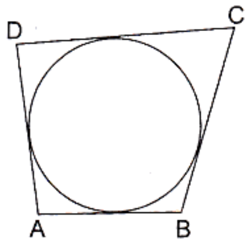
Exercise – 12B

1.



Sol:

We know that when a quadrilateral circumscribes a circle then sum of opposites sides is equal to the sum of other opposite sides.

$\therefore AB + CD = AD + BC$

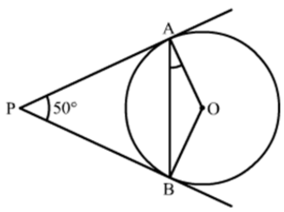
$\Rightarrow 6 + 8 = AD = 9$

$\Rightarrow AD = 5\,cm$

2.

Sol:

Construction: Join OB



We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OBP = \angle OAP = 90°$

Now, In quadrilateral AOBP

$\angle AOB + \angle OBP + \angle APB + \angle OAP = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle AOB + 90° + 50° + 90° = 360°$

$\Rightarrow 230° + \angle BOC = 360°$

$\Rightarrow \angle AOB = 130°$

Now, In isosceles triangle AOB

$\angle AOB + \angle OAB + \angle OBA = 180°$ [Angle sum property of a triangle]

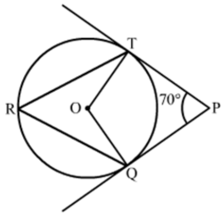
$\Rightarrow 130° + 2\angle OAB = 180°$ $[\because \angle OAB = \angle OBA]$

$\Rightarrow \angle OAB = 25°$

3.

Sol:

Construction: Join OQ and OT



We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OTP = \angle OQP = 90°$

Now, In quadrilateral OQPT

$\angle QOT + \angle OTP + \angle OQP + \angle TPO = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle QOT + 90° + 90° + 70° = 360°$

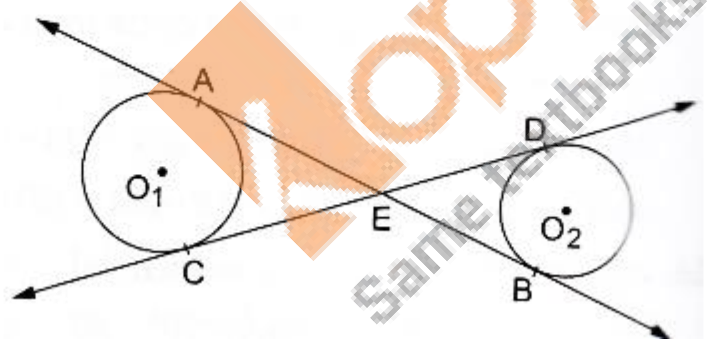
$\Rightarrow 250° + \angle QOT = 360°$

$\Rightarrow \angle QOT = 110°$

We know that the angle subtended by an arc at the center is double the angle subtended by the arc at any point on the remaining part of the circle.

$\therefore \angle TRQ = \frac{1}{2}(\angle QOT) = 55°$

4.



Sol:

We know that tangent segments to a circle from the same external point are congruent.

So, we have

$EA = EC$ for the circle having center O_1

and

$ED = EB$ for the circle having center O_1

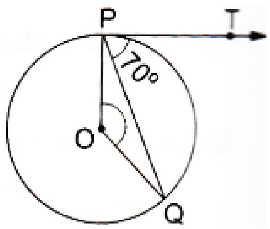
Now, Adding ED on both sides in EA = EC. we get

$EA + ED = EC + ED$

$\Rightarrow EA + EB = EC + ED$

$\Rightarrow AB = CD$

5.



Sol:

We know that the radius and tangent are perpendicular at their point of contact.

$\therefore \angle OPT = 90°$

Now, $\angle OPQ = \angle OPT - \angle TPQ = 90° - 70° = 20°$

Since, OP = OQ as both are radius

$\therefore \angle OPQ = \angle OQP = 20°$ (Angles opposite to equal sides are equal)

Now, In isosceles Δ POQ

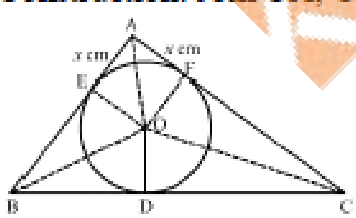
$\angle POQ + \angle OPQ + \angle OQP = 180°$ (Angle sum property of a triangle)

$\Rightarrow \angle POQ = 180° - 20° = 140°$

6.

Sol:

Construction: Join OA, OB, OC, OE $\perp$ AB at E and OF $\perp$ AC at F



We know that tangent segments to a circle from the same external point are congruent

Now, we have

$AE = AF$, $BD = BE = 4cm$ and $CD = CF = 3cm$

Now,

$Area(\Delta ABC) = Area(\Delta BOC) + Area(\Delta AOB) + Area(\Delta AOC)$

$\Rightarrow 21 = \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AB \times OE + \frac{1}{2} \times AC \times OF$

$\Rightarrow 42 = 7 \times 2 + (4 + x) \times 2 + (3 + x) \times 2$

$\Rightarrow 21 = 7 + 4 + x + 3 + x$

$\Rightarrow 21 = 14 + 2x$

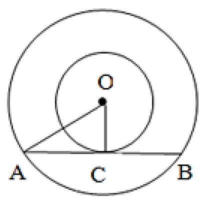
$\Rightarrow 2x = 7$

$\Rightarrow x = 3.5\,cm$

$\therefore AB = 4 + 3.5 = 7.5\,cm \text{ and } AC = 3 + 3.5 = 6.5\,cm$

7.

Sol:



Given Two circles have the same center O and AB is a chord of the larger circle touching the smaller circle at C; also. $OA = 5$ cm and $OC = 3$ cm

In $\Delta OAC, OA^2 = OC^2 + AC^2$

$\therefore AC^2 = OA^2 - OC^2$

$\Rightarrow AC^2 = 5^2 - 3^2$

$\Rightarrow AC^2 = 25 - 9$

$\Rightarrow AC^2 = 16$

$\Rightarrow AC = 4\,cm$

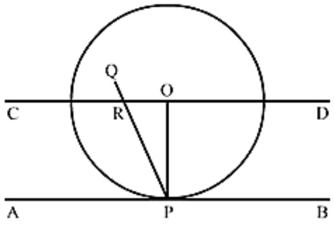
$\therefore AB = 2AC$ (Since perpendicular drawn from the center of the circle bisects the chord)

$\therefore AB = 2 \times 4 = 8\,cm$

The length of the chord of the larger circle is 8 cm.

8.

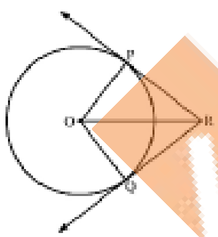
Sol:



Let AB be the tangent to the circle at point P with center O.
To prove: PQ passes through the point O.
Construction: Join OP.
Through O, draw a straight line CD parallel to the tangent AB.
Proof: Suppose that PQ doesn't passes through point O.
PQ intersect CD at R and also intersect AB at P
AS, CD || AB. PQ is the line of intersection.
$\angle ORP = \angle RPA$ (Alternate interior angles)
but also.
$\angle RPA = 90^\circ (OP \perp AB)$
$\Rightarrow \angle ORP = 90^\circ$
$\angle ROP + \angle OPA = 180^\circ$ (Co interior angles)
$\Rightarrow \angle ROP + 90^\circ = 180^\circ$
$\Rightarrow \angle ROP = 90^\circ$
Thus, the ΔORP has 2 right angles i.e., $\angle ORP$ and $\angle ROP$ which is not possible
Hence, our supposition is wrong
$\therefore$ PQ passes through the point O.

9.

Sol:



Construction Join PO and OQ
In ΔPOR *and* ΔQOR
$OP = OQ$ (Radii)
$RP = RQ$ (Tangents from the external point are congruent)
$OR = OR$ (Common)
By SSS congruency, $\Delta POR \cong \Delta QOR$
$\angle PRO = \angle QRO$ (C.P.C.T)

Now, $\angle PRO + \angle QRO = \angle PRQ$

$\Rightarrow 2\angle PRO = 120°$

$\Rightarrow \angle PRO = 60°$

Now. In ΔPOR

$\cos 60° = \frac{PR}{OR}$

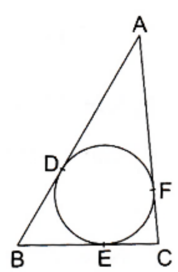
$\Rightarrow \frac{1}{2} = \frac{PR}{OR}$

$\Rightarrow OR = 2PR$

$\Rightarrow OR = PR + PR$

$\Rightarrow OR = PR + RQ$

10.



Sol:

We know that tangent segments to a circle from the same external point are congruent

Now, we nave

$AD = AF, BD = BE$ and $CE = CF$

Now $AD + BD = 14cm$ .....(1)

$AF + FC = 12cm$

$\Rightarrow AD + FC = 12cm$ ....(2)

$BE + EC = 8cm$

$\Rightarrow BD + FC = 8cm$ ....(3)

Adding all these we get

$AD + BD + AD + FC + BD + FC = 342$

$\Rightarrow 2(AD + BD + FC) = 34$

$\Rightarrow AD + BO + FC = 17cm$ ....(4)

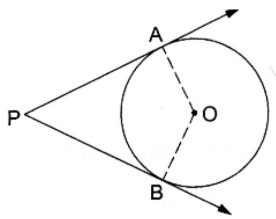
Solving (1) and (4), we get

$FC = 3cm$

Solving (2) and (4), we get

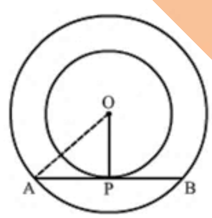
$BD = 5cm = BE$
Solving (3) and (4), we get
and $AD = 9cm$

11.



