Exercise – 11A

1. Show that each of the progressions given below is an AP. Find the first term, common difference and next term of each.

(i) 9, 15, 21, 27,.... (ii) 11, 6, 1, – 4,.... (iii) $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, \dots$ (iv) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$ (v) $\sqrt{20}, \sqrt{45}, \sqrt{80}, \sqrt{125}, \dots$ Sol: The given progression 9, 15, 21, 27,..... (i) Clearly, 15 - 9 = 21 - 15 = 27 - 21 = 6 (Constant) Thus, each term differs from its preceding term by 6. So, the given progression is an KIACK 3 AP. First term = 9Common difference =6Next term of the AP = 27 + 6 = 33The given progression 11, 6, 1, – 4,..... 100 (ii) Clearly, 6 - 11 = 1 - 6 = -4 - 1 = -5 (Constant) Thus, each term differs from its preceding term by 6. So, the given progression is an AP. First term = 11Common difference = -5Next term of the AP = -4 + (-5) = -9The given progression $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, \dots$ (iii) Clearly, $\frac{-5}{6} - (-1) = \frac{-2}{3} - (\frac{-5}{6}) = \frac{-1}{2} - (\frac{-2}{3}) = \frac{1}{6}$ (Constant) Thus, each term differs from its preceding term by $\frac{1}{6}$. So, the given progression is an AP. First term = -1Common difference = $\frac{1}{6}$ Next tern of the $AP = \frac{-1}{2} + \frac{1}{6} = \frac{-2}{6} = \frac{-1}{3}$

2.

(iv) The given progression $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$ This sequence can be written as $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}$ Clearly, $2\sqrt{2} - \sqrt{2} = 3\sqrt{2} - 2\sqrt{2} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$ (Constant) Thus, each term differs from its preceding term by $\sqrt{2}$, So, the given progression is an AP. First term = $\sqrt{2}$ Common difference = $\sqrt{2}$ Next tern of the $AP = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$ This given progression $\sqrt{20}, \sqrt{45}, \sqrt{80}, \sqrt{125}, \dots$ (v) This sequence can be re-written as $2\sqrt{5}, 3\sqrt{5}, 4\sqrt{5}, 5\sqrt{5}, \dots$ Clearly, $3\sqrt{5} - 2\sqrt{5} = 4\sqrt{5} - 3\sqrt{5} = 5\sqrt{5} - 4\sqrt{5} = \sqrt{5}$ (Constant) Thus, each term differs from its preceding term by $\sqrt{5}$. So, the given progression is an AP. Find: (i) the 20th term of the AP 9,13,17,21,..... (ii) the 35th term of AP 20,17,14,11,..... (iii) the 18th term of the AP $\sqrt{2}$, $\sqrt{18}$, $\sqrt{50}$, $\sqrt{98}$,.... (iv) the 9th term of the AP $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \dots$ (v) the 15the term of the AP $-40, -15, 10, 35, \dots$ Sol: The given *AP* is 9,13,17,21,..... (i) First term, a = 9Common difference, d = 13 - 9 = 4 n^{th} term of the AP, $a_n = a + (n-1)d = 9 + (n-1) \times 4$: 20th term of the AP, $a_{20} = 9 + (20 - 1) \times 4 = 9 + 76 = 85$ The given AP is 20,17,14,11,..... (ii) First term, a = 20Common difference, d = 17 - 20 = -3 n^{th} term of the AP, $a_n = a + (n-1)d = 20 + (n-1) \times (-3)$

3.

∴ 35th term of the AP,
$$a_{35} = 20 + (35-1) \times (-3) = 20 - 102 = -82$$

(iii) The given AP is $\sqrt{2}, \sqrt{18}, \sqrt{50}, \sqrt{98}, \dots$
First term, $a = \sqrt{2}$
Common difference, $d = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$
 n^{at} term of the AP, $a_n = a + (n-1)d = \sqrt{2} + (n-1) \times 2\sqrt{2}$
∴ 18th term of the AP, $a_{ns} = \sqrt{2} + (18-1) \times 2\sqrt{2} = \sqrt{2} + 34\sqrt{2} = 35\sqrt{2} = \sqrt{2450}$
(iv) The given AP is $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \dots$...
First term, $a = \frac{3}{4}$
Common difference, $d = \frac{5}{4} - \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$
 n^{at} term of the AP, $a_n = a + (n-1)d = \frac{3}{4} + (n-1) \times (\frac{1}{2})$
∴ 9^a term of the AP, $a_q = \frac{3}{4} + (9-1) \times \frac{1}{2} = \frac{3}{4} + 4 = \frac{19}{4}$
(v) The given AP is $-40, -15, 10, 35, \dots$...
First term, $a = -40$
Common difference, $d = -15 - (-40) = 25$
 n^{at} term of the AP, $a_n = a + (n-1)d = -40 + (n-1) \times 25$
∴ 15th term of the AP, $a_1 = -40 + (15-1) \times 25 = -40 + 350 = 310$
Find the 37th term of the AP, $6, 7\frac{3}{4}, 9\frac{1}{2}, 11\frac{1}{4}, \dots$...
First term, $a = 6$ and common difference, $d = 7\frac{3}{4} - 6 \Rightarrow \frac{31}{4} - 6 \Rightarrow \frac{31-24}{4} = \frac{7}{4}$
Now, $T_{57} = a + (37-1)d = a + 36d$
 $= 6 + 36 \times \frac{7}{4} = 6 + 63 = 69$
∴ 37^{ak} term = 69

