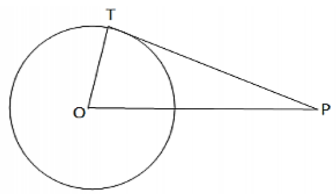
Exercise - 12A

1.

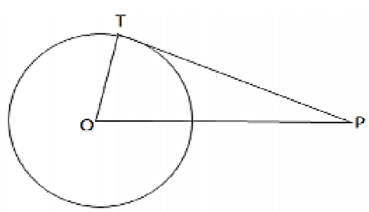
Sol:



Let O be the center of the given circle.
Let P be a point, such that
OP = 17 cm.
Let OT be the radius, where
OT = 5cm
Join TP, where TP is a tangent.
Now, tangent drawn from an external point is perpendicular to the radius at the point of contact.
$\therefore OT \perp PT$
In the right ΔOTP, we have:
$OP^2 = OT^2 + TP^2$ [By Pythagoras' theorem:]
$TP = \sqrt{OP^2 - OT^2}$
$= \sqrt{17^2 - 8^2}$
$= \sqrt{289 - 64}$
$= \sqrt{225}$
$= 15\,cm$
$\therefore$ The length of the tangent is 15 cm.

2.

Sol:



Draw a circle and let P be a point such that OP = 25cm.
Let TP be the tangent, so that TP = 24cm
Join OT where OT is radius.

Now, tangent drawn from an external point is perpendicular to the radius at the point of contact.

$\therefore OT \perp PT$

In the right ΔOTP, we have:

$OP^2 = OT^2 + TP^2$ [By Pythagoras' theorem:]

$OT^2 = \sqrt{OP^2 - TP^2}$

$= \sqrt{25^2 - 24^2}$

$= \sqrt{625 - 576}$

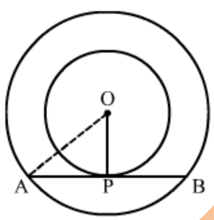
$= \sqrt{49}$

$= 7\,cm$

$\therefore$ The length of the radius is 7cm.

3.

Sol:



We know that the radius and tangent are perpendicular at their point of contact

In right triangle AOP

$AO^2 = OP^2 + PA^2$

$\Rightarrow (6.5)^2 = (2.5)^2 + PA^2$

$\Rightarrow PA^2 = 36$

$\Rightarrow PA = 6\,cm$

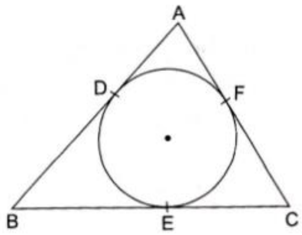
Since, the perpendicular drawn from the center bisects the chord.

$\therefore PA = PB = 6\,cm$

Now, $AB = AP + PB = 6 + 6 = 12\,cm$

Hence, the length of the chord of the larger circle is 12cm.

4.



Sol:

We know that tangent segments to a circle from the same external point are congruent.

Now, we have

$AD = AF$, $BD = BE$ and $CE = CF$

Now, $AD + BD = 12\text{cm}$ …….(1)

$AF + FC = 10$ cm

$\Rightarrow AD + FC = 10$ cm …….(2)

$BE + EC = 8$ cm

$\Rightarrow BD + FC = 8\text{cm}$ …….(3)

Adding all these we get

$AD + BD + AD + FC + BD + FC = 30$

$\Rightarrow 2(AD + BD + FC) = 30$

$\Rightarrow AD + BD + FC = 15\text{cm}$ …….(4)

Solving (1) and (4), we get

$FC = 3$ cm

Solving (2) and (4), we get

$BD = 5$ cm

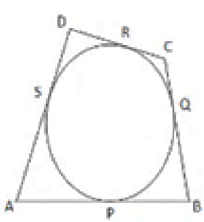
Solving (3) and (4), we get

and $AD = 7$ cm

$\therefore$ $AD = AF = 7$ cm, $BD = BE = 5$ cm and $CE = CF = 3$ cm

5.

Sol:



Let the circle touch the sides of the quadrilateral *AB*, *BC*, *CD* and *DA* at *P*, *Q*, *R* and *S* respectively.