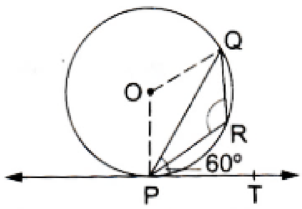
Sol:
We know that the radius and tangent are perpendicular at their point of contact
$\because \angle OBP = \angle OAP = 90°$
Now, In quadrilateral AOBP
$\angle APB + \angle AOB + \angle OBP + \angle OAP = 360°$ [Angle sum property of a quadrilateral]
$\Rightarrow \angle APB + \angle AOB + 90° + 90° = 360°$
$\Rightarrow \angle APB + \angle AOB = 180°$
Since, the sum of the opposite angles of the quadrilateral is $180°$
Hence, AOBP is a cyclic quadrilateral

12.
Sol:



We know that the radius and tangent are perpendicular at their point of contact
Since, the perpendicular drawn from the centre bisect the chord
$\therefore AP = PB = \frac{AB}{2} = 4cm$
In right triangle AOP
$AO^2 = OP^2 + PA^2$
$\Rightarrow 5^2 = OP^2 + 4^2$
$\Rightarrow OP^2 = 9$
$\Rightarrow OP = 3cm$
Hence, the radius of the smaller circle is 3 cm.

13.



Sol:

We know that the radius and tangent are perpendicular at their point of contact

$\therefore \angle OPT = 90°$

Now, $\angle OPQ = \angle OPT - \angle QPT = 90° - 60° = 30°$

Since, $OP = OQ$ as born is radius

$\therefore \angle OPQ = \angle OQP = 30°$ (Angles opposite to equal sides are equal)

Now, In isosceles, POQ

$\angle POQ + \angle OPQ + \angle OQP = 180°$ (Angle sum property of a triangle)

$\Rightarrow \angle POQ = 180° - 30° - 30° = 120°$

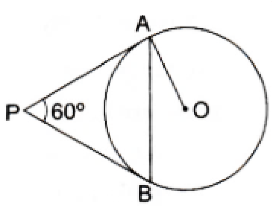
Now, $\angle POQ + \text{reflex } \angle POQ = 360°$ (Complete angle)

$\Rightarrow \text{reflex } \angle POQ = 360° - 120° = 240°$

We know that the angle subtended by an arc at the centre double the angle subtended by the arc at any point on the remaining part of the circle

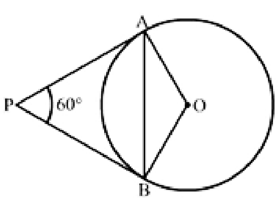
$\therefore \angle PRQ = \frac{1}{2}\left(reflex \angle POQ\right) = 120°$

14.



Sol:

Construction: Join OB



We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OBP = \angle OAP = 90°$

Now, In quadrilateral AOBP

$\angle AOB + \angle OBP + \angle APB + \angle OAP = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle AOB + 90° + 60° + 90° = 360°$

$\Rightarrow 240° + \angle AOB = 360°$

$\Rightarrow \angle AOB = 120°$

Now, In isosceles triangle AOB

$\angle AOB + \angle OAB + \angle OBA = 180°$ [Angle sum property of a triangle]

$\Rightarrow 120° + 2\angle OAB = 180°$ $[\because \angle OAB = \angle OBA]$

$\Rightarrow \angle OAB = 30°$

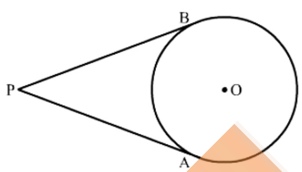
Exercise – Multiple Choice Questions

1.

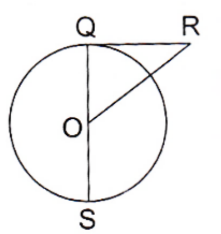
Answer: (b) 2

Sol:

We can draw only two tangents from an external point to a circle.



2.



Answer: (c) 5 cm

Sol:

We know that the radius and tangent are perpendicular at their point of contact

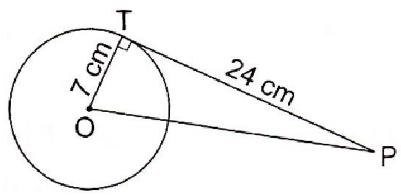
$OQ = \frac{1}{2}QS = 3cm$ [$\because$ Radius is half of diameter]

Now, in right triangle OQR

By using Pythagoras theorem, we have

$OR^2 = RQ^2 + OQ^2$
$= 4^2 + 3^2$
$= 16 + 9$
$= 25$
$\therefore OR^2 = 25$
$\Rightarrow OR = 5cm$

3.



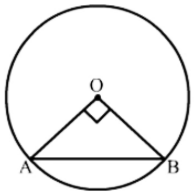
Answer: (c) 25 cm
Sol:
The tangent at any point of a circle is perpendicular to the radius at the point of contact
$\therefore OT \perp PT$
From right – angled triangle *PTO,*
$\therefore OP^2 = OT^2 + PT^2$ [Using Pythagoras' theorem]
$\Rightarrow OP^2 = (7)^2 + (24)^2$
$\Rightarrow OP^2 = 49 + 576$
$\Rightarrow OP^2 = 625$
$\Rightarrow OP = \sqrt{625}$
$\Rightarrow OP = 25cm$

4.
Answer: (d) two diameters
Sol:
Two diameters cannot be parallel as they perpendicularly bisect each other.

5.

Answer: (c) $10\sqrt{2}$

Sol:



In right triangle AOB

By using Pythagoras theorem, we have

$AB^2 = BO^2 + OA^2$

$= 10^2 + 10^2$

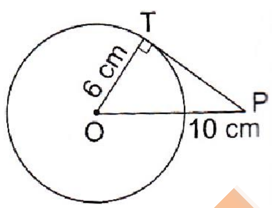
$= 100 + 100$

$= 200$

$\therefore OR^2 = 200$

$\Rightarrow OR = 10\sqrt{2}\, cm$

6.



Answer: (a) 8 cm

Sol:

In right triangle PTO

By using Pythagoras theorem, we have

$PO^2 = OT^2 + TP^2$

$\Rightarrow 10^2 = 6^2 + TP^2$

$\Rightarrow 100 = 36 + TP^2$

$\Rightarrow TP^2 = 64$

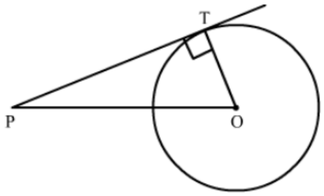
$\Rightarrow TP = 8 cm$

7.

Answer: (a) 10 cm

Sol:

Construction: Join OT.



We know that the radius and tangent are perpendicular at their point of contact

In right triangle PTO

By using Pythagoras theorem, we have

$PO^2 = OT^2 + TP^2$

$\Rightarrow 26^2 = OT^2 + 24^2$

$\Rightarrow 676 = OT^2 + 576$

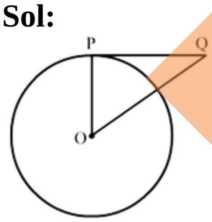
$\Rightarrow TP^2 = 100$

$\Rightarrow TP = 10\,cm$

8.

Answer: 45^0

Sol:



We know that the radius and tangent are perpendicular at their point of contact

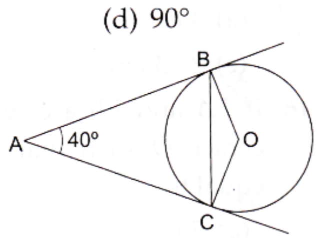
Now, In isosceles right triangle POQ

$\angle POQ + \angle OPQ + \angle OQP = 180°$ [Angle sum property of a triangle]

$\Rightarrow 2\angle OQP + 90° = 180°$

$\Rightarrow \angle OQP = 45°$

9.



Answer: (d)140^0

Sol:

We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OBA = \angle OCA = 90°$

Now, In quadrilateral ABOC

$\angle BAC + \angle OCA + \angle OBA + \angle BOC = 360°$ [Angle sum property of quadrilateral]

$\Rightarrow 40° + 90° + 90° + \angle BOC = 360°$

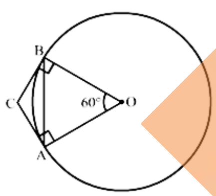
$\Rightarrow 220° + \angle BOC = 360°$

$\Rightarrow \angle BOC = 140°$

10.

Answer: (d)120^0

Sol:



We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OBC = \angle OAC = 90°$

Now, In quadrilateral ABOC

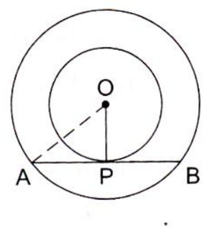
$\angle ACB + \angle OAC + \angle OBC + \angle AOB = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle ACB + 90° + 90° + 60° = 360°$

$\Rightarrow \angle ACB + 240° = 360°$

$\Rightarrow \angle ACB = 120°$

11.