- Find the 25th term of the AP 5, $4\frac{1}{2}$, 4, $3\frac{1}{2}$, 3,..... 4. Sol: The given AP is $5, 4\frac{1}{2}, 4, 3\frac{1}{2}, 3, \dots$ First term = 5Common difference $=4\frac{1}{2}-5 \Rightarrow \frac{9}{2}-5 \Rightarrow \frac{9-10}{2}=-\frac{1}{2}$ $\therefore a = 5 and d = -\frac{1}{2}$ Now, $T_{25} = a + (25 - 1)d = a + 24d$ pool.5, Hisch away $=5+24\times\left(-\frac{1}{2}\right)=5-12=-7$ $\therefore 25^{th}$ term = -7
- Find the nth term of each of the following Aps: 5. (i) 5, 11, 17, 23 ... (ii) 16, 9, 2, -5, Sol: **(i)** (6n − 1) (ii) (23 - 7n)
- If the nth term of a progression is (4n 10) show that it is an AP. Find its 6. (i) first term, (ii) common difference (iii) 16 the term. Sol:

$$T_{n} = (4_{n} - 10) \quad [\text{Given}]$$

$$T_{1} = (4 \times 1 - 10) = -6$$

$$T_{2} = (4 \times 2 - 10) = -2$$

$$T_{3} = (4 \times 3 - 10) = 2$$

$$T_{4} = (4 \times 4 - 10) = 6$$
Clearly, $[-2 - (-6)] = [2 - (-2)] = [6 - 2] = 4$ (Constant)
So, the terms $-6, -2, 2, 6, \dots$ forms an AP.
Thus we have
(i) First term $= -6$
(ii) Common difference $= 4$
(iii) $T_{16} = a + (n-1)d = a + 15d = -6 + 15 \times 4 = 54$

- How many terms are there in the AP 6,1 0, 14, 18,, 174? 7. Sol: In the given AP, a = 6 and d = (10-6) = 4Suppose that there are n terms in the given AP. Then, $T_n = 174$ $\Rightarrow a + (n-1)d = 174$ $\Rightarrow 6 + (n-1) \times 4 = 174$ \Rightarrow 2 + 4*n* = 174 $\Rightarrow 4n = 172$ P. Hisch away \Rightarrow *n* = 43 Hence, there are 43 terms in the given AP. How many terms are there in the AP 41, 38, 35, ...,82
- 8. Sol:

In the given AP, a = 41 and d = (38-41) = -3Suppose that there are n terms in the given AP. Then $T_n = 8$

- $\Rightarrow a + (n-1)d = 8$
- \Rightarrow 41+(n-1)×(-3) = 8
- \Rightarrow 44 3*n* = 8
- \Rightarrow 3*n* = 36

$$\Rightarrow n = 12$$

Hence, there are 12 terms in the given AP.

How many terms are there in the AP 18,15 $\frac{1}{2}$,13,....,-47.? 9.

Sol:

The given AP is $18, 15\frac{1}{2}, 13, \dots, -47$.

First term, a = 18

Common difference, $d = 15\frac{1}{2} - 18 = \frac{31}{2} - 18 = \frac{31 - 36}{2} = -\frac{5}{2}$

Suppose there re n terms in the given AP. Then,

$$a_n = -47$$

$$\Rightarrow 18 + (n-1) \times \left(-\frac{5}{2}\right) = -47 \qquad \left[a_n = a + (n-1)d\right]$$

$$\Rightarrow -\frac{5}{2}(n-1) = -47 - 18 = -65$$

$$\Rightarrow n-1 = -65 \times \left(-\frac{2}{5}\right) = 26$$

$$\Rightarrow n = 26 + 1 = 27$$

Hence, there are 27 terms in the given AP.

10. Which term of term of the AP 3,8,13,18,.... is 88? Sol:

In the given AP, first term, a = 3 and common difference, a = (8-3) = 5.

Let's its n^{th} term be 88

Then,
$$T_n = 88$$

 $\Rightarrow a + (n-1)d = 88$

$$\Rightarrow 3 + (n-1) \times 5 = 88$$

$$\Rightarrow 5n-2=88$$

$$\Rightarrow 5n = 90$$

 $\Rightarrow n = 18$

Hence, the 18th term of the given AP is 88.

11. Which term of AP 72,68,64,60,... is 05 Sol:

In the given AP, first term, a = 72 and common difference, d = (68 - 72) = -4.

