Exercise - 11B

1. Determine k so that (3k -2), (4k – 6) and (k +2) are three consecutive terms of an AP.

Sol:

It is given that $(3k-2),(4k-6)$ and $(k+2)$ are three consecutive terms of an AP.

$\therefore (4k-6)-(3k-2)=(k+2)-(4k-6)$

$\Rightarrow 4k-6-3k+2=k+2-4k+6$

$\Rightarrow k-4=-3k+8$

$\Rightarrow k+3k=8+4$

$\Rightarrow 4k=12$

$\Rightarrow k=3$

Hence, the value of k is 3.

2. Find the value of x for which the numbers (5x + 2), (4x - 1) and (x + 2) are in AP.

Sol:

It is given that $(5x+2),(4x-1)$ and $(x+2)$ are in AP.

$\therefore (4x-1)-(5x+2)=(x+2)-(4x-1)$

$\Rightarrow 4x-1-5x-2=x+2-4x+1$

$\Rightarrow -x-3=-3x+3$

$\Rightarrow 3x-x=3+3$

$\Rightarrow 2x=6$

$\Rightarrow x=3$

Hence, the value of x is 3.

3. If (3y – 1), (3y + 5) and (5y + 1) are three consecutive terms of an AP then find the value of y.

Sol:

It is given that $(3y-1),(3y+5)$ and $(5y+1)$ are three consecutive terms of an AP.

$\therefore (3y+5)-(3y-1)=(5y+1)-(3y+5)$

$\Rightarrow 3y+5-3y+1=5y+1-3y-5$

$\Rightarrow 6=2y-4$

$\Rightarrow 2y=6+4=10$

$\Rightarrow y=5$

Hence, the value of y is 5.

4. Find the value of x for which (x + 2), 2x, ()2x + 3) are three consecutive terms of an AP.

Sol:

Since $(x+2),2x$ and $(2x+3)$ are in AP, we have

$2x-(x+2)=(2x+3)-2x$

$\Rightarrow x-2=3$

$\Rightarrow x=5$

$\therefore x=5$

5. Show that $(a-b)^2,(a^2+b^2)$ and (a^2+b^2) are in AP.

Sol:

The given numbers are $(a-b)^2,(a^2+b^2)$ and $(a+b)^2$.

Now,

$(a^2+b^2)-(a-b)^2=a^2+b^2-(a^2-2ab+b^2)=a^2+b^2-a^2+2ab-b^2=2ab$

$(a+b)^2-(a^2+b^2)=a^2+2ab+b^2-a^2-b^2=2ab$

So, $(a^2+b^2)-(a-b)^2=(a+b)^2-(a^2+b^2)=2ab$ (Constant)

Since each term differs from its preceding term by a constant, therefore, the given numbers are in AP.

6. Find the three numbers in AP whose sum is 15 and product is 80.

Sol:

Let the required numbers be $(a-d),a$ and $(a+d)$.

Then $(a-d)+a+(a+d)=15$

$\Rightarrow 3a = 15$

$\Rightarrow a = 5$

Also, $(a-d).a.(a+d) = 80$

$\Rightarrow a(a^2 - d^2) = 80$

$\Rightarrow 5(25 - d^2) = 80$

$\Rightarrow d^2 = 25 - 16 = 9$

$\Rightarrow d = \pm 3$

Thus, $a = 5$ and $d = \pm 3$

Hence, the required numbers are $(2, 5 \text{ and } 8)$ or $(8, 5 \text{ and } 2)$.

7. The sum of three numbers in AP is 3 and their product is -35. Find the numbers.

Sol:

Let the required numbers be $(a-d), a$ and $(a+d)$.

Then $(a-d) + a + (a+d) = 3$

$\Rightarrow 3a = 3$

$\Rightarrow a = 1$

Also, $(a-d).a.(a+d) = -35$

$\Rightarrow a(a^2 - d^2) = -35$

$\Rightarrow 1.(1 - d^2) = -35$

$\Rightarrow d^2 = 36$

$\Rightarrow d = \pm 6$

Thus, $a = 1$ and $d = \pm 6$

Hence, the required numbers are $(-5, 1 \text{ and } 7)$ or $(7, 1 \text{ and } -5)$.

8. Divide 24 in three parts such that they are in AP and their product is 440.

Sol:

Let the required parts of 24 be $(a-d), a$ and $(a+d)$ such that they are in AP.

