

1. Evaluate:

$$\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

Sol:

On substituting the values of various T-ratios, we get:

$$\begin{aligned} & \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ \\ &= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} \right) = \left(\frac{3}{4} + \frac{1}{4} \right) = \frac{4}{4} = 1 \end{aligned}$$

2. Evaluate:

$$\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$$

Sol:

On substituting the values of various T-ratios, we get:

$$\begin{aligned} & \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ \\ &= \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} \right) = \left(\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} \right) = 0 \end{aligned}$$

3. Evaluate:

$$\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

Sol:

On substituting the values of various T-ratios, we get:

$$\begin{aligned} & \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\ &= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \right) = \left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \right) = \left(\frac{\sqrt{3}+1}{2\sqrt{2}} \right) \end{aligned}$$

4. Evaluate:

$$\frac{\sin 30^\circ}{\cos 45^\circ} + \frac{\cot 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\tan 45^\circ} + \frac{\cos 30^\circ}{\sin 90^\circ}$$

Sol:

$$\frac{\sin 30^\circ}{\cos 45^\circ} + \frac{\cot 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\tan 45^\circ} + \frac{\cos 30^\circ}{\sin 90^\circ}$$

$$= \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)} + \frac{1}{2} - \frac{\left(\frac{\sqrt{3}}{2}\right)}{1} + \frac{\left(\frac{\sqrt{3}}{2}\right)}{1}$$

$$= \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2} + 1}{2}$$

5. Evaluate:

$$\frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

Sol:

$$\frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$\begin{aligned}
 &= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\
 &= \frac{\left(\frac{5}{4} + \frac{4 \times 4}{3} - 1\right)}{\left(\frac{1}{4} + \frac{3}{4}\right)} \\
 &= \frac{\left(\frac{5}{4} + \frac{16}{3} - \frac{1}{1}\right)}{\left(\frac{4}{4}\right)} \\
 &= \frac{(15+64-12)}{4} \\
 &= \frac{67}{12}
 \end{aligned}$$

6. Evaluate:

$$2\cos^2 60^\circ + 3\sin^2 45^\circ - 3\sin^2 30^\circ + 2\cos^2 90^\circ$$

Sol:

On substituting the values of various T-ratios, we get:

$$\begin{aligned}
 &2\cos^2 60^\circ + 3\sin^2 45^\circ - 3\sin^2 30^\circ + 2\cos^2 90^\circ \\
 &= 2 \times \left(\frac{1}{2}\right)^2 + 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 3 \times \left(\frac{1}{2}\right)^2 + 2 \times (0)^2 \\
 &= 2 \times \frac{1}{4} + 3 \times \frac{1}{2} - 3 \times \frac{1}{4} + 0 \\
 &= \left(\frac{1}{2} + \frac{3}{2} - \frac{3}{4}\right) = \left(\frac{2+6-3}{4}\right) = \frac{5}{4}
 \end{aligned}$$

7. Evaluate:

$$\cot^2 30^\circ - 2\cos^2 30^\circ - \frac{3}{4}\sec^2 45^\circ + \frac{1}{4}\operatorname{cosec}^2 30^\circ$$

Sol:

On substituting the values of various T-ratios, we get:

$$\begin{aligned}
 &\cot^2 30^\circ - 2\cos^2 30^\circ - \frac{3}{4}\sec^2 45^\circ + \frac{1}{4}\operatorname{cosec}^2 30^\circ \\
 &= (\sqrt{3})^2 - 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} \times (\sqrt{2})^2 + \frac{1}{4} \times (2)^2 \\
 &= 3 - 2 \times \frac{3}{4} - \frac{3}{4} \times 2 + \frac{1}{4} \times 4 \\
 &= 3 - \frac{3}{2} - \frac{3}{2} + 1 \\
 &= 4 - \left(\frac{3}{2} + \frac{3}{2}\right) \\
 &= 4 - 3 = 1
 \end{aligned}$$

8. Evaluate:

$$(\sin^2 30^\circ + 4 \cot^2 45^\circ - \sec^2 60^\circ) (\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)$$

Sol:

On substituting the values of various T-ratios, we get:

$$(\sin^2 30^\circ + 4 \cot^2 45^\circ - \sec^2 60^\circ) (\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)$$

$$= \left[\left(\frac{1}{2}\right)^2 + 4 \times (1)^2 - (2)^2 \right] \left[(\sqrt{2})^2 \left(\frac{2}{\sqrt{3}}\right)^2 \right]$$

$$= \left(\frac{1}{4} + 4 - 4\right) \left(2 \times \frac{4}{3}\right)$$

$$= \frac{1}{4} \times \frac{8}{3} = \frac{2}{3}$$

9. Evaluate:

$$\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 30^\circ} - 2 \cos^2 45^\circ - \sin^2 0^\circ$$