Let its
$$n^{th}$$
 term be 0.
Then, $T_n = 0$
 $\Rightarrow a + (n-1)d = 0$
 $\Rightarrow 72 + (n-1) \times (-4) = 0$
 $\Rightarrow 76 - 4n = 0$
 $\Rightarrow 4n = 76$
 $\Rightarrow n = 19$

Hence, the 19^{th} term of the given AP is 0.

Which term of the AP $\frac{5}{6}$, 1, 1 $\frac{1}{6}$, 1 $\frac{1}{3}$, is 3? 12. Sol: In the given AP, first term = $\frac{5}{6}$ and common difference, $d = \left(1 - \frac{5}{6} = \frac{1}{6}\right)$ Let its n^{th} term be 3. Now, $T_n = 3$ $\Rightarrow a + (n-1)d = 3$ $\Rightarrow \frac{5}{6} + (n-1) \times \frac{1}{6} = 3$ -81? thooks, there are a set of the set of t $\Rightarrow \frac{2}{3} + \frac{n}{6} = 3$ $\Rightarrow \frac{n}{6} = \frac{7}{3}$ $\Rightarrow n = 14$ Hence, the 14^{th} term of the given AP is 3. Which term of the AP 21, 18, 15, ..., is -81? 13. Sol: The given AP is 21,18,15,..... First term, a = 21Common difference, d = 18 - 21 = -3Suppose n^{th} term of the given AP is -81, then, $a_n = -81$ $\Rightarrow 21 + (n-1) \times (-3) = -81 \qquad \left[a_n = a + (n-1)d\right]$ $\Rightarrow -3(n-1) = -81 - 21 = -102$ $\Rightarrow n-1=\frac{102}{3}=34$ \Rightarrow n = 34 + 1 = 35Hence, the 35th term of the given AP is -81. Which term of the AP 3,8, 13,18,.... Will be 55 more than its 20th term? 14. Sol: Here, a = 3 and d = (8-3) = 5The 20th term is given by $T_{20} = a + (20 - 1)d = a + 19d = 3 + 19 \times 5 = 98$

 \therefore Required term = (98+55) = 153

Then $T_n = 153$

 \Rightarrow 5*n* = 155

Let this be the n^{th} term. \Rightarrow 3+(n-1)×5=153

 \Rightarrow *n* = 31 Hence, the 31st term will be 55 more than 20th term.

Which term of the AP 5, 15, 25, will be 130 more than its 31st term? 15. Sol:

Here, a = 5 and d = (15-5) = 10The 31st term is given by $T_{31} = a + (31 - 1)d = a + 30d = 5 + 30 \times 10 = 305$: Required term = (305 + 130) = 435Let this be the n^{th} term. Then, $T_n = 435$ \Rightarrow 5+(*n*-1)×10 = 435 $\Rightarrow 10n = 440$ $\Rightarrow n = 44$ Hence, the 44th term will be 130 more than its 31st term.

If the 10th term of an AP is 52 and 17th term is 20 more than its 13th term, find the AP 16. Sol:

In the given AP, let the first term be a and the common difference be d.

Then, $T_n = a + (n-1)d$ Now, we have: $T_{10} = a + (10 - 1)d$(1) $\Rightarrow a + 9d = 52$ $T_{13} = a + (13 - 1)d = a + 12d$(2)(3) $T_{17} = a + (17 - 1)d = a + 16d$ But, it is given that $T_{17} = 20 + T_{13}$ i.e., a + 16d = 20 + a + 12d $\Rightarrow 4d = 20$ $\Rightarrow d = 5$ On substituting d = 5 in (1), we get:

 $a + 9 \times 5 = 52$ $\Rightarrow a = 7$ Thus, a = 7 and d = 5 \therefore The terms of the AP are 7,12,17,22,..... Find the middle term of the AP 6, 13, 20,, 216. 17. Sol: The given AP is 6,13,20,.....,216. First term, a = 6Common difference, d = 13 - 6 = 7Suppose these are *n* terms in the given AP. Then, $a_n = 216$ $\Rightarrow 6 + (n-1) \times 7 = 216$ $\begin{bmatrix} a_n = a + (n-1)d \end{bmatrix}$ \Rightarrow 7(n-1) = 216-6 = 210 $\Rightarrow n-1=\frac{210}{7}=30$ \Rightarrow n = 30 + 1 = 31Thus, the given AP contains 31 terms, : Middle term of the given AP $=\left(\frac{31+1}{2}\right)th$ term =16th term $= 6 + (16 - 1) \times 7$ = 6 + 105=111Hence, the middle term of the given AP is 111. 18. Find the middle term of the AP 10, 7, 4,, (-62). Sol: The given AP is 10,7,4,....,-62.