Answer: (c) 16 cm
Sol:
We know that the radius and tangent are perpendicular at their point of contact
In right triangle AOP

$AO^2 = OP^2 + PA^2$

$\Rightarrow 10^2 = 6^2 + PA^2$

$\Rightarrow PA^2 = 64$

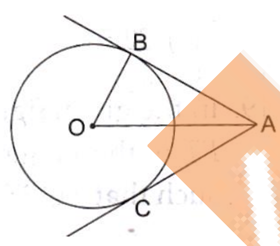
$\Rightarrow PA = 8\,cm$

Since, the perpendicular drawn from the center bisect the chord

$\therefore PA = PB = 8\,cm$

Now, $AB = AP + PB = 8 + 8 = 16\,cm$

12.



Answer: (b) 15
Sol:
We know that the radius and tangent are perpendicular at their point of contact
In right triangle AOB
By using Pythagoras theorem, we have

$OA^2 = AB^2 + OB^2$

$\Rightarrow 17^2 = AB^2 + 8^2$

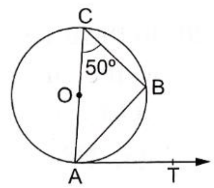
$\Rightarrow 289 = AB^2 + 64$

$\Rightarrow AB^2 = 225$

$\Rightarrow AB = 15\,cm$

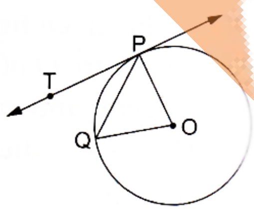
The tangents drawn from the external point are equal
Therefore, the length of AC is 15 cm

13.



Answer: (b)50^0
Sol:
$\angle ABC = 90°$ (Angle in a semicircle)
In ΔABC, we have: $\angle ACB + \angle CAB + \angle ABC = 180°$
$\Rightarrow 50° + \angle CAB + 90° = 180°$
$\Rightarrow \angle CAB = (180° - 140°)$
$\Rightarrow \angle CAB = 40°$
Now, $\angle CAT = 90°$ (Tangents drawn from an external point are perpendicular to the radius at the point of contact)
$\therefore \angle CAB + \angle BAT = 90°$
$\Rightarrow 40° + \angle BAT = 90°$
$\Rightarrow \angle BAT = (90° - 40°)$
$\Rightarrow \angle BAT = 50°$

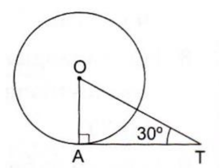
14.



Answer: (a) 35^0
Sol:
We know that the radius and tangent are perpendicular at their point of contact
Since, OP = OQ
$\because POQ$ is a isosceles right triangle
Now, In isosceles right triangle POQ
$\angle POQ + \angle OPQ + \angle OQP = 180°$ [Angle sum proper of a triangle]
$\Rightarrow 70° + 2\angle OPQ = 180°$
$\Rightarrow \angle OPQ = 55°$
Now, $\angle TPQ + \angle OPQ = 90°$

$\Rightarrow \angle TPQ = 35°$

15.



Answer: (c) $2\sqrt{3}$ cm

Sol:

$OA \perp AT$

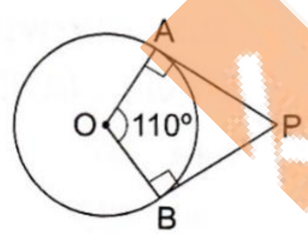
So, $\frac{AT}{OT} = \cos 30°$

$\Rightarrow \frac{AT}{4} = \frac{\sqrt{3}}{2}$

$\Rightarrow AT = \left(\frac{\sqrt{3}}{2} \times 4\right)$

$\Rightarrow AT = 2\sqrt{3}$

16.



Answer: (c) $70°$

Sol:

Given, PA and PB are tangents to a circle with center O, with $\angle AOB = 110°$.

Now, we know that tangents drawn from an external point are perpendicular to the radius at the point of contact.

So, $\angle OAP = 90°$ and $\angle OBP = 90°$

$\Rightarrow \angle OAP + \angle OBP = 90° + 90° = 180°$, which shows that OABP is a cyclic quadrilateral.

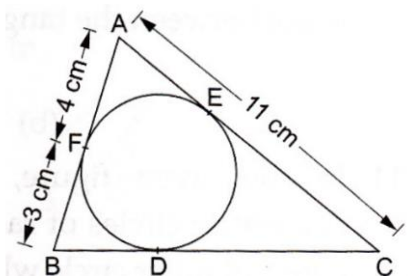
$\therefore \angle AOB + \angle APB = 180°$

$\Rightarrow 110° + \angle APB = 180°$

$\Rightarrow \angle APB = 180° - 110°$

$\Rightarrow \angle APB = 70°$

17.



Answer: (b) 10 cm
Sol:
We know that tangent segments to a circle from the same external point are congruent
Therefore, we have

$AF = AE = 4cm$

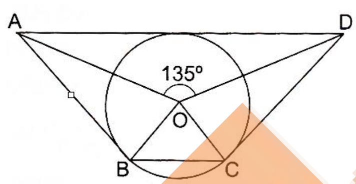
$BF = BD = 3cm$

$EC = AC - AE = 11 - 4 = 7\,cm$

$CD = CE = 7\,cm$

$\therefore BC = BD + DC = 3 + 7 = 10\,cm$

18.

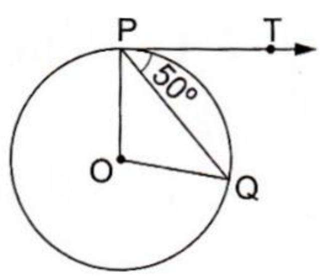


Answer: (b) 45°
Sol:
We know that the sum of angles subtended by opposite sides of a quadrilateral having a circumscribed circle is 180 degrees

$\therefore \angle AOD + \angle BOC = 180°$

$\Rightarrow \angle BOC = 180° - 135° = 45°$

19.



Answer: (a) 100°

Sol:

Given, $\angle QPT = 50°$

And $\angle OPT = 90°$ (Tangents drawn from an external point are perpendicular to the radius at the point of contact)

$\therefore \angle OPQ = (\angle OPT - \angle QPT) = (90° - 50°) = 40°$

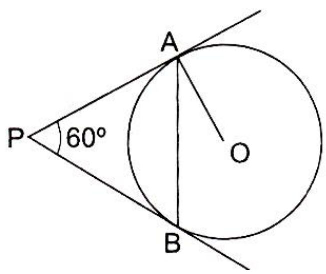
$OP = OQ$ (Radius of the same circle)

$\Rightarrow \angle OQP = \angle OPQ = 40°$

In $\Delta POQ, \angle POQ + \angle OQP + \angle OPQ = 180°$

$\therefore \angle POQ = 180° - (40° + 40°) = 100°$

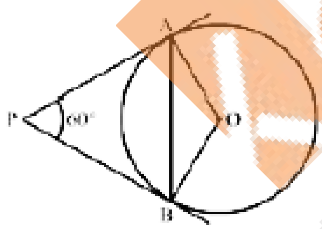
20.



Answer: (b) 30°

Sol:

Construction: Join OB



We know that the radius and tangent are perpendicular at the point of contact

$\therefore \angle OBP = \angle OAP = 90°$

Now, In quadrilateral AOBP

$\angle AOB + \angle OBP + \angle APB + \angle OAP = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle AOB + 90° + 60° + 90° = 360°$

$\Rightarrow 240° + \angle AOB = 360°$

$\Rightarrow \angle AOB = 120°$

Now, In isosceles triangles AOB

$\angle AOB + \angle OAB + \angle OBA = 180°$ [Angle sum property of a triangle]

$\Rightarrow 120° + 2\angle OAB = 180°$ $\qquad [\because \angle OAB = \angle OBA]$

$\Rightarrow \angle OAB = 30°$

21.

Answer: (c) $3\sqrt{3}$ cm

Sol:

Given, PA and PB are tangents to circle with center O and radius 3 cm and $\angle APB = 60°$.

Tangents drawn from an external point are equal; so, PA = PB.

And OP is the bisector of $\angle APB$, which gives $\angle OPB = \angle OPA = 30°$.

$OA \perp PA$. So, from right – angled ΔOPA, we have:

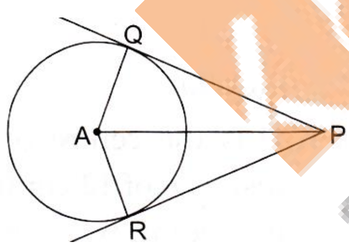
$\frac{OA}{AP} = \tan 30°$

$\Rightarrow \frac{OA}{AP} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{3}{AP} = \frac{1}{\sqrt{3}}$

$= AP = 3\sqrt{3}\, cm$

22.



Answer: (c) 126°

Sol:

We know that the radius and tangent are perpendicular at the point of contact

Now, In ΔPQA

$\angle PQA + \angle QAP + \angle APQ = 180°$ $\qquad$ [Angle sum property of a triangle]

$\Rightarrow 90° + \angle QAP + 27° = 180°$ $\qquad [\because \angle OAB = \angle OBA]$

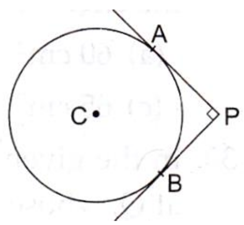
$\Rightarrow \angle QAP = 63°$

In ΔPQA and ΔPRA

$PQ = PR$ $\qquad$ (Tangents draw from same external point are equal)

$QA = RA$ (Radio of the circle)
$AP = AP$ (common)
By SSS congruency
$\Delta PQA \cong \Delta PRA$
$\angle QAP = \angle RAP = 63°$
$\therefore \angle QAR = \angle QAP + \angle RAP = 63° + 63° = 126°$

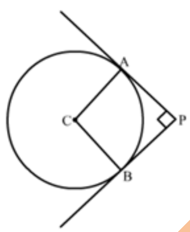
23.



