$\Rightarrow 4n = 185$

$\Rightarrow n = \frac{185}{4} = 46\frac{1}{4}$

But, the number of terms cannot be a fraction.
Hence, 184 is not a term of the given AP.

**23.** Is -150 a term of the AP 11, 8, 5, 2, ……?
**Sol:**
The given AP is 11, 8, 5, 2, ……
Here, $a = 11$ and $d = 8 - 11 = -3$
Let the nth term of the given AP be $-150$. Then,

$a_n = -150$

$\Rightarrow 11 + (n-1)\times(-3) = -150 \qquad [a_n = a + (n-1)d]$

$\Rightarrow -3n + 14 = -150$

$\Rightarrow -3n = -164$

$\Rightarrow n = \frac{164}{3} = 54\frac{2}{3}$

But, the number of terms cannot be a fraction.
Hence, -150 is not a term of the given AP.

**24.** Which term of the AP 121, 117, 113, …. is its first negative term?
**Sol:**
The given AP is 121, 117, 113, …….
Here, $a = 121$ and $d = 117 - 121 = -4$
Let the nth term of the given AP be the first negative term. Then,

$a_n < 0$

$\Rightarrow 121 + (n-1)\times(-4) < 0 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 125 - 4n < 0$

$\Rightarrow -4n < -125$

$\Rightarrow n > \frac{125}{4} = 31\frac{1}{4}$

$\therefore n = 32$

Hence, the 32nd term is the first negative term of the given AP.

**25.** Which term of the AP $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, .....$ is the first negative term?

**Sol:**

The given AP is $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, .....$

Here, $a = 20$ and $d = 19\frac{1}{4} - 20 = \frac{77}{4} - 20 = \frac{77-80}{4} = -\frac{3}{4}$

Let the nth term of the given AP be the first negative term. Then,

$a_n < 0$

$\Rightarrow 20 + (n-1) \times \left(-\frac{3}{4}\right) < 0 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 20 + \frac{3}{4} - \frac{3}{4}n < 0$

$\Rightarrow \frac{83}{4} - \frac{3}{4}n < 0$

$\Rightarrow -\frac{3}{4}n < -\frac{83}{4}$

$\Rightarrow n > \frac{83}{3} = 27\frac{2}{3}$

$\therefore n = 28$

Hence, the 28[th] term is the first negative term of the given AP.

**26.** The 7[th] term of the an AP is -4 and its 13[th] term is -16. Find the AP.

**Sol:**

We have

$T_7 = a + (n-1)d$

$\Rightarrow a + 6d = -4 \qquad ........(1)$

$T_{13} = a + (n-1)d$

$\Rightarrow a + 12d = -16 \qquad .......(2)$

On solving (1) and (2), we get

$a = 8 \text{ and } d = -2$

Thus, first term = 8 and common difference $= -2$

$\therefore$ The term of the AP are $8, 6, 4, 2, .........$

**27.** The 4[th] term of an AP is zero. Prove that its 25[th] term is triple its 11[th] term.

**Sol:**

In the given AP, let the first be a and the common difference be d.

Then, $T_n = a + (n-1)d$

Now, $T_4 = a + (4-1)d$

$\Rightarrow a + 3d = 0$ ......(1)

$\Rightarrow a = -3d$

Again, $T_{11} = a + (11-1)d = a + 10d$

$= -3d + 10d = 7d$ [Using (1)]

Also, $T_{25} = a + (25-1)d = a + 24d = -3d + 24d = 21d$ [Using (1)]

i.e., $T_{25} = 3 \times 7d = (3 \times T_{11})$

Hence, 25th term is triple its 11th term.

**28.** The 8th term of an AP is zero. Prove that its 38th term is triple its 18th term.

**Sol:**

Let a be the first term and d be the common difference of the AP. Then,

$a_8 = 0$ $[a_n = a + (n-1)d]$

$\Rightarrow a + (8-1)d = 0$

$\Rightarrow a + 7d = 0$

$\Rightarrow a = -7d$ .......(1)

Now,

$$\frac{a_{38}}{a_{18}} = \frac{a + (38-1)d}{a + (18-1)d}$$

$$\Rightarrow \frac{a_{38}}{a_{18}} = \frac{-7d + 37d}{-7d + 17d} \quad [From\ (1)]$$

$$\Rightarrow \frac{a_{38}}{a_{18}} = \frac{30d}{10d} = 3$$

$\Rightarrow a_{38} = 3 \times a_{18}$

Hence, the 38th term of the AP id triple its 18th term.