First term, a = 10

Common difference, d = 7 - 10 = -3

Suppose these are *n* terms in the given AP. Then,

$$a_{n} = -62$$

$$\Rightarrow 10 + (n-1) \times (-3) = -62 \qquad [a_{n} = a + (n-1)d]$$

$$\Rightarrow -3(n-1) = -62 - 10 = -72$$

$$\Rightarrow n - 1 = \frac{72}{3} = 24$$

$$\Rightarrow n = 24 + 1 = 25$$
Thus, the given AP contains 25 terms.

$$\therefore \text{ Middle term of the given AP}$$

$$= \left(\frac{25 + 1}{2}\right)th \text{ term}$$

$$= 13th \text{ term}$$

$$= 10 + (13 - 1) \times (-3)$$

$$= 10 - 36$$

$$= -26$$
Hence, the middle term of the given AP is -26.
Find the sum of two middle most terms of the AP $= \frac{4}{3}, -1, \frac{-2}{3}, ..., 4\frac{1}{3}.$
Sol:

19. Find the sum of two middle most terms of the AP

Sol:

The given AP is -

First term, a = -

1

 $\frac{4}{3} = \frac{1}{3}$ Common difference, d = -1-1+

3

Suppose there are n terms in the given AP. Then,

$$a_n = 4\frac{1}{3}$$

$$\Rightarrow -\frac{4}{3} + (n-1) \times \left(\frac{1}{3}\right) = \frac{13}{3} \qquad \left[a_n = a + (n-1)d\right]$$

$$\Rightarrow \frac{1}{3}(n-1) = \frac{13}{3} + \frac{4}{3} = \frac{17}{3}$$

$$\Rightarrow n-1 = 17$$

$$\Rightarrow n = 17 + 1 = 18$$
Thus, the given AP contains 18 terms. So, there are two

o middle terms in the given AP. The middle terms of the given AP are $\left(\frac{18}{2}\right)th$ terms and $\left(\frac{18}{2}+1\right)th$ term i.e. 9th term and 10th term.

3

$$= \left[-\frac{-3}{3} + (9-1) \times \frac{-3}{3} \right] + \left[-\frac{-3}{3} + (10-1) \times \frac{-3}{3} \right]$$
$$= -\frac{4}{3} + \frac{8}{3} - \frac{4}{3} + 3$$
$$= 3$$

Hence, the sum of the middle most terms of the given AP is 3.

20. Find the 8th term from the end of the AP 7, 10, 13,, 184. Sol: Here, a = 7 and d = (10-7) = 3, l = 184 and $n = 8^{th}$ form the end.

Now, nth term from the end = $\left[l - (n-1)d\right]$

 8^{th} term from the end = $\begin{bmatrix} 184 - (8 - 1) \times 3 \end{bmatrix}$

$$= [184 - (8 - 1) \times 3]$$

= $[184 - (7 \times 3)] = (184 - 21) = 163$
om the end is 163.

Hence, the 8th term from the end is 163.

21. Find the 6th term form the end of the AP 17, 14, 11,, (-40). **Sol:**

Here, a = 7 and d = (14-17) = -3, l = (-40) and n = 6Now, nth term from the end $= \lceil l - (n-1)d \rceil$

6th term from the end = $\left[\left(-40 \right) - \left(6 - 1 \right) \times \left(-3 \right) \right]$

$$= [-40+(5\times3)] = (-40+15) = -25$$

Hence, the 6^{th} term from the end is -25.

22. Is 184 a term of the AP 3, 7, 11, 15,? Sol: The given AP is 3,7,11,15,..... Here, a = 3 and d = 7 - 3 = 4Let the nth term of the given AP be 184. Then, $a_n = 184$ $\Rightarrow 3 + (n-1) \times 4 = 184$ $a_n = a + (n-1)d$ $\Rightarrow 4n - 1 = 184$

 $\Rightarrow 4n = 185$ $\Rightarrow n = \frac{185}{4} = 46\frac{1}{4}$ But, the number of terms cannot be a fraction. Hence, 184 is not a term of the given AP. **23.** Is -150 a term of the AP 11, 8, 5, 2,? Sol: The given AP is 11, 8, 5, 2, Here, a = 11 and d = 8 - 11 = -3Let the nth term of the given AP be -150. Then, $a_n = -150$ Hisch away $\left[a_n = a + (n-1)d\right]$ $\Rightarrow 11 + (n-1) \times (-3) = -150$ $\Rightarrow -3n+14 = -150$ $\Rightarrow -3n = -164$ $\Rightarrow n = \frac{164}{3} = 54\frac{2}{3}$

But, the number of terms cannot be a fraction. Hence, -150 is not a term of the given AP.

Which term of the AP 121, 117, 113, is its first negative term? 24. Sol:

The given AP is 121, 117, 113, Here, a = 121 and d = 117 - 121 = -4Let the nth term of the given AP be the first negative term. Then, a < 0

$$a_n < 0$$

$$\Rightarrow 121 + (n-1) \times (-4) < 0 \qquad [a_n = a + (n-1)d]$$

$$\Rightarrow 125 - 4n < 0$$

$$\Rightarrow -4n < -125$$

$$\Rightarrow n > \frac{125}{4} = 31\frac{1}{4}$$

$$\therefore n = 32$$

Hence, the 32^{nd} term is the first negative term of the given AP.