Sol:

On substituting the values of various T-ratios, we get:

$$\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 30^\circ} - 2 \cos^2 45^\circ - \sin^2 0^\circ$$

$$= \frac{4}{(\sqrt{3})^2} + \frac{1}{\left(\frac{1}{2}\right)^2} - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - (0)^2$$

$$= \frac{4}{3} + \frac{1}{\frac{1}{4}} - 2 \times \frac{1}{2} - 0$$

$$= \frac{4}{3} + 4 - 1$$

$$= \frac{4}{3} + 3 = \frac{4+9}{3} = \frac{13}{3}$$

10. Show that:

$$(i) \frac{1 - \sin 60^\circ}{\cos 60^\circ} = \frac{\tan 60^\circ - 1}{\tan 60^\circ + 1}$$

$$(ii) \frac{\cos 30^\circ + \sin 60^\circ}{1 + \sin 30^\circ + \cos 60^\circ} = \cos 30^\circ$$

Sol:

(i)

$$\text{LHS} = \frac{1 - \sin 60^\circ}{\cos 60^\circ} = \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\left(\frac{2 - \sqrt{3}}{2}\right)}{\frac{1}{2}} = \left(\frac{2 - \sqrt{3}}{2}\right) \times 2 = 2 - \sqrt{3}$$

$$\text{RHS} = \frac{\tan 60^\circ - 1}{\tan 60^\circ + 1} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - 1^2} = \frac{3 + 1 - 2\sqrt{3}}{3 - 1} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

Hence, LHS = RHS

$$\therefore \frac{1 - \sin 60^\circ}{\cos 60^\circ} = \frac{\tan 60^\circ - 1}{\tan 60^\circ + 1}$$

(ii)

$$\text{LHS} = \frac{\cos 30^\circ + \sin 60^\circ}{1 + \sin 30^\circ + \cos 60^\circ} = \frac{\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)}{1 + \frac{1}{2} + \frac{1}{2}} = \frac{\frac{\sqrt{3} + \sqrt{3}}{2}}{\frac{2 + 1 + 1}{2}} = \frac{\sqrt{3}}{2}$$

$$\text{Also, RHS} = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Hence, LHS = RHS

$$\therefore \frac{\cos 30^\circ + \sin 60^\circ}{1 + \sin 30^\circ + \cos 60^\circ} = \cos 30^\circ$$

11. Verify each of the following:

(i) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

(ii) $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

(iii) $2 \sin 30^\circ \cos 30^\circ$

(iv) $2 \sin 45^\circ \cos 45^\circ$

Sol:

(i) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right) \times \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

Also, $\sin 30^\circ = \frac{1}{2}$

$$\therefore \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ$$

(ii) $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

$$= \left(\frac{1}{2}\right) \times \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right) \times \left(\frac{1}{2}\right) = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

Also, $\cos 30^\circ = \frac{\sqrt{3}}{2}$

$$\therefore \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ = \cos 30^\circ$$

(iii) $2 \sin 30^\circ \cos 30^\circ$

$$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

Also, $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$$\therefore 2 \sin 30^\circ \cos 30^\circ = \sin 60^\circ$$

(iv) $2 \sin 45^\circ \cos 45^\circ$

$$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$$

Also, $\sin 90^\circ = 1$

$$\therefore 2 \sin 45^\circ \cos 45^\circ = \sin 90^\circ$$

12. If $A = 45^\circ$, verify that:

(i) $\sin 2A = 2 \sin A \cos A$

(ii) $\cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$

Sol:

$$A = 45^\circ$$

$$\Rightarrow 2A = 2 \times 45^\circ = 90^\circ$$

(i) $\sin 2A = \sin 90^\circ = 1$

$$2 \sin A \cos A = 2 \sin 45^\circ \cos 45^\circ = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$$

$$\therefore \sin 2A = 2 \sin A \cos A$$

$$(ii) \cos 2A = \cos 90^\circ = 0$$

$$2 \cos^2 A - 1 = 2 \cos^2 45^\circ - 1 = 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 2 \times \frac{1}{2} - 1 = 1 - 1 = 0$$

$$\text{Now, } 1 - 2 \sin^2 A = 1 - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 1 - 2 \times \frac{1}{2} = 1 - 1 = 0$$

