Exercise - 11C

1. The first three terms of an AP are respectively (3y – 1), (3y + 5) and (5y + 1), find the value of y.

Sol:

The terms $(3y-1),(3y+5)$ and $(5y+1)$ are in AP.

$\therefore (3y+5)-(3y-1)=(5y+1)-(3y+5)$

$\Rightarrow 3y+5-3y+1=5y+1-3y-5$

$\Rightarrow 6=2y-4$

$\Rightarrow 2y=10$

$\Rightarrow y=5$

Hence, the value of y is 5.

2. If $k,(2k-1)$ and $(2k+1)$ are the three successive terms of an AP, find the value of k.

Sol:

It is given that $k,(2k-1)$ and $(2k+1)$ are the three successive terms of an AP.

$\therefore (2k-1)-k=(2k+1)-(2k-1)$

$\Rightarrow k-1=2$

$\Rightarrow k=3$

Hence, the value of k is 3.

3. If 18, a, (b - 3) are in AP, then find the value of (2a – b)

Sol:

It is given that 18, $a,(b-3)$ are in AP.

$\therefore a-18=(b-3)-a$

$\Rightarrow a+a-b=18-3$

$\Rightarrow 2a-b=15$

Hence, the required value is 15.

4. If the numbers a, 9, b, 25 from an AP, find a and b.

Sol:

It is given that the numbers $a,9,b,25$ from an AP.

$\therefore 9-a=b-9=25-b$

So,

$b-9=25-b$

$\Rightarrow 2b=34$

$\Rightarrow b=17$

Also,

$9-a=b-9$

$\Rightarrow a=18-b$

$\Rightarrow a=18-17 \qquad (b=17)$

$\Rightarrow a=1$

Hence, the required values of a and b are 1 and 17, respectively.

5. If the numbers (2n – 1), (3n+2) and (6n -1) are in AP, find the value of n and the numbers

Sol:

It is given that the numbers $(2n-1),(3n+2)$ and $(6n-1)$ are in AP.

$\therefore (3n+2)-(2n-1)=(6n-1)-(3n+2)$

$\Rightarrow 3n+2-2n+1=6n-1-3n-2$

$\Rightarrow n+3=3n-3$

$\Rightarrow 2n=6$

$\Rightarrow n=3$

When, $n=3$

$2n-1=2\times3-1=6-1=5$

$3n+2=3\times3+2=9+2=11$

$6n-1=6\times3-1=18-1=17$

Hence, the required value of n is 3 and the numbers are 5, 11 and 17.

6. How many three-digit natural numbers are divisible by 7?

Sol:

The three digit natural numbers divisible by 7 are $105, 112, 119......, 994$

Clearly, these number are in AP.

Here, $a = 105$ *and* $d = 112 - 105 = 7$

Let this AP contains n terms. Then,

$a_n = 994$

$\Rightarrow 105 + (n-1) \times 7 = 994 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 7n + 98 = 994$

$\Rightarrow 7n = 994 - 98 = 986$

$\Rightarrow n = 128$

Hence, there are 128 three digit numbers divisible by 7.

7. How many three-digit natural numbers are divisible by 9?

Sol:

The three-digit natural numbers divisible by 9 are 108, 117, 126 ,….. 999.

Clearly, these number are in AP.

Here. a = 108 and d = 117 – 108 = 9

Let this AP contains n terms. Then,

$a_n = 999$

$\Rightarrow 108 + (n-1) \times 9 = 999 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 9n + 99 = 999$

$\Rightarrow 9n = 999 - 99 = 900$

$\Rightarrow n = 100$

Hence, there are 100 three-digit numbers divisible by 9.

8. If the sum of first m terms of an AP is ($2m^2 + 3m$) then what is its second term?

Sol:

Let S_m denotes the sum of first m terms of the AP.

$\therefore S_m = 2m^2 + 3m$

$\Rightarrow S_{m-1} = 2(m-1)^2 + 3(m-1) = 2(m^2 - 2m + 1) + 3(m-1) = 2m^2 - m - 1$

Now,

m^{th} term of AP, $a_m = S_m - S_{m-1}$

$\therefore a_m = (2m^2 + 3m) - (2m^2 - m - 1) = 4m + 1$

Putting $m = 2$, we get

$a_2 = 4 \times 2 + 1 = 9$

Hence, the second term of the AP is 9.