25. Which term of the AP $20,19\frac{1}{4},18\frac{1}{2},17\frac{3}{4},\dots$ is the first negative term?

Sol:

The given AP is $20,19\frac{1}{4},18\frac{1}{2},17\frac{3}{4},...$ Here, a = 20 and $d = 19\frac{1}{4} - 20 = \frac{77}{4} - 20 = \frac{77 - 80}{4} = -\frac{3}{4}$

Let the nth term of the given AP be the first negative term. Then, $a_n < 0$

$$\Rightarrow 20 + (n-1) \times \left(-\frac{3}{4}\right) < 0 \qquad \left[a_n = a + (n-1)d\right]$$

$$\Rightarrow 20 + \frac{3}{4} - \frac{3}{4}n < 0$$

$$\Rightarrow \frac{83}{4} - \frac{3}{4}n < 0$$

$$\Rightarrow -\frac{3}{4}n < -\frac{83}{4}$$

$$\Rightarrow n > \frac{83}{3} = 27\frac{2}{3}$$

$$\therefore n = 28$$

Hence, the 28th term is the first negative term of the given AP.

26. The 7^{th} term of the an AP is -4 and its 13^{th} term is -16. Find the AP.

Sol: We have

We have $T_7 = a + (n-1)d$ $\Rightarrow a + 6d = -4$ $T_{13} = a + (n-1)d$ $\Rightarrow a + 12d = -16$ (2)On solving (1) and (2), we get

a = 8 and d = -2

Thus, first term = 8 and common difference = -2

 \therefore The term of the AP are 8, 6, 4, 2,

27. The 4th term of an AP is zero. Prove that its 25th term is triple its 11th term.
Sol:

In the given AP, let the first be a and the common difference be d. Then T = a + (n - 1)d

Then, $T_n = a + (n-1)d$

Now, $T_4 = a + (4-1)d$ $\Rightarrow a + 3d = 0$ (1) $\Rightarrow a = -3d$ Again, $T_{11} = a + (11-1)d = a + 10d$ = -3d + 10d = 7d [Using (1)] Also, $T_{25} = a + (25-1)d = a + 24d = -3d + 24d = 21d$ [Using (1)] i.e., $T_{25} = 3 \times 7d = (3 \times T_{11})$ Hence, 25th term is triple its 11th term.

28. The 8th term of an AP is zero. Prove that its 38th term is triple its 18th term.Sol:

Let a be the first term and d be the common difference of the AP. Then,

$$a_{8} = 0 \qquad \left[a_{n} = a + (n-1)d\right]$$

$$\Rightarrow a + (8-1)d = 0$$

$$\Rightarrow a + 7d = 0$$

$$\Rightarrow a = -7d \qquad \dots\dots(1)$$
Now,
$$\frac{a_{38}}{a_{18}} = \frac{a + (38-1)d}{a + (18-1)d}$$

$$\Rightarrow \frac{a_{38}}{a_{18}} = \frac{-7d + 37d}{-7d + 17d}$$

$$[From (1)]$$

$$\Rightarrow \frac{a_{38}}{a_{18}} = \frac{30d}{10d} = 3$$

$$\Rightarrow a_{38} = 3 \times a_{18}$$

Hence, the 38th term of the AP id triple its 18th term.

29. The 4th term of an AP is 11. The sum of the 5th and 7th terms of this AP is 34. Find its common difference

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$$a_{4} = 11$$

$$\Rightarrow a + (4-1)d = 11 \qquad [a_{n} = a + (n-1)d]$$

$$\Rightarrow a + 3d = 11 \qquad \dots \dots (1)$$

Now,

$$a_{5} + a_{7} = 34 \qquad (Given)$$

 $\Rightarrow (a+4d) + (a+6d) = 34$ $\Rightarrow 2a+10d = 34$ $\Rightarrow a+5d = 17 \qquad \dots \dots (2)$ From (1) and (2), we get 11-3d+5d = 17 $\Rightarrow 2d = 17-11 = 6$ $\Rightarrow d = 3$ Hence, the common difference of the AP is 3.

30. The 9th term of an AP is -32 and the sum of its 11th and 13th terms is -94. Find the common difference of the AP.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$$a_{9} = -32$$

$$\Rightarrow a + (9-1)d = -32 \qquad [a_{n} = a + (n-1)d]$$

$$\Rightarrow a + 8d = -32 \qquad \dots \dots (1)$$
Now,
$$a_{11} + a_{13} = -94 \qquad (Given)$$

$$\Rightarrow (a + 10d) + (a + 12d) = -94$$

$$\Rightarrow 2a + 22d = -94$$

$$\Rightarrow a + 11d = -47 \qquad \dots (2)$$
From (1) and (2), we get
$$-32 - 8d + 11d = -47$$

$$\Rightarrow 3d = -47 + 32 = -15$$

$$\Rightarrow d = -5$$

Hence, the common difference of the AP is -5.