$$\therefore \cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

13. If $A = 30^\circ$, verify that:

$$(i) \sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$(ii) \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$(iii) \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Sol:

$$A = 30^\circ$$

$$\Rightarrow 2A = 2 \times 30^\circ = 60^\circ$$

$$(i) \sin 2A = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\frac{2 \tan A}{1 + \tan^2 A} = \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1 + \frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{4}{3}} = \left(\frac{2}{\sqrt{3}}\right) \times \frac{3}{4} = \frac{\sqrt{3}}{2}$$

$$\therefore \sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$(ii) \cos 2A = \cos 60^\circ = \frac{1}{2}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ} = \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(1 - \frac{1}{3}\right)}{1 + \frac{1}{3}} = \frac{\left(\frac{2}{3}\right)}{\frac{4}{3}} = \left(\frac{2}{3}\right) \times \frac{3}{4} = \frac{1}{2}$$

$$\therefore \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$(iii) \tan 2A = \tan 60^\circ = \sqrt{3}$$

$$\frac{2 \tan A}{1 - \tan^2 A} = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1 - \frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{2}{3}} = \left(\frac{2}{\sqrt{3}}\right) \times \frac{3}{2} = \sqrt{3}$$

$$\therefore \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

14. If $A = 60^\circ$ and $B = 30^\circ$, verify that:

(i) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

(ii) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

Sol:

$A = 60^\circ$ and $B = 30^\circ$

Now, $A + B = 60^\circ + 30^\circ = 90^\circ$

Also, $A - B = 60^\circ - 30^\circ = 30^\circ$

(i) $\sin(A + B) = \sin 90^\circ = 1$

$$\sin A \cos B + \cos A \sin B = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

$$= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}\right) = \left(\frac{3}{4} + \frac{1}{4}\right) = 1$$

$$\therefore \sin(A + B) = \sin A \cos B + \cos A \sin B$$

(ii) $\cos(A + B) = \cos 90^\circ = 0$

$$\cos A \cos B - \sin A \sin B = \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$$

$$= \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}\right) = 0$$

$$\therefore \cos(A + B) = \cos A \cos B - \sin A \sin B$$

15. If $A = 60^\circ$ and $B = 30^\circ$, verify that:

(i) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

(ii) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

(iii) $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Sol:

(i) $\sin(A - B) = \sin 30^\circ = \frac{1}{2}$

$$\sin A \cos B - \cos A \sin B = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$$

$$= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}\right) = \left(\frac{3}{4} - \frac{1}{4}\right) = \frac{2}{4} = \frac{1}{2}$$

$$\therefore \sin(A - B) = \sin A \cos B - \cos A \sin B$$

(ii) $\cos(A - B) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$$\cos A \cos B + \sin A \sin B = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$$

$$= \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}\right) = 2 \times \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$\therefore \cos(A - B) = \cos A \cos B + \sin A \sin B$$

(iii) $\tan(A - B) = \tan 30^\circ = \frac{1}{\sqrt{3}}$

$$\frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ} = \frac{\sqrt{3} - \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\sqrt{3} \times \frac{1}{\sqrt{3}}\right)} = \frac{1}{2} \times \frac{3-1}{\sqrt{3}} = \frac{3}{\sqrt{3}}$$

$$\therefore \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

16. If A and B are acute angles such that $\tan A = \frac{1}{3}$, $\tan B = \frac{1}{2}$ and $\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$, show that $A + B = 45^\circ$.

Sol:

Given:

$$\tan A = \frac{1}{3} \text{ and } \tan B = \frac{1}{2}$$

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

On substituting these values in RHS of the expression, we get:

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\left(\frac{1}{3} + \frac{1}{2}\right)}{\left(1 - \frac{1}{3} \times \frac{1}{2}\right)} = \frac{\left(\frac{5}{6}\right)}{\left(1 - \frac{1}{6}\right)} = \frac{\left(\frac{5}{6}\right)}{\left(\frac{5}{6}\right)} = 1$$

$$\Rightarrow \tan (A + B) = 1 = \tan 45^\circ \quad [\because \tan 45^\circ = 1]$$

$$\therefore A + B = 45^\circ$$

17. Using the formula, $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$, find the value of $\tan 60^\circ$, it being given that $\tan 30^\circ = \frac{1}{\sqrt{3}}$.

