)$, $B(0, 1)$ and $C(2, -3)$ be the give points. Then

$$AB = \sqrt{(0+2)^2 + (1-5)^2} = \sqrt{(2)^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5} \text{ units}$$

$$BC = \sqrt{(2-0)^2 + (-3-1)^2} = \sqrt{(2)^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5} \text{ units}$$

$$AC = \sqrt{(2+2)^2 + (-3-5)^2} = \sqrt{(4)^2 + (-8)^2} = \sqrt{80} = 4\sqrt{5} \text{ units}$$

$$\therefore AB + BC = (2\sqrt{5} + 2\sqrt{5}) \text{ units} = 4\sqrt{5} \text{ units} = AC$$

Hence, the given points are collinear

19. Show that the points A (7, 10), B(-2, 5) and C(3, -4) are the vertices of an isosceles right triangle.

Sol:

The given points are $A(7, 10)$, $B(-2, 5)$ and $C(3, -4)$.

$$AB = \sqrt{(-2-7)^2 + (5-10)^2} = \sqrt{(-9)^2 + (-5)^2} = \sqrt{81+25} = \sqrt{106}$$

$$BC = \sqrt{(3-(-2))^2 + (-4-5)^2} = \sqrt{(5)^2 + (-9)^2} = \sqrt{25+81} = \sqrt{106}$$

$$AC = \sqrt{(3-7)^2 + (-4-10)^2} = \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212}$$

Since, AB and BC are equal, they form the vertices of an isosceles triangle

$$\text{Also, } (AB)^2 + (BC)^2 = (\sqrt{106})^2 + (\sqrt{106})^2 = 212$$

$$\text{and } (AC)^2 = (\sqrt{212})^2 = 212.$$

$$\text{Thus, } (AB)^2 + (BC)^2 = (AC)^2$$

This show that $\triangle ABC$ is right- angled at B.

Therefore, the points $A(7, 10)$, $B(-2, 5)$ and $C(3, -4)$ are the vertices of an isosceles right-angled triangle.

20. Show that the points A (3, 0), B(6, 4) and C(-1, 3) are the vertices of an isosceles right triangle.

Sol:

The given points are A(3,0), B(6,4) and C(-1,3). Now,

$$AB = \sqrt{(3-6)^2 + (0-4)^2} = \sqrt{(-3)^2 + (-4)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5$$

$$BC = \sqrt{(6+1)^2 + (4-3)^2} = \sqrt{(7)^2 + (1)^2}$$

$$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$

$$AC = \sqrt{(3+1)^2 + (0-3)^2} = \sqrt{(4)^2 + (-3)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5$$

$$\therefore AB = AC \text{ and } AB^2 + AC^2 = BC^2$$

Therefore, A(3,0), B(6,4) and C(-1,3) are the vertices of an isosceles right triangle

21. If A(5,2), B(2, -2) and C(-2, t) are the vertices of a right triangle with $\angle B = 90^\circ$, then find the value of t.

Sol:

$$\therefore \angle B = 90^\circ$$

$$\therefore AC^2 = AB^2 + BC^2$$

$$\Rightarrow (5+2)^2 + (2-t)^2 = (5-2)^2 + (2+2)^2 + (2+2)^2 + (-2-t)^2$$

$$\Rightarrow (7)^2 + (t-2)^2 = (3)^2 + (4)^2 + (4)^2 + (t+2)^2$$

$$\Rightarrow 49 + t^2 - 4t + 4 = 9 + 16 + 16 + t^2 + 4t + 4$$

$$\Rightarrow 8 - 4t = 4t$$

$$\Rightarrow 8t = 8$$

$$\Rightarrow t = 1$$

Hence, $t = 1$.