Given, AB = 6cm, BC = 7 cm and CD = 4cm.
Tangents drawn from an external point are equal.

$AP = AS$, $BP = BQ$, $CR = CQ$ and $DR = DS$

Now, $AB + CD\ (AP + BP) + (CR + DR)$

$\Rightarrow AB + CD = (AS + BQ) + (CQ + DS)$

$\Rightarrow AB + CD = (AS + DS) + (BQ + CQ)$

$\Rightarrow AB + CD = AD + BC$

$\Rightarrow AD = (AB + CD) - BC$

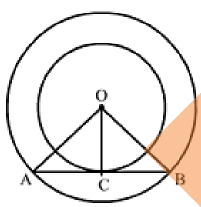
$\Rightarrow AD = (6 + 4) - 7$

$\Rightarrow AD = 3cm.$

$\therefore$ The length of AD is 3 cm.

6.

Sol:
Construction: Join OA, OC and OB



We know that the radius and tangent are perpendicular at their point of contact

$\therefore \angle OCA = \angle OCB = 90°$

Now, In ΔOCA *and* ΔOCB

$\angle OCA = \angle OCB = 90°$

$OA = OB$ (Radii of the larger circle)

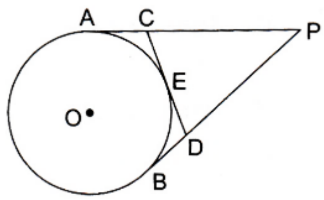
$OC = OC$ (Common)

By RHS congruency

$\Delta OCA \cong \Delta OCB$

$\therefore CA = CB$

7.



Sol:

Given, PA and PB are the tangents to a circle with center O and CD is a tangent at E and PA = 14 cm.

Tangents drawn from an external point are equal.

$\therefore PA = PB, CA = CE$ and $DB = DE$

Perimeter of $\Delta PCD = PC + CD + PD$

$= (PA - CA) + (CE + DE) + (PB - DB)$

$= (PA - CE) + (CE + DE) + (PB - DE)$

$= (PA + PB)$

$= 2PA \ (\because PA = PB)$

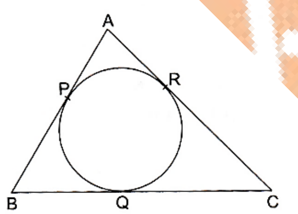
$= (2 \times 14) cm$

$= 28 cm$

=28 cm

$\therefore$ Perimeter of $\Delta PCD = 28 cm.$

8.



Sol:

Given, a circle inscribed in triangle ABC, such that the circle touches the sides of the triangle

Tangents drawn to a circle from an external point are equal.

$\therefore AP = AR = 7cm, CQ = CR = 5cm.$

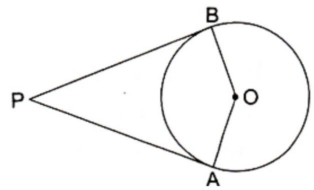
Now, $BP = (AB - AP) = (10 - 7) = 3cm$

$\therefore BP = BQ = 3cm$

$\therefore BC = (BQ + QC)$

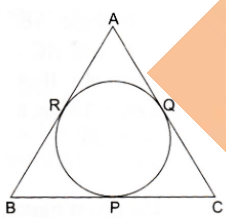
$\Rightarrow BC = 3 + 5$
$\Rightarrow BC = 8$
$\therefore$ The length of BC is 8 cm.

9.



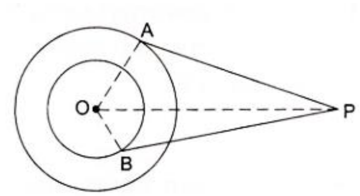
Sol:
Here, $OA = OB$
And $OA \perp AP, OA \perp BP$ (Since tangents drawn from an external point are perpendicular to the radius at the point of contact)
$\therefore \angle OAP = 90°, \angle OBP = 90°$
$\therefore \angle OAP + \angle OBP = 90° + 90° = 180°$
$\therefore \angle AOB + \angle APB = 180°$ (Since, $\angle OAP + \angle OBP + \angle AOB + \angle APB = 360°$)
Sum of opposite angle of a quadrilateral is $180°$.
Hence A, O, B and P are concyclic.