**29.** The 4th term of an AP is 11. The sum of the 5th and 7th terms of this AP is 34. Find its common difference

**Sol:**

Let a be the first term and d be the common difference of the AP. Then,

$a_4 = 11$

$\Rightarrow a + (4-1)d = 11$ $[a_n = a + (n-1)d]$

$\Rightarrow a + 3d = 11$ ........(1)

Now,

$a_5 + a_7 = 34$ (Given)

$\Rightarrow (a+4d)+(a+6d)=34$

$\Rightarrow 2a+10d=34$

$\Rightarrow a+5d=17$ ......(2)

From (1) and (2), we get

$11-3d+5d=17$

$\Rightarrow 2d=17-11=6$

$\Rightarrow d=3$

Hence, the common difference of the AP is 3.

**30.** The 9[th] term of an AP is -32 and the sum of its 11[th] and 13[th] terms is -94. Find the common difference of the AP.

**Sol:**

Let a be the first term and d be the common difference of the AP. Then,

$a_9=-32$

$\Rightarrow a+(9-1)d=-32$ $[a_n=a+(n-1)d]$

$\Rightarrow a+8d=-32$ ........(1)

Now,

$a_{11}+a_{13}=-94$ (Given)

$\Rightarrow (a+10d)+(a+12d)=-94$

$\Rightarrow 2a+22d=-94$

$\Rightarrow a+11d=-47$ ........(2)

From (1) and (2), we get

$-32-8d+11d=-47$

$\Rightarrow 3d=-47+32=-15$

$\Rightarrow d=-5$

Hence, the common difference of the AP is $-5$.

**31.** Determine the nth term of the AP whose 7[th] term is -1 and 16[th] term is 17.

**Sol:**

Let a be the first term and d be the common difference of the AP. Then,

$a_7=-1$

$\Rightarrow a+(7-1)d=-1$ $[a_n=a+(n-1)d]$

$\Rightarrow a+6d=-1$ ..........(1)

Also,

$a_{16}=17$

$\Rightarrow a+15d=17$ ........(2)

From (1) and (2), we get

$-1-6d+15d=17$

$\Rightarrow 9d=17+1=18$

$\Rightarrow d=2$

Putting $d=2$ in (1), we get

$a+6\times 2=-1$

$\Rightarrow a=-1-12=-13$

$\therefore a_n=a+(n-1)d$

$=-13+(n-1)\times 2$

$=2n-15$

Hence, the nth term of the AP is $(2n-15)$.

**32.** If 4 times the 4th term of an AP is equal to 18 times its 18th term then find its 22nd term.

**Sol:**

Let a be the first term and d be the common difference of the AP. Then,

$4\times a_4=18\times a_{18}$ (Given)

$\Rightarrow 4(a+3d)=18(a+17d)$ $\quad [a_n=a+(n-1)d]$

$\Rightarrow 2(a+3d)=9(a+17d)$

$\Rightarrow 2a+6d=9a+153d$

$\Rightarrow 7a=-147d$

$\Rightarrow a=-21d$

$\Rightarrow a+21d=0$

$\Rightarrow a+(22-1)d=0$

$\Rightarrow a_{22}=0$

Hence, the 22nd term of the AP is 0.

**33.** If 10 times the 10th term of an AP is equal to 15 times the 15th term, show that its 25th term is zero.

**Sol:**

Let a be the first term and d be the common difference of the AP. Then,

$10\times a_{10}=15\times a_{15}$ (Given)

$\Rightarrow 10(a+9d)=15(a+14d)$ $\quad [a_n=a+(n-1)d]$

$\Rightarrow 2(a+9d)=3(a+14d)$

$\Rightarrow 2a+18d=3a+42d$

$\Rightarrow a=-24d$

$\Rightarrow a + 24d = 0$

$\Rightarrow a + (25-1)d = 0$

$\Rightarrow a_{25} = 0$

Hence, the 25th term of the AP is 0.