22. Prove that the points A(2, 4), B(2, 6) and $C(2 + \sqrt{3}, 5)$ are the vertices of an equilateral triangle.

Sol:

The given points are A(2,4), B(2,6) and $C(2 + \sqrt{3}, 5)$. Now

$$AB = \sqrt{(2-2)^2 + (4-6)^2} = \sqrt{(0)^2 + (-2)^2}$$

$$= \sqrt{0+4} = 2$$

$$BC = \sqrt{(2-2-\sqrt{3})^2 + (6-5)^2} = \sqrt{(-\sqrt{3})^2 + (1)^2}$$

$$= \sqrt{3+1} = 2$$

$$AC = \sqrt{(2-2-\sqrt{3})^2 + (4-5)^2} = \sqrt{(-\sqrt{3})^2 + (-1)^2}$$

$$= \sqrt{3+1} = 2$$

Hence, the points $A(2,4)$, $B(2,6)$ and $C(2+\sqrt{3},5)$ are the vertices of an equilateral triangle.

- 23.** Show that the points $(-3, -3)$, $(3,3)$ and $C(-3\sqrt{3}, 3\sqrt{3})$ are the vertices of an equilateral triangle.

Sol:

Let the given points be $A(-3, -3)$, $B(3,3)$ and $C(-3\sqrt{3}, 3\sqrt{3})$. Now

$$AB = \sqrt{(-3-3)^2 + (-3-3)^2} = \sqrt{(-6)^2 + (-6)^2}$$

$$= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

$$BC = \sqrt{(3+3\sqrt{3})^2 + (3-3\sqrt{3})^2}$$

$$= \sqrt{9+27+18\sqrt{3}+9+27-18\sqrt{3}} = \sqrt{72} = 6\sqrt{2}$$

$$AC = \sqrt{(-3+3\sqrt{3})^2 + (-3-3\sqrt{3})^2} = \sqrt{(3-3\sqrt{3})^2 + (3+3\sqrt{3})^2}$$

$$= \sqrt{9+27-18\sqrt{3}+9+27+18\sqrt{3}}$$

$$= \sqrt{72} = 6\sqrt{2}$$

Hence, the given points are the vertices of an equilateral triangle.

- 24.** Show that the points $A(-5,6)$, $B(3,0)$ and $C(9,8)$ are the vertices of an isosceles right-angled triangle. Calculate its area.

Sol:

Let the given points be $A(-5,6)$, $B(3,0)$ and $C(9,8)$.

$$AB = \sqrt{(3-(-5))^2 + (0-6)^2} = \sqrt{(8)^2 + (-6)^2} = \sqrt{64+36} = \sqrt{100} = 10 \text{ units}$$

$$BC = \sqrt{(9-3)^2 + (8-0)^2} = \sqrt{(6)^2 + (8)^2} = \sqrt{36+64} = \sqrt{100} = 10 \text{ units}$$

$$AC = \sqrt{(9-(-5))^2 + (8-6)^2} = \sqrt{(14)^2 + (2)^2} = \sqrt{196+4} = \sqrt{200} = 10\sqrt{2} \text{ units}$$

Therefore, $AB = BC = 10 \text{ units}$

$$\text{Also, } (AB)^2 + (BC)^2 = (10)^2 + (10)^2 = 200$$

$$\text{and } (AC)^2 = (10\sqrt{2})^2 = 200$$

$$\text{Thus, } (AB)^2 + (BC)^2 = (AC)^2$$

This show that $\triangle ABC$ is right angled at B .

Therefore, the points $A(-5,6)$, $B(3,0)$ and $C(9,8)$ are the vertices of an isosceles right-angled triangle

$$\text{Also, area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

If AB is the height and BC is the base,

$$\text{Area} = \frac{1}{2} \times 10 \times 10$$

$$= 50 \text{ square units}$$

25. Show that the points $O(0,0)$, $A(3, \sqrt{3})$ and $B(3, -\sqrt{3})$ are the vertices of an equilateral triangle. Find the area of this triangle.

Sol:

The given points are $O(0,0)$, $A(3, \sqrt{3})$ and $B(3, -\sqrt{3})$.

$$OA = \sqrt{(3-0)^2 + \{(\sqrt{3})-0\}^2} = \sqrt{(3)^2 + (\sqrt{3})^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3} \text{ units}$$

$$AB = \sqrt{(3-3)^2 + \{-\sqrt{3}-\sqrt{3}\}^2} = \sqrt{(0)^2 + (2\sqrt{3})^2} = \sqrt{4(3)} = \sqrt{12} = 2\sqrt{3} \text{ units}$$

$$OB = \sqrt{(3-0)^2 + \{-\sqrt{3}-0\}^2} = \sqrt{(3)^2 + (\sqrt{3})^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3} \text{ units}$$

