Exercise – 16A

1. Find the distance between the points

(i) $A(9,3)$ and $B(15,11)$

(ii) $A(7,-4)$ and $B(-5,1)$

(iii) $A(-6,-4)$ and $B(9,-12)$

(iv) $A(1,-3)$ *and* $B(4,-6)$

(v) $P(a+b,a-b)$ *and* $Q(a-b,a+b)$

(vi) $P(a\sin\alpha, a\cos\alpha)$ *and* $Q(a\cos\alpha,-a\sin\alpha)$

Sol:

(i) $A(9,3)$ and $B(15,11)$

The given points are $A(9,3)$ and $B(15,11)$.

Then $(x_1=9, y_1=3)$ *and* $(x_2=15, y_2=11)$

$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(15-9)^2+(11-3)^2}$

$=\sqrt{(15-9)^2+(11-3)^2}$

$=\sqrt{(6)^2+(8)^2}$

$=\sqrt{36+64}$

$=\sqrt{100}$

$=10\,units$

(ii) $A(7,-4)$ and $B(-5,1)$

The given points are $A(7,-4)$ *and* $B(-5,1)$.

Then, $(x_1=7, y_1=-4)$ and $(x_2=-5, y_2=1)$

$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(-5-7)^2+\{1-(-4)\}^2}$

$=\sqrt{(-5-7)^2+(1+4)^2}$

$=\sqrt{(-12)^2+(5)^2}$

$=\sqrt{144+25}$

$=\sqrt{169}$

$= 13 \ units$

$= \sqrt{144+25}$

$= \sqrt{169}$

$= 13 \ units$

(iii) $A(-6,-4)$ and $B(9,-12)$

The given points are $A(-6,-4) \ and \ B(9,-12)$

Then $(x_1 = -6, y_1 = -4) \ and \ (x_2 = 9, y_2 = -12)$

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(9-(-6))^2 + \{-12-(-4)\}^2}$

$= \sqrt{(9+6)^2 + (-12+4)^2}$

$= \sqrt{(15)^2 + (-8)^2}$

$= \sqrt{225+64}$

$= \sqrt{289}$

$= 17 \ units$

(iv) $A(1,-3) \ and \ B(4,-6)$

The given points are $A(1,-3) \ and \ B(4,-6)$

Then $(x_1 = 1, y_1 = -3) \ and \ (x_2 = 4, y_2 = -6)$

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4-1)^2 + \{-6-(-3)\}^2}$

$= \sqrt{(4-1)^2 + (-6+3)^2}$

$= \sqrt{(3)^2 + (-3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= \sqrt{9 \times 2}$

$= 3\sqrt{2} \ units$

(v) $P(a+b, a-b) and \, Q(a-b, a+b)$

The given points are $P(a+b, a-b)$ and $Q(a-b, a+b)$

Then $(x_1 = a+b, y_1 = a-b) \ and \ (x_2 = a-b, y_2 = a+b)$

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\{(a-b)-(a+b)\}^2 + \{(a+b)-(a-b)\}^2}$

$= \sqrt{(a-b-a-b)^2 + (a+b-a+b)^2}$

$= \sqrt{(-2b)^2 + (2b)^2}$

$= \sqrt{4b^2 + 4b^2}$

$= \sqrt{8b^2}$

$= \sqrt{4 \times 2b^2}$

$= 2\sqrt{2}b \text{ units}$

(vi) $P(a\sin\alpha, a\cos\alpha)$ and $Q(a\cos\alpha, -a\sin\alpha)$

The given points are $P(a\sin\alpha, a\cos\alpha)$ and $Q(a\cos a, -a\sin\alpha)$

Then $(x_1 = a\sin\alpha, y_1 = a\cos\alpha)$ and $(x_2 = a\cos a, y_2 = -a\sin\alpha)$

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(a\cos\alpha - a\sin\alpha)^2 + (-a\sin\alpha - a\cos\alpha)^2}$

$= \sqrt{\begin{aligned}&(a^2\cos^2\alpha + a^2\sin^2\alpha - 2a^2\cos\alpha \times \sin\alpha)\\ &+(a^2\sin^2\alpha + a^2\cos^2\alpha + 2a^2\cos\alpha \times \sin\alpha)\end{aligned}}$

$= \sqrt{2a^2\cos^2\alpha + 2a^2\sin^2\alpha}$

$= \sqrt{2a^2(\cos^2\alpha + \sin^2\alpha)}$

$= \sqrt{2a^2(1)}$ (From the identity $\cos^2\alpha + \sin^2\alpha = 1$)

$= \sqrt{2a^2}$

$= \sqrt{2}a \text{ units}$

2. Find the distance of each of the following points from the origin:

(i) $A(5,-12)$ (ii) $B(-5,5)$ (iii) $C(-4,-6)$

Sol:

(i) $A(5,-12)$

Let $O(0,0)$ be the origin

$OA = \sqrt{(5-0)^2 + (-12-0)^2}$

$= \sqrt{(5)^2 + (-12)^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13\ units$

(ii) $B(-5,5)$

Let $O(0,0)$ be the origin.

$OB = \sqrt{(-5-0)^2 + (5-0)^2}$

$= \sqrt{(-5)^2 + (5)^2}$

$= \sqrt{25 + 25}$

$= \sqrt{50}$

$= \sqrt{25 \times 2}$

$= 5\sqrt{2}\ units$

(iii) $C(-4,-6)$

Let $O(0,0)$ be the origin

$OC = \sqrt{(-4-0)^2 + (-6-0)^2}$

$= \sqrt{(-4)^2 + (-6)^2}$

$= \sqrt{16 + 36}$

$= \sqrt{52}$

$= \sqrt{4 \times 13}$

$= 2\sqrt{13}\ units$

3. Find all possible values of x for which the distance between the points $A(x,-1)$ and $B(5,3)$ is 5 units.

Sol:

Given $AB = 5\ units$

Therefore, $(AB)^2 = 25\ units$

$\Rightarrow (5-a)^2 + \{3-(-1)\}^2 = 25$

$\Rightarrow (5-a)^2 + (3+1)^2 = 25$

$\Rightarrow (5-a)^2 + (4)^2 = 25$

$\Rightarrow (5-a)^2 + 16 = 25$

$\Rightarrow (5-a)^2 = 25-16$

$\Rightarrow (5-a)^2 = 9$

$\Rightarrow (5-a) = \pm\sqrt{9}$

$\Rightarrow 5-a = \pm 3$

$\Rightarrow 5-a = 3 \text{ or } 5-a = -3$

$\Rightarrow a = 2 \text{ or } 8$

Therefore, $a = 2 \text{ or } 8$.

4. Find all possible values of y for which distance between the points $A(2,-3)$ and $B(10,y)$ is 10 units.

Sol:

The given points are $A(2,-3)$ and $B(10,y)$

$\therefore AB = \sqrt{(2-10)^2 + (-3-y)^2}$

$= \sqrt{(-8)^2 + (-3-y)^2}$

$= \sqrt{64+9+y^2+6y}$

$\because AB = 10$

$\therefore \sqrt{64+9+y^2+6y} = 10$

$\Rightarrow 73 + y^2 + 6y = 100$ (Squaring both sides)

$\Rightarrow y^2 + 6y - 27 = 0$

$\Rightarrow y^2 + 9y - 3y - 27 = 0$

$\Rightarrow y(y+9) - 3(y+9) = 0$

$\Rightarrow (y+9)(y-3) = 0$

$\Rightarrow y+9 = 0 \text{ or } y-3 = 0$

$\Rightarrow y = -9 \text{ or } y = 3$

Hence, the possible values of y are -9 and 3.

5. Find value of x for which the distance between the points $P(x,4)$ and $Q(9,10)$ is 10 units.

Sol:

The given points are $P(x,4) \text{ and } Q(9,10)$.

$\therefore PQ = \sqrt{(x-9)^2 + (4-10)^2}$

$= \sqrt{(x-9)^2 + (-6)^2}$

$= \sqrt{x^2 - 18x + 81 + 36}$

$= \sqrt{x^2 - 18x + 117}$

$\because PQ = 10$

$\therefore \sqrt{x^2 - 18x + 117} = 10$

$\Rightarrow x^2 - 18x + 117 = 100$ (Squaring both sides)

$\Rightarrow x^2 - 18x + 17 =$

$\Rightarrow x^2 - 17x - x + 17 = 0$

$\Rightarrow x(x-17) - 1(x-17) = 0$

$\Rightarrow (x-17)(x-1) = 0$

$\Rightarrow x - 17 = 0 \text{ or } x - 1 = 0$

$\Rightarrow x = 17 \text{ or } x = 1$

Hence, the values of x are 1 and 17.

6. If the point $A(x, 2)$ is equidistant form the points $B(8, -2)$ and $C(2, -2)$, find the value of x. Also, find the value of x. Also, find the length of AB.

Sol:

As per the question

$AB = AC$

$\Rightarrow \sqrt{(x-8)^2 + (2+2)^2} = \sqrt{(x-2)^2 + (2+2)^2}$

Squaring both sides, we get

$(x-8)^2 + 4^2 = (x-2)^2 + 4^2$

$\Rightarrow x^2 - 16x + 64 + 16 = x^2 + 4 - 4x + 16$

$\Rightarrow 16x - 4x = 64 - 4$

$\Rightarrow x = \frac{60}{12} = 5$

Now,

$AB = \sqrt{(x-8)^2 + (2+2)^2}$

$= \sqrt{(5-8)^2 + (2+2)^2}$ $(\because x = 2)$

$= \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

Hence, $x = 5$ and $AB = 5$ *units*.

7. If the point $A(0,2)$ is equidistant form the points $B(3,p)$ and $C(p,5)$ find the value of p. Also, find the length of AB.

Sol:

As per the question

$AB = AC$

$\Rightarrow \sqrt{(0-3)^2+(2-p)^2} = \sqrt{(0-p)^2+(2-5)^2}$

$\Rightarrow \sqrt{(-3)^2+(2-p)^2} = \sqrt{(-p)^2+(-3)^2}$

Squaring both sides, we get

$(-3)^2+(2-p)^2 = (-p)^2+(-3)^2$

$\Rightarrow 9+4+p^2-4p = p^2+9$

$\Rightarrow 4p = 4$

$\Rightarrow p = 1$

Now,

$AB = \sqrt{(0-3)^2+(2-p)^2}$

$= \sqrt{(-3)^2+(2-1)^2} \qquad (\because p = 1)$

$= \sqrt{9+1}$

$= \sqrt{10}$ *units*

Hence, $p = 1$ *and* $AB = \sqrt{10}$ *units*

8. Find the point on the a-axis which is equidistant from the points (2, -5) and (-2, 9).

Sol:

Let (x, 0) be the point on the *x* axis. Then as per the question, we have

$\sqrt{(x-2)^2+(0+5)^2} = \sqrt{(x+2)^2+(0-9)^2}$

$\Rightarrow \sqrt{(x-2)^2+(5)^2} = \sqrt{(x+2)^2+(-9)^2}$

$\Rightarrow (x-2)^2+(5)^2 = (x+2)^2+(-9)^2 \qquad$ (Squaring both sides)

$\Rightarrow x^2-4x+4+25 = x^2+4x+4+81$

$\Rightarrow 8x = 25-81$

$\Rightarrow x = -\frac{56}{8} = -7$

Hence, the point on the *x*-axis is $(-7,0)$.

9. Find the points on the x-axis, each of which is at a distance of 10 units from the point A(11, -8).

Sol:

Let $P(x, 0)$ be the point on the x-axis. Then as per the question we have

$AP = 10$

$\Rightarrow \sqrt{(x-11)^2 + (0+8)^2} = 10$

$\Rightarrow (x-11)^2 + 8^2 = 100$ (Squaring both sides)

$\Rightarrow (x-11)^2 = 100 - 64 = 36$

$\Rightarrow x - 11 = \pm 6$

$\Rightarrow x = 11 \pm 6$

$\Rightarrow x = 11-6, 11+6$

$\Rightarrow x = 5, 17$

Hence, the points on the x-axis are $(5,0)$ and $(17,0)$.

10. Find the points on the y-axis which is equidistant form the points $A(6,5)$ and $B(-4,3)$

Sol:

Let P $(0, y)$ be a point on the y-axis. Then as per the question, we have

$AP = BP$

$\Rightarrow \sqrt{(0-6)^2 + (y-5)^2} = \sqrt{(0+4)^2 + (y-3)^2}$

$\Rightarrow \sqrt{(6)^2 + (y-5)^2} = \sqrt{(4)^2 + (y-3)^2}$

$\Rightarrow (6)^2 + (y-5)^2 = (4)^2 + (y-3)^2$ (Squaring both sides)

$\Rightarrow 36 + y^2 - 10y + 25 = 16 + y^2 - 6y + 9$

$\Rightarrow 4y = 36$

$\Rightarrow y = 9$

Hence, the point on the y-axis is $(0,9)$.

11. If the points $P(x, y)$ is point equidistant from the points $A(5,1)$ and $B(-1,5)$, Prove that 3x=2y. **Sol:**

As per the question, we have

$AP = BP$

$\Rightarrow \sqrt{(x-5)^2 + (y-1)^2} = \sqrt{(x+1)^2 + (y-5)^2}$

$\Rightarrow (x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2$ (Squaring both sides)

$\Rightarrow x^2 - 10x + 25 + y^2 - 2y + 1 = x^2 + 2x + 1 + y^2 - 10y + 25$

$\Rightarrow -10x-2y=2x-10y$

$\Rightarrow 8y=12x$

$\Rightarrow 3x=2y$

Hence, $3x=2y$

12. If $p(x,y)$ is point equidistant from the points $A(6,-1)$ and $B(2,3)$, show that x – y = 3

Sol:

The given points are $A(6,-1)$ and $B(2,3)$. The point $P(x, y)$ equidistant from the points A and B So, $PA = PB$

Also, $(PA)^2=(PB)^2$

$\Rightarrow (6-x)^2+(-1-y)^2=(2-x)^2+(3-y)^2$

$\Rightarrow x^2-12x+36+y^2+2y+1=x^2-4x+4+y^2-6y+9$

$\Rightarrow x^2+y^2-12x+2y+37=x^2+y^2-4x-6y+13$

$\Rightarrow x^2+y^2-12x+2y-x^2-y^2+4x+6y=13-37$

$\Rightarrow -8x+8y=-24$

$\Rightarrow -8(x-y)=-24$

$\Rightarrow x-y=\frac{-24}{-8}$

$\Rightarrow x-y=3$

Hence proved.

13. Find the co-ordinates of the point equidistant from three given points $A(5,3), B(5,-5)$ and $C(1,-5)$

Sol:

Let the required point be $P(x,y)$. Then $AP=BP=CP$

That is, $(AP)^2=(BP)^2=(CP)^2$

This means $(AP)^2=(BP)^2$

$\Rightarrow (x-5)^2+(y-3)^2=(x-5)^2+(y+5)^2$

$\Rightarrow x^2-10x+25+y^2-6y+9=x^2-10x+25+y^2+10y+25$

$\Rightarrow x^2-10x+y^2-6y+34=x^2-10x+y^2+10y+50$

$\Rightarrow x^2-10x+y^2-6y-x^2+10x-y^2-10y=50-34$

$\Rightarrow -16y=16$

$\Rightarrow y=-\frac{16}{16}=-1$

And $(BP)^2 = (CP)^2$

$\Rightarrow (x-5)^2 + (y+5)^2 = (x-1)^2 + (y+5)^2$

$\Rightarrow x^2 - 10x + 25 + y^2 + 10y + 25 = x^2 - 2x + 1 + y^2 + 10y + 25$

$\Rightarrow x^2 - 10x + y^2 + 10y + 50 = x^2 - 2x + y^2 + 10y + 26$

$\Rightarrow x^2 - 10x + y^2 + 10y - x^2 + 2x - y^2 - 10y = 26 - 50$

$\Rightarrow -8x = -24$

$\Rightarrow x = \frac{-24}{-8} = 3$

Hence, the required point is $(3, -1)$.

14. If the points $A(4,3)$ and $B(x,5)$ lies on a circle with the centre $O(2,3)$. Find the value of x.

Sol:

Given, the points $A(4,3)$ and $B(x,5)$ lie on a circle with center $O(2,3)$.