31. Determine the nth term of the AP whose 7th term is -1 and 16th term is 17.Sol:

Let a be the first term and d be the common difference of the AP. Then, $a_7 = -1$ $\Rightarrow a + (7-1)d = -1$ $[a_n = a + (n-1)d]$ $\Rightarrow a + 6d = -1$ (1) Also, $a_{16} = 17$ $\Rightarrow a + 15d = 17$ (2) From (1) and (2), we get -1-6d+15d = 17 $\Rightarrow 9d = 17+1=18$ $\Rightarrow d = 2$ Putting d = 2 in (1), we get $a+6\times 2 = -1$ $\Rightarrow a = -1-12 = -13$ $\therefore a_n = a + (n-1)d$ $= -13 + (n-1) \times 2$ = 2n-15Hence, the nth term of the AP is (2n-15).

32. If 4 times the 4th term of an AP is equal to 18 times its 18th term then find its 22nd term.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

 $4 \times a_4 = 18 \times a_{18} \quad \text{(Given)}$ $\Rightarrow 4(a+3d) = 18(a+17d)$

$$\Rightarrow 4(a+3d) = 18(a+17d)$$

$$\Rightarrow 2(a+3d) = 9(a+17d)$$

$$\Rightarrow 2a+6d = 9a+153d$$

$$\Rightarrow 7a = -147d$$

$$\Rightarrow a = -21d$$

$$\Rightarrow a + 21d = 0$$

$$\Rightarrow a + (22-1)d = 0$$

$$\Rightarrow a_{22} = 0$$

Hence, the 22^{nd} term of the AP is 0.

33. If 10 times the 10th term of an AP is equal to 15 times the 15th term, show that its 25th term is zero.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$$10 \times a_{10} = 15 \times a_{15} \text{ (Given)}$$

$$\Rightarrow 10(a+9d) = 15(a+14d) \qquad [a_n = a + (n-1)d]$$

$$\Rightarrow 2(a+9d) = 3(a+14d)$$

$$\Rightarrow 2a+18d = 3a+42d$$

$$\Rightarrow a = -24d$$

 $\Rightarrow a + 24d = 0$ $\Rightarrow a + (25 - 1)d = 0$ $\Rightarrow a_{25} = 0$ Hence, the 25th term of the AP is 0.

34. Find the common difference of an AP whose first term is 5 and the sum of its first four terms is half the sum of the next four terms.

Sol: Let the common difference of the AP be d. First term, a = 5Now, $a_1 + a_2 + a_3 + a_4 = \frac{1}{2}(a_5 + a_6 + a_7 + a_8)$ (Given) $\Rightarrow a + (a + d) + (a + 2d) + (a + 3d) = \frac{1}{2}[(a + 4d) + (a + 5d) + (a + 6d) + (a + 7d)]$ $[a_n = a + (n-1)d]$ $\Rightarrow 4a + 6d = \frac{1}{2}(4a + 22d)$ $\Rightarrow 8a + 12d = 4a + 22d$ $\Rightarrow 22d - 12d = 8a - 4a$ $\Rightarrow 10d = 4a$ $\Rightarrow d = \frac{2}{5}a$ $\Rightarrow d = \frac{2}{5}x5 = 2$ (a = 5)

Hence, the common difference of the AP is 2.

35. The sum of the 2nd and 7th terms of an AP is 30. If its 15th term is 1 less than twice its 8th term, find AP.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

Also, $a_{15} = 2a_8 - 1$ (Given)

 $\Rightarrow a+14d = 2(a+7d)-1$ $\Rightarrow a + 14d = 2a + 14d - 1$ $\Rightarrow -a = -1$ $\Rightarrow a = 1$ Putting a = 1 in (1), we get $2 \times 1 + 7d = 30$ $\Rightarrow 7d = 30 - 2 = 28$ $\Rightarrow d = 4$ So, $a_2 = a + d = 1 + 4 = 5$ $a_3 = a + 2d = 1 + 2 \times 4 = 9$ Hence, the AP is 1, 5, 9, 13,

36. For what value of n, the nth terms of the arithmetic progressions 63, 65, 67, ... and 3, 10, 17, ... are equal?

Sol:

Let the term of the given progressions be t_n and T_n , respectively

The first AP is 63, 65, 67,...

Let its first term be a and common difference be d. R textbook

Then a = 63 and d = (65 - 63) = 2

So, its nth term is given by

$$t_n = a + (n-1)d$$

$$\Rightarrow$$
 63+(n-1)×2

$$\Rightarrow 61 + 2n$$

The second AP is 3, 10, 17,... Let its first term be A and common difference be D. Then A = 3 and D = (10 - 3) = 7So, its nth term is given by $T_n = A + (n-1)D$

$$\Rightarrow$$
 3+(*n*-1)×

$$\Rightarrow$$
 7*n*-4

Now,
$$t_n = T_n$$

$$\Rightarrow 61 + 2n = 7n - 4$$

7

$$\Rightarrow 65 = 5n$$

$$\Rightarrow n = 13$$

Hence, the 13 terms of the Al's are the same.

