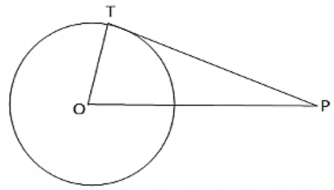
Exercise - 12A

1. Find the length of tangent drawn to a circle with radius 8 cm form a point 17 cm away from the center of the circle

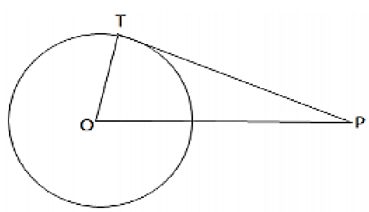
Sol:



Let O be the center of the given circle.
Let P be a point, such that
OP = 17 cm.
Let OT be the radius, where
OT = 5cm
Join TP, where TP is a tangent.
Now, tangent drawn from an external point is perpendicular to the radius at the point of contact.
$\therefore OT \perp PT$
In the right ΔOTP, we have:
$OP^2 = OT^2 + TP^2$ [By Pythagoras' theorem:]
$TP = \sqrt{OP^2 - OT^2}$
$= \sqrt{17^2 - 8^2}$
$= \sqrt{289 - 64}$
$= \sqrt{225}$
$= 15\,cm$
$\therefore$ The length of the tangent is 15 cm.

2. A point P is 25 cm away from the center of a circle and the length of tangent drawn from P to the circle is 24 cm. Find the radius of the circle.

Sol:



Draw a circle and let P be a point such that OP = 25cm.
Let TP be the tangent, so that TP = 24cm
Join OT where OT is radius.

Now, tangent drawn from an external point is perpendicular to the radius at the point of contact.

$\therefore OT \perp PT$

In the right ΔOTP, we have:

$OP^2 = OT^2 + TP^2$ [By Pythagoras' theorem:]

$OT^2 = \sqrt{OP^2 - TP^2}$

$= \sqrt{25^2 - 24^2}$

$= \sqrt{625 - 576}$

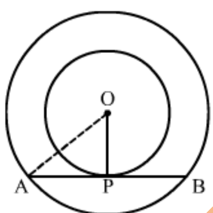
$= \sqrt{49}$

$= 7\,cm$

$\therefore$ The length of the radius is 7cm.

3. Two concentric circles are of radii 6.5 cm and 2.5 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Sol:



We know that the radius and tangent are perpendicular at their point of contact

In right triangle AOP

$AO^2 = OP^2 + PA^2$

$\Rightarrow (6.5)^2 = (2.5)^2 + PA^2$

$\Rightarrow PA^2 = 36$

$\Rightarrow PA = 6\,cm$

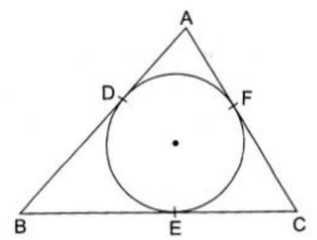
Since, the perpendicular drawn from the center bisects the chord.

$\therefore PA = PB = 6\,cm$

Now, $AB = AP + PB = 6 + 6 = 12\,cm$

Hence, the length of the chord of the larger circle is 12cm.

4. In the given figure, a circle inscribed in a triangle ABC, touches the sides AB, BC and AC at points D, E and F Respectively. If AB= 12cm, BC=8cm and AC = 10cm, find the length of AD, BE and CF.



Sol:

We know that tangent segments to a circle from the same external point are congruent.

Now, we have

$AD = AF$, $BD = BE$ and $CE = CF$

Now, $AD + BD = 12\text{cm}$ …….(1)

$AF + FC = 10 \text{ cm}$

$\Rightarrow AD + FC = 10 \text{ cm}$ …….(2)

$BE + EC = 8 \text{ cm}$

$\Rightarrow BD + FC = 8\text{cm}$ …….(3)

Adding all these we get

$AD + BD + AD + FC + BD + FC = 30$

$\Rightarrow 2(AD + BD + FC) = 30$

$\Rightarrow AD + BD + FC = 15\text{cm}$ …….(4)

Solving (1) and (4), we get

$FC = 3 \text{ cm}$

Solving (2) and (4), we get

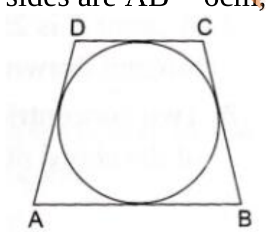
$BD = 5 \text{ cm}$

Solving (3) and (4), we get

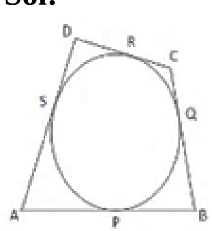
and $AD = 7 \text{ cm}$

$\therefore$ $AD = AF = 7 \text{ cm}$, $BD = BE = 5 \text{ cm}$ and $CE = CF = 3 \text{ cm}$

5. In the given figure, a circle touches all the four sides of a quadrilateral ABCD whose three sides are AB = 6cm, BC=7cm and CD=4 cm. Find AD.