Answer:(b)117^0

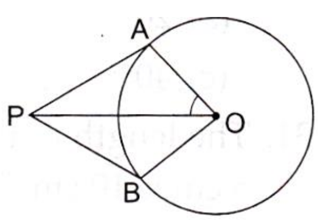
Sol:

Construction: Join CA and CB



We know that the radius and tangent are perpendicular at their point of contact
$\because \angle CAP = \angle CBP = 90°$
Since, in quadrilateral ACBP all the angles are right angles
$\therefore$ $ACPB$ is a rectangle
Now, we know that the pair of opposite sides are equal in rectangle
$\therefore CB = AP$ and $CA = BP$
Therefore, $CB = AP = 4cm$ and $CA = BP = 4cm$

24.



Answer:(b)50^0

Sol:

Given, PA and PB are two tangents to a circle with center O and $\angle APB = 80°$

$\therefore \angle APO = \frac{1}{2}\angle APB = 40°$

[Since they are equally inclined to the line segment joining the center to that point and $\angle OAP = 90°$]

[Since tangents drawn from an external point are perpendicular to the radius at the point of contact]

Now, in triangle *AOP*:

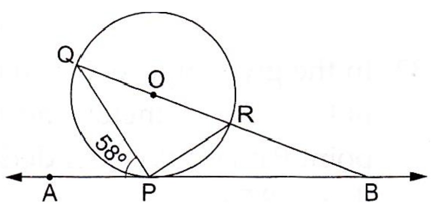
$\angle AOP + \angle OAP + \angle APO = 180°$

$\Rightarrow \angle AOP + 90° + 40° = 180°$

$\Rightarrow \angle AOP = 180° - 130°$

$\Rightarrow \angle AOP = 50°$

25.



Answer: (a) 32°

Sol:

We know that a chord passing through the center is the diameter of the circle.

$\because \angle QPR = 90°$ (Angle in a semi circle is 90°)

By using alternate segment theorem

We have $\angle APQ = \angle PRQ = 58°$

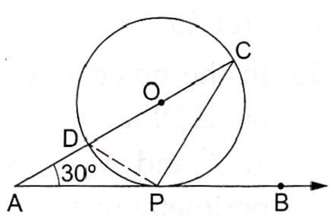
Now, In Δ PQR

$\angle PQR + \angle PRQ + \angle QPR = 180°$ [Angle sum properly of a triangle]

$\Rightarrow \angle PQR + 58° + 90° = 180°$

$\Rightarrow \angle PQR = 32°$

26.



Answer: (b) 90°

Sol:

We know that a chord passing through the center is the diameter of the circle.

$\because \angle DPC = 90°$ (Angle in a semicircle is 90°)

Now, In ΔCDP

$\angle CDP + \angle DCP + \angle DPC = 180°$ [Angle sum property of a triangle]

$\Rightarrow \angle CDP + \angle DCP + 90° = 180°$

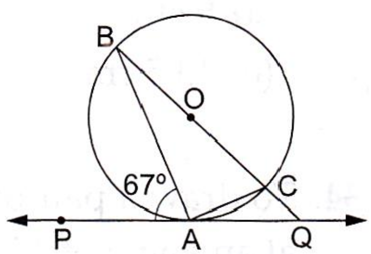
$\Rightarrow \angle CDP + \angle DCP = 90°$

By using alternate segment theorem

We have $\angle CDP = \angle CPB$

$\therefore \angle CPB + \angle ACP = 90°$

27.



Answer: (d)

Sol:

We know that a chord passing through the center is the diameter of the circle.

$\because BAC = 90°$ (Angle in a semicircle is $90°$)

By using alternate segment theorem

We have $\angle PAB = \angle ACB = 67°$

Now, In ABC

$\angle ABC + \angle ACB + \angle BAC = 180°$ [Angle sum property of a triangle]

$\Rightarrow \angle ABC + 67° + 90° = 180°$

$\Rightarrow \angle ABC = 23°$

Now, $\angle BAQ = 180° - \angle PAB$ [Linear pair angles]

$= 180° - 67°$

$= 113°$

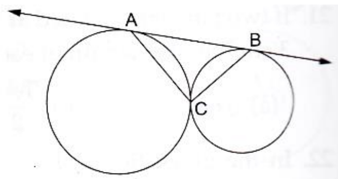
Now, In ΔABQ

$\angle ABQ + \angle AQB + \angle BAQ = 180°$ [Angle sum property of a triangle]

$\Rightarrow 23° + \angle AQB + 113° = 180°$

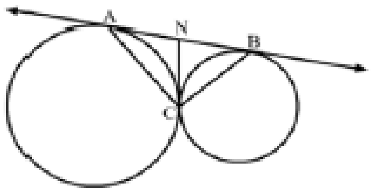
$\Rightarrow \angle AQB = 44°$

28.



Answer: (c) $90°$

Sol:



We know that tangent segments to a circle from the same external point are congruent

Therefore, we have

$NA = NC$ and $NC = NB$

We also know that angle opposite to equal sides is equal

$\therefore \angle NAC = \angle NCA$ and $\angle NBC = \angle NCB$

Now, $\angle ANC + \angle BNC = 180°$ [Linear pair angles]

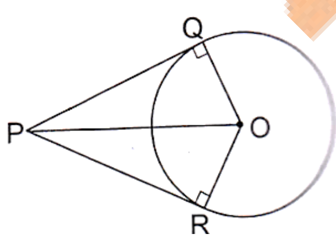
$\Rightarrow \angle NBC + \angle NCB + \angle NAC + \angle NCA = 180°$ [Exterior angle property]

$\Rightarrow 2\angle NCB + 2\angle NCA = 180°$

$\Rightarrow 2(\angle NCA + \angle NCA) = 180°$

$\Rightarrow \angle ACB = 90°$

29.



Answer: (a) $60\,cm^2$

Sol:

Given,

$OQ = OR = 5\,cm, OP = 13\,cm.$

$\angle OQP = \angle ORP = 90°$ (Tangents drawn from an external point are perpendicular to the radius at the point of contract)

From right – angled ΔPOQ:

$$PQ^2 = \left(OP^2 - OQ^2\right)$$

$$\Rightarrow PQ^2 = \left(OP^2 - OQ^2\right)$$

$$\Rightarrow PQ^2 = 13^2 - 5^2$$

$$\Rightarrow PQ^2 = 169 - 25$$

$$\Rightarrow PQ = 144$$

$$\Rightarrow PQ = \sqrt{144}$$

$$\Rightarrow PQ = 12\,cm$$

$$\therefore ar\left(\Delta OQP\right) = \frac{1}{2} \times PQ \times OQ$$

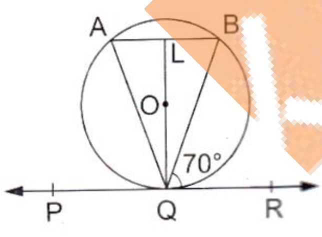
$$\Rightarrow ar\left(\Delta OQP\right) = \left(\frac{1}{2} \times 12 \times 5\right) cm^2$$

$$\Rightarrow ar\left(\Delta OQP\right) = 30 cm^2$$

Similarly, $ar\left(\Delta ORP\right) = 30\,cm^2$

$$\therefore ar\left(quad.PQOR\right) = \left(30+30\right) cm^2 = 60\,cm^2$$

30.



Answer: (c)40^0

Sol:

Since, $AB \parallel PR, BQ$ is transversal

$\angle BQR = \angle ABQ = 70°$ [Alternative angles]

$OQ \perp PQR$ (Tangents drawn from an external point are perpendicular to the radius at the point of contract)

and $AB \parallel PQR$

$\therefore QL \perp AB$; so, $OL \perp AB$

$\therefore OL$ bisects chord AB [Perpendicular drawn from the center bisects the chord]

From ΔQLA and QLB:

$\angle QLA = \angle QLB = 90°$

$LA = LB$ (OL bisects chord AB)

QL is the common side.

$\therefore \Delta QLA \cong \Delta QLB$ [By SAS congruency]

$\therefore \angle QAL = \angle QBL$

$\Rightarrow \angle QAB = \angle QBA$

$\therefore \Delta AQB$ is isosceles

$\therefore \angle LQA = \angle LQR$

$\angle LQP = \angle LQR = 90°$

$\angle LQB = (90° - 70°) = 20°$

$\therefore \angle LQA = \angle LQB = 20°$

$\Rightarrow \angle LQA = \angle LQB = 20°$

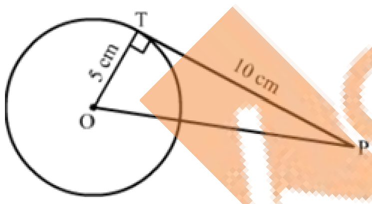
$\Rightarrow \angle AQB = \angle LQA + \angle LQB$

$= 40°$

31.

Answer: (d) $\sqrt{125}\ cm$

Sol:



We know that the radius and tangent are perpendicular at their point of contact

In right triangle PTO

By using Pythagoras theorem, we have

$PO^2 = OT^2 + TP^2$

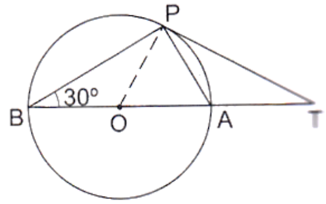
$\Rightarrow PO^2 = 5^2 + 10^2$

$\Rightarrow PO^2 = 25 + 100$

$\Rightarrow PO^2 = 125$

$\Rightarrow PO = \sqrt{125 cm}$

32.