37. The 17th term of AP is 5 more than twice its 8th term. If the 11th term of the AP is 43, find its nth term.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$$a_{17} = 2a_8 + 5 \quad (\text{Given})$$

$$\therefore a + 16d = 2(a + 7d) + 5 \quad [a_n = a + (n-1)d]$$

$$\Rightarrow a + 16d = 2a + 14d + 5$$

$$\Rightarrow a - 2d = -5 \qquad \dots\dots\dots(1)$$

Also,

$$a_{11} = 43 \quad (\text{Given})$$

$$\Rightarrow a + 10d = 43 \qquad \dots\dots\dots(2)$$

From (1) and (2), we get

$$-5 + 2d + 10d = 43$$

$$\Rightarrow 12d = 43 + 5 = 48$$

$$\Rightarrow d = 4$$

Putting $d = 4$ in (1), we get

$$a - 2 \times 4 = -5$$

$$\Rightarrow a = -5 + 8 = 3$$

$$\therefore a_n = a + (n-1)d$$

$$= 3 + (n-1) \times 4$$

$$= 4n - 1$$

Hence, the nth term of the AP is $(4n = 1)$

Hence, the nth term of the AP is (4n-1).
38. The 24th term of an AP is twice its 10th term. Show that its 72nd term is 4 times its 15th term. Sol:

Let a be the first term and d be the common difference of the AP. Then,

$$a_{24} = 2a_{10} \quad (Given)$$

$$\Rightarrow a + 23d = 2(a + 9d) \qquad [a_n = a + (n - 1)d]$$

$$\Rightarrow a + 23d = 2a + 18d$$

$$\Rightarrow 2a - a = 23d - 18d$$

$$\Rightarrow a = 5d \qquad \dots\dots\dots(1)$$
Now,
$$\frac{a_{72}}{a_{15}} = \frac{a + 71d}{a + 14d}$$

 $\Rightarrow \frac{a_{72}}{a_{15}} = \frac{5d + 71d}{5d + 14d} \qquad [From (1)]$ $\Rightarrow \frac{a_{72}}{a_{15}} = \frac{76d}{19d} = 4$ $\Rightarrow a_{72} = 4 \times a_{15}$ Hence, the 72nd term of the AP is 4 times its 15th term.

39. The 19th term of an AP is equal to 3 times its 6th term. If its 9th term is 19, find the AP. **Sol:**

Let a be the first term and d be the common difference of the AP. Then,

$$a_{19} = 3a_{6} \qquad (Given)$$

$$\Rightarrow a + 18d = 3(a + 5d) \qquad \left[a_{n} = a + (n-1)d\right]$$

$$\Rightarrow a + 18d = 3a + 15d$$

$$\Rightarrow 3a - a = 18d - 15d$$

$$\Rightarrow 2a = 3d \qquad \dots \dots (1)$$
Also,

$$a_{9} = 19 \qquad (Given)$$

$$\Rightarrow a + 8d = 19 \qquad \dots \dots (2)$$
From (1) and (2), we get

$$\frac{3d}{2} + 8d = 19$$

$$\Rightarrow \frac{3d + 16d}{2} = 19$$

$$\Rightarrow 19d = 38$$

$$\Rightarrow d = 2$$
Putting $d = 2$ in (1), we get

$$2a = 3 \times 2 = 6$$

$$\Rightarrow a = 3$$
So,

$$a_{2} = a + d = 3 + 2 = 5$$

$$a_{3} = a + 2d = 3 + 2 \times 2 = 7, \dots \dots$$
Hence, the AP is 3,5,7,9, \dots …

40. If the pth term of an AP is q and its qth term is p then show that its (p + q)th term is zero.Sol:

In the given AP, let the first be a and the common difference be d.

Then, $T_n = a + (n-1)d$ $\Rightarrow Tp = a + (p-1)d = q$ (i) $\Rightarrow T_q = a + (q-1)d = p$ (ii) On subtracting (i) from (ii), we get: (q-p)d = (p-q) $\Rightarrow d = -1$ Putting d = -1 in (i), we get: a = (p+q-1)Thus, a = (p+q-1) and d = -1Now, $T_{p+q} = a + (p+q-1)d$ = (p+q-1) + (p+q-1)(-1) = (p+q-1) - (p+q-1) = 0Hence, the $(p+q)^{th}$ term is 0 (zero).

41. The first and last terms of an AP are a and I respectively. Show that the sum of the nth term from the beginning and the nth term form the end is (a+1).Sol:

2 3143

In the given AP, first term = a and last term = l. Let the common difference be d. Then, nth term from the beginning is given by $T_n = a + (n-1)d$ (1) Similarly, nth term from the end is given by $T_n = \{l - (n-1)d\}$ (2) Adding (1) and (2), we get $a + (n-1)d + \{l - (n-1)d\}$ = a + (n-1)d + l - (n-1)d

$$= a + 1$$

Hence, the sum of the nth term from the beginning and the nth term from the end (a+1).