Then $OA = OB$

Also $(OA)^2 = (OB)^2$

$\Rightarrow (4-2)^2 + (3-3)^2 = (x-2)^2 + (5-3)^2$

$\Rightarrow (2)^2 + (0)^2 = (x-2)^2 + (2)^2$

$\Rightarrow 4 = (x-2)^2 + 4$

$\Rightarrow (x-2)^2 = 0$

$\Rightarrow x - 2 = 0$

$\Rightarrow x = 2$

Therefore, $x = 2$

15. If the point $C(-2,3)$ is equidistant form the points $A(3,-1)$ and $B(x,8)$, find the value of x. Also, find the distance between BC

Sol:

As per the question, we have

$AC = BC$

$\Rightarrow \sqrt{(-2-3)^2 + (3+1)^2} = \sqrt{(-2-x)^2 + (3-8)^2}$

$\Rightarrow \sqrt{(5)^2 + (4)^2} = \sqrt{(x+2)^2 + (-5)^2}$

$\Rightarrow 25 + 16 = (x+2)^2 + 25$ (Squaring both sides)

$\Rightarrow 25 + 16 = (x+2)^2 + 25$

$\Rightarrow (x+2)^2 = 16$

$\Rightarrow x+2 = \pm 4$

$\Rightarrow x = -2 \pm 4 = -2-4, -2+4 = -6, 2$

Now

$BC = \sqrt{(-2-x)^2 + (3-8)^2}$

$= \sqrt{(-2-2)^2 + (-5)}$

$= \sqrt{16+25} = \sqrt{41}\ units$

Hence, $x = 2$ or -6 and $BC = \sqrt{41}\ units$

16. If the point $P(2,2)$ is equidistant from the points $A(-2,k)$ and $B(-2k,-3)$, find k. Also, find the length of AP.

Sol:

As per the question, we have

$AP = BP$

$\Rightarrow \sqrt{(2+2)^2 + (2+k)^2} = \sqrt{(2+2k)^2 + (2+3)^2}$

$\Rightarrow \sqrt{(4)^2 + (2-k)^2} = \sqrt{(2+2k)^2 + (5)^2}$

$\Rightarrow 16+4+k^2-4k = 4+4k^2+8k+25$ (Squaring both sides)

$\Rightarrow k^2+4k+3 = 0$

$\Rightarrow (k+1)(k+3) = 0$

$\Rightarrow k = -3, -1$

Now for $k = -1$

$AP = \sqrt{(2+2)^2 + (2-k)^2}$

$= \sqrt{(4)^2 + (2+1)^2}$

$= \sqrt{16+9} = 5\ units$

For $k = -3$

$AP = \sqrt{(2+2)^2 + (2-k)^2}$

$= \sqrt{(4)^2 + (2+3)^2}$

$= \sqrt{16+25} = \sqrt{41}\ units$

Hence, $k = -1, -3; AP = 5\ units\ \ for\ k = -1\ and\ AP = \sqrt{41}\ units\ \ for\ k = -3.$

17. If the point (x, y) is equidistant form the points $(a+b, b-a)$ and $(a-b, a+b)$, prove that bx = ay.

Sol:

As per the question, we have

$$\sqrt{(x-a-b)^2+(y-b+a)^2}=\sqrt{(x-a+b)^2+(y-a-b)^2}$$

$$\Rightarrow (x-a-b)^2+(y-b+a)^2=(x-a+b)^2+(y-a-b)^2 \quad \text{(Squaring both sides)}$$

$$\Rightarrow x^2+(a+b)^2-2x(a+b)+y^2+(a-b)^2-2y(a-b)=x^2+(a-b)^2-2x(a-b)+y^2$$
$$+(a+b)^2-2y(a+b)$$

$$\Rightarrow -x(a+b)-y(a-b)=-x(a-b)-y(a+b)$$

$$\Rightarrow -xa-xb-ay+by=-xa+bx-ya-by$$

$$\Rightarrow by=bx$$

Hence, $bx=ay$.

18. Using the distance formula, show that the given points are collinear:

(i) (1, -1), (5, 2) and (9, 5) (ii) (6, 9), (0, 1) and (-6, -7)

(iii) (-1, -1), (2, 3) and (8, 11) (iv) (-2, 5), (0,1) and (2, -3)

Sol:

(i) Let $A(1,-1), B(5,2)$ and $C(9,5)$ be the give points. Then

$$AB=\sqrt{(5-1)^2+(2+1)^2}=\sqrt{4^2+3^2}=\sqrt{25}=5 \text{ units}$$

$$BC=\sqrt{(9-5)^2+(5-2)^2}=\sqrt{4^2+3^2}=\sqrt{25}=5 \text{ units}$$

$$AC=\sqrt{(9-1)^2+(5+1)^2}=\sqrt{8^2+6^2}=\sqrt{100}=10 \text{ units}$$

$$\therefore AB+BC=(5+5) \text{ units}=10 \text{ units}=AC$$

Hence, the given points are collinear

(ii) Let $A(6,9), B(0,1)$ and $C(-6,-7)$ be the give points. Then

$$AB=\sqrt{(0-6)^2+(1-9)^2}=\sqrt{(-6)^2+(-8)^2}=\sqrt{100}=10 \text{ units}$$

$$BC=\sqrt{(-6-0)^2+(-7-1)^2}=\sqrt{(-6)^2+(-8)^2}=\sqrt{100}=10 \text{ units}$$

$$AC=\sqrt{(-6-6)^2+(-7-9)^2}=\sqrt{(-12)^2+(16)^2}=\sqrt{400}=20 \text{ units}$$

$$\therefore AB+BC=(10+10) \text{ units}=20 \text{ units}=AC$$

Hence, the given points are collinear

(iii) Let $A(-1,-1), B(2,3)$ and $C(8,11)$ be the give points. Then

$AB = \sqrt{(2+1)^2 + (3+1)^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5\ units$

$BC = \sqrt{(8-2)^2 + (11-3)^2} = \sqrt{(6)^2 + (8)^2} = \sqrt{100} = 10\ units$

$AC = \sqrt{(8+1)^2 + (11+1)^2} = \sqrt{(9)^2 + (12)^2} = \sqrt{225} = 15\ units$

$\therefore AB + BC = (5+10)units = 15\ units = AC$

Hence, the given points are collinear

(iv) Let $A(-2,5), B(0,1)$ and $C(2,-3)$ be the give points. Then

$AB = \sqrt{(0+2)^2 + (1-5)^2} = \sqrt{(2)^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}\ units$

$BC = \sqrt{(2-0)^2 + (-3-1)^2} = \sqrt{(2)^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}\ units$

$AC = \sqrt{(2+2)^2 + (-3-5)^2} = \sqrt{(4)^2 + (-8)^2} = \sqrt{80} = 4\sqrt{5}\ units$

$\therefore AB + BC = (2\sqrt{5} + 2\sqrt{5})units = 4\sqrt{5}\ units = AC$

Hence, the given points are collinear

19. Show that the points A (7, 10), B(-2, 5) and C(3, -4) are the vertices of an isosceles right triangle.

Sol:

The given points are $A(7,10), B(-2,5)\ and\ C(3,-4)$.

$AB = \sqrt{(-2-7)^2 + (5-10)^2} = \sqrt{(-9)^2 + (-5)^2} = \sqrt{81+25} = \sqrt{106}$

$BC = \sqrt{(3-(-2))^2 + (-4-5)^2} = \sqrt{(5)^2 + (-9)^2} = \sqrt{25+81} = \sqrt{106}$

$AC = \sqrt{(3-7)^2 + (-4-10)^2} = \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212}$

Since, AB and BC are equal, they form the vertices of an isosceles triangle

Also, $(AB)^2 + (BC)^2 = (\sqrt{106})^2 + (\sqrt{106})^2 = 212$

and $(AC)^2 = (\sqrt{212})^2 = 212.$

Thus, $(AB)^2 + (BC)^2 = (AC)^2$

This show that ΔABC is right- angled at B.

Therefore, the points $A(7,10), B(-2,5)\ and\ C(3,-4)$ are the vertices of an isosceles right-angled triangle.

20. Show that the points A (3, 0), B(6, 4) and C(-1, 3) are the vertices of an isosceles right triangle.

Sol:

The given points are $A(3,0), B(6,4)$ and $C(-1,3)$. Now,

$AB = \sqrt{(3-6)^2 + (0-4)^2} = \sqrt{(-3)^2 + (-4)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

$BC = \sqrt{(6+1)^2 + (4-3)^2} = \sqrt{(7)^2 + (1)^2}$

$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$

$AC = \sqrt{(3+1)^2 + (0-3)^2} = \sqrt{(4)^2 + (-3)^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$

$\because AB = AC \text{ and } AB^2 + AC^2 = BC^2$

Therefore, $A(3,0), B(6,4)$ and $C(-1,3)$ are die vertices of an isosceles right triangle

21. If A(5,2), B(2, -2) and C(-2, t) are the vertices of a right triangle with $\angle B = 90°$, then find the value of t.

Sol:

$\because \angle B = 90°$

$\therefore AC^2 = AB^2 + BC^2$

$\Rightarrow (5+2)^2 + (2-t)^2 = (5-2)^2 + (2+2)^2 + (2+2)^2 + (-2-t)^2$

$\Rightarrow (7)^2 + (t-2)^2 = (3)^2 + (4)^2 + (4)^2 + (t+2)^2$

$\Rightarrow 49 + t^2 - 4t + 4 = 9 + 16 + 16 + t^2 + 4t + 4$

$\Rightarrow 8 - 4t = 4t$

$\Rightarrow 8t = 8$

$\Rightarrow t = 1$

Hence, $t = 1$.

22. Prove that the points A(2, 4), b(2, 6) and $C(2+\sqrt{3}, 5)$ are the vertices of an equilateral triangle.

Sol:

The given points are $A(2,4), B(2,6)$ *and* $C(2+\sqrt{3},5)$.*Now*

$AB = \sqrt{(2-2)^2 + (4-6)^2} = \sqrt{(0)^2 + (-2)^2}$

$= \sqrt{0+4} = 2$

$BC = \sqrt{\left(2-2-\sqrt{3}\right)^2 + (6-5)^2} = \sqrt{\left(-\sqrt{3}\right)^2 + (1)^2}$

$= \sqrt{3+1} = 2$

$AC = \sqrt{\left(2-2-\sqrt{3}\right)^2 + (4-5)^2} = \sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2}$

$= \sqrt{3+1} = 2$

Hence, the points $A(2,4), B(2,6)$ and $C\left(2+\sqrt{3},5\right)$ are the vertices of an equilateral triangle.

23. Show that the points (-3, -3),(3,3) and $C\left(-3\sqrt{3},3\sqrt{3}\right)$ are the vertices of an equilateral triangle.

Sol:

Let the given points be $A(-3,-3), B(3,3)$ *and* $C\left(-3\sqrt{3},3\sqrt{3}\right)$. Now

$AB = \sqrt{(-3-3)^2 + (-3-3)^2} = \sqrt{(-6)^2 + (-6)^2}$

$= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

$BC = \sqrt{\left(3+3\sqrt{3}\right)^2 + \left(3-3\sqrt{3}\right)^2}$

$= \sqrt{9+27+18\sqrt{3}+9+27-18\sqrt{3}} = \sqrt{72} = 6\sqrt{2}$

$AC = \sqrt{\left(-3+3\sqrt{3}\right)^2 + \left(-3-3\sqrt{3}\right)^2} = \sqrt{\left(3-3\sqrt{3}\right)^2 + \left(3+3\sqrt{3}\right)^2}$

$= \sqrt{9+27-18\sqrt{3}+9+27+18\sqrt{3}}$

$= \sqrt{72} = 6\sqrt{2}$

Hence, the given points are the vertices of an equilateral triangle.

24. Show that the points A(-5,6), B(3,0) and C(9,8) are the vertices of an isosceles right-angled triangle. Calculate its area.

Sol:

Let the given points be $A(-5,6) B(3,0)$ *and* $C(9,8)$.

$AB = \sqrt{(3-(-5))^2 + (0-6)^2} = \sqrt{(8)^2 + (-6)^2} = \sqrt{64+36} = \sqrt{100} = 10 \text{ units}$

$BC = \sqrt{(9-3)^2 + (8-0)^2} = \sqrt{(6)^2 + (8)^2} = \sqrt{36+64} = \sqrt{100} = 10 \text{ units}$

$AC = \sqrt{(9-(-5))^2 + (8-6)^2} = \sqrt{(14)^2 + (2)^2} = \sqrt{196+4} = \sqrt{200} = 10\sqrt{2} \text{ units}$

Therefore, $AB = BC = 10 \text{ units}$

Also, $(AB)^2+(BC)^2=(10)^2+(10)^2=200$

and $(AC)^2=\left(10\sqrt{2}\right)^2=200$

Thus, $(AB)^2+(BC)^2=(AC)^2$

This show that ΔABC is right angled at B.

Therefore, the points $A(-5,6)B(3,0)$ *and* $C(9,8)$ are the vertices of an isosceles right-angled triangle

Also, area of a triangle $=\frac{1}{2}\times base\times height$

If AB is the height and BC is the base,

Area $=\frac{1}{2}\times10\times10$

= 50 square units

25. Show that the points O(0,0), $A\left(3,\sqrt{3}\right)$ and $B\left(3,-\sqrt{3}\right)$ are the vertices of an equilateral triangle. Find the area of this triangle.

Sol:

The given points are $O(0,0)A\left(3,\sqrt{3}\right)$ *and* $B\left(3,-\sqrt{3}\right)$.

$$OA=\sqrt{(3-0)^2+\left\{\left(\sqrt{3}\right)-0\right\}^2}=\sqrt{(3)^2+\left(\sqrt{3}\right)^2}=\sqrt{9+3}=\sqrt{12}=2\sqrt{3}\ units$$

$$AB=\sqrt{(3-3)^2+\left(-\sqrt{3}-\sqrt{3}\right)^2}=\sqrt{(0)+\left(2\sqrt{3}\right)^2}=\sqrt{4(3)}=\sqrt{12}=2\sqrt{3}\ units$$

$$OB=\sqrt{(3-0)^2+\left(-\sqrt{3}-0\right)^2}=\sqrt{(3)^2+\left(\sqrt{3}\right)^2}=\sqrt{9+3}=\sqrt{12}=2\sqrt{3}\ units$$

Therefore, $OA=AB=OB=2\sqrt{3}\ units$

Thus, the points $O(0,0)$ $A\left(3,\sqrt{3}\right)$ and $B\left(3,-\sqrt{3}\right)$ are the vertices of an equilateral triangle

Also, the area of the triangle $OAB=\frac{\sqrt{3}}{4}\times(side)^2$

$=\frac{\sqrt{3}}{4}\times\left(2\sqrt{3}\right)^2$

$=\frac{\sqrt{3}}{4}\times12$

$=3\sqrt{3}$ square units.

26. Show that the following points are the vertices of a square:
(i) A (3,2), B(0,5), C(-3,2) and D(0,-1)
(ii) A (6,2), B(2,1), C(1,5) and D(5,6)
(iii) A (0,-2), B(3,1), C(0,4) and D(-3,1)
Sol:
(i) The given points are $A(3,2), B(0,5), C(-3,2)$ and $D(0,-1)$.

$$AB=\sqrt{(0-3)^2+(5-2)^2}=\sqrt{(-3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}\ units$$

$$BC=\sqrt{(-3-0)^2+(2-5)^2}=\sqrt{(-3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}\ units$$

$$CD=\sqrt{(0+3)^2+(-1-2)^2}=\sqrt{(3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}\ units$$

$$DA=\sqrt{(0-3)^2+(-1-2)^2}=\sqrt{(-3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}\ units$$

Therefore, $AB=BC=CD=DA=3\sqrt{2}\ units$

Also, $AC=\sqrt{(-3-3)^2+(2-2)^2}=\sqrt{(-6)^2+(0)^2}=\sqrt{36}=6\ units$

$$BD=\sqrt{(0-0)^2+(-1-5)^2}=\sqrt{(0)^2+(-6)^2}=\sqrt{36}=6\ units$$

Thus, diagonal $AC=$ diagonal BD
Therefore, the given points from a square.

(ii) The given points are $A(6,2), B(2,1), C(1,5)\ and\ D(5,6)$

$$AB=\sqrt{(2-6)^2+(1-2)^2}=\sqrt{(-4)^2+(-1)^2}=\sqrt{16+1}=\sqrt{17}\ units$$

$$BC=\sqrt{(1-2)^2+(5-1)^2}=\sqrt{(-1)^2+(-4)^2}=\sqrt{1+16}=\sqrt{17}\ units$$

$$CD=\sqrt{(5-1)^2+(6-5)^2}=\sqrt{(4)^2+(1)^2}=\sqrt{16+1}=\sqrt{17}\ units$$

$$DA=\sqrt{(5-6)^2+(6-2)^2}=\sqrt{(1)^2+(4)^2}=\sqrt{1+16}=\sqrt{17}\ units$$

Therefore, $AB=BC=CD=DA=17\ units$

Also, $AC=\sqrt{(1-6)^2+(5-2)^2}=\sqrt{(-5)^2+(3)^2}=\sqrt{25+9}=\sqrt{34}\ units$

$$BD=\sqrt{(5-2)^2+(6-1)^2}=\sqrt{(3)^2+(5)^2}=\sqrt{9+25}=\sqrt{34}\ units$$

Thus, diagonal $AC=$ diagonal BD
Therefore, the given points from a square.

(iii) The given points are $P(0,-2), Q(3,1), R(0,4)\ and\ S(-3,1)$

$$PQ=\sqrt{(3-0)^2+(1+2)^2}=\sqrt{(3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}\ units$$

$$QR=\sqrt{(0-3)^2+(4-1)^2}=\sqrt{(-3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}\ units$$

$RS=\sqrt{(-3-0)^2+(1-4)^2}=\sqrt{(-3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}\ units$

$SP=\sqrt{(-3-0)^2+(1+2)^2}=\sqrt{(-3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}\ units$

Therefore, $PQ=QS=RS=SP=3\sqrt{2}\ units$

Also, $PR=\sqrt{(0-0)^2+(4+2)^2}=\sqrt{(0)^2+(6)^2}=\sqrt{36}=6\ units$

$QS=\sqrt{(-3-3)^2+(1-1)^2}=\sqrt{(-6)^2+(0)^2}=\sqrt{36}=6\ units$

Thus, diagonal $PR=$ diagonal QS

Therefore, the given points from a square.

27. Show that the points $A(-3,2), B(-5,-5), C(2,-3)\ and\ D(4,4)$. are the vertices of a rhombus. Find the area of this rhombus

Sol:

The given points are $A(-3,2), B(-5,-5), C(2,-3)\ and\ D(4,4)$.

$AB=\sqrt{(-5+3)^2+(-5-2)^2}=\sqrt{(-2)^2+(-7)^2}=\sqrt{4+49}=\sqrt{53}\ units$

$BC=\sqrt{(2+5)^2+(-3+5)^2}=\sqrt{(7)^2+(2)^2}=\sqrt{49+4}=\sqrt{53}\ units$

$CD=\sqrt{(4-2)^2+(4+3)^2}=\sqrt{(2)^2+(7)^2}=\sqrt{4+49}=\sqrt{53}\ units$

$DA=\sqrt{(4+3)^2+(4-2)^2}=\sqrt{(7)^2+(2)^2}=\sqrt{49+4}=\sqrt{53}\ units$

Therefore, $AB=BC=CD=DA=\sqrt{53}\ units$

Also, $AC=\sqrt{(2+3)^2+(-3-2)^2}=\sqrt{(5)^2+(-5)^2}=\sqrt{25+25}=\sqrt{50}=\sqrt{25\times2}=5\sqrt{2}\ units$

$BD=\sqrt{(4+5)^2+(4+5)^2}=\sqrt{(9)^2+(9)^2}=\sqrt{81+81}=\sqrt{162}=\sqrt{81\times2}=9\sqrt{2}\ units$

Thus, diagonal AC is not equal to diagonal BD.