**34.** Find the common difference of an AP whose first term is 5 and the sum of its first four terms is half the sum of the next four terms.

**Sol:**

Let the common difference of the AP be d.

First term, $a = 5$

Now,

$a_1 + a_2 + a_3 + a_4 = \frac{1}{2}(a_5 + a_6 + a_7 + a_8)$ (Given)

$\Rightarrow a + (a+d) + (a+2d) + (a+3d) = \frac{1}{2}[(a+4d) + (a+5d) + (a+6d) + (a+7d)]$

$[a_n = a + (n-1)d]$

$\Rightarrow 4a + 6d = \frac{1}{2}(4a + 22d)$

$\Rightarrow 8a + 12d = 4a + 22d$

$\Rightarrow 22d - 12d = 8a - 4a$

$\Rightarrow 10d = 4a$

$\Rightarrow d = \frac{2}{5}a$

$\Rightarrow d = \frac{2}{5} \times 5 = 2$ $(a = 5)$

Hence, the common difference of the AP is 2.

**35.** The sum of the 2nd and 7th terms of an AP is 30. If its 15th term is 1 less than twice its 8th term, find AP.

**Sol:**

Let a be the first term and d be the common difference of the AP. Then,

$a_2 + a_7 = 30$ (Given)

$\therefore (a+d) + (a+6d) = 30$ $[a_n = a + (n-1)d]$

$\Rightarrow 2a + 7d = 30$ .............(1)

Also,

$a_{15} = 2a_8 - 1$ (Given)

$\Rightarrow a+14d=2(a+7d)-1$

$\Rightarrow a+14d=2a+14d-1$

$\Rightarrow -a=-1$

$\Rightarrow a=1$

Putting $a$ = 1in (1), we get

$2\times1+7d=30$

$\Rightarrow 7d=30-2=28$

$\Rightarrow d=4$

So,

$a_2=a+d=1+4=5$

$a_3=a+2d=1+2\times4=9$..........

Hence, the AP is 1, 5, 9, 13, …….

**36.** For what value of n, the nth terms of the arithmetic progressions 63, 65, 67, … and 3, 10, 17, … are equal?

**Sol:**

Let the term of the given progressions be $t_n$ and $T_n$, respectively.

The first AP is 63, 65, 67,...

Let its first term be a and common difference be *d*.

Then a = 63 and d = (65 - 63) = 2

So, its nth term is given by

$t_n=a+(n-1)d$

$\Rightarrow 63+(n-1)\times2$

$\Rightarrow 61+2n$

The second AP is 3, 10, 17,...

Let its first term be A and common difference be D.

Then A = 3 and D = (10 - 3) = 7

So, its nth term is given by

$T_n=A+(n-1)D$

$\Rightarrow 3+(n-1)\times7$

$\Rightarrow 7n-4$

Now, $t_n=T_n$

$\Rightarrow 61+2n=7n-4$

$\Rightarrow 65=5n$

$\Rightarrow n=13$

Hence, the l3 terms of the Al's are the same.

**37.** The 17th term of AP is 5 more than twice its 8th term. If the 11th term of the AP is 43, find its nth term.

**Sol:**

Let a be the first term and d be the common difference of the AP. Then,

$a_{17} = 2a_8 + 5$ (Given)

$\therefore a + 16d = 2(a + 7d) + 5 \quad [a_n = a + (n-1)d]$

$\Rightarrow a + 16d = 2a + 14d + 5$

$\Rightarrow a - 2d = -5 \quad .........(1)$

Also,

$a_{11} = 43$ (Given)

$\Rightarrow a + 10d = 43 \quad ........(2)$

From (1) and (2), we get

$-5 + 2d + 10d = 43$

$\Rightarrow 12d = 43 + 5 = 48$

$\Rightarrow d = 4$

Putting $d = 4$ in (1), we get

$a - 2 \times 4 = -5$

$\Rightarrow a = -5 + 8 = 3$

$\therefore a_n = a + (n-1)d$

$= 3 + (n-1) \times 4$

$= 4n - 1$

Hence, the nth term of the AP is $(4n-1)$.