Answer: (b) 30°

Sol:

We know that a chord passing through the center is the diameter of the circle

$\because \angle BPA = 90°$ (Angle in a semicircle is 90°)

By using alternate segment theorem

We have $\angle APT = \angle ABP = 30°$

Now, In ΔABP

$\angle PBA + \angle BPA + \angle BAP = 180°$ [Angle sum property of a triangle]

$\Rightarrow 30° + 90° + \angle BAP = 180°$

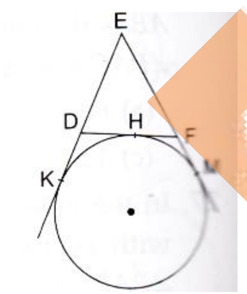
$\Rightarrow \angle BAP = 60°$

Now, $\angle BAP = \angle APT + \angle PTA$

$\Rightarrow 60° = 30° + \angle PTA$

$\Rightarrow \angle PTA = 30°$

33.



Answer: (d) 18 cm

Sol:

We know that tangent segments to a circle from the same external point are congruent.

Therefore, we have

$EK = EM = 9cm$

Now, $EK + EM = 18cm$

$\Rightarrow ED + DK + EF + FM = 18cm$

$\Rightarrow ED + DH + EF + HF = 18cm$ $(\because DK = DH \ \ and \ \ FM = FH)$

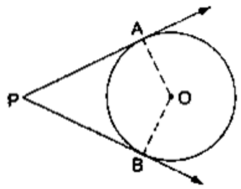
$\Rightarrow ED + DF + EF = 18cm$

$\Rightarrow$ Perimeter of $\Delta EDF = 18cm$

34.

Answer: (b) 135^0

Sol:



Suppose PA and PB are two tangents we want to draw which inclined at an angle of $45°$

We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OBP = \angle OAP = 90°$

Now, in quadrilateral AOBP

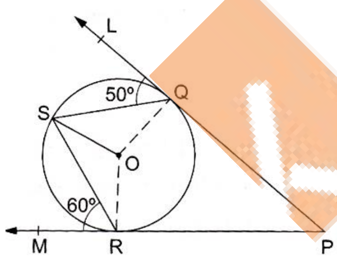
$\angle AOB + \angle OBP + \angle OAP + \angle APB = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle AOB + 90° + 90° + 45° = 360°$

$\Rightarrow \angle AOB + 225° = 360°$

$\Rightarrow \angle AOB = 135°$

35.



Answer:(d)70^0

Sol:

PQL is a tangent OQ is the radius; so, $\angle OQL = 90°$

$\therefore \angle OQS = (90° - 50°) = 40°$

Now, $OQ = OS$ (Radius of the same circle)

$\Rightarrow \angle OSQ = \angle OQS = 40°$

Similarly, $\angle ORS = (90° - 60°) = 30°$,

And, $OR = OS$ (Radius of the same circle)

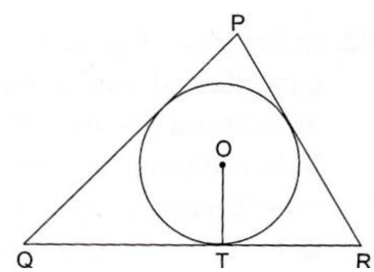
$\Rightarrow \angle OSR = \angle ORS = 30^\circ$

$\therefore \angle QSR = \angle OSQ + \angle OSR$

$\Rightarrow \angle QSR = (40^\circ + 30^\circ)$

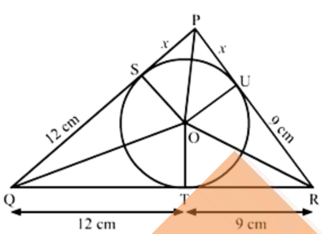
$\Rightarrow \angle QSR = 70^\circ$

36.



Answer: (c) 22.5 cm

Sol:



We know that tangent segments to a circle from the same external point are congruent.
Therefore, we have

$PS = PU = x$

$QT = QS = 12\,cm$

$RT = RU = 9\,cm$

Now,

$Ar(\Delta PQR) = Ar(\Delta POR) + Ar(\Delta QOR) + Ar(\Delta POQ)$

$\Rightarrow 189 = \frac{1}{2} \times OU \times PR + \frac{1}{2} \times OT \times QR + \frac{1}{2} \times OS \times PQ$

$\Rightarrow 378 = 6 \times (x+9) + 6 \times (21) + 6 \times (12 + x)$

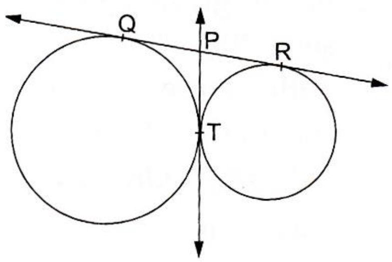
$\Rightarrow 63 = x + 9 + 21 + x + 12$

$\Rightarrow 2x = 21$

$\Rightarrow x = 10.5\,cm$

Now, $PQ = QS + SP = 12 + 10.5 + 10.5 = 22.5\,cm$

37.



Answer: (d) 7.6 cm

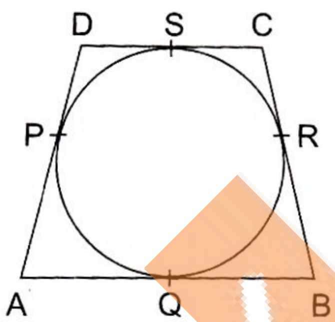
Sol:

We know that tangent segments to a circle from the same external point are congruent.
Therefore, we have

$PT = PO = 3.8$ cm and $PT = PR$ 3.8 cm

$\therefore QR = QP + PR = 3.8 + 3.8 = 7.6 cm$

38.



Answer: (a) 9 cm

Sol:

Tangents drawn from an external point to a circle are equal.

So, $AQ = AP = 5 cm$

$CR = CS = 3 cm$

And $BR = (BC - CR)$

$\Rightarrow BR = (7-3) cm$

$\Rightarrow BR = 4 cm$

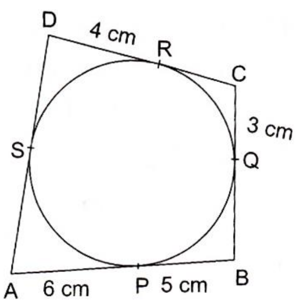
$BQ = BR = 4 cm$

$\therefore AB = (AQ + BQ)$

$\Rightarrow AB = (5+4) cm$

$\Rightarrow AB = 9 cm$

39.



Answer: (c) 36 cm

Sol:

Given, $AP = 6cm, BP = 5cm, CQ = 3cm$ and $DR = 4cm$

Tangents drawn from an external point to a circle are equal

So, $AP = AS = 6cm, BP = BQ = 5cm, CQ = CR = 3cm, DR = DS = 4cm.$

$\therefore AB = AP + BP = 6 + 5 = 11cm$

$BC = BQ + CQ = 5 + 3 = 8cm$

$CD = CR + DR = 3 + 4 = 7cm$

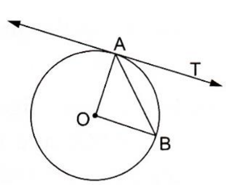
$AD = AS + DS = 6 + 4 = 10cm$

$\therefore$ Perimeter of quadrilateral $ABCD = AB + BC + CD + DA$

$= (11 + 8 + 7 + 10)cm$

$= 36cm$

40.



Answer:(b)50^0

Sol:

Given: AO and BC are the radius of the circle

Since, AO = BO

$\therefore \Delta AOB$ is an isosceles triangle

Now, in ΔAOB

$\angle AOB + \angle OBA + \angle OAB = 180°$ (Angle sum property of triangle)

$\Rightarrow 100° + \angle OAB + \angle OAB = 180°$ $(\angle OBA = \angle OAB)$

$\Rightarrow 2\angle OAB = 80°$

$\Rightarrow \angle OAB = 40°$

We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OAT = 90°$

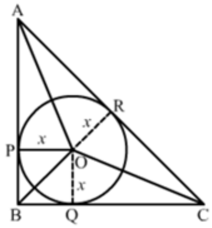
$\Rightarrow \angle OAB + \angle BAT = 90°$

$\Rightarrow \angle BAT = 90° - 40° = 50°$

41.

Answer: (b) 2 cm

Sol:



In right triangle ABC

By using Pythagoras theorem we have

$AC^2 = AB^2 + BC^2$

$= 5^2 + 12^2$

$= 25 + 144$

$= 169$

$\therefore AC^2 = 169$

$\Rightarrow AC = 13cm$

Now,

$Ar(\Delta ABC) = Ar(\Delta AOB) + Ar(\Delta BOC) + Ar(\Delta AOC)$

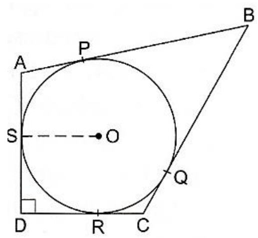
$\Rightarrow \frac{1}{2} \times AB \times BC = \frac{1}{2} \times OP \times AB + \frac{1}{2} \times OQ \times BC + \frac{1}{2} \times OR \times AC$

$\Rightarrow 5 \times 12 = x \times 5 + x \times 12 + x \times 13$

$\Rightarrow 60 = 30x$

$\Rightarrow x = 2cm$

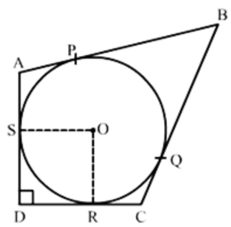
42.