10.



Sol:
We know that tangent segments to a circle from the same external point are congruent
Now, we have
$AR = AO$, $BR = BP$ and $CP = CQ$
Now, $AB = AC$
$\Rightarrow AR + RB = AQ + QC$
$\Rightarrow AR + RB = AR + OC$
$\Rightarrow RB = QC$
$\Rightarrow BP = CP$
Hence, P bisects BC at P.

11.



Sol:

Given, O is the center of two concentric circles of radii OA = 6 cm and OB = 4 cm.
PA and PB are the two tangents to the outer and inner circles respectively and PA = 10 cm.

Now, tangent drawn from an external point is perpendicular to the radius at the point of contact.

$\therefore \angle OAP = \angle OBP = 90°$

$\therefore$ From right – angled $\Delta OAP, OP^2 = OA^2 + PA^2$

$\Rightarrow OP = \sqrt{OA^2 + PA^2}$

$\Rightarrow OP = \sqrt{6^2 + 10^2}$

$\Rightarrow OP = \sqrt{136} cm.$

$\therefore$ From right – angled $\Delta OAP, OP^2 = OB^2 + PB^2$

$\Rightarrow PB = \sqrt{OP^2 - OB^2}$

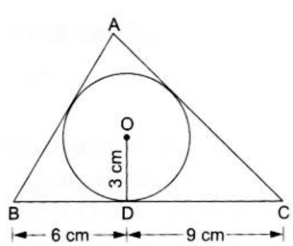
$\Rightarrow PB = \sqrt{136 - 16}$

$\Rightarrow PB = \sqrt{120} cm$

$\Rightarrow PB = 10.9 cm.$

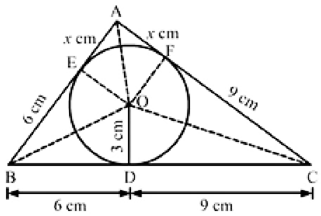
$\therefore$ The length of PB is 10.9 cm.

12.



Sol:

Construction: Join $OA, OB, OC, OE \perp AB$ at E and $OF \perp AC$ at F



We know that tangent segments to a circle from me same external point are congruent

Now, we have

$AE = AF$, $BD = BE$ =6 cm and $CD = CF$ = 9 cm

Now,

$$Area(\Delta ABC) = Area(\Delta BOC) + Area(\Delta AOB) + Area(\Delta AOC)$$

$$\Rightarrow 54 = \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AB \times OE + \frac{1}{2} \times AC \times OF$$

$$\Rightarrow 108 = 15 \times 3 + (6 + x) \times 3 + (9 + x) \times 3$$

$$\Rightarrow 36 = 15 + 6 + x + 9 + x$$

$$\Rightarrow 36 = 30 + 2x$$

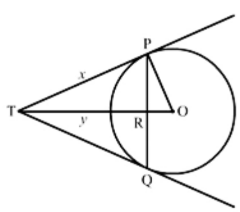
$$\Rightarrow 2x = 6$$

$$\Rightarrow x = 3cm$$

$$\therefore AB = 6 + 3 = 9cm \text{ and } AC = 9 + 3 = 12cm$$

13.

Sol:



Let TR = y and TP= x

We know that the perpendicular drawn from the center to me chord bisects It.