Therefore, $OA = AB = OB = 2\sqrt{3} \text{ units}$

Thus, the points $O(0,0)$, $A(3, \sqrt{3})$ and $B(3, -\sqrt{3})$ are the vertices of an equilateral triangle

$$\text{Also, the area of the triangle } OAB = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (2\sqrt{3})^2$$

$$= \frac{\sqrt{3}}{4} \times 12$$

$$= 3\sqrt{3} \text{ square units.}$$

26. Show that the following points are the vertices of a square:

- (i) A (3,2), B(0,5), C(-3,2) and D(0,-1)
- (ii) A (6,2), B(2,1), C(1,5) and D(5,6)
- (iii) A (0,-2), B(3,1), C(0,4) and D(-3,1)

Sol:

- (i) The given points are A(3,2), B(0,5), C(-3,2) and D(0,-1).

$$AB = \sqrt{(0-3)^2 + (5-2)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$BC = \sqrt{(-3-0)^2 + (2-5)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$CD = \sqrt{(0+3)^2 + (-1-2)^2} = \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$DA = \sqrt{(0-3)^2 + (-1-2)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

Therefore, $AB = BC = CD = DA = 3\sqrt{2} \text{ units}$

$$\text{Also, } AC = \sqrt{(-3-3)^2 + (2-2)^2} = \sqrt{(-6)^2 + (0)^2} = \sqrt{36} = 6 \text{ units}$$

$$BD = \sqrt{(0-0)^2 + (-1-5)^2} = \sqrt{(0)^2 + (-6)^2} = \sqrt{36} = 6 \text{ units}$$

Thus, diagonal $AC = \text{diagonal } BD$

Therefore, the given points form a square.

- (ii) The given points are A(6,2), B(2,1), C(1,5) and D(5,6)

$$AB = \sqrt{(2-6)^2 + (1-2)^2} = \sqrt{(-4)^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17} \text{ units}$$

$$BC = \sqrt{(1-2)^2 + (5-1)^2} = \sqrt{(-1)^2 + (4)^2} = \sqrt{1+16} = \sqrt{17} \text{ units}$$

$$CD = \sqrt{(5-1)^2 + (6-5)^2} = \sqrt{(4)^2 + (1)^2} = \sqrt{16+1} = \sqrt{17} \text{ units}$$

$$DA = \sqrt{(5-6)^2 + (6-2)^2} = \sqrt{(1)^2 + (4)^2} = \sqrt{1+16} = \sqrt{17} \text{ units}$$

Therefore, $AB = BC = CD = DA = \sqrt{17} \text{ units}$

$$\text{Also, } AC = \sqrt{(1-6)^2 + (5-2)^2} = \sqrt{(-5)^2 + (3)^2} = \sqrt{25+9} = \sqrt{34} \text{ units}$$

$$BD = \sqrt{(5-2)^2 + (6-1)^2} = \sqrt{(3)^2 + (5)^2} = \sqrt{9+25} = \sqrt{34} \text{ units}$$

Thus, diagonal $AC = \text{diagonal } BD$

Therefore, the given points form a square.

- (iii) The given points are P(0,-2), Q(3,1), R(0,4) and S(-3,1)

$$PQ = \sqrt{(3-0)^2 + (1+2)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$QR = \sqrt{(0-3)^2 + (4-1)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$RS = \sqrt{(-3-0)^2 + (1-4)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$SP = \sqrt{(-3-0)^2 + (1+2)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

Therefore, $PQ = QS = RS = SP = 3\sqrt{2}$ units

$$\text{Also, } PR = \sqrt{(0-0)^2 + (4+2)^2} = \sqrt{(0)^2 + (6)^2} = \sqrt{36} = 6 \text{ units}$$

$$QS = \sqrt{(-3-3)^2 + (1-1)^2} = \sqrt{(-6)^2 + (0)^2} = \sqrt{36} = 6 \text{ units}$$

Thus, diagonal PR = diagonal QS

Therefore, the given points form a square.