Therefore ABCD is a quadrilateral with equal sides and unequal diagonals

Hence, ABCD a rhombus

Area of a rhombus $=\frac{1}{2}\times$ (product of diagonals)

$=\frac{1}{2}\times(5\sqrt{2})\times(9\sqrt{2})$

$=\frac{45(2)}{2}$

$=45$ square units.

28. Show that the points A(3,0), B(4,5), C(-1,4) and D(-2,-1) are the vertices of a rhombus. Find its area.

Sol:

The given points are $A(3,0), B(4,5), C(-1,4)$ and $D(-2,-1)$

$AB = \sqrt{(3-4)^2 + (0-5)^2} = \sqrt{(-1)^2 + (-5)^2}$

$= \sqrt{1+25} = \sqrt{26}$

$BC = \sqrt{(4+1)^2 + (5-4)^2} = \sqrt{(5)^2 + (1)^2}$

$= \sqrt{25+1} = \sqrt{26}$

$CD = \sqrt{(-1+2)^2 + (4+1)^2} = \sqrt{(1)^2 + (5)^2}$

$= \sqrt{1+25} = \sqrt{26}$

$AD = \sqrt{(3+2)^2 + (0+1)^2} = \sqrt{(5)^2 + (1)^2}$

$= \sqrt{25+1} = \sqrt{26}$

$AC = \sqrt{(3+1)^2 + (0-4)^2} = \sqrt{(4)^2 + (-4)^2}$

$= \sqrt{16+16} = 4\sqrt{2}$

$BD = \sqrt{(4+2)^2 + (5+1)^2} = \sqrt{(6)^2 + (6)^2}$

$= \sqrt{36+36} = 6\sqrt{2}$

$\because AB = BC = CD = AD = 6\sqrt{2}$ *and* $AC \neq BD$

Therefore, the given points are the vertices of a rhombus

Area $(\Delta ABCD) = \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24$ *sq. units*

Hence, the area of the rhombus is 24 sq. units.

29. Show that the points A(6,1), B(8,2), C(9,4) and D(7,3) are the vertices of a rhombus. Find its area.

Sol:

The given points are $A(6,1), B(8,2), C(9,4)$ and $D(7,3)$.

$AB = \sqrt{(6-8)^2 + (1-2)^2} = \sqrt{(-2)^2 + (-1)^2}$

$= \sqrt{4+1} = \sqrt{5}$

$BC = \sqrt{(8-9)^2 + (2-4)^2} = \sqrt{(-1)^2 + (-2)^2}$

$= \sqrt{1+4} = \sqrt{5}$

$CD = \sqrt{(9-7)^2 + (4-3)^2} = \sqrt{(2)^2 + (1)^2}$

$= \sqrt{4+1} = \sqrt{5}$

$AD = \sqrt{(7-6)^2 + (3-1)^2} = \sqrt{(1)^2 + (2)^2}$

$= \sqrt{1+4} = \sqrt{5}$

$AC = \sqrt{(6-9)^2 + (1-4)^2} = \sqrt{(-3)^2 + (-3)^2}$

$= \sqrt{9+9} = 3\sqrt{2}$

$BD = \sqrt{(8-7)^2 + (2-3)^2} = \sqrt{(1)^2 + (-1)^2}$

$= \sqrt{1+1} = \sqrt{2}$

$\because AB = BC = CD = AD = \sqrt{5}$ *and* $AC \neq BD$

Therefore, the given points are the vertices of a rhombus. Now

Area $(\Delta ABCD) = \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 3\sqrt{2} \times \sqrt{2} = 3\, sq.\ units$

Hence, the area of the rhombus is 3 sq. units.

30. Show that the points A(2,1), B(5,2), C(6,4) and D(3,3) are the angular points of a parallelogram. Is this figure a rectangle?

Sol:

The given points are $A(2,1), B(5,2), C(6,4)$ *and* $D(3,3)$

$AB = \sqrt{(5-2)^2 + (2-1)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$ *units*

$BC = \sqrt{(6-5)^2 + (4-2)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5}$ *units*

$CD = \sqrt{(3-6)^2 + (3-4)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$ *units*

$AD = \sqrt{(3-2)^2 + (3-1)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5}$ *units*

Thus, $AB = CD = \sqrt{10}$ units and $BC = AD = \sqrt{5}$ units

So, quadrilateral *ABCD* is a parallelogram

Also, $AC = \sqrt{(6-2)^2 + (4-1)^2} = \sqrt{(4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$ *units*

$BD = \sqrt{(3-5)^2 + (3-2)^2} = \sqrt{(-2)^2 + (1)^2} = \sqrt{4+1} = \sqrt{5}$ *units*

But diagonal AC is not equal to diagonal BD.

Hence, the given points do not form a rectangle.

31. Show hat A(1,2), B(4,3),C(6,6) and D(3,5) are the vertices of a parallelogram. Show that ABCD is not rectangle.

Sol:

The given vertices are $A(1,2), B(4,3), C(6,6)$ and $D(3,5)$.

$$AB = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{(-3)^2 + (-1)^2}$$
$$= \sqrt{9+1} = \sqrt{10}$$
$$BC = \sqrt{(4-6)^2 + (3-6)^2} = \sqrt{(-2)^2 + (-3)^2}$$
$$= \sqrt{4+9} = \sqrt{13}$$
$$CD = \sqrt{(6-3)^2 + (6-5)^2} = \sqrt{(3)^2 + (1)^2}$$
$$= \sqrt{9+1} = \sqrt{10}$$
$$AD = \sqrt{(1-3)^2 + (2-5)^2} = \sqrt{(-2)^2 + (-3)^2}$$
$$= \sqrt{4+9} = \sqrt{13}$$
$$\because AB = CD = \sqrt{10} \text{ units and } BC = AD = \sqrt{13} \text{ units}$$

Therefore, *ABCD* is a parallelogram

$$AC = \sqrt{(1-6)^2 + (2-6)^2} = \sqrt{(-5)^2 + (-4)^2}$$
$$= \sqrt{25+16} = \sqrt{41}$$
$$BD = \sqrt{(4-3)^2 + (3-5)^2} = \sqrt{(1)^2 + (-2)^2}$$
$$= \sqrt{1+4} = \sqrt{5}$$

Thus, the diagonal *AC* and *BD* are not equal and hence *ABCD* is not a rectangle

32. Show that the following points are the vertices of a rectangle.

(i) A (-4,-1), B(-2,-4), C(4,0) and D(2,3)

(ii) A (2, -2), B(14,10), C(11,13) and D(-1,1)

(iii) A (0,-4), B(6,2), C(3,5) and D(-3,-1)

Sol:

(i) The given points are $A(-4,-1), B(-2,-4) C(4,0)$ *and* $D(2,3)$

$$AB = \sqrt{\{-2-(-4)\}^2 + \{-4-(-1)\}^2} = \sqrt{(2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13} \text{ units}$$
$$BC = \sqrt{\{4-(-2)\}^2 + \{0-(-4)\}^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13} \text{ units}$$
$$CD = \sqrt{(2-4)^2 + (3-0)^2} = \sqrt{(-2)^2 + (3)^2} = \sqrt{4+9} = \sqrt{13} \text{ units}$$
$$AD = \sqrt{\{2-(-4)\}^2 + \{3-(-1)\}^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13} \text{ units}$$

Thus, $AB = CD = \sqrt{13}\ units\ and\ BC = AD = 2\sqrt{13}\ units$

Also, $AC = \sqrt{\{4-(-4)\}^2 + \{0-(-1)\}^2} = \sqrt{(8)^2 + (1)^2} = \sqrt{64+1} = \sqrt{65}\ units$

$BD = \sqrt{\{2-(-2)\}^2 + \{3-(-4)\}^2} = \sqrt{(4)^2 + (7)^2} = \sqrt{16+49} = \sqrt{65}\ units$

Also, diagonal *AC* = diagonal *BD*

Hence, the given points form a rectangle

(ii) The given points are $A(2,-2), B(14,10) C(11,13)\ and\ D(-1,1)$

$AB = \sqrt{(14-2)^2 + \{10-(-2)\}^2} = \sqrt{(12)^2 + (12)^2} = \sqrt{144+144} = \sqrt{288} = 12\sqrt{2}\ units$

$BC = \sqrt{(11-14)^2 + (13-10)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}\ units$

$CD = \sqrt{(-1-11)^2 + (1-13)^2} = \sqrt{(-12)^2 + (-12)^2} = \sqrt{144+144} = \sqrt{288} = 12\sqrt{2} units$

$AD = \sqrt{(-1-2)^2 + \{1-(-2)\}^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}\ units$

Thus, $AB = CD = 12\sqrt{2}\ units\ and\ BC = AD = 3\sqrt{2}\ units$

Also,

$AC = \sqrt{(11-2)^2 + \{13-(-2)\}^2} = \sqrt{(9)^2 + (15)^2} = \sqrt{81+225} = \sqrt{306} = 3\sqrt{34}\ units$

$BD = \sqrt{(-1-14)^2 + (1-10)^2} = \sqrt{(-15)^2 + (-9)^2} = \sqrt{81+225} = \sqrt{306} = 3\sqrt{34}\ units$

Also, diagonal *AC* = diagonal *BD*

Hence, the given points from a rectangle

(iii) The given points are $A(0,-4), B(6,2) C(3,5)\ and\ D(-3,-1)$.

$AB = \sqrt{(6-0)^2 + \{2-(-4)\}^2} = \sqrt{(6)^2 + (6)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}\ units$

$BC = \sqrt{(3-6)^2 + (5-2)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}\ units$

$CD = \sqrt{(-3-3)^2 + (-1-5)^2} = \sqrt{(-6)^2 + (-6)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}\ units$

$AD = \sqrt{(-3-0)^2 + \{-1-(-4)\}^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}\ units$

Thus, AB = CD = $\sqrt{10}$ units and BC = AD = $\sqrt{5}$ units

Also, $AC = \sqrt{(3-0)^2 + \{5-(-4)\}^2} = \sqrt{(3)^2 + (9)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}\ units$

$BD = \sqrt{(-3-6)^2 + (-1-2)^2} = \sqrt{(-9)^2 + (-3)^2} = \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}\ units$

Also, diagonal *AC* = diagonal *BD*

Hence, the given points from a rectangle

Exercise – 16B

1. Find the coordinates of the point which divides the join of $A(-1,7)$ *and* $B(4,-3)$. in the ratio $2:3$

Sol:

The end points of AB are $A(-1,7)$ *and* $B(4,-3)$.

Therefore, $(x_1=-1, y_1=7)$ *and* $(x_2=4, y_2=-3)$

Also, $m=2$ *and* $n=3$

Let the required point be $P(x,y)$.

By section formula, we get

$$x=\frac{(mx_2+nx_1)}{(m+n)}, y=\frac{(my_2+ny_1)}{(m+n)}$$

$$\Rightarrow x=\frac{\{2\times4+3\times(-1)\}}{2+3}, y=\frac{\{2\times(-3)+3\times7\}}{2+3}$$

$$\Rightarrow x=\frac{8-3}{5}, y=\frac{-6+21}{5}$$

$$\Rightarrow x=\frac{5}{5}, y=\frac{15}{5}$$

Therefore, $x=1$ *and* $y=3$

Hence, the coordinates of the required point are $(1,3)$.

2. Find the co-ordinates of the point which divides the join of A(-5, 11) and B(4,-7) in the ratio 7 : 2

Sol:

The end points of *AB* are $A(-5,11)$ *and* $B(4,-7)$.

Therefore, $(x_1=-5, y_1=11)$ *and* $(x_2=4, y_2=-7)$

Also, $m=7$ *and* $n=2$

Let the required point be $P(x,y)$.

By section formula, we get

$$x=\frac{(mx_2+nx_1)}{(m+n)}, y=\frac{(my_2+ny_1)}{(m+n)}$$

$$\Rightarrow x=\frac{\{7\times4+2\times(-5)\}}{7+2}, y=\frac{\{7\times(-7)+2\times11\}}{7+2}$$

$$\Rightarrow x=\frac{28-10}{9}, y=\frac{-49+22}{9}$$

$\Rightarrow x = \frac{18}{9}, y = -\frac{27}{9}$

Therefore, $x = 2$ *and* $y = -3$

Hence, the required point are $P(2,-3)$.

3. If the coordinates of points A and B are (-2, -2) and (2, -4) respectively. Find the coordinates of the point P such that AP=$\frac{3}{7}$ AB, where P lies on the segment AB.

Sol:

The coordinates of the points *A* and *B*are $(-2,-2)$ and $(2,-4)$ respectively, where

$AP = \frac{3}{7}AB$ *and* P lies on the line segment *AB*. So

$AP + BP = AB$

$\Rightarrow AP + BP = \frac{7AP}{3}$ $\qquad \because AP = \frac{3}{7}AB$

$\Rightarrow BP = \frac{7AP}{3} - AP$

$\Rightarrow \frac{AP}{BP} = \frac{3}{4}$

Let (*x*, *y*) be the coordinates of *P* which divides *AB* in the ratio 3 : 4 internally Then

$x = \frac{3\times 2 + 4\times(-2)}{3+4} = \frac{6-8}{7} = -\frac{2}{7}$

$y = \frac{3\times(-4) + 4\times(-2)}{3+4} = \frac{-12-8}{7} = -\frac{20}{7}$

Hence, the coordinates of point *P*are $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

4. Point A lies on the line segment PQ joining P(6, -6) and Q(-4, -1) in such a way that $\frac{PA}{PQ} = \frac{2}{5}$. If that point A also lies on the line 3x + k(y + 1) = 0, find the value of k.

Sol:

Let the coordinates of A be (x, y). Here $\frac{PA}{PQ} = \frac{2}{5}$. So,

$PA + AQ = PQ$

$\Rightarrow PA + AQ = \frac{5PA}{2}$ $\qquad \left[\because PA = \frac{2}{5}PQ\right]$

$\Rightarrow AQ = \frac{5PA}{2} - PA$

$\Rightarrow \frac{AQ}{PA} = \frac{3}{2}$

$\Rightarrow \frac{PA}{AQ} = \frac{2}{3}$

Let (x, y) be the coordinates of A, which dives PQ in the ratio 2 : 3 internally Then using section formula, we get

$x = \frac{2\times(-4)+3\times(6)}{2+3} = \frac{-8+18}{5} = \frac{10}{5} = 2$

$y = \frac{2\times(-1)+3\times(-6)}{2+3} = \frac{-2-18}{5} = \frac{-20}{5} = -4$

Now, the point $(2,-4)$ lies on the line $3x+k(y+1)=0$, therefore

$3\times2+k(-4+1)=0$

$\Rightarrow 3k = 6$

$\Rightarrow k = \frac{6}{3} = 2$

Hence, $k = 2$.

5. Points P, Q, R and S divide the line segment joining the points A(1,2) and B(6,7) in five equal parts. Find the coordinates of the points P,Q and R

Sol:

Since, the points *P, Q, R* and *S* divide the line segment joining the points $A(1,2)$ *and* $B(6,7)$ in five equal parts, so

$AP = PQ = QR = R = SB$

Here, point P divides AB in the ratio of 1 : 4 internally So using section formula, we get

Coordinates of $P = \left(\frac{1\times(6)+4\times(1)}{1+4}, \frac{1\times(7)+4\times(2)}{1+4}\right)$

$= \left(\frac{6+4}{5}, \frac{7+8}{5}\right) = (2,3)$

The point Q divides AB in the ratio of 2 : 3 internally. So using section formula, we get

Coordinates of $Q = \left(\frac{2\times(6)+3\times(1)}{2+3}, \frac{2\times(7)+3\times(2)}{2+3}\right)$

$= \left(\frac{12+3}{5}, \frac{14+6}{5}\right) = (3,4)$

The point R divides AB in the ratio of 3 : 2 internally So using section formula, we get

Coordinates of $R = \left(\frac{3\times(6)+2\times(1)}{3+2}, \frac{3\times(7)+2\times(2)}{3+2}\right)$

$= \left(\frac{18+2}{5}, \frac{21+4}{5}\right) = (4,5)$

Hence, the coordinates of the points *P*, *Q* and R are $(2,3),(3,4)$ and $(4,5)$ respectively

6. Points P, Q, and R in that order are dividing line segment joining A (1,6) and B(5, -2) in four equal parts. Find the coordinates of P, Q and R.

Sol:

The given points are $A(1,6)$ and $B(5,-2)$.

Then, $P(x,y)$ is a point that devices the line AB in the ratio $1:3$

By the section formula:

$x = \frac{(mx_2+nx_1)}{(m+n)}, y = \frac{(my_2+ny_1)}{(m+n)}$

$\Rightarrow x = \frac{(1\times5+3\times1)}{1+3}, y = \frac{(1\times(-2)+3\times6)}{1+3}$

$\Rightarrow x = \frac{5+3}{4}, y = \frac{-2+18}{4}$

$\Rightarrow x = \frac{8}{4}, y = \frac{16}{4}$

$\Rightarrow x = 2 \text{ and } y = 4$

Therefore, the coordinates of point P are $(2,4)$

Let Q be the mid-point of AB

Then, $Q(x,y)$

$x = \frac{x_1+x_2}{2}, y = \frac{y_1+y_2}{2}$

$\Rightarrow x = \frac{1+5}{2}, y = \frac{6+(-2)}{2}$

$\Rightarrow x = \frac{6}{2}, y = \frac{4}{2}$

$\Rightarrow x = 3, y = 2$

Therefore, the coordinates of *Q* are $(3,2)$

Let $R(x,y)$ be a point that divides *AB* in the ratio $3:1$

Then, by the section formula:

$$x=\frac{(mx_2+nx_1)}{(m+n)}, y=\frac{(my_2+ny_1)}{(m+n)}$$

$$\Rightarrow x=\frac{(3\times5+1\times1)}{3+1}, y=\frac{(3\times(-2)+1\times6)}{3+1}$$

$$\Rightarrow x=\frac{15+1}{4}, y=\frac{-6+6}{4}$$

$$\Rightarrow x=\frac{16}{4}, y=\frac{0}{4}$$

$$\Rightarrow x=4 \textit{ and } y=0$$

Therefore, the coordinates of R are $(4,0)$.