**38.** The 24th term of an AP is twice its 10th term. Show that its 72nd term is 4 times its 15th term.

**Sol:**

Let a be the first term and d be the common difference of the AP. Then,

$a_{24} = 2a_{10}$ (Given)

$\Rightarrow a + 23d = 2(a + 9d) \quad [a_n = a + (n-1)d]$

$\Rightarrow a + 23d = 2a + 18d$

$\Rightarrow 2a - a = 23d - 18d$

$\Rightarrow a = 5d \quad .......(1)$

Now,

$$\frac{a_{72}}{a_{15}} = \frac{a + 71d}{a + 14d}$$

$\Rightarrow \frac{a_{72}}{a_{15}} = \frac{5d+71d}{5d+14d}$ $\left[From (1)\right]$

$\Rightarrow \frac{a_{72}}{a_{15}} = \frac{76d}{19d} = 4$

$\Rightarrow a_{72} = 4 \times a_{15}$

Hence, the 72nd term of the AP is 4 times its 15th term.

**39.** The 19th term of an AP is equal to 3 times its 6th term. If its 9th term is 19, find the AP.

**Sol:**

Let a be the first term and d be the common difference of the AP. Then,

$a_{19} = 3a_6$ (Given)

$\Rightarrow a+18d = 3(a+5d)$ $\left[a_n = a+(n-1)d\right]$

$\Rightarrow a+18d = 3a+15d$

$\Rightarrow 3a - a = 18d - 15d$

$\Rightarrow 2a = 3d$ ...........(1)

Also,

$a_9 = 19$ (Given)

$\Rightarrow a+8d = 19$ ......(2)

From (1) and (2), we get

$\frac{3d}{2} + 8d = 19$

$\Rightarrow \frac{3d+16d}{2} = 19$

$\Rightarrow 19d = 38$

$\Rightarrow d = 2$

Putting $d = 2$ in (1), we get

$2a = 3 \times 2 = 6$

$\Rightarrow a = 3$

So,

$a_2 = a + d = 3 + 2 = 5$

$a_3 = a + 2d = 3 + 2 \times 2 = 7, ........$

Hence, the AP is $3, 5, 7, 9, ........$

**40.** If the pth term of an AP is q and its qth term is p then show that its (p + q)th term is zero.

**Sol:**

In the given AP, let the first be a and the common difference be d.

Then, $T_n = a+(n-1)d$

$\Rightarrow Tp = a+(p-1)d = q$ .......$(i)$

$\Rightarrow T_q = a+(q-1)d = p$ ........$(ii)$

On subtracting (i) from (ii), we get:

$(q-p)d = (p-q)$

$\Rightarrow d = -1$

Putting $d = -1$ in (i), we get:

$a = (p+q-1)$

Thus, $a = (p+q-1)$ *and* $d = -1$

Now, $T_{p+q} = a+(p+q-1)d$

$= (p+q-1)+(p+q-1)(-1)$

$= (p+q-1)-(p+q-1) = 0$

Hence, the $(p+q)^{th}$ term is 0 (zero).

**41.** The first and last terms of an AP are a and l respectively. Show that the sum of the nth term from the beginning and the nth term form the end is ( a + l ).

**Sol:**

In the given AP, first term = *a* and last term = *l*.

Let the common difference be *d*.

Then, nth term from the beginning is given by

$T_n = a+(n-1)d$ .....(1)

Similarly, nth term from the end is given by

$T_n = \{l-(n-1)d\}$ ......(2)

Adding (1) and (2), we get

$a+(n-1)d+\{l-(n-1)d\}$

$= a+(n-1)d+l-(n-1)d$

$= a+1$

Hence, the sum of the nth term from the beginning and the nth term from the end $(a+1)$.

**42.** How many two-digit number are divisible by 6?

**Sol:**

The two digit numbers divisible by 6 are 12, 18, 24,……, 96

Clearly, these number are in AP.

Here, $a = 12$ and $d = 18-12 = 6$

Let this AP contains n terms. Then,

$a_n = 96$

$\Rightarrow 12 + (n-1) \times 6 = 96$ $[a_n = a + (n-1)d]$

$\Rightarrow 6n + 6 = 96$

$\Rightarrow 6n = 96 - 6 = 90$

$\Rightarrow n = 15$

Hence, these are 15 two-digit numbers divisible by 6.