Then $(a-d) + a + (a+d) = 24$

$\Rightarrow 3a = 24$

$\Rightarrow a = 8$

Also, $(a-d).a.(a+d) = 440$

$\Rightarrow a(a^2 - d^2) = 440$

$\Rightarrow 8(64 - d^2) = 440$

$\Rightarrow d^2 = 64 - 55 = 9$

$\Rightarrow d = \pm 3$

Thus, $a = 8$ and $d = \pm 3$

Hence, the required parts of 24 are $(5, 8, 11)$ *or* $(11, 8, 5)$.

9. The sum of three consecutive terms of an AP is 21 and the sum of the squares of these terms is 165. Find these terms

Sol:

Let the required terms be $(a-d), a$ and $(a+d)$.

Then $(a-d)+a+(a+d)=21$

$\Rightarrow 3a = 21$

$\Rightarrow a = 7$

Also, $(a-d)^2 + a^2 + (a+d)^2 = 165$

$\Rightarrow 3a^2 + 2d^2 = 165$

$\Rightarrow (3 \times 49 + 2d^2) = 165$

$\Rightarrow 2d^2 = 165 - 147 = 18$

$\Rightarrow d^2 = 9$

$\Rightarrow d = \pm 3$

Thus, $a = 7$ and $d = \pm 3$

Hence, the required terms are $(4, 7, 10)$ *or* $(10, 7, 4)$.

10. The angles of quadrilateral are in whose AP common difference is $10°$. Find the angles.

Sol:

Let the required angles be $(a-15)°, (a-5)°, (a+5)°$ *and* $(a+15)°$, as the common difference is 10 (given).

Then $(a-15)° + (a-5)° + (a+5)° + (a+15)° = 360°$

$\Rightarrow 4a = 360$

$\Rightarrow a = 90$

Hence, the required angles of a quadrilateral are

$(90-15)°, (90-5)°, (90+5)°$ *and* $(90+15)°$; *or* $75°, 85°, 95°$ *and* $105°$.

11. Find four numbers in AP whose sum is 8 and the sum of whose squares is 216.

Sol:

(4, 6, 8, 10) or (10, 8, 6, 4)

12. Divide 32 into four parts which are the four terms of an AP such that the product of the first and fourth terms is to product of the second and the third terms as 7:15.

Sol:

Let the four parts in AP be $(a-3d),(a-d),(a+d)$ *and* $(a+3d)$. Then,

$(a-3d)+(a-d)+(a+d)+(a+3d)=32$

$\Rightarrow 4a=32$

$\Rightarrow a=8$ ....(1)

Also,

$(a-3d)(a+3d):(a-d)(a+d)=7:15$

$\Rightarrow \frac{(8-3d)(8+3d)}{(8-d)(8+d)}=\frac{7}{15}$ [*From* (1)]

$\Rightarrow \frac{64-9d^2}{64-d^2}=\frac{7}{15}$

$\Rightarrow 15(64-9d^2)=7(64-d^2)$

$\Rightarrow 960-135d^2=448-7d^2$

$\Rightarrow 135d^2-7d^2=960-448$

$\Rightarrow 128d^2=512$

$\Rightarrow d^2=4$

$\Rightarrow d=\pm 2$

When $a=8$ and $d=2$,

$a-3d=8-3\times 2=8-6=2$

$a-d=8-2=6$

$a+d=8+2=10$

$a+3d=8+3\times 2=8+6=14$

When $a=8$ and $d=-2$,

$a-3d=8-3\times(-2)=8+6=14$

$a-d=8-(-2)=8+2=10$

$a+d=8-2=6$

$a+3d=8+3\times(-2)=8-6=2$

Hence, the four parts are 2, 6, 10 and 14.

13. The sum of first three terms of an AP is 48. If the product of first and second terms exceeds 4 times the third term by 12. Find the AP.

Sol:

Let the first three terms of the AP be $(a-d), a$ *and* $(a+d)$. Then,

$(a-d)+a+(a+d)=48$

$\Rightarrow 3a=48$

$\Rightarrow a=16$

Now,

$(a-d)\times a=4(a+d)+12$ (Given)

$\Rightarrow (16-d)\times 16=4(16+d)+12$

$\Rightarrow 256-16d=64+4d+12$

$\Rightarrow 16d+4d=256-76$

$\Rightarrow 20d=180$

$\Rightarrow d=9$

When $a=16$ and $d=9$,

$a-d=16-9=7$

$a+d=16+9=25$

Hence, the first three terms of the AP are 7, 16, and 25.