Hence, the coordinates of point P, Q and R are $(2,4),(3,2)$ *and* $(4,0)$ respectively.

7. The line segment joining the points A(3,-4) and B(1,2) is trisected at the points P(p, -2) and $Q\left(\frac{5}{3},q\right)$. Find the values of p and q.

Sol:

Let P and Q be the points of trisection of AB.
Then, P divides AB in the radio 1:2
So, the coordinates of P are

$$x=\frac{(mx_2+nx_1)}{(m+n)}, y=\frac{(my_2+ny_1)}{(m+n)}$$

$$\Rightarrow x=\frac{\{1\times1+2\times(3)\}}{1+2}, y=\frac{\{1\times2+2\times(-4)\}}{1+2}$$

$$\Rightarrow x=\frac{1+6}{3}, y=\frac{2-8}{3}$$

$$\Rightarrow x=\frac{7}{3}, y=-\frac{6}{3}$$

$$\Rightarrow x=\frac{7}{3}, y=-2$$

Hence, the coordinates of P are $\left(\frac{7}{3},-2\right)$

But $(p,-2)$ are the coordinates of P.

So, $p=\frac{7}{3}$

Also, Q divides the line AB in the ratio 2:1
So, the coordinates of Q are

$$x = \frac{(mx_2 + mx_1)}{(m+n)}, y = \frac{(my_2 + my_1)}{(m+n)}$$

$$\Rightarrow x = \frac{(2\times1+1\times3)}{2+1}, y = \frac{\{2\times2+1\times(-4)\}}{2+1}$$

$$\Rightarrow x = \frac{2+3}{3}, y = \frac{4-4}{3}$$

$$\Rightarrow x = \frac{5}{3}, y = 0$$

Hence, coordinates of Q are $\left(\frac{5}{3}, 0\right)$.

But the given coordinates of Q are $\left(\frac{5}{3}, q\right)$.

So, $q = 0$

Thus, $p = \frac{7}{3}$ *and* $q = 0$

8. Find the coordinates of the midpoints of the line segment joining
(i) A(3,0) and B(-5, 4) (ii) P(-11,-8) and Q(8,-2)

Sol:

(i) The given points are $A(3,0)$ *and* $B(-5,4)$.

Let (x, y) be the midpoint of AB. Then:

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

$$\Rightarrow x = \frac{3+(-5)}{2}, y = \frac{0+4}{2}$$

$$\Rightarrow x = \frac{-2}{2}, y = \frac{4}{2}$$

$$\Rightarrow x = -1, y = 2$$

Therefore, $(-1, 2)$. are the coordinates of midpoint of AB.

(ii) The given points are $P(-11,-8)$ *and* $Q(8,-2)$.

Let (x, y) be the midpoint of PQ. Then:

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

$$\Rightarrow x = \frac{-11+8}{2}, y = \frac{-8--2}{2}$$

$\Rightarrow x = -\frac{3}{2}, y = -\frac{10}{2}$

$\Rightarrow x = -\frac{3}{2}, y = -5$

Therefore, $\left(-\frac{3}{2}, -5\right)$ are the coordinates of midpoint of PQ.

9. If (2, p) is the midpoint of the line segment joining the points A(6, -5) and B(-2,11) find the value of p.

Sol:

The given points are $A(6,-5)$ *and* $B(-2,11)$.

Let (x, y) be the midpoint of AB. Then,

$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$

$\Rightarrow x = \frac{6+(-2)}{2}, y = \frac{-5+11}{2}$

$\Rightarrow x = \frac{6-2}{2}, y = \frac{-5+11}{2}$

$\Rightarrow x = \frac{4}{2}, y = \frac{6}{2}$

$\Rightarrow x = 2, y = 3$

So, the midpoint of AB is $(2,3)$.

But it is given that midpoint of AB is $(2, p)$.

Therefore, the value of $p = 3$.

10. The midpoint of the line segment joining A (2a, 4) and B (-2, 3b) is C (1, 2a+1). Find the values of a and b.

Sol:

The points are $A(2a,4)$ *and* $B(-2,3b)$.

Let $C(1, 2a+1)$ be the mid-point of AB. Then:

$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$

$\Rightarrow 1 = \frac{2a(-2)}{2}, 2a+1 = \frac{4+3b}{2}$

$\Rightarrow 2 = 2a-2, 4a+2 = 4+3b$

$\Rightarrow 2a = 2+2, 4a-3b = 4-2$

$\Rightarrow a = \frac{4}{2}, 4a - 3b = 2$

$\Rightarrow a = 2, 4a - 3b = 2$

Putting the value of a in the equation $4a + 3b = 2$, we get:

$4(2) - 3b = 2$

$\Rightarrow -3b = 2 - 8 = -6$

$\Rightarrow b = \frac{6}{3} = 2$

Therefore, $a = 2$ and $b = 2$.

11. The line segment joining $A(-2, 9)$ and $B(6, 3)$ is a diameter of a circle with center C. Find the coordinates of C.

Sol:

The given points are $A(-2, 9)$ *and* $B(6, 3)$

Then, $C(x, y)$ is the midpoint of *AB*.

$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$

$\Rightarrow x = \frac{-2 + 6}{2}, y = \frac{9 + 3}{2}$

$\Rightarrow x = \frac{4}{2}, y = \frac{12}{2}$

$\Rightarrow x = 2, y = 6$

Therefore, the coordinates of point *C* are $(2, 6)$.

12. Find the coordinates of a point *A*, where *AB* is a diameter of a circle with center $C(2, -3)$ and the other end of the diameter is $B(1, 4)$.

Sol:

$C(2, -3)$ is the center of the given circle. Let $A(a, b)$ *and* $B(1, 4)$ be the two end-points of the given diameter *AB*. Then, the coordinates of C are

$x = \frac{a + 1}{2}, y = \frac{b + 4}{2}$

It is given that $x = 2$ and $y = -3$.

$\Rightarrow 2 = \frac{a + 1}{2}, -3 = \frac{b + 4}{2}$

$\Rightarrow 4 = a + 1, -6 = b + 4$

$\Rightarrow a = 4 - 1, b = -6 - 4$

$\Rightarrow a = 3, b = -10$

Therefore, the coordinates of point A are $(3, -10)$.

13. In what ratio does the point P(2,5) divide the join of A (8,2) and B(-6, 9)?

Sol:

Let the point $P(2,5)$ divide AB in the ratio $k:1$.

Then, by section formula, the coordinates of P are

$$x = \frac{-6k+8}{k+1}, y = \frac{9k+2}{k+1}$$

It is given that the coordinates of P are $(2,5)$.

$$\Rightarrow 2 = \frac{-6k+8}{k+1}, 5 = \frac{9k+2}{k+1}$$

$$\Rightarrow 2k+2 = -6k+8, 5k+5 = 9k+2$$

$$\Rightarrow 2k+6k = 8-2, 5-2 = 9k-5k$$

$$\Rightarrow 8k = 6, 4k = 3$$

$$\Rightarrow k = \frac{6}{8}, k = \frac{3}{4}$$

$\Rightarrow k = \frac{3}{4}$ in each case..

Therefore, the point $P(2,5)$ divides AB in the ratio $3:4$

14. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and B(2, -5).

Sol:

Let $k:1$ be the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ *and* $(2, -5)$. Then

$$\left(\frac{3}{4}, \frac{5}{12}\right) = \left(\frac{k(2)+\frac{1}{2}}{k+1}, \frac{k(-5)+\frac{2}{2}}{k+1}\right)$$

$$\Rightarrow \frac{k(2)+\frac{1}{2}}{k+1} = \frac{3}{4} \text{ and } \frac{k(-5)+\frac{3}{2}}{k+1} = \frac{5}{12}$$

$\Rightarrow 8k+2=3k+3 \ and \ -60k+18=5k+5$

$\Rightarrow k=\frac{1}{5} \ and \ k=\frac{1}{5}$

Hence, the required ratio is $1:5$.

15. Find the ratio in which the point P(m, 6) divides the join of A(-4, 3) and B(2, 8) Also, find the value of m.

Sol:

Let the point $P(m,6)$ divide the line AB in the ratio $k:1$.

Then, by the section formula:

$x=\frac{mx_2+nx_1}{m+n}, y=\frac{my_2+ny_1}{m+n}$

The coordinates of P are $(m,6)$.

$m=\frac{2k-4}{k+1}, 6=\frac{8k+3}{k+1}$

$\Rightarrow m(k+1)=2k-4, 6k+6=8k+3$

$\Rightarrow m(k+1)=2k-4, 6-3=8k-6k$

$\Rightarrow m(k+1)=2k-4, 2k=3$

$\Rightarrow m(k+1)=2k-4, k=\frac{3}{2}$

Therefore, the point P divides the line AB in the ratio $3:2$

Now, putting the value of k in the equation $m(k+1)=2k-4$, we get:

$m\left(\frac{3}{2}+1\right)=2\left(\frac{3}{2}\right)-4$

$\Rightarrow m\left(\frac{3+2}{2}\right)=3-4$

$\Rightarrow \frac{5m}{2}=-1 \Rightarrow 5m=-2 \Rightarrow m=-\frac{2}{5}$

Therefore, the value of $m=-\frac{2}{5}$

So, the coordinates of P are $\left(-\frac{2}{5},6\right)$.

16. Find the ratio in which the pint (-3, k) divide the join of A(-5, -4) and B(-2, 3),Also, find the value of k.

Sol:

Let the point $P(-3,k)$ divide the line AB in the ratio s : 1

Then, by the section formula:

$$x=\frac{mx_1+nx_1}{m+n}, y=\frac{my_2+ny_1}{m+n}$$

The coordinates of P are $(-3,k)$.

$$-3=\frac{-2s-5}{s+1}, k=\frac{3s-4}{s+1}$$

$$\Rightarrow -3s-3=-2s-5, k(s+1)=3s-4$$

$$\Rightarrow -3s+2s=-5+3, k(s+1)=3s-4$$

$$\Rightarrow -s=-2, k(s+1)=3s-4$$

$$\Rightarrow s=2, k(s+1)=3s-4$$

Therefore, the point P divides the line AB in the ratio 2 : 1.

Now, putting the value of s in the equation $k(s+1)=3s-4$, we get:

$$k(2+1)=3(2)-4$$

$$\Rightarrow 3k=6-4$$

$$\Rightarrow 3k=2\Rightarrow k=\frac{2}{3}$$

Therefore, the value of $k=\frac{2}{3}$

That is, the coordinates of P are $\left(-3,\frac{2}{3}\right)$.

17. In what ratio is the line segment joining A(2, -3) and B(5, 6) divide by the x-axis? Also, find the coordinates of the pint of division.

Sol:

Let AB be divided by the x-axis in the ratio $k:1$ at the point P.

Then, by section formula the coordination of P are

$$P=\left(\frac{5k+2}{k+1},\frac{6k-3}{k+1}\right)$$

But P lies on the x-axis; so, its ordinate is 0.

Therefore, $\frac{6k-3}{k+1}=0$

$$\Rightarrow 6k-3=0\Rightarrow 6k=3\Rightarrow k=\frac{3}{6}\Rightarrow k=\frac{1}{2}$$

Therefore, the required ratio is $\frac{1}{2}:1$, which is same as 1 : 2

Thus, the *x*-axis divides the line *AB* li the ratio 1 : 2 at the point *P*.

Applying $k=\frac{1}{2}$, we get the coordinates of point.

$$P\left(\frac{5k+1}{k+1},0\right)$$

$$=P\left(\frac{5\times\frac{1}{2}+2}{\frac{1}{2}+1},0\right)$$

$$=p\left(\frac{\frac{5+4}{2}}{\frac{5+2}{2}},0\right)$$

$$=P\left(\frac{9}{3},0\right)$$

$$=P(3,0)$$

Hence, the point of intersection of *AB* and the x-axis is $P(3,0)$

18. In what ratio is the line segment joining the points A(-2, -3) and B(3,7) divided by the y-axis? Also, find the coordinates of the point of division.

Sol:

Let *AB* be divided by the *x*-axis in the ratio $k:1$ at the point *P*.

Then, by section formula the coordination of *P* are

$$P=\left(\frac{3k-2}{k+1},\frac{7k-3}{k+1}\right)$$

But *P* lies on the *y*-axis; so, its abscissa is 0.

Therefore, $\frac{3k-2}{k+1}=0$

$$\Rightarrow 3k-2=0\Rightarrow 3k=2\Rightarrow k=\frac{2}{3}\Rightarrow k=\frac{2}{3}$$

Therefore, the required ratio is $\frac{2}{3}:1$, which is same as 2 : 3

Thus, the *x*-axis divides the line *AB* in the ratio 2 : 3 at the point *P*.

Applying $k=\frac{2}{3}$, we get the coordinates of point.

$$P\left(0,\frac{7k-3}{k+1}\right)$$

$= P\left(0, \dfrac{7\times\frac{2}{3}-3}{\frac{2}{3}+1}\right)$

$= p\left(0, \dfrac{\frac{14-9}{3}}{\frac{2+3}{3}}\right)$

$= P\left(0, \dfrac{5}{5}\right)$

$= P(0,1)$

Hence, the point of intersection of *AB* and the *x*-axis is $P(0,1)$.

19. In what ratio does the line $x-y-2=0$ divide the line segment joining the points $A(3,-1)$ *and* $B(8,9)$?

Sol:

Let the line $x-y-2=0$ divide the line segment joining the points $A(3,-1)$ and $B(8,9)$ in the ratio $k:1$ at P.

Then, the coordinates of P are

$P\left(\dfrac{8k+3}{k+1}, \dfrac{9k-1}{k+1}\right)$

Since, P lies on the line $x-y-2=0$, we have:

$\left(\dfrac{8k+3}{k+1}\right)-\left(\dfrac{9k-1}{k+1}\right)-2=0$

$\Rightarrow 8k+3-9k+1-2k-2=0$

$\Rightarrow 8k-9k-2k+3+1-2=0$

$\Rightarrow -3k+2=0$

$\Rightarrow -3k=-2$

$\Rightarrow k=\dfrac{2}{3}$

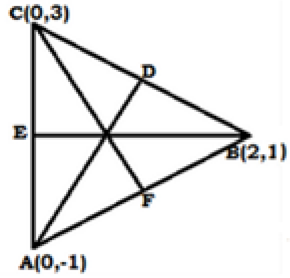
So, the required ratio is $\dfrac{2}{3}:1$, which is equal to $2:3$.

20. Find the lengths of the medians of a ΔABC whose vertices are $A(0,-1), B(2,1)$ and $C(0,3)$.

Sol:

The vertices of ΔABC are $A(0,-1), B(2,1)$ and $C(0,3)$.

Let AD, BE and CF be the medians of ΔABC.



Let D be the midpoint of BC. So, the coordinates of D are

$$D\left(\frac{2+0}{2},\frac{1+3}{2}\right) \text{i.e. } D\left(\frac{2}{2},\frac{4}{2}\right) \text{i.e. } D(1,2)$$

Let E be the midpoint of AC. So the coordinate of E are

$$E\left(\frac{0+0}{2},\frac{-1+3}{2}\right) \text{i.e. } E\left(\frac{0}{2},\frac{0}{2}\right) \text{i.e. } E(0,1)$$

Let F be the midpoint of AB. So, the coordinates of F are

$$F\left(\frac{0+2}{2},\frac{-1+1}{2}\right) \text{i.e. } F\left(\frac{2}{2},\frac{0}{2}\right) \text{i.e. } F(1,0)$$

$$AD=\sqrt{(1-0)^2+(2-(-1))^2}=\sqrt{(1)^2+(3)^2}=\sqrt{1+9}=\sqrt{10} \text{ units.}$$

$$BE=\sqrt{(0-2)^2+(1-1)^2}=\sqrt{(-2)^2+(0)^2}=\sqrt{4+0}=\sqrt{4}=2 \text{ units.}$$

$$CF=\sqrt{(1-0)^2+(0-3)^2}=\sqrt{(1)^2+(-3)^2}=\sqrt{1+9}=\sqrt{10} \text{ units.}$$

Therefore, the lengths of the medians: $AD=\sqrt{10}$ units, $BE=2$ units and $CF=\sqrt{10}$ units.

21. Find the centroid of ΔABC whose vertices are A(-1, 0) B(5, -2) and C(8,2)

Sol:

Here, $(x_1=-1, y_1=0), (x_2=5, y_2=-2)$ and $(x_3=8, y_3=2)$

Let $G(x,y)$ be the centroid of the ΔABC. Then,

$$x=\frac{1}{3}(x_1+x_2+x_3)=\frac{1}{3}(-1+5+8)=\frac{1}{3}(12)=4$$

$$y=\frac{1}{3}(y_1+y_2+y_3)=\frac{1}{3}(0-2+2)=\frac{1}{3}(0)=0$$

Hence, the centroid of ΔABC is $G(4,0)$.

22. If G(-2, 1) is the centroid of a ΔABC and two of its vertices are A(1, -6) and B(-5, 2) , find the third vertex of the triangle.

Sol:

Two vertices of ΔABC are $A(1,-6)$ and $B(-5,2)$. Let the third vertex be C(a, b).

Then the coordinates of its centroid are

$$C\left(\frac{1-5+a}{3},\frac{-6+2+b}{3}\right)$$

$$C\left(\frac{-4+a}{3},\frac{-4+b}{3}\right)$$

But it is given that $G(-2,1)$ is the centroid. Therefore,

$$-2=\frac{-4+a}{3},1=\frac{-4+b}{3}$$

$$\Rightarrow -6=-4+a,3=-4+b$$

$$\Rightarrow -6+4=a,3+4=b$$

$$\Rightarrow a=-2,b=7$$

Therefore, the third vertex of ΔABC is $C(-2,7)$.

23. Find the third vertex of a ΔABC if two of its vertices are $B(-3,1)$ and $C(0,-2)$, and its centroid is at the origin

Sol:

Two vertices of ΔABC are $B(-3,1)$ and $C(0,-2)$. Let the third vertex be $A(a, b)$.