42. How many two-digit number are divisible by 6?

Sol:

The two digit numbers divisible by 6 are 12, 18, 24,...., 96 Clearly, these number are in AP. Here, a = 12 and d = 18 - 12 = 6

Let this AP contains n terms. Then,

 $a_n = 96$ $\Rightarrow 12 + (n-1) \times 6 = 96 \qquad [a_n = a + (n-1)d]$ $\Rightarrow 6n + 6 = 96$ $\Rightarrow 6n = 96 - 6 = 90$ $\Rightarrow n = 15$ Hence, these are 15 two-digit numbers divisible by 6.

43. How many two-digits numbers are divisible by 3?

Sol:

The two-digit numbers divisible by 3 are 12, 15, 18, ..., 99. Clearly, these number are in AP. Hitch away Here, a = 12 and d = 15 - 12 = 3Let this AP contains n terms. Then, $a_{n} = 99$ $\Rightarrow 12 + (n-1) \times 3 = 99 \qquad \left[a_n = a + (n-1)d\right]$ $\Rightarrow 3n+9 = 99$ $\Rightarrow 3n + 9 = 99$ $\Rightarrow 3n = 99 - 9 = 90$ $\Rightarrow n = 30$ Hence, there are 30 two-digit numbers divisible by 3. 44. How many three-digit numbers are divisible by 9? Sol: The three-digit numbers divisible by 9 are 108, 117, 126,...., 999. Clearly, these number are in AP. Here. a = 108 and d = 117 - 108 = 9

Let this AP contains n terms. Then.

 $a_n = 999$

 $\Rightarrow 108 + (n-1) \times 9 = 999 \qquad \qquad \left[a_n = a + (n-1)d\right]$ $\Rightarrow 9n + 99 = 999$

 $\Rightarrow 9n = 999 - 99 = 900$

$$\Rightarrow n = 100$$

Hence: there are 100 three-digit numbers divisible by 9.

45. Hoe many numbers are there between 101 and 999, which are divisible by both 2 and 5?Sol:

The numbers which are divisible by both 2 and 5 are divisible by 10 also. Now, the numbers between 101 and 999 which are divisible 10 are 110, 120, 130, ..., 990. Clearly, these number are in AP Here, a = 110 and d = 120 - 110 = 10Let this AP contains n terms. Then, $a_n = 990$ $\Rightarrow 110 + (n-1) \times 10 = 990$ $\left[a_n = a + (n-1)d\right]$ $\Rightarrow 10n + 100 = 990$ $\Rightarrow 10n = 990 - 100 = 890$ $\Rightarrow n = 89$ Hence, there are 89 numbers between 101 and 999 which are divisible by both 2 and 5.

46. In a flower bed, there are 43 rose plants in the first row, 41 in second, 39 in the third, and so on. There are 11 rose plants in the last row. How many rows are there in the flower bed? Sol:

The numbers of rose plants in consecutive rows are 43, 41, 39,..., 11. Difference of rose plants between two consecutive rows = (41 - 43) = (39 - 41) = -2[Constant] So, the given progression is an AP Here, first term = 43 Common difference = -2Last term 11 Let *n* be the last term, then we have: $T_n = a + (n-1)d$ $\Rightarrow 11 = 43 + (n-1)(-2)$ $\Rightarrow 11 = 45 - 2n$ $\Rightarrow 34 = 2n$ $\Rightarrow n = 17$ Hence, the 17th term is 11 or there are 17 rows in the flower bed.

47. A sum of ₹2800 is to be used to award four prizes. If each prize after the first is ₹200 less than the preceding prize, find the value of each of the prizes.

Sol:

Let the amount of the first prize be ₹a

Since each prize after the first is ₹200 less than the preceding prize, so the amounts of the four prizes are in AP.

Amount of the second prize $= \Re (a - 200)$

Amount of the third prize $= \neq (a - 2 \times 200) = (a - 400)$

Amount of the fourth prize $= = (a - 3 \times 200) = (a - 600)$

Now,

Total sum of the four prizes = 2,800 $\therefore \exists a + \exists (a - 200) + \exists (a - 400) + \exists (a - 600) = \exists 2,800$ $\Rightarrow 4a - 1200 = 2800$ $\Rightarrow 4a = 2800 + 1200 = 4000$ $\Rightarrow a = 1000$ $\therefore \text{ Amount of the first prize} = \exists 1,000$ Amount of the second prize = $\exists (1000 - 200) = \exists 800$ Amount of the third prize = $\exists (1000 - 400) = \exists 600$ Amount of the fourth prize = $\exists (1000 - 600) = \exists 400$ Hence, the value of each of the prizes is $\exists 1,000, \exists 800, \exists 600 \text{ and } \exists 400$.