9. What is the sum of first n terms of the AP a, 3a, 5a, …..

Sol:

The given AP is $3a, 5a, \ldots\ldots$

Here,

First term, A = a

Common difference, $D = 3a - a = 2a$

$\therefore$ Sum of the n terms, S_n

$= \frac{n}{2}\left[2 \times a + (n-1) \times 2a\right]$ $\left\{S_n = \frac{n}{2}\left[2A + (n-1)D\right]\right\}$

$= \frac{n}{2}(2a + 2an - 2a)$

$= \frac{n}{2} \times 2an$

$= an^2$

Hence, the required sum is an^2.

10. What is the 5th term form the end of the AP 2, 7, 12, …., 47?

Sol:

The given AP is 2, 7, 12, ..., 47.

Let us re-write the given AP in reverse order i.e. 47, 42, .., 12, 7, 2.

Now, the 5th term from the end of the given AP is equal to the 5th term from beginning of the AP 47, 42,.... ,12, 7, 2.

Consider the AP 47, 42,..., 12, 7, 2.

Here, $a = 47$ and $d = 42 - 47 = -5$

5th term of this AP

= 47 + (5 – 1) × (–5)

= 47 – 20

= 27

Hence, the 5th term from the end of the given AP is 27.

11. If a_n denotes the nth term of the AP 2, 7, 12, 17, … find the value of $(a_{30} - a_{20})$.

Sol:

The given AP is 2, 7, 12, 17,………

Here, $a = 2$ and $d = 7 - 2 = 5$

$\therefore a_{30} - a_{20}$

$= \left[2+(30-1)\times 5\right] - \left[2+(20-1)\times 5\right] \qquad \left[a_n = a+(n-1)d\right]$

$= 147 - 97$

$= 50$

Hence, the required value is 50.

12. The nth term of an AP is (3n +5). Find its common difference.

Sol:

We have

$T_n = (3n+5)$

Common difference $= T_2 - T_1$

$T_1 = 3\times 1 + 5 = 8$

$T_2 = 3\times 2 + 5 = 11$

$d = 11 - 8 - 3$

Hence, the common difference is 3.

13. The nth term of an AP is (7 – 4n). Find its common difference.

Sol:

We have

$T_n = (7-4n)$

Common difference $= T_2 - T_1$

$T_1 = 7 - 4\times 1 = 3$

$T_2 = 7 - 4\times 2 = -1$

$d = -1 - 3 = -4$

Hence, the common difference is -4.

14. Write the next term for the AP $\sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

Sol:

The given AP is $\sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

On simplifying the terms, we get:

$2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, ...$

Here, $a = 2\sqrt{2}$ and $d = \left(3\sqrt{2} - 2\sqrt{2}\right) = \sqrt{2}$

$\therefore$ Next term, $T_4 = a + 3d = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2} = \sqrt{50}$

15. Write the next term of the AP $\sqrt{2}, \sqrt{8}, \sqrt{18}, \ldots\ldots\ldots$

Sol:

The given AP is $\sqrt{2}, \sqrt{8}, \sqrt{18}, \ldots\ldots\ldots$

On simplifying the terms, we get:

$\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, \ldots\ldots$

Here, $a = \sqrt{2}$ and $d = \left(2\sqrt{2} - \sqrt{2}\right) = \sqrt{2}$

$\therefore$ Next term, $T_4 = a + 3d = \sqrt{2} + 3\sqrt{2} = 4\sqrt{2} = \sqrt{32}$

16. Which term of the AP 21, 18, 15, … is zero?

Sol:

In the given AP, first term, $a = 21$ and common difference, $d = (18 - 21) = -3$

Let's its n^{th} term be 0.

Then, $T_n = 0$

$\Rightarrow a + (n-1)d = 0$

$\Rightarrow 21 + (n-1) \times (-3) = 0$

$\Rightarrow 24 - 3n = 0$

$\Rightarrow 3n = 24$

$\Rightarrow n = 8$

Hence, the 8^{th} term of the given AP is 0.

17. Find the sum of the first n natural numbers.

Sol:

The first n natural numbers are 1, 2, 3, 4, 5,……..,n

Here, a = 1 and d = (2 – 1) = 1

Sum of n terms of an AP is given by

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \left(\frac{n}{2}\right) \times \left[2 \times 1 + (n-1) \times 1\right]$

$= \left(\frac{n}{2}\right) \times [2 + n - 1] = \left(\frac{n}{2}\right) \times (n+1) = \frac{n(n+1)}{2}$

18. Find the sum of first n even natural numbers.

Sol:

The first *n* even natural numbers are 2, 4, 6, 8, 10, …., *n*.