Then, the coordinates of its centroid are

$$\left(\frac{-3+0+a}{3},\frac{1-2+b}{3}\right)$$

i.e., $\left(\frac{-3+a}{3},\frac{-1+b}{3}\right)$

But it is given that the centroid is at the origin, that is $G(0,0)$. Therefore

$$0=\frac{-3+a}{3},0=\frac{-1+b}{3}$$

$$\Rightarrow 0=-3+a,0=-1+b$$

$$\Rightarrow 3=a,1=b$$

$$\Rightarrow a=3,b=1$$

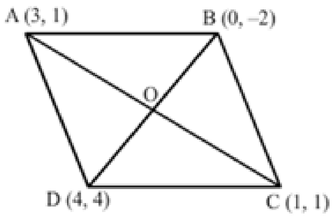
Therefore, the third vertex of ΔABC is $A(3,1)$.

24. Show that the points $A(3,1), B(0,-2), C(1,1)$ and $D(4,4)$ are the vertices of parallelogram ABCD.

Sol:

The points are $A(3,1), B(0,-2), C(1,1)$ and $D(4,4)$

Join *AC* and *BD*, intersecting at *O*.



We know that the diagonals of a parallelogram bisect each other.

Midpoint of $AC = \left(\frac{3+1}{2}, \frac{1+1}{2}\right) = \left(\frac{4}{2}, \frac{2}{2}\right) = (2,1)$

Midpoint of $BD = \left(\frac{0+4}{2}, \frac{-2+4}{2}\right) = \left(\frac{4}{2}, \frac{2}{2}\right) = (2,1)$

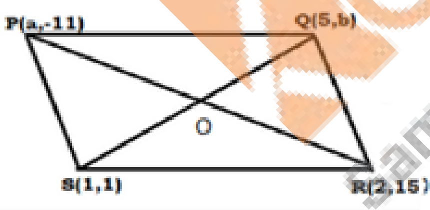
Thus, the diagonals *AC* and *BD* have the same midpoint

Therefore, *ABCD* is a parallelogram.

25. If the points $P(a,-11), Q(5,b), R(2,15)$ *and* $S(1,1)$. are the vertices of a parallelogram PQRS, find the values of a and b.

Sol:

The points are $P(a,-11), Q(5,b), R(2,15)$ *and* $S(1,1)$.



Join *PR* and Q*S*, intersecting at *O*.

We know that the diagonals of a parallelogram bisect each other

Therefore, *O* is the midpoint of *PR* as well as *QS*.

Midpoint of $PR = \left(\frac{a+2}{2}, \frac{-11+15}{2}\right) = \left(\frac{a+2}{2}, \frac{4}{2}\right) = \left(\frac{a+2}{2}, 2\right)$

Midpoint of $QS = \left(\frac{5+1}{2}, \frac{b+1}{2}\right) = \left(\frac{6}{2}, \frac{b+1}{2}\right) = \left(3, \frac{b+1}{2}\right)$

Therefore, $\frac{a+2}{2} = 3, \frac{b+1}{2} = 2$

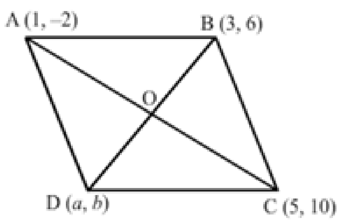
$\Rightarrow a+2=6, b+1=4$
$\Rightarrow a=6-2, b=4-1$
$\Rightarrow a=4 \text{ and } b=3$

26. If three consecutive vertices of a parallelogram $ABCD$ are $A(1,-2), B(3,6) \text{ and } C(5,10)$, find its fourth vertex *D*.

Sol:

Let $A(1,-2), B(3,6) \text{ and } C(5,10)$ be the three vertices of a parallelogram *ABCD* and the fourth vertex be $D(a,b)$.

Join *AC* and *BD* intersecting at *O*.



We know that the diagonals of a parallelogram bisect each other

Therefore, *O* is the midpoint of *AC* as well as *BD*.

Midpoint of $AC=\left(\frac{1+5}{2},\frac{-2+10}{2}\right)=\left(\frac{6}{2},\frac{8}{2}\right)=(3,4)$

Midpoint of $BD=\left(\frac{3+a}{2},\frac{6+b}{2}\right)$

Therefore, $\frac{3+a}{2}=3$ and $\frac{6+b}{2}=4$

$\Rightarrow 3+a=6 \text{ and } 6+b=8$
$\Rightarrow a=6-3 \text{ and } b=8-6$
$\Rightarrow a=3 \text{ and } b=2$

Therefore, the fourth vertex is $D(3,2)$.

27. In what ratio does y-axis divide the line segment joining the points (-4, 7) and (3, -7)?

Sol:

Let *y*-axis divides the e segment pining the points $(-4,7)$ and $(3,-7)$ in the ratio $k:1$. Then

$0=\frac{3k-4}{k+1}$

$\Rightarrow 3k=4$

$\Rightarrow k=\frac{4}{3}$

Hence, the required ratio is $4:3$

28. If the point $P\left(\frac{1}{2}, y\right)$ lies on the line segment joining the points A(3, -5) and B(-7, 9) then find the ratio in which P divides AB. Also, find the value of y.

Sol:

Let the point $P\left(\frac{1}{2}, y\right)$ divides the line segment joining the points $A(3,-5)$ *and* $B(-7,9)$ in the ratio $k:1$. Then

$$\left(\frac{1}{2}, y\right) = \left(\frac{k(-7)+3}{k+1}, \frac{k(9)-3}{k+1}\right)$$

$$\Rightarrow \frac{-7k+3}{k+1} = \frac{1}{2} \text{ and } \frac{9k-5}{k+1} = y$$

$$\Rightarrow k+1 = -14k+6 \Rightarrow k = \frac{1}{3}$$

Now, substituting $k = \frac{1}{3}$ in $\frac{9k-5}{k+1} = y$, we get

$$\frac{\frac{9}{3}-5}{\frac{1}{3}+1} = y \Rightarrow y = \frac{9-15}{1+3} = -\frac{3}{2}$$

Hence, required ratio is $1:3$ and $y = -\frac{3}{2}$.

29. Find the ratio which the line segment joining the pints A(3, -3) and B(-2,7) is divided by x-axis Also, find the point of division.

Sol:

The line segment joining the points $A(3,-3)$ *and* $B(-2,7)$ is divided by *x*-axis. Let the required ratio be $k:1$. So,

$$0 = \frac{k(7)-3}{k+1} \Rightarrow k = \frac{3}{7}$$

Now,

$$\text{Point of division} = \left(\frac{k(-2)+3}{k+1}, \frac{k(7)-3}{k+1}\right)$$

$$= \left(\frac{\frac{3}{7}\times(-2)+3}{\frac{3}{7}+1}, \frac{\frac{3}{7}\times(7)-3}{\frac{3}{7}+1}\right) \qquad \left(\because k = \frac{3}{7}\right)$$

$$=\left(\frac{-6+21}{3+7},\frac{21-21}{3+7}\right)$$

$$=\left(\frac{3}{2},0\right)$$

Hence, the required ratio is $3:7$ and the point of division is $\left(\frac{3}{2},0\right)$

30. The base QR of a n equilateral triangle PQR lies on x-axis. The coordinates of the point Q are (-4, 0) and origin is the midpoint of the base. Find the coordinates of the points P and R.

Sol:

Let $(x,0)$ be the coordinates of R. Then

$$0=\frac{-4+x}{2}\Rightarrow x=4$$

Thus, the coordinates of R are $(4,0)$.

Here, $PQ=QR=PR$ and the coordinates of P lies on $y-axis$. Let the coordinates of P be $(0,y)$. Then,

$$PQ=QR\Rightarrow PQ^2=QR^2$$

$$\Rightarrow(0+4)^2+(y-0)^2=8^2$$

$$\Rightarrow y^2=64-16=48$$

$$\Rightarrow y=\pm4\sqrt{3}$$

Hence, the required coordinates are $R(4,0)$ *and* $P\left(0,4\sqrt{3}\right)$ *or* $P\left(0,-4\sqrt{3}\right)$.

31. The base BC of an equilateral triangle ABC lies on y-axis. The coordinates of point C are (0, -3). The origin is the midpoint of the base. Find the coordinates of the points A and B. Also, find the coordinates of another point D such that ABCD is a rhombus.

Sol:

Let $(0,y)$ be the coordinates of B. Then

$$0=\frac{-3+y}{2}\Rightarrow y=3$$

Thus, the coordinates of B are $(0,3)$

Here. $AB=BC=AC$ and by symmetry the coordinates of A lies on x-axis Let the coordinates of A be (x, 0). Then

$$AB=BC\Rightarrow AB^2=BC^2$$

$$\Rightarrow(x-0)^2+(0-3)^2=6^2$$

$\Rightarrow x^2 = 36 - 9 = 27$

$\Rightarrow x = \pm 3\sqrt{3}$

If the coordinates of point A are $\left(3,\sqrt{3},0\right)$. then the coordinates of D are $\left(-3\sqrt{3},0\right)$.

If the coordinates of point A are $\left(-3\sqrt{3},0\right)$. then the coordinates of D are $\left(-3\sqrt{3},0\right)$.

Hence the required coordinates are $A\left(3\sqrt{3},0\right), B\left(0,3\right)$ and $D\left(-3\sqrt{3},0\right)$ or

$A\left(-3\sqrt{3},0\right), B\left(0,3\right)$ *and* $D\left(3\sqrt{3},0\right)$.

32. Find the ratio in which the point (-1, y) lying on the line segment joining points A(-3, 10) and (6, -8) divides it. Also, find the value of y.

Sol:

Let k be the ratio in which $P(-1, y)$ divides the line segment joining the points

$A(-3,10)$ *and* $B(6,-8)$

Then,

$$(-1, y) = \left(\frac{k(6)-3}{k+1}, \frac{k(-8)+10}{k+1}\right)$$

$$\Rightarrow \frac{k(6)-3}{k+1} = -1 \text{ and } y = \frac{k(-8)+10}{k+1}$$

$$\Rightarrow k = \frac{2}{7}$$

Substituting $k = \frac{2}{7}$ in $y = \frac{k(-8)+10}{k+1}$, we get

$$y = \frac{\frac{-8\times 2}{7}+10}{\frac{2}{7}+1} = \frac{-16+70}{9} = 6$$

Hence, the required ratio is $2:7$ *and* $y = 6$.

33. ABCD is rectangle formed by the points A(-1, -1), B(-1, 4), C(5, 4) and D(5, -1). If P,Q,R and S be the midpoints of AB, BC, CD and DA respectively, Show that PQRS is a rhombus.

Sol:

Here, the points P, Q, R and S are the midpoint of AB, BC, CD *and* DA respectively. Then

Coordinates of $P = \left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$

Coordinates of $Q = \left(\frac{-1+5}{2}, \frac{4+4}{2}\right) = (2,4)$

Coordinates of $R = \left(\frac{5+5}{2}, \frac{4-1}{2}\right) = \left(5, \frac{3}{2}\right)$

Coordinates of $S = \left(\frac{-1+5}{2}, \frac{-1-1}{2}\right) = (2,-1)$

Now,

$$PQ = \sqrt{(2+1)^2 + \left(4-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$QR = \sqrt{(5-2)^2 + \left(\frac{3}{2}-4\right)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$RS = \sqrt{(5-2)^2 + \left(\frac{3}{2}+1\right)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$SP = \sqrt{(2+1)^2 + \left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$PR = \sqrt{(5-1)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} = \sqrt{36} = 6$$

$$QS = \sqrt{(2-2)^2 + (-1-4)^2} = \sqrt{25} = 5$$

Thus, $PQ = QR = RS = SP$ and $PR \neq QS$ therefore $PQRS$ is a rhombus.

34. The midpoint P of the line segment joining points A(-10, 4) and B(-2, 0) lies on the line segment joining the points C(-9, -4) and D(-4, y). Find the ratio in which P divides CD. Also, find the value of y.

Sol:

The midpoint of AB is $\left(\frac{-10-2}{2}, \frac{4+10}{2}\right) = P(-6,2)$.

Let k be the ratio in which P divides CD. So

$$(-6,2) = \left(\frac{k(-4)-9}{k+1}, \frac{k(y)-4}{k+1}\right)$$

$$\Rightarrow \frac{k(-4)-9}{k+1} = -6 \text{ and } \frac{k(y)-4}{k+1} = 2$$

$$\Rightarrow k = \frac{3}{2}$$

Now, substituting $k=\frac{3}{2}$ in $\frac{k(y)-4}{k+1}=2$, we get

$$\frac{y\times\frac{3}{2}-4}{\frac{3}{2}+1}=2$$

$$\Rightarrow \frac{3y-8}{5}=2$$

$$\Rightarrow y=\frac{10+8}{3}=6$$

Hence, the required ratio is $3:2$ and $y=6$.

Exercise – 16C

1. Find the area of ΔABC whose vertices are:

(i) $A(1,2), B(-2,3)$ *and* $C(-3,-4)$

(ii) $A(-5,7), B(-4,-5)$ *and* $C(4,5)$

(iii) $A(3,8), B(-4,2)$ *and* $C(5,-1)$

(iv) $A(10,-6), B(2,5)$ *and* $C(-1,-3)$

Sol:

(i) $A(1,2), B(-2,3)$ *and* $C(-3,-4)$ are the vertices of ΔABC. Then,

$(x_1=1, y_1=2), (x_2=-2, y_2=3)$ *and* $(x_3=-3, y_3=-4)$

Area of triangle ABC

$$=\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$$

$$=\frac{1}{2}\{1(3-(-4))+(-2)(-4-2)+(-3)(2-3)\}$$

$$=\frac{1}{2}\{1(3+4)-2(-6)-3(-1)\}$$

$$=\frac{1}{2}\{7+12+3\}$$

$$=\frac{1}{2}(22)$$

$=11\,sq.\ units$

(ii) $A(-5,7), B(-4,-5)$ *and* $C(4,5)$ are the vertices of ΔABC. Then,

$(x_1=-5, y_1=7), (x_2=-4, y_2=-5)$ *and* $(x_3=4, y_3=5)$

Area of triangle *ABC*

$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

$= \frac{1}{2}\{(-5)(-5-5) + (-4)(5-7) + 4(7-(5))\}$

$= \frac{1}{2}\{(-5)(-10) - 4(-2) + 4(12)\}$

$= \frac{1}{2}\{50 + 8 + 48\}$

$= \frac{1}{2}(106)$

$= 53$ *sq. units*

(iii) $A(3,8), B(-4,2)$ *and* $C(5,-1)$ are verticals of ΔABC. Then,

$(x_1 = 3, y_1 = 8), (x_2 = -4, y_2 = 2)$ *and* $(x_3 = 5, y_3 = -1)$

Area of triangle *ABC*

$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

$= \frac{1}{2}\{3(2-(-1)) + (-4)(-1-8) + 5(8-2)\}$

$= \frac{1}{2}\{3(2+1) - 4(-9) + 5(6)\}$

$= \frac{1}{2}\{9 + 36 + 30\}$

$\Rightarrow \frac{1}{2}(75)$

$= 37.5$ *sq. units*

(iv) $A(10,-6), B(2,5)$ *and* $C(-1,-3)$ are the vertex of ΔABC. Then,

$(x_1 = 10, y_1 = -6), (x_2 = 2, y_2 = 5)$ *and* $(x_3 = -1, y_3 = 3)$

Area of triangle *ABC*

$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

$= \frac{1}{2}\{10(5-3) + 2(3-(-6)) + (-1)(-6-5)\}$

$= \frac{1}{2}\{10(2) + 2(9) - 1(-11)\}$

$= \frac{1}{2}\{20 + 18 + 11\}$

$= \frac{1}{2}(49)$

$= 24.5\, sq.\ units$

2. Find the area of a quadrilateral ABCD whose vertices area A(3, -1), B(9, -5) C(14, 0) and D(9, 19).

Sol:

By joining *A* and *C*, we get two triangles *ABC* and *ACD*.

Let

$A(x_1, y_1) = A(3, -1), B(x_2, y_2) = B(9, -5), C(x_3, y_3) = C(14, 0)\ and\ D(x_4, y_4) = D(9, 19)$

Then,

Area of $\Delta ABC = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

$= \frac{1}{2}\left[3(-5 - 0) + 9(0 + 1) + 14(-1 + 5)\right]$

$= \frac{1}{2}\left[-15 + 9 + 56\right] = 25\, sq.\ units$

Area of $\Delta ACD = \frac{1}{2}\left[x_1(y_3 - y_4) + x_3(y_4 - y_1) + x_4(y_1 - y_3)\right]$

$= \frac{1}{2}\left[3(0 - 19) + 14(19 + 1) + 9(-1 - 0)\right]$

$= \frac{1}{2}\left[-57 + 280 - 9\right] = 107\, sq.\ units$

So, the area of the quadrilateral is $25 + 107 = 132\, sq.\ units.$

3. Find the area of quadrilateral PQRS whose vertices are P(-5, -3), Q(-4,-6),R(2, -3) and S(1,2).

Sol:

By joining *P* and *R*, we get two triangles *PQR* and *PRS*.

Let $P(x_1, y_1) = P(-5, -3), Q(x_2, y_2) = Q(-4, -6), R(x_3, y_3) = R(2, -3)$ and. Then

$S(x_4, y_4) = S(1, 2)$

Area of $\Delta PQR = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

$= \frac{1}{2}\left[-5(-6 + 3) - 4(-3 + 3) + 2(-3 + 6)\right]$

$= \frac{1}{2}\left[15 - 0 + 6\right] = \frac{21}{2}\ sq.\ units$

Area of $\Delta PRS = \frac{1}{2}\left[x_1(y_3-y_4)+x_3(y_4-y_1)+x_4(y_1-y_3)\right]$

$=\frac{1}{2}\left[-5(-3-2)+2(2+3)+1(-3+3)\right]$

$=\frac{1}{2}[25+10+0]=\frac{35}{2}\ sq.\ units$

So, the area of the quadrilateral $PQRS$ is $\frac{21}{2}+\frac{35}{2}=28\ sq.\ units\ sq.\ units$

4. Find the area of quadrilateral ABCD whose vertices are A(-3, -1), B(-2,-4) C(4,-1) and D(3,4)

Sol:

By joining A and C, we get two triangles ABC and ACD.