27. Show that the points $A(-3, 2)$, $B(-5, -5)$, $C(2, -3)$ and $D(4, 4)$ are the vertices of a rhombus. Find the area of this rhombus

Sol:

The given points are $A(-3, 2)$, $B(-5, -5)$, $C(2, -3)$ and $D(4, 4)$.

$$AB = \sqrt{(-5+3)^2 + (-5-2)^2} = \sqrt{(-2)^2 + (-7)^2} = \sqrt{4+49} = \sqrt{53} \text{ units}$$

$$BC = \sqrt{(2+5)^2 + (-3+5)^2} = \sqrt{(7)^2 + (2)^2} = \sqrt{49+4} = \sqrt{53} \text{ units}$$

$$CD = \sqrt{(4-2)^2 + (4+3)^2} = \sqrt{(2)^2 + (7)^2} = \sqrt{4+49} = \sqrt{53} \text{ units}$$

$$DA = \sqrt{(4+3)^2 + (4-2)^2} = \sqrt{(7)^2 + (2)^2} = \sqrt{49+4} = \sqrt{53} \text{ units}$$

Therefore, $AB = BC = CD = DA = \sqrt{53}$ units

$$\text{Also, } AC = \sqrt{(2+3)^2 + (-3-2)^2} = \sqrt{(5)^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2} \text{ units}$$

$$BD = \sqrt{(4+5)^2 + (4+5)^2} = \sqrt{(9)^2 + (9)^2} = \sqrt{81+81} = \sqrt{162} = \sqrt{81 \times 2} = 9\sqrt{2} \text{ units}$$

Thus, diagonal AC is not equal to diagonal BD .

Therefore ABCD is a quadrilateral with equal sides and unequal diagonals

Hence, ABCD a rhombus

$$\text{Area of a rhombus} = \frac{1}{2} \times (\text{product of diagonals})$$

$$= \frac{1}{2} \times (5\sqrt{2}) \times (9\sqrt{2})$$

$$= \frac{45(2)}{2}$$

$$= 45 \text{ square units.}$$

28. Show that the points A(3,0), B(4,5), C(-1,4) and D(-2,-1) are the vertices of a rhombus. Find its area.

Sol:

The given points are A(3,0), B(4,5), C(-1,4) and D(-2,-1)

$$AB = \sqrt{(3-4)^2 + (0-5)^2} = \sqrt{(-1)^2 + (-5)^2}$$

$$= \sqrt{1+25} = \sqrt{26}$$

$$BC = \sqrt{(4+1)^2 + (5-4)^2} = \sqrt{(5)^2 + (1)^2}$$

$$= \sqrt{25+1} = \sqrt{26}$$

$$CD = \sqrt{(-1+2)^2 + (4+1)^2} = \sqrt{(1)^2 + (5)^2}$$

$$= \sqrt{1+25} = \sqrt{26}$$

$$AD = \sqrt{(3+2)^2 + (0+1)^2} = \sqrt{(5)^2 + (1)^2}$$

$$= \sqrt{25+1} = \sqrt{26}$$

$$AC = \sqrt{(3+1)^2 + (0-4)^2} = \sqrt{(4)^2 + (-4)^2}$$

$$= \sqrt{16+16} = 4\sqrt{2}$$

$$BD = \sqrt{(4+2)^2 + (5+1)^2} = \sqrt{(6)^2 + (6)^2}$$

$$= \sqrt{36+36} = 6\sqrt{2}$$

$$\therefore AB = BC = CD = AD = 6\sqrt{2} \text{ and } AC \neq BD$$

Therefore, the given points are the vertices of a rhombus

$$\text{Area } (\Delta ABCD) = \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24 \text{ sq. units}$$

Hence, the area of the rhombus is 24 sq. units.

