Exercise - 10D

1.

Sol:

(i) The given equation is $2x^2 - 8x + 5 = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = 2, b = -8$ *and* $c = 5$.

$\therefore$ Discriminant, $D = b^2 - 4ac = (-8)^2 - 4 \times 2 \times 5 = 64 - 40 = 24 > 0$

Hence, the given equation has real and unequal roots.

(ii) The given equation is $3x^2 - 2\sqrt{6}x + 2 = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = 3, b = -2\sqrt{6}$ *and* $c = 2$.

$\therefore$ Discriminant, $D = b^2 - 4ac = \left(-2\sqrt{6}\right)^2 - 4 \times 3 \times 2 = 24 - 24 = 0$

Hence, the given equation has real and equal roots.

(iii) The given equation is $5x^2 - 4x + 1 = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = 5, b = -4$ *and* $c = 1$.

$\therefore$ Discriminant, $D = b^2 - 4ac = (-4)^2 - 4 \times 5 \times 1 = 16 - 20 = -4 < 0$

Hence, the given equation has no real roots.

(iv) The given equation is

$5x(x-2)+6=0$

$\Rightarrow 5x^2-10x+6=0$

This is of the form $ax^2+bx+c=0$, where $a=5, b=-10$ *and* $c=6$.

Discriminant, $D=b^2-4ac=(-10)^2-4\times5\times6=100-120=-20<0$

Hence, the given equation has no real roots.

(v) The given equation is $12x^2-4\sqrt{15}x+5=0$

This is of the form $ax^2+bx+c=0$, where $a=12, b=-4\sqrt{15}$ *and* $c=5$.

Discriminant, $D=b^2-4ac=\left(-4\sqrt{15}\right)^2-4\times12\times5=240-240=0$

Hence, the given equation has real and equal roots.

(vi) The given equation is $x^2-x+2=0$.

This is of the form $ax^2+bx+c=0$, where $a=1, b=-1$ *and* $c=2$.

Discriminant, $D=b^2-4ac=(-1)^2-4\times1\times2=1-8=-7<0$

Hence, the given equation has no real roots.

2.

Sol:

The given equation is $2\left(a^2+b^2\right)x^2+2(a+b)x+1=0$.

$\therefore D=\left[2(a+b)\right]^2-4\times2\left(a^2+b^2\right)\times1$

$=4\left(a^2+2ab+b^2\right)-8\left(a^2+b^2\right)$

$=4a^2+8ab+4b^2-8a^2-8b^2$

$=-4a^2+8ab-4b^2$

$=-4\left(a^2-2ab+b^2\right)$

$=-4(a-b)^2<0$

Hence, the given equation has no real roots.

3.

Sol:

Given:

$x^2+px-q^2=0$

Here,

$a=1, b=p$ *and* $c=-q^2$

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Discriminant D is given by:

$D = (b^2 - 4ac)$

$= p^2 - 4 \times 1 \times (-q^2)$

$= (p^2 + 4q^2) > 0$

$D > 0$ for all real values of p and q.

Thus, the roots of the equation are real.

4.

Sol:

Given:

$3x^2 + 2kx + 27 = 0$

Here,

$a = 3, b = 2k \text{ and } c = 27$

It is given that the roots of the equation are real and equal; therefore, we have:

$D = 0$

$\Rightarrow (2k)^2 - 4 \times 3 \times 27 = 0$

$\Rightarrow 4k^2 - 324 = 0$

$\Rightarrow 4k^2 = 324$

$\Rightarrow k^2 = 81$

$\Rightarrow k = \pm 9$

$\therefore k = 9 \text{ or } k = -9$

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5.

Sol:

The given equation is

$kx(x - 2\sqrt{5}) + 10 = 0$

$\Rightarrow kx^2 - 2\sqrt{5}kx + 10 = 0$

This is of the form $ax^2 + bx + c = 0$, where $a = k, b = -2\sqrt{5}k \text{ and } c = 10$.

$\therefore D = b^2 - 4ac = (-2\sqrt{5}k)^2 - 4 \times k \times 10 = 20k^2 - 40k$

The given equation will have real and equal roots if $D = 0$.