**43.** How many two-digits numbers are divisible by 3?

**Sol:**

The two-digit numbers divisible by 3 are 12, 15, 18, ..., 99.

Clearly, these number are in AP.

Here, $a = 12$ and $d = 15 - 12 = 3$

Let this AP contains n terms. Then,

$a_n = 99$

$\Rightarrow 12 + (n-1) \times 3 = 99$ $[a_n = a + (n-1)d]$

$\Rightarrow 3n + 9 = 99$

$\Rightarrow 3n = 99 - 9 = 90$

$\Rightarrow n = 30$

Hence, there are 30 two-digit numbers divisible by 3.

**44.** How many three-digit numbers are divisible by 9?

**Sol:**

The three-digit numbers divisible by 9 are 108, 117, 126,...., 999.

Clearly, these number are in AP.

Here. $a$ = 108 and $d$ = 117 – 108 = 9

Let this AP contains n terms. Then.

$a_n = 999$

$\Rightarrow 108 + (n-1) \times 9 = 999$ $[a_n = a + (n-1)d]$

$\Rightarrow 9n + 99 = 999$

$\Rightarrow 9n = 999 - 99 = 900$

$\Rightarrow n = 100$

Hence: there are 100 three-digit numbers divisible by 9.

**45.** Hoe many numbers are there between 101 and 999, which are divisible by both 2 and 5?

**Sol:**

The numbers which are divisible by both 2 and 5 are divisible by 10 also.

Now, the numbers between 101 and 999 which are divisible 10 are 110, 120, 130, .., 990.

Clearly, these number are in AP

Here, $a = 110$ and $d = 120 - 110 = 10$
Let this AP contains n terms. Then,
$a_n = 990$
$\Rightarrow 110 + (n-1) \times 10 = 990$   $[a_n = a + (n-1)d]$
$\Rightarrow 10n + 100 = 990$
$\Rightarrow 10n = 990 - 100 = 890$
$\Rightarrow n = 89$
Hence, there are 89 numbers between 101 and 999 which are divisible by both 2 and 5.

**46.** In a flower bed, there are 43 rose plants in the first row, 41 in second, 39 in the third, and so on. There are 11 rose plants in the last row. How many rows are there in the flower bed?
**Sol:**
The numbers of rose plants in consecutive rows are 43, 41, 39,..., 11.
Difference of rose plants between two consecutive rows = (41 – 43) = (39 – 41) = –2 [Constant]
So, the given progression is an AP
Here, first term = 43
Common difference = –2
Last term 11
Let $n$ be the last term, then we have:
$T_n = a + (n-1)d$
$\Rightarrow 11 = 43 + (n-1)(-2)$
$\Rightarrow 11 = 45 - 2n$
$\Rightarrow 34 = 2n$
$\Rightarrow n = 17$
Hence, the 17$^{th}$ term is 11 or there are 17 rows in the flower bed.

**47.** A sum of ₹2800 is to be used to award four prizes. If each prize after the first is ₹200 less than the preceding prize, find the value of each of the prizes.
**Sol:**
Let the amount of the first prize be ₹a
Since each prize after the first is ₹200 less than the preceding prize, so the amounts of the four prizes are in AP.
Amount of the second prize = ₹ $(a - 200)$
Amount of the third prize = ₹ $(a - 2 \times 200) = (a - 400)$
Amount of the fourth prize = ₹ $(a - 3 \times 200) = (a - 600)$
Now,

Total sum of the four prizes = 2,800

$\therefore$ ₹a+₹$(a-200)+$₹$(a-400)+$₹$(a-600)=$₹2,800

$\Rightarrow 4a-1200=2800$

$\Rightarrow 4a=2800+1200=4000$

$\Rightarrow a=1000$

$\therefore$ Amount of the first prize = ₹1,000

Amount of the second prize $=$₹$(1000-200)$=₹800

Amount of the third prize $=$₹$(1000=400)=$₹600

Amount of the fourth prize $=$₹$(1000-600)$ = ₹400

Hence, the value of each of the prizes is ₹1,000, ₹800, ₹600 and ₹400.