29. Show that the points A(6,1), B(8,2), C(9,4) and D(7,3) are the vertices of a rhombus. Find its area.

Sol:

The given points are A(6,1), B(8,2), C(9,4) and D(7,3).

$$AB = \sqrt{(6-8)^2 + (1-2)^2} = \sqrt{(-2)^2 + (-1)^2}$$

$$= \sqrt{4+1} = \sqrt{5}$$

$$BC = \sqrt{(8-9)^2 + (2-4)^2} = \sqrt{(-1)^2 + (-2)^2}$$

$$= \sqrt{1+4} = \sqrt{5}$$

$$CD = \sqrt{(9-7)^2 + (4-3)^2} = \sqrt{(2)^2 + (1)^2}$$

$$= \sqrt{4+1} = \sqrt{5}$$

$$AD = \sqrt{(7-6)^2 + (3-1)^2} = \sqrt{(1)^2 + (2)^2}$$

$$= \sqrt{1+4} = \sqrt{5}$$

$$AC = \sqrt{(6-9)^2 + (1-4)^2} = \sqrt{(-3)^2 + (-3)^2}$$

$$= \sqrt{9+9} = 3\sqrt{2}$$

$$BD = \sqrt{(8-7)^2 + (2-3)^2} = \sqrt{(1)^2 + (-1)^2}$$

$$= \sqrt{1+1} = \sqrt{2}$$

$$\therefore AB = BC = CD = AD = \sqrt{5} \text{ and } AC \neq BD$$

Therefore, the given points are the vertices of a rhombus. Now

$$\text{Area } (\Delta ABCD) = \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 3\sqrt{2} \times \sqrt{2} = 3 \text{ sq. units}$$

Hence, the area of the rhombus is 3 sq. units.

30. Show that the points A(2,1), B(5,2), C(6,4) and D(3,3) are the angular points of a parallelogram. Is this figure a rectangle?

Sol:

The given points are A(2,1), B(5,2), C(6,4) and D(3,3)

$$AB = \sqrt{(5-2)^2 + (2-1)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$

$$BC = \sqrt{(6-5)^2 + (4-2)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5} \text{ units}$$

$$CD = \sqrt{(3-6)^2 + (3-4)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$

$$AD = \sqrt{(3-2)^2 + (3-1)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5} \text{ units}$$

Thus, $AB = CD = \sqrt{10}$ units and $BC = AD = \sqrt{5}$ units

So, quadrilateral ABCD is a parallelogram

$$\text{Also, } AC = \sqrt{(6-2)^2 + (4-1)^2} = \sqrt{(4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5 \text{ units}$$

$$BD = \sqrt{(3-5)^2 + (3-2)^2} = \sqrt{(-2)^2 + (1)^2} = \sqrt{4+1} = \sqrt{5} \text{ units}$$

But diagonal AC is not equal to diagonal BD.

Hence, the given points do not form a rectangle.

31. Show that A(1,2), B(4,3), C(6,6) and D(3,5) are the vertices of a parallelogram. Show that ABCD is not rectangle.

Sol:

The given vertices are A(1,2), B(4,3), C(6,6) and D(3,5).

$$AB = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{(-3)^2 + (-1)^2}$$

$$= \sqrt{9+1} = \sqrt{10}$$

$$BC = \sqrt{(4-6)^2 + (3-6)^2} = \sqrt{(-2)^2 + (-3)^2}$$

$$= \sqrt{4+9} = \sqrt{13}$$

$$CD = \sqrt{(6-3)^2 + (6-5)^2} = \sqrt{(3)^2 + (1)^2}$$

$$= \sqrt{9+1} = \sqrt{10}$$

$$AD = \sqrt{(1-3)^2 + (2-5)^2} = \sqrt{(-2)^2 + (-3)^2}$$

$$= \sqrt{4+9} = \sqrt{13}$$

$$\therefore AB = CD = \sqrt{10} \text{ units and } BC = AD = \sqrt{13} \text{ units}$$

Therefore, ABCD is a parallelogram

$$AC = \sqrt{(1-6)^2 + (2-6)^2} = \sqrt{(-5)^2 + (-4)^2}$$

$$= \sqrt{25+16} = \sqrt{41}$$

$$BD = \sqrt{(4-3)^2 + (3-5)^2} = \sqrt{(1)^2 + (-2)^2}$$

$$= \sqrt{1+4} = \sqrt{5}$$

Thus, the diagonal AC and BD are not equal and hence ABCD is not a rectangle

32. Show that the following points are the vertices of a rectangle.

- (i) A (-4,-1), B(-2,-4), C(4,0) and D(2,3)
- (ii) A (2, -2), B(14,10), C(11,13) and D(-1,1)
- (iii) A (0,-4), B(6,2), C(3,5) and D(-3,-1)