Let $A(x_1,y_1)=A(-3,-1), B(x_2,y_2)=B(-2,-4), C(x_3,y_3)=C(4,-1)$ and. Then $D(x_4,y_4)=D(3,4)$

Area of $\Delta ABC=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

$=\frac{1}{2}\left[-3(-4+1)-2(-1+1)+4(-1+4)\right]$

$=\frac{1}{2}[9-0+12]=\frac{21}{2}\ sq.\ units$

Area of $\Delta ACD=\frac{1}{2}\left[x_1(y_3-y_4)+x_3(y_4-y_1)+x_4(y_1-y_3)\right]$

$=\frac{1}{2}\left[-3(-1-4)+4(4+1)+3(-1+1)\right]$

$=\frac{1}{2}[15+20+0]=\frac{35}{2}\ sq.\ units$

So, the area of the quadrilateral $ABCD$ is $\frac{21}{2}+\frac{35}{2}=28\ sq.\ units\ sq.\ units$

5. Find the area of quadrilateral ABCD whose vertices are A(-5, 7), B(-4, -5) C(-1,-6) and D(4,5)

Sol:

By joining A and C, we get two triangles ABC and ACD.

Let $A(x_1,y_1)=A(-5,7), B(x_2,y_2)=B(-4,-5), C(x_3,y_3)=C(-1,-6)$ and.

$D(x_4,y_4)=D(4,5)$

Then

Area of $\Delta ABC = \frac{1}{2}\left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right]$

$= \frac{1}{2}\left[-5(-5+6) - 4(-6-7) - 1(7+5)\right]$

$= \frac{1}{2}\left[-5+52-12\right] = \frac{35}{2}$ *sq. units*

Area of $\Delta ACD = \frac{1}{2}\left[x_1\left(y_3 - y_4\right) + x_3\left(y_4 - y_1\right) + x_4\left(y_1 - y_3\right)\right]$

$= \frac{1}{2}\left[-5(-6-5) - 1(5-7) + 4(7+6)\right]$

$= \frac{1}{2}\left[55+2+52\right] = \frac{109}{2}$ *sq. units*

So, the area of the quadrilateral *ABCD* is $\frac{35}{2} + \frac{109}{2} = 72$ *sq. units*

6. Find the area of the triangle formed by joining the midpoints of the sides of the triangle whose vertices are A(2,1) B(4,3) and C(2,5)

Sol:

The verticals of the triangle are $A(2,1), B(4,3)$ *and* $C(2,5)$.

Coordinates of midpoint of $AB = P\left(x_1, y_1\right) = \left(\frac{2+4}{2}, \frac{1+3}{2}\right) = (3,2)$

Coordinates of midpoint of $BC = Q\left(x_2, y_2\right) = \left(\frac{4+2}{2}, \frac{3+5}{2}\right) = (3,4)$

Coordinates of midpoint of $AC = R\left(x_3, y_3\right) = \left(\frac{2+2}{2}, \frac{1+5}{2}\right) = (2,3)$

Now,

Area of $\Delta PQR = \frac{1}{2}\left[x_2\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right]$

$= \frac{1}{2}\left[3(4-3) + 3(3-2) + 2(2-4)\right]$

$= \frac{1}{2}\left[3+3-4\right] = 1$ *sq. unit*

Hence, the area of the quadrilateral triangle is 1 sq. unit.

7. A(7, -3), B(5,3) and C(3,-1) are the vertices of a ΔABC and AD is its median. Prove that the median AD divides ΔABC into two triangles of equal areas.

Sol:

The vertices of the triangle are $A(7,-3), B(5,3), C(3,-1)$.

Coordinates of $D = \left(\frac{5+3}{2}, \frac{3-1}{2}\right) = (4,1)$

For the area of the triangle *ADC*, let

$A(x_1, y_1) = A(7,-3), D(x_2, y_2) = D(4,1)$ *and* $C(x_3, y_3) = C(3,-1)$. Then

Area of $\Delta ADC = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

$= \frac{1}{2}\left[7(1+1) + 4(-1+3) + 3(-3-1)\right]$

$= \frac{1}{2}[14 + 8 - 12] = 5 \ sq.\ unit$

Now, for the area of triangle *ABD*, let

$A(x_1, y_1) = A(7,-3), B(x_2, y_2) = B(5,3)$ *and* $D(x_3, y_3) = D(4,1)$. Then

Area of $\Delta ADC = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

$= \frac{1}{2}\left[7(3-1) + 5(1+3) + 4(-3-3)\right]$

$= \frac{1}{2}[14 + 20 - 24] = 5 \ sq.\ unit$

Thus, Area $(\Delta ADC) = \text{Area}(\Delta ABD) = 5 \ sq.\ units$

Hence, *AD* divides ΔABC into two triangles of equal areas.

8. Find the area of ΔABC with A(1, -4) and midpoints of sides through A being (2, -1) and (0, -1).

Sol:

Let (x_2, y_2) *and* (x_3, y_3) be the coordinates of *B* and *C* respectively. Since, the coordinates of *A* are $(1,-4)$, therefore

$\frac{1+x_2}{2} = 2 \Rightarrow x_2 = 3$

$\frac{-4+y_2}{2} = -1 \Rightarrow y_2 = 2$

$\frac{1+x_2}{2} = 0 \Rightarrow x_3 = -1$

$\frac{-4+y_3}{2} = -1 \Rightarrow y_3 = 2$

Let $A(x_1, y_1) = A(1,-4), B(x_2, y_2) = B(3,2)$ *and* $C(x_3, y_3) = C(-1,2)$ Now

Area $(\Delta ABC) = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

$$=\frac{1}{2}\left[1(2-2)+3(2+4)-1(-4-2)\right]$$
$$=\frac{1}{2}\left[0+18+6\right]$$
$$=12\ sq.\ units$$

Hence, the area of the triangle ΔABC is 12 sq. units

9. A(6,1) , B(8,2) and C(9,4) are the vertices of a parallelogram ABCD. If E is the midpoint of DC, find the area of ΔADE

Sol:

Let (x, y) be the coordinates of D and (x', y') be thee coordinates of E. since, the diagonals of a parallelogram bisect each other at the same point, therefore

$$\frac{x+8}{2}=\frac{6+9}{2}\Rightarrow x=7$$
$$\frac{y+2}{2}=\frac{1+4}{2}\Rightarrow y=3$$

Thus, the coordinates of D are $(7,3)$

E is the midpoint of DC, therefore

$$x'=\frac{7+9}{2}\Rightarrow x'=8$$
$$y'=\frac{3+4}{2}\Rightarrow y'=\frac{7}{2}$$

Thus, the coordinates of E are $\left(8,\frac{7}{2}\right)$

Let $A(x_1, y_1)=A(6,1), E(x_2, y_2)=E\left(8,\frac{7}{2}\right)$ *and* $D(x_3, y_3)=D(7,3)$ Now

Area $(\Delta ABC)=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

$$=\frac{1}{2}\left[6\left(\frac{7}{2}-3\right)+8(3-1)+7\left(1-\frac{7}{2}\right)\right]$$
$$=\frac{1}{2}\left[\frac{3}{2}\right]$$
$$=\frac{3}{4}\ sq.\ unit$$

Hence, the area of the triangle ΔADE is $\frac{3}{4}$ sq. units

10. If the vertices of ΔABC be A(1, -3) B(4, p) and C(-9, 7) and its area is 15 square units, find the values of p.

Sol:

Let $A(x_1, y_1) = A(1, -3), B(x_2, y_2) = B(4, p)$ *and* $C(x_3, y_3) = C(-9, 7)$ Now

$$\text{Area}(\Delta ABC) = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$\Rightarrow 15 = \frac{1}{2}\left[1(p-7) + 4(7+3) - 9(-3-p)\right]$$

$$\Rightarrow 15 = \frac{1}{2}[10p + 60]$$

$$\Rightarrow |10p + 60| = 30$$

Therefore

$\Rightarrow 10p + 60 = -30$ or 30

$\Rightarrow 10p = -90$ or -30

$\Rightarrow p = -9$ or -3

Hence, $p = -9$ or $p = -3$.

11. Find the value of k so that the area of the triangle with vertices A (k+1, 1), B(4, -3) and C(7, -k) is 6 square units.

Sol:

Let $A(x_1, y_1) = A(k+1, 1), B(x_2, y_2) = B(4, -3)$ and $C(x_3, y_3) = C(7, -k)$ now

$$\text{Area } (\Delta ABC) = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$\Rightarrow 6 = \frac{1}{2}\left[(k+1)(-3+k) + 4(-k-1) + 7(1+3)\right]$$

$$\Rightarrow 6 = \frac{1}{2}\left[k^2 - 2k - 3 - 4k - 4 + 28\right]$$

$$\Rightarrow k^2 - 6k + 9 = 0$$

$$\Rightarrow (k-3)^2 = 0 \Rightarrow k = 3$$

Hence, $k = 3$.

12. For what value of k(k>0) is the area of the triangle with vertices (-2, 5), (k, -4) and (2k+1, 10) equal to 53 square units?

Sol:

Let $A(x_1 = -2, y_1 = 5), B(x_2 = k, y_2 = -4)$ and $C(x_3 = 2k+1, y_3 = 10)$ be the vertices of the triangle, So

Area $(\Delta ABC)=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

$\Rightarrow 53=\frac{1}{2}\left[(-2)(-4-10)+k(10-5)+(2k+1)(5+4)\right]$

$\Rightarrow 53=\frac{1}{2}\left[28+5k+9(2k+1)\right]$

$\Rightarrow 28+5k+18k+9=106$

$\Rightarrow 37+23k=106$

$\Rightarrow 23k=106-37=69$

$\Rightarrow k=\frac{69}{23}=3$

Hence, $k=3$.

13. Show that the following points are collinear:
(i) A(2,-2), B(-3, 8) and C(-1, 4)
(ii) A(-5,1), B(5, 5) and C(10, 7)
(iii) A(5,1), B(1, -1) and C(11, 4)
(iv) A(8,1), B(3, -4) and C(2, -5)

Sol:

(i) Let $A(x_1=2, y_1=-2), B(x_2=-3, y_2=8)$ and $C(x_3=-1, y_3=4)$ be the given points.

Now $x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)$

$=2(8-4)+(-3)(4+2)+(-1)(-2-8)$

$=8-18+10$

$=0$

Hence, the given points are collinear.

(ii) Let $A(x_1=-5, y_1=1), B(x_2=5, y_2=5)$ and $C(x_3=10, y_3=7)$ be the given points.

Now $x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)$

$=(-5)(5-7)+5(7-1)+10(1-5)$

$=-5(-2)+5(6)+10(-4)$

$=10+30-40$

$=0$

Hence, the given points are collinear.

(iii) Let $A(x_1=5, y_1=1), B(x_2=1, y_2=-1)$ and $C(x_3=11, y_3=4)$ be the given points.

Now $x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)$

$= 5(-1-4)+1(4-1)+11(1+1)$
$= -25+3+22$
$= 0$
Hence, the given points are collinear.

(iv) Let $A(x_1 = 8, y_1 = 1), B(x_2 = 3, y_2 = -4)$ and $C(x_3 = 2, y_3 = -5)$ be the given points.
Now $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$
$= 8(-4+5)+3(-5-1)+2(1+4)$
$= 8-18+10$
$= 0$
Hence, the given points are collinear.

14. Find the value of x for which points A(x, 2), B(-3, -4) and C(7, -5) are collinear.
Sol:
Let $A(x_1, y_1) = A(x, 2), B(x_2, y_2) = B(-3, -4)$ *and* $C(x_3, y_3) = C(7, -5)$. So the condition for three collinear points is
$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$
$\Rightarrow x(-4+5) - 3(-5-2) + 7(2+4) = 0$
$\Rightarrow x + 21 + 42 = 0$
$\Rightarrow x = -63$
Hence, $x = -63$.

15. For what value of x are the points A(-3, 12), B(7, 6) and C(x, 9) collinear.
Sol:
$A(-3, 12), B(7, 6)$ *and* $C(x, 9)$ are the given points. Then:
$(x_1 = -3, y_1 = 12), (x_2 = 7, y_2 = 6)$ and $(x_3 = x, y_3 = 9)$
It is given that points *A*, *B* and *C* are collinear. Therefore,
$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$
$\Rightarrow (-3)(6-9) + 7(9-12) + x(12-6) = 0$
$\Rightarrow (-3)(-3) + 7(-3) + x(6) = 0$
$\Rightarrow 9 - 21 + 6x = 0$
$\Rightarrow 6x - 12 = 0$
$\Rightarrow 6x = 12$
$\Rightarrow x = \frac{12}{6} = 12$
Therefore, when $x = 2$, the given points are collinear

16. For what value of y, are the points P(1, 4), Q(3,y) and R(-3, 16) are collinear?

Sol:

$P(1,4), Q(3,y)$ *and* $R(-3,16)$ are the given points. Then:

$(x_1 = 1, y_1 = 4), (x_2 = 3, y_2 = y)$ *and* $(x_3 = -3, y_3 = 16)$

It is given that the points *P*, *Q* and *R* are collinear.

Therefore,

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$\Rightarrow 1(y-16) + 3(16-4) + (-3)(4-y) = 0$

$\Rightarrow 1(y-16) + 3(12) - 3(4-y) = 0$

$\Rightarrow y - 16 + 36 - 12 + 3y = 0$

$\Rightarrow 8 + 4y = 0$

$\Rightarrow 4y = -8$

$\Rightarrow y = -\frac{8}{4} = -2$

When, $y = -2$, the given points are collinear.

17. Find the value of y for which the points A(-3, 9), B(2,y) and C(4,-5) are collinear.

Sol:

Let $A(x_1 = -3, y_1 = 9), B(x_2 = 2, y_2 = y)$ and $C(x_3 = 4, y_3 = -5)$ be the given points

The given points are collinear if

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_2 - y_2) = 0$

$\Rightarrow (-3)(y+5) + 2(-5-9) + 4(9-y) = 0$

$\Rightarrow -3y - 15 - 28 + 36 - 4y = 0$

$\Rightarrow 7y = 36 - 43$

$\Rightarrow y = -1$

18. For what values of k are the points A(8, 1) B(3, -2k) and C(k, -5) collinear.

Sol:

Let $A(x_1 = 8, y_1 = 1), B(x_2 = 3, y_2 = -2k)$ and $C(x_3 = k, y_3 = -5)$ be the given points

The given points are collinear if

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$\Rightarrow 8(-2k+5) + 3(-5-1) + k(1+2k) = 0$

$\Rightarrow -16k + 40 - 18 + k + 2k^2 = 0$

$\Rightarrow 2k^2 - 15k + 22 = 0$

$\Rightarrow 2k^2 - 11k - 4k + 22 = 0$

$\Rightarrow k(2k-11) - 2(2k-11) = 0$

$\Rightarrow (k-2)(2k-11)=0$

$\Rightarrow k=2 \ or \ k=\frac{11}{22}$

Hence, $k=2 \ or \ k=\frac{11}{22}$.

19. Find a relation between x and y, if the points A(2, 1), B(x, y) and C(7,5) are collinear.

Sol:

Let $A(x_1=2, y_1=1), B(x_2=x, y_2=y) \ and \ C(x_3=7, y_3=5)$ be the given points

The given points are collinear if

$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$

$\Rightarrow 2(y-5)+x(5-1)+7(1-y)=0$

$\Rightarrow 2y-10+4x+7-7y=0$

$\Rightarrow 4x-5y-3=0$

Hence, the required relation is $4x-5y-3=0$.

20. Find a relation between x and y, if the points A(x, y), B(-5, 7) and C(-4, 5) are collinear.

Sol:

Let $A(x_1=x, y_1=y), B(x_2=-5, y_2=7) \ and \ C(x_3=-4, y_3=5)$ be the given points

The given points are collinear if

$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$

$\Rightarrow x(7-5)+(-5)(5-y)+(-4)(y-7)=0$

$\Rightarrow 7x-5x-25+5y-4y+28=0$

$\Rightarrow 2x+y+3=0$

Hence, the required relation is $2x+y+3=0$

21. Prove that the points $A(a,0), B(0,b) \ and \ C(1,1)$ are collinear, if $\left(\frac{1}{a}+\frac{1}{b}\right)=1$.

Sol:

Consider the points $A(a,0), B(0,b) \ and \ C(1,1)$.

Here, $(x_1=a, y_1=0).(x_2=0, y_2=b) \ and \ (x_3=1, y_3=1)$.

It is given that the points are collinear. So,

$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$

$\Rightarrow a(b-1)+0(1-0)+1(0-b)=0$

$\Rightarrow ab-a-b=0$

Dividing the equation by *ab*:

$\Rightarrow 1-\frac{1}{b}-\frac{1}{a}=0$

$\Rightarrow 1-\left(\frac{1}{a}+\frac{1}{b}\right)=0$

$\Rightarrow \left(\frac{1}{a}+\frac{1}{b}\right)=1$

Therefore, the given points are collinear if $\left(\frac{1}{a}+\frac{1}{b}\right)=1$.

22. If the points P(-3, 9), Q(a, b) and R(4, -5) are collinear and a+b=1, find the value of a and b.

Sol:

Let $A(x_1=3,\ y_1=9)$, $B(x_2=a,\ y_3=b)$ and $C(x_3=4,\ \ y=-5)$ be the given points.

The given pots are collinear if

$x_1(y_2-y_3)+x_3(y_3-y_1)+x_3(y_1-y_2)=0$

$\Rightarrow (-3)(b+5)+a(-5-9)+4(9-b)=0$

$\Rightarrow -3b-15-14a+36-4b=0$

$\Rightarrow 2a+b=3$

Now solving $a+b=1$ and $2a+b=3$, we get $a=2$ and $b\ =-1$.

Hence, a = 2 and $b\ =-1$.

23. Find the area of ΔABC with vertices A(0, -1), B(2,1) and C(0, 3). Also, find the area of the triangle formed by joining the midpoints of its sides. Show that the ratio of the areas of two triangles is 4:1.