Sol:

$$A = 30^\circ$$

$$\Rightarrow 2A = 2 \times 30^\circ = 60^\circ$$

By substituting the value of the given T-ratio, we get:

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\Rightarrow \tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1 - \frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{2}{3}} = \left(\frac{2}{\sqrt{3}}\right) \times \frac{3}{2} = \sqrt{3}$$

$$\therefore \tan 60^\circ = \sqrt{3}$$

18. Using the formula, $\cos A = \sqrt{\frac{1 + \cos 2A}{2}}$, find the value of $\cos 30^\circ$, it being given that $\cos 60^\circ = \frac{1}{2}$.

Sol:

$$A = 30^\circ$$

$$\Rightarrow 2A = 2 \times 30^\circ = 60^\circ$$

By substituting the value of the given T-ratio, we get:

$$\cos A = \sqrt{\frac{1 + \cos 2A}{2}}$$

$$\cos 30^\circ = \sqrt{\frac{1 + \cos 60^\circ}{2}} = \sqrt{\frac{1 + \frac{1}{2}}{2}} = \sqrt{\frac{\frac{3}{2}}{2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$\therefore \cos A = \frac{\sqrt{3}}{2}$$

19. Using the formula, $\sin A = \sqrt{\frac{1 - \cos 2A}{2}}$, find the value of $\sin 30^\circ$, it being given that $\cos 60^\circ = \frac{1}{2}$.

Sol:

$$A = 30^\circ$$

$$\Rightarrow 2A = 2 \times 30^\circ = 60^\circ$$

By substituting the value of the given T-ratio, we get:

$$\sin A = \sqrt{\frac{1 - \cos 2A}{2}}$$

$$\sin 30^\circ = \sqrt{\frac{1 - \cos 60^\circ}{2}} = \sqrt{\frac{1 - \frac{1}{2}}{2}} = \sqrt{\frac{\frac{1}{2}}{2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\therefore \sin 30^\circ = \frac{1}{2}$$

20. In the adjoining figure, $\triangle ABC$ is a right-angled triangle in which $\angle B = 90^\circ$, $\angle 30^\circ$ and $AC = 20\text{cm}$.

Find (i) BC , (ii) AB .

Sol:

From the given right-angled triangle, we have:

$$\frac{BC}{AC} = \sin 30^\circ$$

$$\Rightarrow \frac{BC}{20} = \frac{1}{2}$$

$$\Rightarrow BC = \frac{20}{2} = 10\text{cm}$$

$$\text{Also, } \frac{AB}{AC} = \cos 30^\circ$$

$$\Rightarrow \frac{AB}{20} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow AB = \left(20 \times \frac{\sqrt{3}}{2}\right) = 10\sqrt{3}\text{ cm}$$

$$\therefore BC = 10\text{cm and } AB = 10\sqrt{3}\text{ cm}$$

21. In the adjoining figure, $\triangle ABC$ is right-angled at B and $\angle A = 30^\circ$. If $BC = 6\text{cm}$, find (i) AB , (ii) AC .

Sol:

From the given right-angled triangle, we have:

$$\frac{BC}{AB} = \tan 30^\circ$$

$$\Rightarrow \frac{6}{AB} = \frac{1}{\sqrt{3}}$$

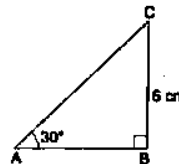
$$\Rightarrow AB = 6\sqrt{3}\text{cm}$$

$$\text{Also, } \frac{BC}{AC} = \sin 30^\circ$$

$$\Rightarrow \frac{6}{AC} = \frac{1}{2}$$

$$\Rightarrow AC = (2 \times 6) = 12\text{ cm}$$

$$\therefore AB = 6\sqrt{3}\text{ cm and } AC = 12\text{ cm}$$



22. In the adjoining figure, $\triangle ABC$ is right-angled at B and $\angle A = 45^\circ$. If $AC = 3\sqrt{2}$ cm, find (i) BC, (ii) AB.

Sol:

From the right-angled $\triangle ABC$, we have:

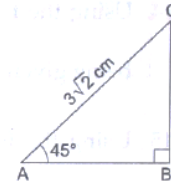
$$\frac{BC}{AC} = \sin 45^\circ$$

$$\Rightarrow \frac{BC}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow BC = 3 \text{ cm}$$

$$\text{Also, } \frac{AB}{AC} = \cos 45^\circ$$

$$\Rightarrow \frac{AB}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow AB = 3 \text{ cm}$$

$$\therefore BC = 3 \text{ cm and } AB = 3 \text{ cm}$$



23. If $\sin(A + B) = 1$ and $\cos(A - B) = 1$, $0^\circ \leq (A + B) \leq 90^\circ$ and $A > B$, then find A and B.

Sol:

Here, $\sin(A + B) = 1$

$$\Rightarrow \sin(A + B) = 90^\circ \quad [\because \sin 90^\circ = 1]$$

$$\Rightarrow (A + B) = 90^\circ \quad \dots\dots(i)$$

Also, $\cos(A - B) = 1$

$$\Rightarrow \cos(A - B) = 0^\circ \quad [\because \cos 0^\circ = 1]$$

$$\Rightarrow A - B = 0^\circ \quad \dots(ii)$$

Solving (i) and (ii), we get:

$$A = 45^\circ \text{ and } B = 45^\circ$$

24. If $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$, $0^\circ \leq (A + B) \leq 90^\circ$ and $A > B$, then find A and B.

Sol:

Here, $\sin(A - B) = \frac{1}{2}$

$$\Rightarrow \sin(A - B) = 30^\circ \quad [\because \sin 30^\circ = \frac{1}{2}]$$

$$\Rightarrow (A - B) = 30^\circ \quad \dots\dots(i)$$

Also, $\cos(A + B) = \frac{1}{2}$

$$\Rightarrow \cos(A + B) = \cos 60^\circ \quad [\because \cos 60^\circ = \frac{1}{2}]$$

$$\Rightarrow A + B = 60^\circ \quad \dots(ii)$$

Solving (i) and (ii), we get:

$$A = 45^\circ \text{ and } B = 15^\circ$$

25. If $\tan(A - B) = \frac{1}{\sqrt{3}}$ and $\tan(A + B) = \sqrt{3}$, $0^\circ \leq (A + B) \leq 90^\circ$ and $A > B$, then find A and B.

Sol:

Here, $\tan(A - B) = \frac{1}{\sqrt{3}}$

$$\Rightarrow \tan(A - B) = \tan 30^\circ \quad [\because \tan 30^\circ = \frac{1}{\sqrt{3}}]$$

$$\Rightarrow (A - B) = 30^\circ \quad \dots\dots(i)$$

Also, $\tan(A + B) = \sqrt{3}$

$$\Rightarrow \tan (A + B) = \tan 60^{\circ} \quad [:: \tan 60^{\circ} = \sqrt{3}]$$

$$\Rightarrow A + B = 60^{\circ} \quad \dots\dots(ii)$$

Solving (i) and (ii), we get:

$$A = 45^{\circ} \text{ and } B = 15^{\circ}$$

26. If $3x = \operatorname{cosec} \theta =$ and $\frac{3}{x} = \cot \theta$ find the value of $3\left(x^2 - \frac{1}{x^2}\right)$

Sol:

$$\begin{aligned} & 3\left(x^2 - \frac{1}{x^2}\right) \\ &= \frac{9}{3}\left(x^2 - \frac{1}{x^2}\right) \\ &= \frac{1}{3}\left(9x^2 - \frac{9}{x^2}\right) \\ &= \frac{1}{3}\left[(3x^2) - \left(\frac{3}{x}\right)^2\right] \\ &= \frac{1}{3}[(\operatorname{cosec} \theta)^2 - (\cot \theta)^2] \\ &= \frac{1}{3}(\operatorname{cosec}^2 \theta - \cot^2 \theta) \\ &= \frac{1}{3}(1) = \frac{1}{3} \end{aligned}$$

27. If $\sin (A+B) = \sin A \cos B + \cos A \sin B$ and $\cos (A-B) = \cos A \cos B + \sin A \sin B$

(i) $\sin (75^{\circ})$

(ii) $\cos (15^{\circ})$

Sol:

Let $A = 45^{\circ}$ and $B = 30^{\circ}$

(i) As, $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$\Rightarrow \sin (45^{\circ} + 30^{\circ}) = \sin 45^{\circ} \cos 30^{\circ} + \cos 45^{\circ} \sin 30^{\circ}$$

$$\Rightarrow \sin (75^{\circ}) = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$\Rightarrow \sin (75^{\circ}) = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$\therefore \sin (75^{\circ}) = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) As, $\cos (A - B) = \cos A \cos B + \sin A \sin B$

$$\Rightarrow \cos (45^{\circ} - 30^{\circ}) = \cos 45^{\circ} \cos 30^{\circ} + \sin 45^{\circ} \sin 30^{\circ}$$

$$\Rightarrow \cos (15^{\circ}) = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$\Rightarrow \cos (15^{\circ}) = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$\therefore \cos (15^{\circ}) = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Disclaimer: $\cos 15^{\circ}$ can also be written by taking $A = 60^{\circ}$ and $B = 45^{\circ}$.