$\therefore 20k^2 - 40k = 0$

$\Rightarrow 20k(k-2) = 0$

$\Rightarrow k = 0 \text{ or } k - 2 = 0$

$\Rightarrow k = 0 \text{ or } k = 2$

But, for $k = 0,$ we get $10 = 0,$ which is not true

Hence, 2 is the required value of *k*.

6.

Sol:

The given equation is $4x^2 + px + 3 = 0.$

This is of the form $ax^2 + bx + c = 0,$ where $a = 4, b = p \text{ and } c = 3.$

$\therefore D = b^2 - 4ac = p^2 - 4 \times 4 \times 3 = p^2 - 48$

The given equation will have real and equal roots if $D = 0.$

$\therefore p^2 - 48 = 0$

$\Rightarrow p^2 = 48$

$\Rightarrow p = \pm\sqrt{48} = \pm 4\sqrt{3}$

Hence, $4\sqrt{3}$ and $-4\sqrt{3}$ are the required values of *p*.

7.

Sol:

The given equation is $9x^2 - 3kx + k = 0.$

This is of the form $ax^2 + bx + c = 0,$ where $a = 9, b = -3k \text{ and } c = k.$

$\therefore D = b^2 - 4ac = (-3k)^2 - 4 \times 9 \times k = 9k^2 - 36k$

The given equation will have real and equal roots if $D = 0.$

$\therefore 9k^2 - 36k = 0$

$\Rightarrow 9k(k-4) = 0$

$\Rightarrow k = 0 \text{ or } k - 4 = 0$

$\Rightarrow k = 0 \text{ or } k = 4$

But, $k \neq 0$ (Given)

Hence, the required values of *k* is 4.

8.

Sol:

The given equation is $(3k+1)x^2+2(k+1)x+1=0.$

This is of the form $ax^2+bx+c=0,$ where $a=3k+1, b=2(k+1)$ *and* $c=1.$

$\therefore D=b^2-4ac$

$=[2(k+1)]^2-4\times(3k+1)\times1$

$=4(k^2+2k+1)-4(3k+1)$

$=4k^2+8k+4-12k-4$

$=4k^2-4k$

The given equation will have real and equal roots if $D=0.$

$\therefore 4k^2-4k=0$

$\Rightarrow 4k(k-1)=0$

$\Rightarrow k=0$ *or* $k-1=0$

$\Rightarrow k=0$ *or* $k=1$

Hence, 0 and 1are the required values of k.

9.

Sol:

The given equation is $(2p+1)x^2-(7p+2)x+(7p-3)=0.$

This is of the form $ax^2+bx+c=0,$ where $a=2p+1, b=-(7p+2)$ *and* $c=7p-3.$

$\therefore D=b^2-4ac$

$=-[-(7p+2)]^2-4\times(2p+1)\times(7p-3)$

$=(49p^2+28p+4)-4(14p^2+p-3)$

$=49p^2+28p+4-56p^2-4p+12$

$=-7p^2+24p+16$

The given equation will have real and equal roots if $D=0.$

$\therefore -7p^2+24p+16=0$

$\Rightarrow 7p^2-24p-16=0$

$\Rightarrow 7p^2-28p+4p-16=0$

$\Rightarrow 7p(p-4)+4(p-4)=0$

$\Rightarrow (p-4)(7p+4)=0$

$\Rightarrow p-4=0$ *or* $7p+4=0$

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$\Rightarrow p = 4 \text{ or } p = -\frac{4}{7}$

Hence, 4 and $-\frac{4}{7}$ are the required values of p.

10.

Sol:

The given equation is $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0.$

This is of the form $ax^2 + bx + c = 0,$ where $a = p+1, b = -6(p+1) \text{ and } c = 3(p+9).$

$\therefore D = b^2 - 4ac$

$= [-6(p+1)]^2 - 4 \times (p+1) \times 3(p+9)$

$= 12(p+1)[3(p+1) - (p+9)]$

$= 12(p+1)(2p-6)$

The given equation will have real and equal roots if $D = 0.$

$\therefore 12(p+1)(2p-6) = 0$

$\Rightarrow p+1 = 0 \text{ or } 2p-6 = 0$

$\Rightarrow p = -1 \text{ or } p = 3$

But, $p \neq -1$ (Given)

Thus, the value of p is 3

Putting $p = 3,$ the given equation becomes $4x^2 - 24x + 36 = 0$

$4x^2 - 24x + 36 = 0$

$\Rightarrow 4(x^2 - 6x + 9) = 0$

$\Rightarrow (x-3)^2 = 0$

$\Rightarrow x - 3 = 0$

$\Rightarrow x = 3$

Hence, 3 is the repeated root of this equation.