Sol:

Let $A(x_1=0, y_1=-1), B(x_2=2, y_2=1)$ and $C(x_3=0, y_3=3)$ be the given points. Then

$Area(\Delta ABC)=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

$=\frac{1}{2}\left[0(1-3)+2(3+1)+0(-1-1)\right]$

$=\frac{1}{2}\times 8=4\ sq.\ units$

So, the area of the triangle $\Delta ABC\ is\ 4\ sq.\ units$

Let $D(a_1,b_1), E(a_2,b_2)$ and $F(a_3,b_3)$ be the midpoints of AB, BC and AC respectively

Then

$a_1=\frac{0+2}{2}=1\quad b_1=\frac{-1+1}{2}=0$

$a_2 = \frac{2+0}{2} = 1 \quad b_2 = \frac{1+3}{2} = 2$

$a_3 = \frac{0+0}{2} = 0 \quad b_3 = \frac{-1+3}{2} = 1$

Thus, the coordinates of *D*, *E* and *F*are $D(a_1 = 1, b_1 = 0)$, $E(a_2 = 1, b_2 = 2)$ and $F(a_3 = 0,\ b_3 = 1)$.Now

Area $(\Delta DEF) = \frac{1}{2}\left[a_1(b_2 - b_2) + a_2(b_3 - b_1) + a_3(b_1 - b_2)\right]$

$= \frac{1}{2}\left[1(2-1) + 1(1-0) + 0(0-2)\right]$

$= \frac{1}{2}[1+1+0] = 1\ sq.\ unit$

So, the area of the triangle ΔDEF is 1 sq. unit.

Hence, $\Delta ABC : \Delta DEF = 4:1$.

Exercise – 16D

1. Points A(-1, y) and B(5,7) lie on the circle with centre O(2, -3y).Find the value of y.

Sol:

The given points are $A(-1, y)$, $8(5,7)$ and $O(2, -3y)$.

Here, *AO* and *BO* are the radii of the circle. So

$AO = BO \Rightarrow AO^2 = BO^2$

$\Rightarrow (2+1)^2 + (-3y - y)^2 = (2-5)^2 + (-3y-7)^2$

$\Rightarrow 9 + (4y)^2 = (-3)^2 + (3y+7)^2$

$\Rightarrow 9 + 16y^2 = 9 + 9y^2 + 49 + 42y$

$\Rightarrow 7y^2 - 42y - 49 = 0$

$\Rightarrow y^2 - 6y - 7 = 0$

$\Rightarrow y^2 - 7y + y - 7 = 0$

$\Rightarrow y(y-7) + 1(y-7) = 0$

$\Rightarrow (y-7)(y+1) = 0$

$\Rightarrow y = -1\ or\ y = 7$

Hence, $y = 7\ or\ y = -1$.

2. If the point A(0,2) is equidistant from the points B(3,p) and C(p, 5), find p.

Sol:

The given ports are $A(0,2), B(3, p)$ and $C(p, 5)$.

$AB = AC \Rightarrow AB^2 = AC^2$

$\Rightarrow (3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$

$\Rightarrow 9 + p^2 - 4p + 4 = p^2 + 9$

$\Rightarrow 4p = 4 \Rightarrow p = 1$

Hence, $p = 1$.

3. ABCD is a rectangle whose three vertices are A(4,0), C(4,3) and D(0,3). Find the length of one its diagonal.

Sol:

The given vertices are $B(4, 0)$, $C(4, 3)$ and $D(0, 3)$ Here, BD one of the diagonals So

$BD = \sqrt{(4-0)^2 + (0-3)^2}$

$= \sqrt{(4)^2 + (-3)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$

Hence, the length of the diagonal is 5 units.

4. If the point P(k-1, 2) is equidistant from the points A(3,k) and B(k,5), find the value of k.

Sol:

The given points are $P(k-1, 2), A(3, k)$ *and* $B(k, 5)$.

$\because AP = BP$

$\therefore AP^2 = BP^2$

$\Rightarrow (k-1-3)^2 + (2-k)^2 = (k-1-k)^2 + (2-5)^2$

$\Rightarrow (k-4)^2 + (2-k)^2 = (-1)^2 + (-3)^2$

$\Rightarrow k^2 - 8y + 16 + 4 + k^2 - 4k = 1 + 9$

$\Rightarrow k^2 - 6y + 5 = 0$

$\Rightarrow (k-1)(k-5) = 0$

$\Rightarrow k = 1$ *or* $k = 5$

Hence, $k = 1$ *or* $k = 5$

5. Find the ratio in which the point P(x,2) divides the join of A(12, 5) and B(4, -3).

Sol:

Let k be the ratio in which the point $P(x, 2)$ divides the line joining the points

$A(x_1 = 12,\ y_1 = 5)$ *and* $B(x_2 = 4, y_2 = -3)$. Then

$x=\frac{k\times4+12}{k+1}$ *and* $2=\frac{k\times(-3)+5}{k+1}$

Now,

$2=\frac{k\times(-3)+5}{k+1}\Rightarrow 2k+2=-3k+5\Rightarrow k=\frac{3}{5}$

Hence, the required ratio is $3:5$.

6. Prove that the diagonals of a rectangle ABCD with vertices A(2,-1), B(5,-1) C(5,6) and D(2,6) are equal and bisect each other.

Sol:

The vertices of the rectangle *ABCD* are $A(2,-1), B(5,-1), C(5,6)$ and $D(2,6)$. Now

Coordinates of midpoint of $AC=\left(\frac{2+5}{2},\frac{-1+6}{2}\right)=\left(\frac{7}{2},\frac{5}{2}\right)$

Coordinates of midpoint of $BD=\left(\frac{5+2}{2},\frac{-1+6}{2}\right)=\left(\frac{7}{2},\frac{5}{2}\right)$

Since, the midpoints of *AC* and *BD* coincide, therefore the diagonals of rectangle *ABCD* bisect each other

7. Find the lengths of the medians AD and BE of ΔABC whose vertices are A(7,-3), B(5,3) and C(3,-1)

Sol:

The given vertices are $A(7,-3), B(5,3)$ *and* $C(3,-1)$.

Since *D* and *E* are the midpoints of *BC* and *AC* respectively. therefore

Coordinates of $D=\left(\frac{5+3}{2},\frac{3-1}{2}\right)=(4,1)$

Coordinates of $E=\left(\frac{7+3}{2},\frac{-3-1}{2}\right)=(5,-2)$

Now

$AD=\sqrt{(7-4)^2+(-3-1)^2}=\sqrt{9+16}=5$

$BE=\sqrt{(5-5)^2+(3+2)^2}=\sqrt{0+25}=5$

Hence, *AD* = *BE* = 5 units.

8. If the point C(k,4) divides the join of A(2,6) and B(5,1) in the ratio 2:3 then find the value of k.

Sol:

Here, the point $C(k,4)$ divides the join of $A(2,6)$ and $B(5,1)$ in ratio $2:3$. So

$k = \frac{2\times5+3\times2}{2+3}$

$= \frac{10+6}{5}$

$= \frac{16}{5}$

Hence, $k = \frac{16}{5}$.

9. Find the point on x-axis which is equidistant from points A(-1,0) and B(5,0)

Sol:

Let $P(x,0)$ be the point on $x-axis.$ Then

$AP = BP \Rightarrow AP^2 = BP^2$

$\Rightarrow (x+1)^2 + (0-0)^2 = (x-5)^2 + (0-0)^2$

$\Rightarrow x^2 + 2x + 1 = x^2 - 10x + 25$

$\Rightarrow 12x = 24 \Rightarrow x = 2$

Hence, $x = 2$

10. Find the distance between the points $A\left(\frac{-8}{5}, 2\right)$ and $B\left(\frac{2}{5}, 2\right)$

Sol:

The given points are $A\left(\frac{-8}{5}, 2\right)$ and $B\left(\frac{2}{5}, 2\right)$

Then, $\left(x_1 = \frac{-8}{5}, y_1 = 2\right)$ *and* $\left(x_2 = \frac{2}{5}, y_2 = 2\right)$

Therefore,

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left\{\frac{2}{5} - \left(\frac{-8}{5}\right)\right\}^2 + (2-2)^2}$

$= \sqrt{(2)^2 + (0)^2}$

$= \sqrt{4+0}$

$= \sqrt{4}$

$= 2$ *units.*

11. Find the value of a, so that the point $(3,a)$ lies on the line represented by $2x-3y=5$.

Sol:

The points $(3,a)$ lies on the line $2x-3y=5.$

If point $(3,a)$ lies on the line $2x-3y=5,$ then $2x-3y=5$

$\Rightarrow (2\times3)-(3\times a)=5$

$\Rightarrow 6-3a=5$

$\Rightarrow 3a=1$

$\Rightarrow a=\frac{1}{3}$

Hence, the value of a is $\frac{1}{3}$.

12. If the points $A(4,3)$ *and* $B(x,5)$ lie on the circle with center $O(2,3)$, find the value of x.

Sol:

The given points $A(4, 3)$ and $B(x, 5)$ lie on the circle with center $O(2, 3)$.

Then, $OA = OB$

$\Rightarrow \sqrt{(x-2)^2+(5-3)^2}=\sqrt{(4-2)^2+(3-3)^2}$

$\Rightarrow (x-2)^2+2^2=2^2+0^2$

$\Rightarrow (x-2)^2=(2^2-2^2)$

$\Rightarrow (x-2)^2=0$

$\Rightarrow x-2=0$

$\Rightarrow x=2$

Hence, the value of $x=2$

13. If $P(x,y)$ is equidistant from the points $A(7,1)$ *and* $B(3,5)$, find the relation between x and y.

Sol:

Let the point $P(x, y)$ be equidistant from the points $A(7, 1)$ and $B(3, 5)$

Then,

$PA=PB$

$\Rightarrow PA^2=PB^2$

$\Rightarrow (x-7)^2+(y-1)^2=(x-3)^2+(y-5)^2$

$\Rightarrow x^2+y^2-14x-2y+50=x^2+y^2-6x-10y+34$

$\Rightarrow 8x-8y=16$

$\Rightarrow x-y=2$

14. If the centroid of ΔABC having vertices $A(a,\ b),\ B(b,\ c)$ and $C(c,\ a)$ is the origin, then find the value of $(a+b+c)$.

Sol:

The given points are $A(a,b), B(b,c)$ *and* $C(c,a)$

Here,

$(x_1=a, y_1=b), (x_2=b, y_2=c)$ *and* $(x_3=c, y_3=a)$

Let the centroid be (x, y).

Then,

$x=\frac{1}{3}(x_1+x_2+x_3)$

$=\frac{1}{3}(a+b+c)$

$=\frac{a+b+c}{3}$

$y=\frac{1}{3}(y_1+y_2+y_3)$

$=\frac{1}{3}(b+c+a)$

$=\frac{a+b+c}{3}$

But it is given that the centroid of the triangle is the origin.

Then, we have

$\frac{a+b+c}{3}=0$

$\Rightarrow a+b+c=0$

15. Find the centroid of ΔABC whose vertices are $A(2,2), B(-4,-4)$ *and* $C(5,-8)$.

Sol:

The given points are $A(2,2), B(-4,-4)$ *and* $C(5,-8)$.

Here, $(x_1=2, y_1=2), (x_2=-4, y_2=-4)$ *and* $(x_3=5, y_3=-8)$

Let $G(x, y)$ be the centroid of ΔABC Then,

$x=\frac{1}{3}(x_1+x_2+x_3)$

$=\frac{1}{3}(2-4+5)$

$=1$

$y=\frac{1}{3}(y_1+y_2+y_3)$

$=\frac{1}{3}(2-4-8)$

$=\frac{-10}{3}$

Hence, the centroid of ΔABC is $G\left(1,\frac{-10}{3}\right)$.

16. In what ratio does the point $C(4,5)$ divides the join of $A(2,3)$ *and* $B(7,8)$?

Sol:

Let the required ratio be *k : 1*

Then, by section formula, the coordinates of C are

$C\left(\frac{7k+2}{k+1},\frac{8k+3}{k+1}\right)$

Therefore,

$\frac{7k+2}{k+1}=4$ *and* $\frac{8k+3}{k+1}=5$ $\quad [\because C(4,5) \text{ is given}]$

$\Rightarrow 7k+2=4k+4$ and $8k+3=5k+5 \Rightarrow 3k=2$

$\Rightarrow k=\frac{2}{3}$ in each case

So, the required ratio is $\frac{2}{3}:1$, which is same as $2:3$.

17. If the points $A(2,3)$, $B(4,k)$ *and* $C(6,-3)$ are collinear, find the value of *k*.

Sol:

The given points are $A(2,3)$, $B(4,k)$ *and* $C(6,-3)$

Here, $(x_1=2, y_1=3), (x_2=4, y_2=k)$ and $(x_3=6, y_3=-3)$

It is given that the points *A*, *B* and *C* are collinear. Then,

$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$

$\Rightarrow 2(k+3)+4(-3-3)+6(3-k)=0$

$\Rightarrow 2k+6-24+18-6k=0$

$\Rightarrow -4k=0$

$\Rightarrow k=0$

Exercise – Multiple Choice Questions

1. The distance of the point P(-6,8) from the origin is
(a) 8 (b) $2\sqrt{7}$ (c) 6 (d) 10
Answer: (d) 10
Sol:
The distance of a point (x, y) from the origin $O(0,0)$ is $\sqrt{x^2 + y^2}$
Let $P(x=-6, y=8)$ be the gen point. Then
$OP = \sqrt{x^2 + y^2}$
$= \sqrt{(-6)^2 + 8^2}$
$= \sqrt{36+64}$
$= \sqrt{100} = 10$

2. The distance of the point (-3, 4) from x-axis is
(a) 3 (b) -3 (c) 4 (d) 5
Answer: (c) 4
Sol:
The distance of a point (*x*. *y*) from $x-axis$ is $|y|$.
Here, the point is $(-3,4)$. So, its distance from *x*-axis is $|4| = 4$

3. The point on x-axis which is equidistant from the points A(-1, 0) and B(5,0) is
(a) (0,2) (b) (2,0) (c) (3,0) (d) (0,3)
Answer: (b) (2,0)
Sol:
Let $P(x,0)$ the point on *x*-axis, then
$AP = BP \Rightarrow AP^2 = BP^2$
$\Rightarrow (x+1)^2 + (0-0)^2 = (x-5)^2 + (0-0)^2$
$\Rightarrow x^2 + 2x + 1 = x^2 - 10x + 25$
$\Rightarrow 12x = 24 \Rightarrow x = 2$
Thus, the required point is (2, 0).

4. If R(5,6) is the midpoint of the line segment AB joining the points A(6,5) and B(4,4) then y equals
(a) 5 (b) 7 (c) 12 (d) 6
Answer: (b) 7

Sol:

Since $R(5,6)$ is the midpoint of the line segment AB joining the points $A(6,5)$ *and* $B(4,y)$, therefore

$$\frac{5+y}{2}=6$$
$$\Rightarrow 5+y=12$$
$$\Rightarrow y=12-5=7$$

5. If the point C(k,4) divides the join of the points A(2,6) and B(5,1) in the ratio 2:3 then the value of k is

(a) 16 (b) $\frac{28}{5}$ (c) $\frac{16}{5}$ (d) $\frac{8}{5}$

Answer: (c) $\frac{16}{5}$

Sol:

The point $C(k,4)$ dives the join of the points $A(2,6)$ *and* $B(5,1)$ in the ratio $2:3.$ So

$$k=\frac{2\times5+3\times2}{2+3}=\frac{10+6}{5}=\frac{16}{5}$$

6. The perimeter of the triangle with vertices (0,4), (0,0) and (3,0) is

(a) $\left(7+\sqrt{5}\right)$ (b) 5 (c) 10 (d) 12

Answer: (d) 12

Sol:

Let $A(0,4),\ B(0,0)$ and $C(3,0)$ be the given vertices. So

$$AB=\sqrt{(0-0)^2+(4-0)^2}=\sqrt{16}=4$$
$$BC=\sqrt{(0-3)^2+(0-0)^2}=\sqrt{9}=3$$
$$AC=\sqrt{(0-3)^2+(4-0)^2}=\sqrt{9+16}=5$$

Therefore

$$AB+BC+AC=4+3+5=12.$$

7. If A(1,3) B(-1,2) C(2,5) and D(x,4) are the vertices of a ||gm ABCD then the value of x is

(a) 3 (b) 4 (c) 0 (d) $\frac{3}{2}$

Answer: (b) 4

Sol:

The diagonals of a parallelogram bisect each other. The vertices of the 11gm *ABCD* are $A(1,3), B(-1,2)$ and $C(2,5)$ *and* $D(x,4)$

Here, *AC* and *BD* are the diagonals. So

$$\frac{1+2}{2}=\frac{-1+x}{2}$$

$$\Rightarrow x-1=3$$

$$\Rightarrow x=1+3=4$$

8. If the points A(x,2), B(-3, -4) and C(7, -5) are collinear then the value of x is

(a) -63 (b) 63 (c) 60 (d) -60

Answer: (a) -63

Sol:

Let $A(x_1=x,\ y_1=2), B(x_2=-3,\ y_2=-4)$ and $C(x_3=7, y_3=-5)$ be collinear points. Then

$$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$$

$$\Rightarrow x(-4+5)+(-3)(-5-2)+7(2+4)=0$$

$$\Rightarrow x+21+42=0$$

$$\Rightarrow x=-63$$

9. The area of a triangle with vertices A(5,0), B(8,0) and C(8,4) in square units is

(a) 20 (b) 12 (c) 6 (d) 16

Answer: (c) 6

Sol:

Let $A(x_1=5, y_1=0), B(x_2=8, y_2=0)$ and $C(x_3=8, y_3=4)$ be the vertices of the triangle.