$$\therefore PR = RQ$$

Now, $PR + RQ = 4.8$

$$\Rightarrow PR + PR = 4.8$$

$$\Rightarrow PR = 2.4$$

Now, in right triangle POR

By Using Pythagoras theorem, we have

$PO^2 = OR^2 + PR^2$

$\Rightarrow 3^2 = OR^2 + (2.4)^2$

$\Rightarrow OR^2 = 3.24$

$\Rightarrow OR = 1.8$

Now, in right triangle TPR

By Using Pythagoras theorem, we have

$TP^2 = TR^2 + PR^2$

$\Rightarrow x^2 = y^2 + (2.4)^2$

$\Rightarrow x^2 = y^2 + 5.76 \qquad ....(1)$

Again, In right triangle TPQ

By Using Pythagoras theorem, we have

$TO^2 = TP^2 + PO^2$

$\Rightarrow (y+1.8)^2 = x^2 + 3^2$

$\Rightarrow y^2 + 3.6y + 3.24 = x^2 + 9$

$\Rightarrow y^2 + 3.6y = x^2 + 5.76 \qquad ......(2)$

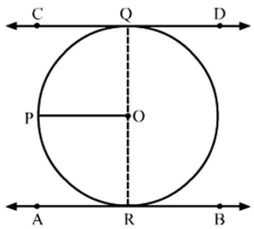
Solving (1) and (2), we get

$x = 4cm \text{ and } y = 3.2cm$

$\therefore TP = 4cm$

14.

Sol:



Suppose CD and AB are two parallel tangents of a circle with center O

Construction: Draw a line parallel to CD passing through O i.e. OP

We know that the radius and tangent are perpendicular at their point of contact.

$\angle OQC = \angle ORA = 90°$

Now, $\angle OQC + \angle POQ = 180°$ (co-interior angles)

$\Rightarrow \angle POQ = 180° - 90° = 90°$

Similarly, Now, $\angle ORA + \angle POR = 180°$ (co-interior angles)

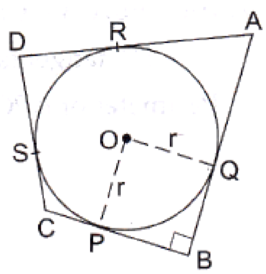
$\Rightarrow \angle POQ = 180° - 90° = 90°$

Now, $\angle POR + \angle POQ = 90° + 90° = 180°$

Since, $\angle POR$ *and* $\angle POQ$ are linear pair angles whose sum is 180°

Hence, QR is a straight line passing through center O.

15.



Sol:

We know that tangent segments to a circle from the same external point are congruent

Now, we have

DS = DR, AR = AQ

Now AD = 23 cm

$\Rightarrow AR + RD = 23$

$\Rightarrow AR = 23 - RD$

$\Rightarrow AR = 23 - 5 \ [\therefore DS = DR = 5]$

$\Rightarrow AR = 18 cm$

Again, AB = 29 cm

$\Rightarrow AQ + QB = 29$

$\Rightarrow QB = 29 - AQ$

$\Rightarrow QB = 29 - 18 \qquad [\because AR = AQ = 18]$

$\Rightarrow QB = 11 cm$

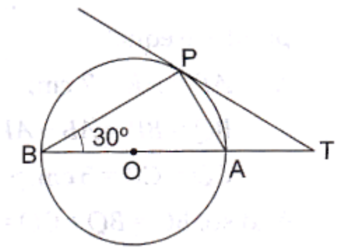
Since all the angles are in a quadrilateral BQOP are right angles and $OP = BQ$

Hence, BQOP is a square.

We know that all the sides of square are equal.

Therefore, BQ = PO = 11 cm

16.



Sol:

AB is the chord passing through the center

So, AB is the diameter

Since, angle in a semicircle is a right angle

$\therefore \angle APB = 90°$

By using alternate segment theorem

We have $\angle APB = \angle PAT = 30°$

Now, in ΔAPB

$\angle BAP + \angle APB + \angle BAP = 180°$ (Angle sum property of triangle)

$\Rightarrow \angle BAP = 180° - 90° - 30° = 60°$

Now, $\angle BAP = \angle APT + \angle PTA$ (Exterior angle property)

$\Rightarrow 60° = 30° + \angle PTA$

$\Rightarrow \angle PTA = 60° - 30° = 30°$

We know that sides opposite to equal angles are equal

$\therefore AP = AT$

In right triangle ABP

$\sin \angle ABP = \dfrac{AP}{BA}$

$\Rightarrow \sin 30° = \dfrac{AT}{BA}$

$\Rightarrow \dfrac{1}{2} = \dfrac{AT}{BA}$

$\therefore BA : AT = 2 : 1$