11.

Sol:

It is given that -5 is a root of the quadratic equation $2x^2 + px - 15 = 0.$

$\therefore 2(-5)^2 + p \times (-5) - 15 = 0$

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$\Rightarrow -5p + 35 = 0$

$\Rightarrow p = 7$

The roots of the equation $px^2 + px + k = 0 = 0$ are equal.

$\therefore D = 0$

$\Rightarrow p^2 - 4pk = 0$

$\Rightarrow (7)^2 - 4 \times 7 \times k = 0$

$\Rightarrow 49 - 28k = 0$

$\Rightarrow k = \dfrac{49}{28} = \dfrac{7}{4}$

Thus, the value of k is $\dfrac{7}{4}$.

12.

Sol:

It is given that 3 is a root of the quadratic equation $x^2 - x + k = 0$.

$\therefore (3)^2 - 3 + k = 0$

$\Rightarrow k + 6 = 0$

$\Rightarrow k = -6$

The roots of the equation $x^2 + 2kx + \left(k^2 + 2k + p\right) = 0$ are equal.

$\therefore D = 0$

$\Rightarrow (2k)^2 - 4 \times 1 \times \left(k^2 + 2k + p\right) = 0$

$\Rightarrow 4k^2 - 4k^2 - 8k - 4p = 0$

$\Rightarrow -8k - 4p = 0$

$\Rightarrow p = \dfrac{8k}{-4} = -2k$

$\Rightarrow p = -2 \times (-6) = 12$

Hence, the value of p is 12.

13.

Sol:

It is given that -4 is a root of the quadratic equation $x^2 + 2x + 4p = 0$.

$\therefore (-4)^2 + 2\times(-4) + 4p = 0$

$\Rightarrow 16 - 8 + 4p = 0$

$\Rightarrow 4p + 8 = 0$

$\Rightarrow p = -2$

The equation $x^2 + px(1+3k) + 7(3+2k) = 0$ has real roots.

$\therefore D = 0$

$\Rightarrow \left[p(1+3k)\right]^2 - 4\times1\times7(3+2k) = 0$

$\Rightarrow \left[-2(1+3k)\right]^2 - 28(3+2k) = 0$

$\Rightarrow 4(1+6k+9k^2) - 28(3+2k) = 0$

$\Rightarrow 4(1+6k+9k^2-21-14k) = 0$

$\Rightarrow 9k^2 - 8k - 20 = 0$

$\Rightarrow 9k^2 - 18k + 10k - 20 = 0$

$\Rightarrow 9k(k-2) + 10(k-2) = 0$

$\Rightarrow (k-2)(9k+10) = 0$

$\Rightarrow k - 2 = 0 \ or \ 9k + 10 = 0$

$\Rightarrow k = 2 \ or \ k = -\frac{10}{9}$

Hence, the required value of k is 2 or $-\frac{10}{9}$.

14.

Sol:

Given:

$(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$

Here,

$a = (1+m^2), b = 2mc \ and \ c = (c^2 - a^2)$

It is given that the roots of the equation are equal; therefore, we have:

$D = 0$

$\Rightarrow (b^2 - 4ac) = 0$

$\Rightarrow (2mc)^2 - 4\times(1+m^2)\times(c^2 - a^2) = 0$

$\Rightarrow 4m^2c^2 - 4\left(c^2 - a^2 + m^2c^2 - m^2a^2\right) = 0$

$\Rightarrow 4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$

$\Rightarrow -4c^2 + 4a^2 + 4m^2a^2 = 0$

$\Rightarrow a^2 + m^2a^2 = c^2$

$\Rightarrow a^2\left(1 + m^2\right) = c^2$

$\Rightarrow c^2 = a^2\left(1 + m^2\right)$

Hence proved.

15.