Sol:

- (i) The given points are A(-4,-1), B(-2,-4), C(4,0) and D(2,3)

$$AB = \sqrt{\{-2 - (-4)\}^2 + \{-4 - (-1)\}^2} = \sqrt{(2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13} \text{ units}$$

$$BC = \sqrt{\{4 - (-2)\}^2 + \{0 - (-4)\}^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13} \text{ units}$$

$$CD = \sqrt{(2-4)^2 + (3-0)^2} = \sqrt{(-2)^2 + (3)^2} = \sqrt{4+9} = \sqrt{13} \text{ units}$$

$$AD = \sqrt{\{2 - (-4)\}^2 + \{3 - (-1)\}^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13} \text{ units}$$

Thus, $AB = CD = \sqrt{13}$ units and $BC = AD = 2\sqrt{13}$ units

Also, $AC = \sqrt{\{4 - (-4)\}^2 + \{0 - (-1)\}^2} = \sqrt{(8)^2 + (1)^2} = \sqrt{64 + 1} = \sqrt{65}$ units

$BD = \sqrt{\{2 - (-2)\}^2 + \{3 - (-4)\}^2} = \sqrt{(4)^2 + (7)^2} = \sqrt{16 + 49} = \sqrt{65}$ units

Also, diagonal $AC =$ diagonal BD

Hence, the given points form a rectangle

- (ii) The given points are $A(2, -2), B(14, 10), C(11, 13)$ and $D(-1, 1)$

$AB = \sqrt{(14 - 2)^2 + \{10 - (-2)\}^2} = \sqrt{(12)^2 + (12)^2} = \sqrt{144 + 144} = \sqrt{288} = 12\sqrt{2}$ units

$BC = \sqrt{(11 - 14)^2 + (13 - 10)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$ units

$CD = \sqrt{(-1 - 11)^2 + (1 - 13)^2} = \sqrt{(-12)^2 + (-12)^2} = \sqrt{144 + 144} = \sqrt{288} = 12\sqrt{2}$ units

$AD = \sqrt{(-1 - 2)^2 + \{1 - (-2)\}^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$ units

Thus, $AB = CD = 12\sqrt{2}$ units and $BC = AD = 3\sqrt{2}$ units

Also,

$AC = \sqrt{(11 - 2)^2 + \{13 - (-2)\}^2} = \sqrt{(9)^2 + (15)^2} = \sqrt{81 + 225} = \sqrt{306} = 3\sqrt{34}$ units

$BD = \sqrt{(-1 - 14)^2 + (1 - 10)^2} = \sqrt{(-15)^2 + (-9)^2} = \sqrt{81 + 225} = \sqrt{306} = 3\sqrt{34}$ units

Also, diagonal $AC =$ diagonal BD

Hence, the given points form a rectangle

- (iii) The given points are $A(0, -4), B(6, 2), C(3, 5)$ and $D(-3, -1)$.

$AB = \sqrt{(6 - 0)^2 + \{2 - (-4)\}^2} = \sqrt{(6)^2 + (6)^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$ units

$BC = \sqrt{(3 - 6)^2 + (5 - 2)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$ units

$CD = \sqrt{(-3 - 3)^2 + (-1 - 5)^2} = \sqrt{(-6)^2 + (-6)^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$ units

$AD = \sqrt{(-3 - 0)^2 + \{-1 - (-4)\}^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$ units

Thus, $AB = CD = \sqrt{10}$ units and $BC = AD = \sqrt{5}$ units

Also, $AC = \sqrt{(3 - 0)^2 + \{5 - (-4)\}^2} = \sqrt{(3)^2 + (9)^2} = \sqrt{9 + 81} = \sqrt{90} = 3\sqrt{10}$ units

$BD = \sqrt{(-3 - 6)^2 + (-1 - 2)^2} = \sqrt{(-9)^2 + (-3)^2} = \sqrt{81 + 9} = \sqrt{90} = 3\sqrt{10}$ units

Also, diagonal $AC =$ diagonal BD

Hence, the given points form a rectangle