Here, $a = 2$ and $d = (4-2) = 2$

Sum of n terms of an AP is given by

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \left(\frac{n}{2}\right) \times \left[2 \times 2 + (n-1) \times 2\right]$

$= \left(\frac{n}{2}\right) \times [4 + 2n - 2] = \left(\frac{n}{2}\right) \times (2n+2) = n(n+1)$

Hence, the required sum is $n(n+1)$.

19. The first term of an AP is p and its common difference is q. Find its 10th term.

Sol:

Here, $a = p$ and $d = q$

Now, $T_n = a + (n-1)d$

$\Rightarrow T_n = p + (n-1)q$

$\therefore T_{10} = p + 9q$

20. If $\frac{4}{5}, a, 2$ are in AP, find the value of a.

Sol:

If 45, a and 2 are three consecutive terms of an AP, then we have:

$a - 45 = 2 - a$

$\Rightarrow 2a = 2 + 45$

$\Rightarrow 2a = 145$

$\Rightarrow a = 75$

21. If (2p +1), 13, (5p -3) are in AP, find the value of p.

Sol:

Let $(2p+1), 13, (5p-3)$ be three consecutive terms of an AP.

Then $13 - (2p+1) = (5p-3) - 13$

$\Rightarrow 7p = 28$

$\Rightarrow p = 4$

$\therefore$ When $p = 4, (2p+1), 13$ and $(5p-3)$ from three consecutive terms of an AP.

22. If (2p – 1), 7, 3p are in AP, find the value of p.

Sol:

Let $(2p-1), 7$ and $3p$ be three consecutive terms of an AP.

Then $7-(2p-1)=3p-7$

$\Rightarrow 5p=15$

$\Rightarrow p=3$

$\therefore$ When $p=3, (2p-1), 7$ and $3p$ form three consecutive terms of an AP.

23. If the sum of first p terms of an AP is (ap^2+bp), find its common difference.

Sol:

Let S_p denotes the sum of first p terms of the AP.

$\therefore S_p = ap^2+bp$

$\Rightarrow S_{p-1} = a(p-1)^2+b(p-1)$

$= a(p^2-2p+1)+b(p-1)$

$= ap^2-(2a-b)p+(a-b)$

Now,

p^{th} term of $AP, a_p = S_p - S_{p-1}$

$=(ap^2+bp)-\left[ap^2-(2a-b)p+(a-b)\right]$

$= ap^2+bp-ap^2+(2a-b)p-(a-b)$

$= 2ap-(a-b)$

Let d be the common difference of the AP.

$\therefore d = a_p - a_{p-1}$

$=\left[2ap-(a-b)\right]=\left[2a(p-1)-(a-b)\right]$

$= 2ap-(a-b)-2a(p-1)+(a-b)$

$= 2a$

Hence, the common difference of the AP is 2a.

24. If the sum of first n terms is $(3n^2+5n)$, find its common difference.

Sol:

Let S_n denotes the sum of first n terms of the AP.

$\therefore S_n = 3n^2+5n$

$\Rightarrow S_{n-1} = 3(n-1)^2+5(n-1)$

$= 3(n^2 - 2n + 1) + 5(n-1)$

$= 3n^2 - n - 2$

Now,

n^{th} term of $AP, a_n = S_n - S_{n-1}$

$= (3n^2 + 5n) - (3n^2 - n - 2)$

$= 6n + 2$

Let d be the common difference of the AP.

$\therefore d = a_n - a_{n-1}$

$= (6n+2) - [6(n-1) + 2]$

$= 6n + 2 - 6(n-1) - 2$

$= 6$

Hence, the common difference of the AP is 6.

25. Find an AP whose 4th term is 9 and the sum of its 6th and 13th terms is 40.

Sol:

Let *a* be the first term and d be the common difference of the AP. Then,

$a_4 = 9$

$\Rightarrow a + (4-1)d = 9 \qquad [a_n = a + (n-1)d]$

$\Rightarrow a + 3d = 9 \qquad ....(1)$

Now,

$a_6 + a_{13} = 40 \qquad$ (Given)

$\Rightarrow (a + 5d) + (a + 12d) = 40$

$\Rightarrow 2a + 17d = 40 \qquad ....(2)$

From (1) and (2), we get

$2(9 - 3d) + 17d = 40$

$\Rightarrow 18 - 6d + 17d = 40$

$\Rightarrow 11d = 40 - 18 = 22$

$\Rightarrow d = 2$

Putting $d = 2$ in (1), we get

$a + 3 \times 2 = 9$

$\Rightarrow a = 9 - 6 = 3$

Hence, the AP is 3, 5, 7, 9, 11,…….