Sol:

Given:

$\left(c^2 - ab\right)x^2 - 2\left(a^2 - bc\right)x + \left(b^2 - ac\right) = 0$

Here,

$a = \left(c^2 - ab\right), b = -2\left(a^2 - bc\right), c = \left(b^2 - ac\right)$

It is given that the roots of the equation are real and equal; therefore, we have:

$D = 0$

$\Rightarrow \left(b^2 - 4ac\right) = 0$

$\Rightarrow \left\{-2\left(a^2 - bc\right)\right\}^2 - 4 \times \left(c^2 - ab\right) \times \left(b^2 - ac\right) = 0$

$\Rightarrow 4\left(a^4 - 2a^2bc + b^2c^2\right) - 4\left(b^2c^2 - ac^3 - ab^3 + a^2bc\right) = 0$

$\Rightarrow a^4 - 2a^2bc + b^2c^2 - b^2c^2 + ac^3 + ab^3 - a^2bc = 0$

$\Rightarrow a^4 - 3a^2bc + ac^3 + ab^3 = 0$

$\Rightarrow a\left(a^3 - 3abc + c^3 + b^3\right) = 0$

Now,

$a = 0 \ or \ a^3 - 3abc + c^3 + b^3 = 0$

$a = 0 \ or \ a^3 + b^3 + c^3 = 3abc$

16. Sol:

Given:

$2x^2 + px + 8 = 0$

Here,

$a=2, b=p \ and \ c=8$

Discriminant D is given by:

$D=\left(b^2-4ac\right)$

$=p^2-4\times2\times8$

$=\left(p^2-64\right)$

If $D\ge0$, the roots of the equation will be real

$\Rightarrow\left(p^2-64\right)\ge0$

$\Rightarrow\left(p+8\right)\left(p-8\right)\ge0$

$\Rightarrow p\ge8 \ and \ p\le-8$

Thus, the roots of the equation are real for $p\ge8$ and $p\le-8$.

17.

Sol:

Given:

$\left(\alpha-12\right)x^2+2\left(\alpha-12\right)x+2=0$

Here,

$a=\left(\alpha=12\right), b=2\left(\alpha-12\right) \ and \ c=2$

It is given that the roots of the equation are equal; therefore, we have

$D=0$

$\Rightarrow\left(b^2-4ac\right)=0$

$\Rightarrow\left\{2\left(\alpha-12\right)\right\}^2-4\times\left(\alpha-12\right)\times2=0$

$\Rightarrow4\left(\alpha^2-24\alpha+144\right)-8\alpha+96=0$

$\Rightarrow4\alpha^2-96\alpha+576-8\alpha+96=0$

$\Rightarrow4\alpha^2-104\alpha+672=0$

$\Rightarrow\alpha^2-26\alpha+168=0$

$\Rightarrow\alpha^2-14\alpha-12\alpha+168=0$

$\Rightarrow\alpha\left(\alpha-14\right)-12\left(\alpha-14\right)=0$

$\Rightarrow\left(\alpha-14\right)\left(\alpha-12\right)=0$

$\therefore\alpha=14 \ or \ \alpha=12$

If the value of   is 12, the given equation becomes non-quadratic.

Therefore, the value of   will be 14 for the equation to have equal roots.

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18.

Sol:

Given:

$9x^2 + 8kx + 16 = 0$

Here,

$a = 9, b = 8k \text{ and } c = 16$

It is given that the roots of the equation are real and equal; therefore, we have:

$D = 0$

$\Rightarrow (b^2 - 4ac) = 0$

$\Rightarrow (8k)^2 - 4 \times 9 \times 16 = 0$

$\Rightarrow 64k^2 - 576 = 0$

$\Rightarrow 64k^2 = 576$

$\Rightarrow k^2 = 9$

$\Rightarrow k = \pm 3$

$\therefore k = 3 \text{ or } k = -3$

19.