Answer: (d) 21 cm

Sol:

Construction: Join OR



We know that tangent segments to a circle from the same external point are congruent.

Therefore, we have

$BP = BQ = 27\,cm$

$CQ = CR$

Now, $BC = 38cm$

$\Rightarrow BQ + QC = 38$

$\Rightarrow QC = 38 - 27 = 11cm$

Since, all the angles in quadrilateral DROS are right angles.

Hence, DROS is a rectangle.

We know that opposite sides of rectangle are equal

$\therefore OS = RD = 10cm$

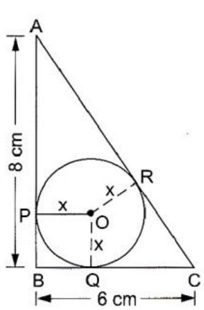
Now, $CD = CR + RD$

$= CQ + RD$

$= 11 + 10$

$= 21cm$

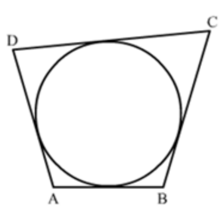
43.



Answer: (a) 2 cm
Sol:
Given, $AB = 8\,cm, BC = 6\,cm$
Now, in ΔABC :
$AC^2 = AB^2 + BC^2$
$\Rightarrow AC^2 = (8^2 + 6^2)$
$\Rightarrow AC^2 = (64 + 36)$
$\Rightarrow AC^2 = 100$
$\Rightarrow AC = \sqrt{100}$
$\Rightarrow AC = 10\,cm$
$PBQO$ is a square
$CR = CQ$ (Since the lengths of tangents drawn from an external point are equal)
$\therefore CQ = (BC - BQ) = (6 - x)\,cm$
Similarly, $AR = AP = (AB = BP) = (8 - x)\,cm$
$\therefore AC = (AR + CR) = [(8 - x) + (6 - x)]\,cm$
$\Rightarrow 10 = (14 - 2x)\,cm$
$\Rightarrow 2x = 4$
$\Rightarrow x = 2\,cm$
$\therefore$ The radius of the circle is 2 cm.

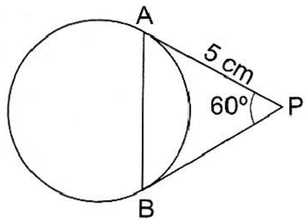
44.

Answer: (a) 3 cm
Sol:



We know that when a quadrilateral circumscribes a circle then sum of opposes sides is equal to the sum of other opposite sides
$\therefore AB + DC = AD + BC$
$\Rightarrow 6 + 4 = AD + 7$
$\Rightarrow AD = 3\,cm$

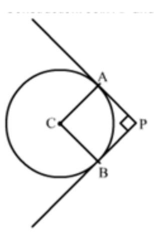
45.



Answer: (b) 5 cm
Sol:
The lengths of tangents drawn from a point to a circle are equal
So, $PA = PB$ and therefore, $\angle PAB = \angle PBA = x$ (say).
Then, in ΔPAB:
$\angle PAB + \angle PBA + \angle APB = 180°$
$\Rightarrow x + x + 60° = 180°$
$\Rightarrow 2x = 180° - 60°$
$\Rightarrow 2x = 120°$
$\Rightarrow x = 60°$
$\therefore$ Each angle of ΔPAB is $60°$ and therefore, it is an equilateral triangle.
$\therefore AB = PA = PB = 5cm$
$\therefore$ The length of the chord AB is $5cm$.

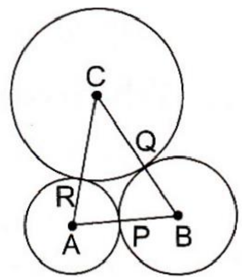
46.

Answer: (c) 5 cm
Sol:
Construction: Join AF and AE



We know that the radius and tangent are perpendicular at their point of contact
$\because \angle AED = \angle AFD = 90°$
Since, in quadrilateral AEDF all the angles are right angles
$\therefore AEDF$ is a rectangle
Now, we know that the pair of opposite sides is equal in rectangle
$\therefore AF = DE = 5cm$
Therefore, the radius of the circle is 5 cm

47.



Answer: (b) 2 cm

Sol:

Given, $AB = 5$ cm, $BC = 7$ cm and $CA = 6$ cm.

Let, $AR = AP = x$ cm.

$BQ = BP = y$ cm

$CR = CQ = z$ cm

(Since the length of tangents drawn from an external point arc equal)

Then, $AB = 5$ cm

$\Rightarrow AP + PB = 5cm$

$\Rightarrow x + y = 5$ ....(i)

Similarly, $y + z = 7$ ......(ii)

and $z + x = 6$ ....(iii)

Adding (i), (ii) and (iii), we get:

$(x+y)+(y+z)+(z+x)=18$

$\Rightarrow 2(x+y+z)=18$

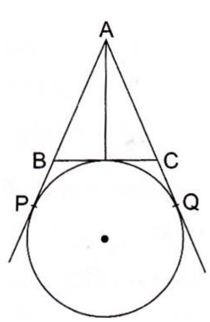
$\Rightarrow (x+y+z)=9$ ....(iv)

Now, (iv) – (ii):

$\Rightarrow x = 2$

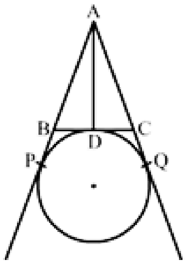
$\therefore$ The radius of the circle with center A is 2 cm.

48.



Answer: (d) 7.5 cm

Sol:



We know that tangent segments to a circle from the same external point are congruent

Therefore, we have

$AP = AQ$

$BP = BD$

$CQ = CD$

Now, $AB + BC + AC = 5 + 4 + 6 = 15$

$\Rightarrow AB + BD + DC + AC = 15cm$

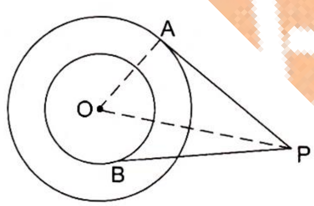
$\Rightarrow AB + BP + CQ + AC = 15cm$

$\Rightarrow AP + AQ = 15cm$

$\Rightarrow 2AP = 15cm$

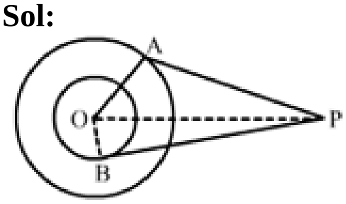
$\Rightarrow AP = 7.5cm$

49.



Answer: (c) $4\sqrt{10}\ cm$

Sol:



Given, $OP = 5cm, PA = 12cm$

Now, join O and B

Then, $OB = 3cm.$

Now, $\angle OAP = 90°$ (Tangents drawn from an external point are perpendicular to the radius at the point of contract)

Now, in ΔOAP :

$OP^2 = OA^2 + PA^2$

$\Rightarrow OP^2 = 5^2 + 12^2$

$\Rightarrow OP^2 = 25 + 144$

$\Rightarrow OP^2 = 169$

$\Rightarrow OP = \sqrt{169}$

$\Rightarrow OP = 13$

Now, in ΔOBP :

$PB^2 = OP^2 - OB^2$

$\Rightarrow PB^2 = 13^2 - 3^2$

$\Rightarrow PB^2 = 169 - 9$

$\Rightarrow PB^2 = 160$

$\Rightarrow PB = \sqrt{160}$

$\Rightarrow PB = 4\sqrt{10}cm$

50.

Answer: (d) A circle can have more than two parallel tangents. parallel to a given line.
Sol:
A circle can have more than two parallel tangents. parallel to a given line.
This statement is false because there can only be two parallel tangents to the given line in a circle.

51.

Answer: (d) A straight line can meet a circle at one point only.

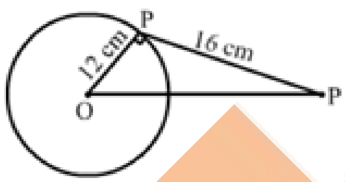
Sol:
A straight be can meet a circle at one point only
This statement is not true because a straight line that is not a tangent but a secant cuts the circle at two points.

52.

Answer: (d) A tangent to the circle can be drawn form a point inside the circle.
Sol:
A tangent to the circle can be drawn from a point Inside the circle.
This statement is false because tangents are the lines drawn from an external point to the circle that touch the circle at one point.

53.

Answer: (a) Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
Sol:



(a) Both Assertion (A) and Reason (R) are true and Reason (R) s a correct explanation of Assertion (A)
In $\Delta OPQ, \angle OPQ = 90°$

$$\therefore OQ^2 = OP^2 + PQ^2$$
$$\Rightarrow OQ = \sqrt{OP^2 + PQ^2}$$
$$= \sqrt{12^2 + 16^2}$$
$$= \sqrt{144 + 256}$$
$$= \sqrt{400}$$
$$= 20\,cm$$

54.