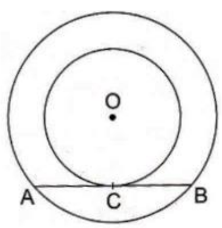
Sol:



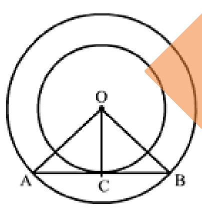
Let the circle touch the sides of the quadrilateral *AB*, *BC*, *CD* and *DA* at *P*, *Q*, *R* and *S* respectively.

Given, AB = 6cm, BC = 7 cm and CD = 4cm.
Tangents drawn from an external point are equal.
$\therefore AP = AS, BP = BQ, CR = CQ$ and $DR = DS$
Now, $AB + CD (AP + BP) + (CR + DR)$
$\Rightarrow AB + CD = (AS + BQ) + (CQ + DS)$
$\Rightarrow AB + CD = (AS + DS) + (BQ + CQ)$
$\Rightarrow AB + CD = AD + BC$
$\Rightarrow AD = (AB + CD) - BC$
$\Rightarrow AD = (6 + 4) - 7$
$\Rightarrow AD = 3 cm.$
$\therefore$ The length of AD is 3 cm.

6. In the given figure, the chord AB of the larger of the two concentric circles, with center O, touches the smaller circle at C. Prove that AC = CB.

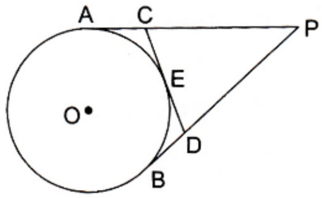


Sol:
Construction: Join OA, OC and OB



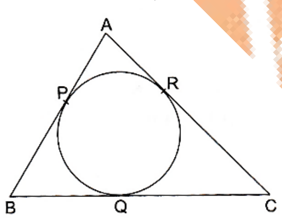
We know that the radius and tangent are perpendicular at their point of contact
$\therefore \angle OCA = \angle OCB = 90°$
Now, In ΔOCA and ΔOCB
$\angle OCA = \angle OCB = 90°$
$OA = OB$ (Radii of the larger circle)
$OC = OC$ (Common)
By RHS congruency
$\Delta OCA \cong \Delta OCB$
$\therefore CA = CB$

7. From an external point P, tangents PA and PB are drawn to a circle with center O. If CD is the tangent to the circle at a point E and PA = 14cm, find the perimeter of ΔPCD.



Sol:
Given, PA and PB are the tangents to a circle with center O and CD is a tangent at E and PA = 14 cm.
Tangents drawn from an external point are equal.
$\therefore PA = PB, CA = CE$ and $DB = DE$
Perimeter of $\Delta PCD = PC + CD + PD$
$= (PA - CA) + (CE + DE) + (PB - DB)$
$= (PA - CE) + (CE + DE) + (PB - DE)$
$= (PA + PB)$
$= 2PA \ (\because PA = PB)$
$= (2 \times 14) cm$
$= 28 cm$
=28 cm
$\therefore$ Perimeter of $\Delta PCD = 28 cm.$

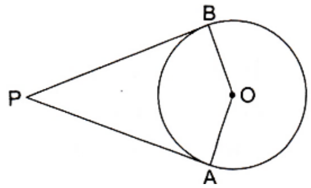
8. A circle is inscribed in a ΔABC touching AB, BC and AC at P, Q and R respectively. If AB = 10 cm, AR=7cm and CR=5cm, find the length of BC.



Sol:
Given, a circle inscribed in triangle ABC, such that the circle touches the sides of the triangle
Tangents drawn to a circle from an external point are equal.
$\therefore AP = AR = 7cm, CQ = CR = 5cm.$
Now, $BP = (AB - AP) = (10 - 7) = 3cm$
$\therefore BP = BQ = 3cm$
$\therefore BC = (BQ + QC)$

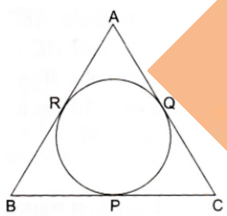
$\Rightarrow BC = 3+5$
$\Rightarrow BC = 8$
$\therefore$ The length of BC is 8 cm.