Then,

$$Area(\Delta ABC)=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$=\frac{1}{2}\left[5(0-4)+8(4-0)+8(0-0)\right]$$

$$=\frac{1}{2}\left[-20+32+0\right]$$

$$=6\ sq.\ units$$

10. The area of ΔABC with vertices A(a,0), O(0,0) and B(0,b) in square units is

(a) ab (b) $\frac{1}{2}ab$ (c) $\frac{1}{2}a^2b^2$ (d) $\frac{1}{2}b^2$

Answer: (b) $\frac{1}{2}ab$

Sol:

Let $A(x_1 = a, y_1 = 0), O(x_2 = 0, y_2 = 0)$ *and* $B(x_3 = 0, y_3 = b)$ be the given vertices. So

$$Area(\Delta ABO) = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|a(0-b) + 0(b-0) + 0(0-0)\right|$$

$$= \frac{1}{2}\left|-ab\right|$$

$$= \frac{1}{2}ab$$

11. If $P\left(\frac{a}{2}, 4\right)$ is the midpoint of the line segment joining the points A(-6, 5) and B(-2,3) then the value of a is

(a) -8 (b) 3 (c) -4 (d) 4

Answer: (a) -8

Sol:

The point $P\left(\frac{a}{2}, 4\right)$ is the midpoint of the line segment joining the points $A(-6,5)$ and $B(-2,3)$.

So $\frac{a}{2} = \frac{-6-2}{2}$

$\Rightarrow \frac{a}{2} = -4$

$\Rightarrow a = -8$

12. ABCD is a rectangle whose three vertices are B(4,0), C(4,3) and D(0,3) The length of one of its diagonals is

(a) 5 (b) 4 (c) 3 (d) 245

Answer: (a) 5

Sol:

Here, *AC* and *BD* are two diagonals of the rectangle *ABCD*. So

$$BD = \sqrt{(4-0)^2 + (0-3)^2}$$

$$= \sqrt{(4)^2 + (-3)^2}$$

$$= \sqrt{16+9}$$

$$= \sqrt{25}$$

= 5 units

13. The coordinates of the point P dividing the line segment joining the points A(1,3), and B(4,6) in the ratio 2:1 is
(a) (2,4) (b) (3,5) (c) (4,2) (d) (5,3)
Answer: (b) (3,5)
Sol:
Here, the point P divides the me segment pining the points $A(1,3)$ and $B(4,6)$ in the ratio $2:1$. Then,

Coordinates of $P=\left(\frac{2\times4+1\times1}{2+1},\frac{2\times6+1\times3}{2+1}\right)$

$=\left(\frac{8+1}{3},\frac{12+3}{3}\right)$

$=\left(\frac{9}{3},\frac{15}{3}\right)$

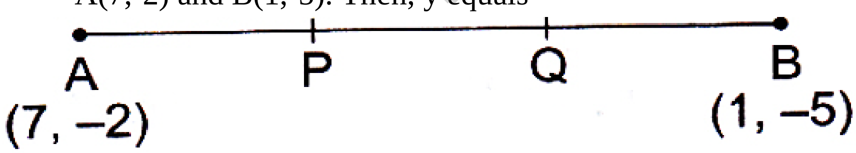
$=(3,5)$

14. If the coordinates of one end of a diameter of a circle are (2,3) and the coordinates of its centre are (-2,5), then the coordinates of the other end of the diameter are
(a) (-6,7) (b) (6.-7) (c) (4,2) (d) (5,3)
Answer: (a) (-6,7)
Sol:
Let (x,y) be the coordinates of the other end of the diameter. Then

$-2=\frac{2+x}{2}\Rightarrow x=-6$

$5=\frac{3+y}{2}\Rightarrow y=7$

15. In the given figure P(5,-3) and Q(3,y) are the points of trisection of the line segment joining A(7,-2) and B(1,-5). Then, y equals



(a) 2 (b) 4 (c) -4 (d) $\frac{-5}{2}$

Answer: (c) -4
Sol:
Here, $AQ:BQ=2:1$. Then,

$y=\frac{2\times(-5)+1\times(-2)}{2+1}$

$= \frac{-10-2}{3}$
$= -4$

16. The midpoint of segment AB is P(0,4). If the coordinates of B are (-2, 3), then the coordinates of A are
(a) (2,5) (b) (-2,-5) (c) (2,9) (d) (-2,11)
Answer: (a) (2,5)
Sol:
Let (x, y) be the coordinates of A. then,

$$0 = \frac{-2+x}{2} \Rightarrow x = 2$$
$$4 = \frac{3+y}{2} \Rightarrow y = 8-3 = 5$$

Thus, the coordinates of A are $(2,5)$.

17. The point P which divides the line segment joining the points A(2,-5) and B(5,2) in the ratio 2:3 lies in the quadrant
(a) I (b) II (c) III (d) IV
Answer: (d) IV
Sol:
Let (x, y) be the coordinates of P. Then,

$$x = \frac{2\times5+3\times2}{2+3} = \frac{10+6}{5} = \frac{16}{5}$$
$$y = \frac{2\times2+3\times(-5)}{2+3} = \frac{4-15}{5} = \frac{-11}{5}$$

Thus, the coordinates of point P are $\left(\frac{16}{5}, \frac{-11}{5}\right)$ and so it lies in the fourth quadrant

18. If A(-6,7) and B(-1,-5) are two given points then the distance 2AB is
(a) 13 (b) 26 (c) 169 (d) 238
Answer: (b) 26
Sol:
The given points are $A(-6,7)$ *and* $B(-1,-5)$. So

$$AB = \sqrt{(-6+1)^2 + (7+5)^2}$$
$$= \sqrt{(-5)^2 + (12)^2}$$

$= \sqrt{25+144}$
$= \sqrt{169}$
$= 13$
Thus, $2AB = 26.$

19. Which point on x-axis is equidistant from the points A(7,6) and B(-3,4)
(a) (0,4) (b) (-4,0) (c) (3,0) (d) (0,3)
Answer: (c) (3,0)
Sol:
Let $p(x,0)$ be the point on $x-axis.$ Then as per the question
$AP = BP \Rightarrow AP^2 = BP^2$
$\Rightarrow (x-7)^2 + (0-6)^2 = (x-3)^2 + (0-4)^2$
$\Rightarrow x^2 - 14x + 49 + 36 = x^2 + 6x + 9 + 16$
$\Rightarrow 60 = 20x$
$\Rightarrow x = \frac{60}{20} = 3$
Thus, the required point is $(3,0)$.

20. The distance of P(3,4) from the x-axis is
(a) 3 units (b) 4 units (c) 5 units (d) 1 unit
Answer: (b) 4 units
Sol:
The *y*-coordinate the distance of the point from the *x*-axis
Here, the *y*-coordinate is 4.

21. In what ratio does the x-axis divide the join of A(2, -3) and B(5,6)?
(a) 2:3 (b) 3:5 (c) 1:2 (d) 2:1
Answer: (c) 1 :2
Sol:
Let *AB* be divided by the *x*-axis in the ratio $k:1$ at the point *P*.
Then, by section formula, the coordinates of *P* are
$P\left(\frac{5k+2}{k+1}, \frac{6k-3}{k+1}\right)$
But *P* lies on the *x*-axes so, its ordinate is 0.
$\frac{6k-3}{k+1} = 0$
$\Rightarrow 6k - 3 = 0$

$\Rightarrow 6k = 3$

$\Rightarrow k = \frac{1}{2}$

Hence, the required ratio is $\frac{1}{2}:1$ which is same as $1:2$.

22. In what ratio does the y-axis divide the join of P(-4,2) and Q(8,3)?
(a) 3:1 (b) 1:3 (c) 2:1 (d) 1:2
Answer: (d) 1:2
Sol:
Let AB be divided by the *y*-axis in the ratio $k:1$ at the point *P*.
Then, by section formula, the coordinates of *P* are

$P\left(\frac{8k-4}{k+1}, \frac{3k+2}{k+1}\right)$

But, *P* lies on the *y*-axis, so, its abscissa is 0.

$\Rightarrow \frac{8k-4}{k+1} = 0$

$\Rightarrow 8k - 4 = 0$

$\Rightarrow 8k = 4$

$\Rightarrow k = \frac{1}{2}$

Hence, the required ratio is $\frac{1}{2}:1$, which is same as $1:2$.

23. If P(-1,1) is the midpoint of the line segment joining A(-3,b) and B(1, b+4) then b=?
(a) 1 (b) -1 (c) 2 (d) 0
Answer: (b) -1
Sol:
The given ports are $A(-3, b)$ *and* $B(1, b+4)$.

Then, $(x_1 = -3, y_1 = b)$ *and* $(x_2 = 1, y_2 = b+4)$

Therefore,

$x = \frac{[(-3)+1]}{2}$

$= \frac{-2}{2}$

$= -1$

And

$y = \frac{\left[b + \left(b + 4\right)\right]}{2}$

$= \frac{2b + 4}{2}$

$= b + 2$

But the midpoint is $P\left(-1,1\right)$.

Therefore,

$b + 2 = 1$

$\Rightarrow b = -1$

24. The line 2x+y-4=0 divide the line segment joining A(2,-2) and B(3,7) in the ratio
(a) 2:5 (b) 2:9 (c) 2:7 (d) 2:3

Answer: (b) 2:9

Sol:

Let the line $2x + y - 4 = 0$ divide the line segment in the ratio $k:1$ at the point *P*.

Then, by section formula the coordinates of *P* are

$P\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

Since *P* lies on the line $2x + y - 4 = 0$, we have

$\frac{2\left(3k+2\right)}{k+1} + \frac{7k-2}{k+1} - 4 = 0$

$\Rightarrow \left(6k+4\right) + \left(7k-2\right) - \left(4k+4\right) = 0$

$\Rightarrow 9k = 2$

$\Rightarrow k = \frac{2}{9}$

Hence, the required ratio is $\frac{2}{9}:1$ which is same as $2:9$.

25. If A(4,2), B(6,5) and C(1,4) be the vertices of ΔABC and AD is a median, then the coordinates of D are

(a) $\left(\frac{5}{2},3\right)$ (b) $\left(5,\frac{7}{2}\right)$ (c) $\left(\frac{7}{2},\frac{9}{2}\right)$ (d) none of these

Answer: (c) $\left(\frac{7}{2},\frac{9}{2}\right)$

Sol:

D is the midpoint of *BC*

So, the coordinates of *D* are

$D\left(\frac{6+1}{2}.\frac{5+4}{2}\right)\left[B(6,5)\ and\ C(1,4) \Rightarrow (x_1=6, y_1=5)\ and\ (x_2=1, y_2=4)\right]$

i.e., $D\left(\frac{7}{2},\frac{9}{2}\right)$

26. If A(-1,0), B(5,-2) and C(8,2) are the vertices of ΔABC then its centroid is
(a) (12,0) (b) (6,0) (c) (0,6) (d) (4,0)
Answer: (d) (4,0)
Sol:
The given point are $A(-1,0), B(5,-2)\ and\ C(8,2)$.

Here, $(x_1=-1, y=0),(x_2=5, y=-2)\ and\ (x_3=8, y_3=2)$

Let $G(x,y)$ be the centroid of ΔABC. Then,

$x=\frac{1}{3}(x_1+x_2+x_3)$

$=\frac{1}{3}(-1+5+8)$

$=4$

and

$y=\frac{1}{3}(y_1+y_2+y_3)$

$=\frac{1}{3}(0-2+2)$

$=0$

Hence, the centroid of ΔABC is $G(4,0)$.

27. Two vertices of ΔABC are A(-1,4) and B(5,2) and its centroid is G(0,-3). Then the coordinates of C are
(a) (4,3) (b) (4,15) (c) (-4,-15) (d) (-15, -4)
Answer: (c) (-4,-15)
Sol:
Two vertices of ΔABC are $A(-1,4)\ and\ B(5,2)$.

Let the third vertex be $C(a,b)$.

Then, the coordinates of its centroid are

$G\left(\frac{-1+5+a}{3},\frac{4+2+b}{3}\right)$

$i.e.,\ G\left(\frac{4+a}{3},\frac{6+b}{3}\right)$

But it is given that the centroid is $G(0,-3)$.

Therefore,

$\frac{4+a}{3}=0 \; and \; \frac{6+b}{3}=-3$

$\Rightarrow 4+a=0 \; and \; 6+b=-9$

$\Rightarrow a=-4 \; and \; b=-15$

Hence, the third vertex of ΔABC is $C(-4,-15)$.

28. The points A(-4,0), B(4,0) and C(0,3) are the vertices of a triangle, which is
(a) isosceles (b) equilateral (c) scalene (d) right-angled

Answer: (a) isosceles

Sol:

Let $A(-4,0), 8(4,0)$ and $C(0,3)$ be the given points. Then,

$AB=\sqrt{(4+4)^2+(0-0)^2}$

$=\sqrt{(8)^2+(0)^2}$

$=\sqrt{64+0}$

$=\sqrt{64}$

$=8 \; units$

$BC=\sqrt{(0-4)^2+(3-0)^2}$

$=\sqrt{(-4)^2+(3)^2}$

$=\sqrt{16+9}$

$=\sqrt{25}$

$=5 \; units$

$AC=\sqrt{(0+4)^2+(3-0)^2}$

$=\sqrt{(4)^2+(3)^2}$

$=\sqrt{16+9}$

$=\sqrt{25}$

$=5 \; units$

$BC=AC=5 \; units$

Therefore, ΔABC is isosceles

29. The points P(0,6), Q(-5,3) and R(3,1) are the vertices of a triangle, which is
(a) equilateral (b) isosceles (c) scalene (d) right-angled
Ans: (d) right - angled
Sol:
Let $P(0,6), Q(-5,3)$ *and* $R(3,1)$ be the given points. Then,

$PQ = \sqrt{(-5-0)^2 + (3-6)^2}$

$= \sqrt{(-5)^2 + (-3)^2}$

$= \sqrt{25+9}$

$= \sqrt{34}$ *units*

$QR = \sqrt{(3+5)^2 + (1-3)^2}$

$= \sqrt{(8)^2 + (-2)^2}$

$= \sqrt{64+4}$

$= \sqrt{68}$

$= 2\sqrt{17}$ *units*

$PR = \sqrt{(3-0)^2 + (1-6)^2}$

$= \sqrt{(3)^2 + (-5)^2}$

$= \sqrt{9+25}$

$= \sqrt{34}$ *units*

$PQ^2 + PR^2 \Rightarrow \left\{\left(\sqrt{34}\right)^2 + \left(\sqrt{34}\right)^2\right\} = 68$

$QR^2 \Rightarrow \left(2\sqrt{17}\right)^2 = 68$

Thus, $PQ^2 + PR^2 = QR^2$

Therefore, ΔPQR is right-angled.

30. If the points A(2,3), B(5,k) and C(6,7) are collinear then
(a) k =4 (b) k = 6 (c) k = $\frac{-3}{2}$ (d) k = $\frac{11}{4}$
Ans: (b) k = 6
Sol:
The given points are $A(2,3), B(5,k)$ *and* $C(6,7)$.

Here, $(x_1 = 2, y_1 = 3), (x_2 = 5, y_2 = k)$ *and* $(x_3 = 6, y_3 = 7)$.

Points *A*, *B* and *C* are collinear. Then,

$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$

$\Rightarrow 2(k-7)+5(7-3)+6(3-k)=0$

$\Rightarrow 2k-14+20+18-6k=0$

$\Rightarrow -4k=-24$

$\Rightarrow k=6$

31. If the point $A(1,2), O(0,0)$ *and* $C(a,b)$ are collinear, then

(a) a = b (b) a = 2b (c) 2a = b (d) a + b = 0

Ans: (c) 2a = b

Sol:

The given points are $A(1,2), O(0,0)$ *and* $C(a,b)$

Here, $(x_1=1, y_1=2), (x_2=0, y_2=0)$ *and* $(x_3=a, y_3=b)$.

Point *A*, *O* and *C* are collinear

$\Rightarrow x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$

$\Rightarrow 1(0-b)+0(b-2)+a(2-0)=0$

$\Rightarrow -b+2a=0$

$\Rightarrow 2a=b$

32. The area of ΔABC with vertices A(3,0) , B(7,0) and C(8,4) is

(a) 14 sq units (b) 28 sq units (c) 8 sq units (d) 6 sq units

Ans: (c) 8 sq units

Sol:

The given points are $A(3,0), B(7,0)$ *and* $C(8,4)$.

Here, $(x_1=3, y_1=0), (x_2=7, y_2=0)$ *and* $(x_3=8, y_3=4)$

Therefore,

$$Area\ of\ \Delta ABC=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$=\frac{1}{2}\left[3(0-4)+7(4-0)+8(0-0)\right]$$

$$=\frac{1}{2}\left[-12+28+0\right]$$

$$=\left(\frac{1}{2}\times 16\right)$$

$$=8\ sq.\ units$$

33. AOBC is rectangle whose three vertices are A(0,3), O(0,0) and B(5,0). The length of each of its diagonals is

(a) 5 units (b) 3 units (c) 4 units (d) $\sqrt{34}$ units

Ans: (c) 4 units

Sol:

$A(0,3), O(0,0)$ *and* $B(5,0)$ are the three vertices of a rectangle; let C be the fourth vertex

Then, the length of the diagonal,

$AB = \sqrt{(5-0)^2 + (0-3)^2}$

$= \sqrt{(5)^2 + (-3)^2}$

$= \sqrt{25+9}$

$= \sqrt{34}$ *units*

Since, the diagonals al rectangle is equal.

Hence, the length of its diagonals is $\sqrt{34}$ units.

34. If the distance between the points A(4,p) and B(1,0) is 5 then

(a) p = 4 only (b) p = -4 only (c) p = $\pm$ 4 (d) p = 0

Ans: (c) p = $\pm$ 4

Sol:

The given points are $A(4, p)$ *and* $B(1,0)$ *and* $AB = 5$.

Then, $(x_1 = 4, y_1 = p)$ *and* $(x_2 = 1, y_2 = 0)$

Therefore,

$AB = 5$

$\Rightarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5$

$\Rightarrow \sqrt{(1-4)^2 + (0-p)^2} = 5$

$\Rightarrow (-3)^2 + (-p)^2 = 25$

$\Rightarrow 9 + p^2 = 25$

$\Rightarrow p^2 = 16$

$\Rightarrow p = \pm\sqrt{16}$

$\Rightarrow p = \pm 4$