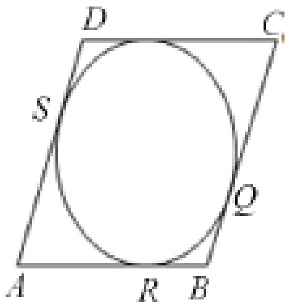
Answer: (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

Sol:

Assertion -

We know that It two tangents are drawn to a circle from an external pout, they subtend equal angles at the center

Reason:



Given, a parallelogram $ABCD$ circumscribes a circle with center O

$AB = BC = CD = AD$

We know that the tangents drawn from an external point to circle are equal

$\therefore AP = AS$ ....(i) [tangents from A]

$BP = BQ$ ....(ii) [tangents from B]

$CR = CQ$ ....(iii) [tangents from C]

$DR = DS$ ....(iv) [tangents from D]

$\therefore AB + CD = AP + BP + CR + DR$

$= AS + BQ + CQ + DS$ [from (i), (ii), (iii) and (iv)]

$= (AS + DS) + (BQ + CQ)$

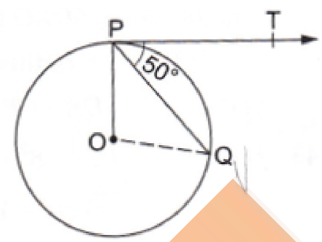
$= AD + BC$

55.

The correct answer is (a) / (b) / (c) / (d).
Answer: (d) Assertion (A) is false and Reason (R) is true.

Exercise - Formative Assessment

1.



Answer: (b) 100°
Sol:
Given, $\angle QPT = 50°$
Now, $\angle OPT = 90°$ (Since tangents drawn from an external point are perpendicular to the radius at point of contact)
$\therefore \angle OPQ = (\angle OPT - \angle QPT) = (90° - 50°) = 40°$
$OP = OQ$ (Radii of the same circle)

$\Rightarrow \angle OPQ = \angle OQP = 40°$

In ΔPOQ

$\angle POQ + \angle OPQ + \angle OQP = 180°$

$\Rightarrow \angle POQ + 40° + 40° = 180°$

$\Rightarrow \angle POQ = 180° - (40° + 40°)$

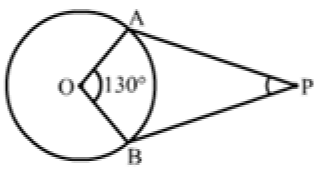
$\Rightarrow \angle POQ = 180° - 80°$

$\Rightarrow \angle POQ = 100°$

2.

Answer: (c) 50^0

Sol:



OA and *OB* are the two radii of a circle with center *O*.

Also, *AP* and *BP* are the tangents to the circle.

Given, $\angle AOB = 130°$

Now, $\angle OAB = \angle OBA = 90°$ (Since tangents drawn from an external point are perpendicular to the radius at point of contact)

In quadrilateral *OAPB,*

$\angle AOB + \angle OAB + \angle OBA + \angle APB = 360°$

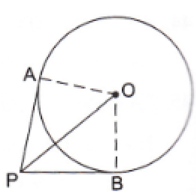
$\Rightarrow 130° + 90° + 90° + \angle APB = 360°$

$\Rightarrow \angle APB = 360° - (130° + 90° + 90°)$

$\Rightarrow \angle APB = 360° - 310°$

$\Rightarrow \angle APB = 50°$

3.



Answer: (b) 50°

Sol:

From ΔOPA and ΔOPB

$OA = OB$ (Radii of the same circle)

OP (Common side)

$PA = PB$ (Since tangents drawn from an external point to a circle are equal)

$\therefore \Delta OPA \cong \Delta OPB$ (SSS rule)

$\therefore \angle APO = \angle BPO$

$\therefore \angle APO = \frac{1}{2}\angle APB = 40°$

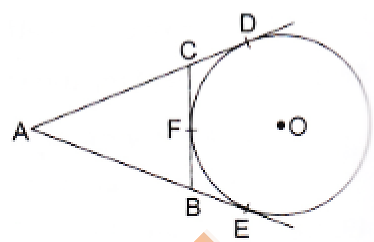
And $\angle OAP = 90°$ (Since tangents drawn from an external point are perpendicular to the radius at point of contact)

Now, in $\Delta OAP, \angle AOP + \angle OAP + \angle APO = 180°$

$\Rightarrow \angle AOP + 90° + 40° = 180°$

$\Rightarrow \angle AOP = 180° - 130° = 50°$

4.



Answer: (b) 10cm

Sol:

Since the tangents from an external point are equal, we have

$AD = AE, CD = CF, BE = BF$

Perimeter of $\Delta ABC = AC + AB + CB$

$= (AD - CD) + (CF + BF) + (AE - BE)$

$= (AD - CF) + (CF + BF) + (AE - BF)$

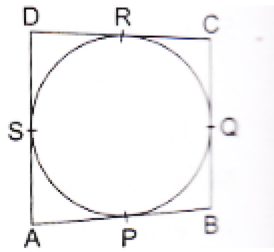
$= AD + AE$

$= 2AE$

$= 2 \times 5$

$= 10\,cm$

5.



Sol:

We know that tangent segments to a circle from the same external point are congruent

Now, we have

$CR = CQ, AS = AP$ and $BQ = BP$

Now, $BC = 7\,cm$

$\Rightarrow CQ + BQ = 7$

$\Rightarrow BQ = 7 - CQ$

$\Rightarrow BQ = 7 - 3 \qquad [\because CQ = CR = 3]$

$\Rightarrow BQ = 4\,cm$

Again, $AB = AP + PB$

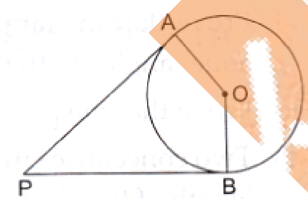
$= AP = BQ$

$= 5 + 4 \qquad [\because AS = AP = 5]$

$= 9\,cm$

Hence, the value of x 9cm

6.



Sol:

Here, OA = OB

And $OA \perp AP, OA \perp BP,$ (Since tangents drawn from an external point arc perpendicular to the radius at the point of contact)

$\therefore \angle OAP = 90°, \angle OBP = 90°$

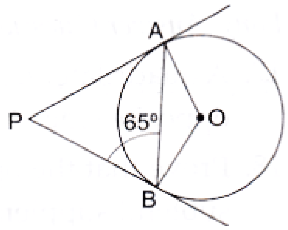
$\therefore \angle OAP + \angle OBP = 90° + 90° = 180°$

$\therefore \angle AOB + \angle APB = 180°$ (Since, $\angle OAP + \angle OBP + \angle AOB + \angle APB = 360°$)

Sum of opposite angle of a quadrilateral is 180°.

Hence A, O, B and P are concyclic.

7.



Sol:

We know that tangents drawn from the external port are congruent

$\therefore PA = PB$

Now, In isosceles triangle APB

$\angle APB + \angle PBA = \angle PAB = 180°$ [Angle sum property of a triangle]

$\Rightarrow \angle APB + 65° + 65° = 180°$ $[\because \angle PBA = \angle PAB = 65°]$

$\Rightarrow \angle APB = 50°$

We know that the radius and tangent are perpendicular at their port of contact

$\therefore \angle OBP = \angle OAP = 90°$

Now, In quadrilateral $AOBP$

$\angle AOB + \angle OBP + \angle APB + \angle OAP = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle AOB + 90° + 50° + 90° = 360°$

$\Rightarrow 230° + \angle BOC = 360°$

$\Rightarrow \angle AOB = 130°$

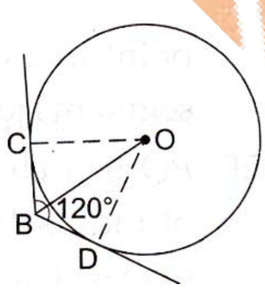
Now, In isosceles triangle AOB

$\angle AOB + \angle OAB + \angle OBA = 180°$ [Angle sum property of a triangle]

$\Rightarrow 130° + 2\angle OAB = 180°$ $[\because \angle OAB = \angle OBA]$

$\Rightarrow \angle OAB = 25°$

8.



Ans:

Sol:

Here, OB is the bisector of $\angle CBD$.

(Two tangents are equally inclined to the line segment joining the center to that point)

$\therefore \angle CBO = \angle DBO = \frac{1}{2}\angle CBD = 60°$

$\therefore From\ \Delta BOD, \angle BOD = 30°$

Now, from right – angled ΔBOD,

$\Rightarrow \frac{BD}{OB} = \sin 30°$

$\Rightarrow OB = 2BD$

$\Rightarrow OB = 2BC$ (Since tangents from an external point are equal. i.e., *BC* = *BD*)

$\therefore OB = 2BC$

9.

Sol:

(i) A line intersecting a circle at two district points is called a <u>secant</u>

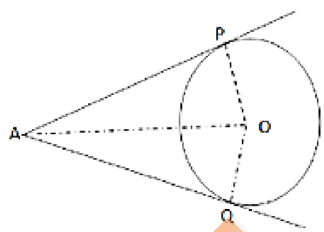
(ii) A circle can have <u>two </u>parallel tangents at the most

(iii) The common point of a tangent to a circle and the circle is called the <u>point of contact.</u>

(iv) A circle can have <u>infinite</u> tangents

10.

Sol:



Given two tangents AP and AQ are drawn from a point A to a circle with center O.

To prove: *AP = AQ*

Join *OP*, *OQ* and *OA*.

AP is tangent at *P* and *OP* is the radius.

$\therefore OP \perp AP$ (Since tangents drawn from an external point are perpendicular to the radius at the point of contact)

Similarly, $OQ \perp AQ$

In the right ΔOPA and ΔOQA, we have:

$OP = OQ$ [radii of the same circle]

$\angle OPA = \angle OQA (= 90°)$

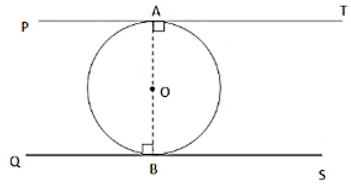
$OA = OA$ [Common side]

$\therefore \Delta OPA \cong \Delta OQA$ [By R.H.S – Congruence]

Hence, $AP = AQ$

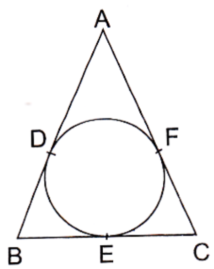
11.