Sol:

(i) The given equation is $kx^2 + 6x + 1 = 0.$

$\therefore D = 6^2 - 4 \times k \times 1 = 36 - 4k$

The given equation has real and distinct roots if $D > 0.$

$\therefore 36 - 4k > 0$

$\Rightarrow 4k < 36$

$\Rightarrow k < 9$

(ii) The given equation is $x^2 - kx + 9 = 0.$

$\therefore D = (-k)^2 - 4 \times 1 \times 9 = k^2 - 36$

The given equation has real and distinct roots if $D > 0.$

$\therefore k^2 - 36 > 0$

$\Rightarrow (k - 6)(k + 6) > 0$

$\Rightarrow k < -6 \text{ or } k > 6$

(iii) The given equation is $9x^2 + 3kx + 4 = 0.$

$\therefore D = (3k)^2 - 4 \times 9 \times 4 = 9k^2 - 144$

The given equation has real and distinct roots if $D > 0.$

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$\therefore 9k^2 - 144 > 0$

$\Rightarrow 9\left(k^2 - 16\right) > 0$

$\Rightarrow (k-4)(k+4) > 0$

$\Rightarrow k < -4 \; or \; k > 4$

(iv) The given equation is $5x^2 - kx + 1 = 0$.

$\therefore D = (-k)^2 - 4 \times 5 \times 1 = k^2 - 20$

The given equation has real and distinct roots if $D > 0$.

$\therefore k^2 - 20 > 0$

$\Rightarrow k^2 - \left(2\sqrt{5}\right)^2 > 0$

$\Rightarrow \left(k - 2\sqrt{5}\right)\left(k + 2\sqrt{5}\right) > 0$

$\Rightarrow k < -2\sqrt{5} \; or \; k > 2\sqrt{5}$

20.

Sol:

The given equation is $(a-b)x^2 + 5(a+b)x - 2(a-b) = 0$.

$\therefore D = \left[5(a+b)\right]^2 - 4 \times (a-b) \times \left[-2(a-b)\right]$

$= 25(a+b)^2 + 8(a-b)^2$

Since a and b are real and $a \neq b$, so $(a-b)^2 > 0 \; and \; (a+b)^2 > 0$.

$\therefore 8(a-b)^2 > 0$ ....(1) (Product of two positive numbers is always positive)

Also, $25(a+b)^2 > 0$ ....(2) (Product of two positive numbers is always positive)

Adding (1) and (2), we get

$25(a+b)^2 + 8(a-b)^2 > 0$ (Sum of two positive numbers is always positive)

$\Rightarrow D > 0$

Hence, the roots of the given equation are real and unequal.

21.

Sol:

It is given that the roots of the equation $\left(a^2 + b^2\right)x^2 - 2(ac + bd)x + \left(c^2 + d^2\right) = 0$ are equal.

$\therefore D=0$

$\Rightarrow \left[-2\left(ac+bd\right)\right]^2-4\left(a^2+b^2\right)\left(c^2+d^2\right)=0$

$\Rightarrow 4\left(a^2c^2+b^2d^2+2abcd\right)-4\left(a^2c^2+a^2d^2+b^2c^2+b^2d^2\right)=0$

$\Rightarrow 4\left(a^2c^2+b^2d^2+2abcd-a^2c^2-a^2d^2-b^2c^2-b^2d^2\right)=0$

$\Rightarrow \left(-a^2d^2+2abcd-b^2c^2\right)=0$

$\Rightarrow -\left(a^2d^2-2abcd+b^2c^2\right)=0$

$\Rightarrow \left(ad-bc\right)^2=0$

$\Rightarrow ad-bc=0$

$\Rightarrow ad=bc$

$\Rightarrow \frac{a}{b}=\frac{c}{d}$

Hence proved.

22.

Sol:

It is given that the roots of the equation $ax^2+2bx+c=0$ are real.

$\therefore D_1=\left(2b\right)^2-4\times a\times c\geq 0$

$\Rightarrow 4\left(b^2-ac\right)\geq 0$

$\Rightarrow b^2-ac\geq 0$ ....(1)

Also, the roots of the equation $bx^2-2\sqrt{ac}x+b=0$ are real.

$\therefore D_2=\left(-2\sqrt{ac}\right)^2-4\times b\times b\geq 0$

$\Rightarrow 4\left(ac-b^2\right)\geq 0$

$\Rightarrow -4\left(b^2-ac\right)\geq 0$

$\Rightarrow b^2-ac\geq 0$ ....(2)

The roots of the given equations are simultaneously real if (1) and (2) holds true together. This is possible if

$b^2-ac=0$

$\Rightarrow b^2=ac$

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