Sol:



Here, PT and QS are the tangents to the circle with center O and AB is the diameter
Now, radius of a circle is perpendicular to the tangent at the point of contact
$\therefore OA \perp AT$ and $OB \perp BS$ (Since tangents drawn from an external point are perpendicular to the radius at point of contact)
$\therefore \angle OAT = \angle OBQ = 90°$
But $\angle OAT$ and $\angle OBQ$ are alternate angles.
$\therefore AT$ is parallel to BS.

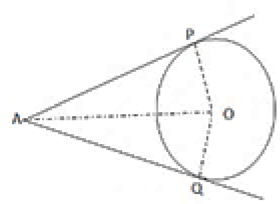
12.



Sol:
Given, $AB = AC$
We know that the tangents from an external point are equal
$\therefore AD = AF, BD = BE \;\; and \;\; CF = CE$ ....(i)
Now, $AB = AC$
$\Rightarrow AD + DB = AF + FC$
$\Rightarrow AF + DB = AF + FC$ $\quad [from(i)]$
$\Rightarrow DB = FC$
$\Rightarrow BE = CE$ $\quad [from(i)]$
Hence proved.

13.

Sol:



Given: A circle with center O and a point A outside it. Also, AP and AQ are the two tangents to the circle

To prove: $\angle AOP = \angle AOQ$.

Proof : In ΔAOP and ΔAOQ, we have

$AP = AQ$ [tangents from an external point are equal]

$OP = OQ$ [radii of the same circle]

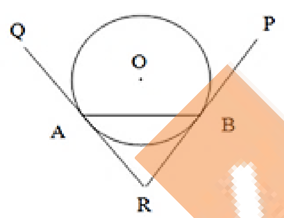
$OA = OA$ [common side]

$\therefore \Delta AOP \cong \Delta AOQ$ [by SSS – congruence]

Hence, $\angle AOP = \angle AOQ$ (c.p.c.t).

14.

Sol:



Let RA and RB be two tangents to the circle with center O and let AB be a chord of the circle.

We have to prove that $\angle RAB = \angle RDA$.

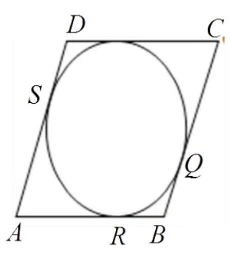
$\therefore$ Now, RA

$= RB$ (Since tangents drawn from an external point to a circle are equal)

In. $\Delta RAB, \angle RAB = \angle RDA$ (Since opposite sides are equal, their base angles are also equal)

15.

Sol:



Given, a parallelogram $ABCD$ circumscribes a circle with center O

$AB = BC = CD = AD$

We know that the tangents drawn from an external point to circle are equal

$\therefore AP = AS$ ....(i) [tangents from A]

$BP = BQ$ ....(ii) [tangents from B]

$CR = CQ$ ....(iii) [tangents from C]

$DR = DS$ ....(iv) [tangents from D]

$\therefore AB + CD = AP + BP + CR + DR$

$= AS + BQ + CQ + DS$ [from (i), (ii), (iii) and (iv)]

$= (AS + DS) + (BQ + CQ)$

$= AD + BC$

Thus, $(AB + CD) = (AD + BC)$

$\Rightarrow 2AB = 2AD$ [$\because$ opposite sides of a parallelogram are equal]

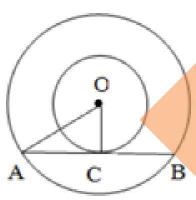
$\Rightarrow AB = AD$

$\therefore CD = AB = AD = BC$

Hence, $ABCD$ is a rhombus.

16.

Sol:



Given: Two circles have the same center O and AB is a chord of the larger circle touching the smaller circle at C. also, $OA = 5$ cm ad OC 3 cm

In $\Delta OAC, OA^2 = OC^2 + AC^2$

$\therefore AC^2 = OA^2 - OC^2$

$\Rightarrow AC^2 = 5^2 - 3^2$

$\Rightarrow AC^2 = 25 - 9$

$\Rightarrow AC^2 = 16$

$\Rightarrow AC = 4\,cm$

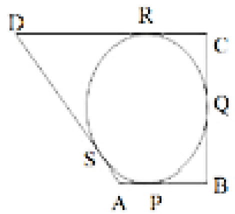
$\therefore AB = 2\,AC$ (Since perpendicular drawn from the center of the circle bisects the chord)

$\therefore AB = 2 \times 4 = 8\,cm$

The length of the chord of the larger circle is 8cm.

17.

Sol:



We know that the tangents drawn from an external point to circle are equal.

$\therefore AP = AS$ ....(i) [tangents from A]

$BP = BQ$ ....(ii) [tangents from B]

$CR = CQ$ ....(iii) [tangents from C]

$DR = DS$ ....(iv) [tangents from D]

$\therefore AB + CD = (AP + BP) + (CR + DR)$

$= (AS + BQ) + (CQ + DS)$ [using (i), (ii), (iii) and (iv)]

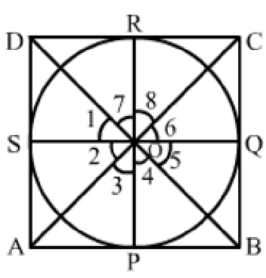
$= (AS + DS) + (BQ + CQ)$

$= AD + BC$

Hence, $(AB + CD) = (AD + BC)$

18.

Sol:



Given, a quadrilateral $ABCD$ circumference a circle with center O.

To prove: $\angle AOB + \angle COD = 180°$

And $\angle AOD + \angle BOC = 180°$

Join: OP, OQ, OR and OS.

We know that the tangents drawn from an external point of a circle subtend equal angles at the center.

$\therefore \angle 1 = \angle 7, \angle 2 = \angle 3, \angle 4 = \angle 5$ and $\angle 6 = \angle 8$

And $\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6+\angle 7+\angle 8=360^\circ$ [angles at a point]

$\Rightarrow(\angle 1+\angle 7)+(\angle 3+\angle 2)+(\angle 4+\angle 5)+(\angle 6+\angle 8)=360^\circ$

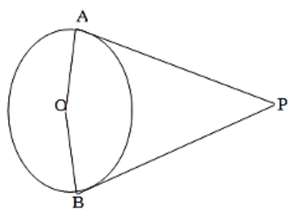
$2\angle 1+2\angle 2+2\angle 6+2\angle 5=360^\circ$

$\Rightarrow \angle 1+\angle 2+\angle 5+\angle 6=180^\circ$

$\Rightarrow \angle AOB+\angle COD=180^\circ$ and $\angle AOD+\angle BOC=180^\circ$

19.

Ans:
Sol:



Given, PA and PB are the tangents drawn from a point P to a circle with center O. Also, the line segments OA and OB are drawn.

To prove: $\angle APB+\angle AOB=180^\circ$

We know that the tangent to a circle is perpendicular to the radius through the point of contact

$\therefore PA\perp OA$

$\Rightarrow \angle OAP=90^\circ$

$PB\perp OB$

$\Rightarrow \angle OBP=90^\circ$

$\therefore \angle OAP+\angle OBP=(90^\circ+90^\circ)=180^\circ$ ....(i)

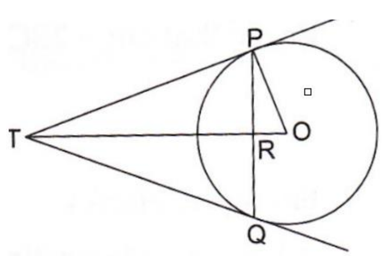
But we know that the sum of all the angles of a quadrilateral is 360°.

$\therefore \angle OAP+\angle OBP+\angle APB+\angle AOB=360^\circ$ ....(ii)

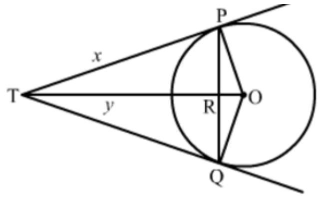
From (i) and (ii), we get:

$\angle APB+\angle AOB=180^\circ$

20.



Sol:



Let TR = y and TP = x

We know that the perpendicular drawn from the center to the chord bisects it.

$\therefore PR = RQ$

Now, PR + RQ = 16

PR + PR = 16

$\Rightarrow PR = 8$

Now, in right triangle POR

By Using Pythagoras theorem, we have

$PO^2 = OR^2 + PR^2$

$\Rightarrow 10^2 = OR^2 + (8)^2$

$\Rightarrow OR^2 = 36$

$\Rightarrow OR = 6$

Now, in right triangle TPR

By Using Pythagoras theorem, we have

$TP^2 = TR^2 + PR^2$

$\Rightarrow x^2 = y^2 + (8)^2$

$\Rightarrow x^2 = y^2 + 64 \qquad ....(1)$

Again, in right triangle TPQ

By Using Pythagoras theorem, we have

$TO^2 = TP^2 + PO^2$

$\Rightarrow (y+6)^2 = x^2 + 10^2$

$\Rightarrow y^2 + 12y + 36 = x^2 + 100$

$\Rightarrow y^2 + 12y = x^2 + 64 \qquad ....(2)$

Solving (1) and (2), we get

$x = 10.67$

$\therefore TP = 10.67\,cm$