9. In the given figure, PA and PB are the tangent segemtns to a circle with centre O. Show that he points A, O, B and P are concyclic.



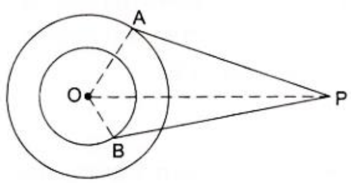
Sol:
Here, $OA = OB$
And $OA \perp AP, OA \perp BP$ (Since tangents drawn from an external point are perpendicular to the radius at the point of contact)
$\therefore \angle OAP = 90°, \angle OBP = 90°$
$\therefore \angle OAP + \angle OBP = 90° + 90° = 180°$
$\therefore \angle AOB + \angle APB = 180°$ (Since, $\angle OAP + \angle OBP + \angle AOB + \angle APB = 360°$)
Sum of opposite angle of a quadrilateral is $180°$.
Hence A, O, B and P are concyclic.

10. In the given figure, an isosceles triangle ABC, with AB = AC, circumscribes a circle. Prove that point of contact P bisects the base BC.



Sol:
We know that tangent segments to a circle from the same external point are congruent
Now, we have
$AR = AO$, $BR = BP$ and $CP = CQ$
Now, $AB = AC$
$\Rightarrow AR + RB = AQ + QC$
$\Rightarrow AR + RB = AR + OC$
$\Rightarrow RB = QC$
$\Rightarrow BP = CP$
Hence, P bisects BC at P.

11. In the given figure, O is the centre of the two concentric circles of radii 4 cm and 6cm respectively. AP and PB are tangents to the outer and inner circle respectively. If PA = 10cm, find the length of PB up to one place of the decimal.



Sol:

Given, O is the center of two concentric circles of radii OA = 6 cm and OB = 4 cm. PA and PB are the two tangents to the outer and inner circles respectively and PA = 10 cm.

Now, tangent drawn from an external point is perpendicular to the radius at the point of contact.

$\therefore \angle OAP = \angle OBP = 90°$

$\therefore$ From right – angled $\Delta OAP, OP^2 = OA^2 + PA^2$

$\Rightarrow OP = \sqrt{OA^2 + PA^2}$

$\Rightarrow OP = \sqrt{6^2 + 10^2}$

$\Rightarrow OP = \sqrt{136} cm.$

$\therefore$ From right – angled $\Delta OAP, OP^2 = OB^2 + PB^2$

$\Rightarrow PB = \sqrt{OP^2 - OB^2}$

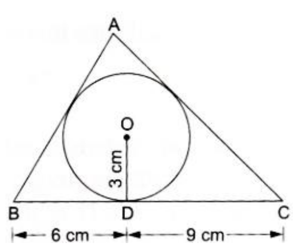
$\Rightarrow PB = \sqrt{136 - 16}$

$\Rightarrow PB = \sqrt{120} cm$

$\Rightarrow PB = 10.9\, cm.$

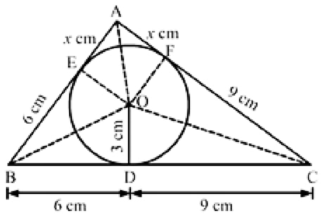
$\therefore$ The length of PB is 10.9 cm.

12. In the given figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm such that the segments BC and DC into which BC is divided by the point of contact D, are of lengths 6cm and 9cm respectively. If the area of $\Delta ABC = 54 cm^2$ then find the lengths of sides AB and AC.



Sol:

Construction: Join $OA, OB, OC, OE \perp AB$ at E and $OF \perp AC$ at F



We know that tangent segments to a circle from me same external point are congruent
Now, we have
AE = *AF*, *BD* = *BE* =6 cm and *CD* = *CF* = 9 cm
Now,

$$Area(\Delta ABC) = Area(\Delta BOC) + Area(\Delta AOB) + Area(\Delta AOC)$$

$$\Rightarrow 54 = \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AB \times OE + \frac{1}{2} \times AC \times OF$$

$$\Rightarrow 108 = 15 \times 3 + (6 + x) \times 3 + (9 + x) \times 3$$

$$\Rightarrow 36 = 15 + 6 + x + 9 + x$$

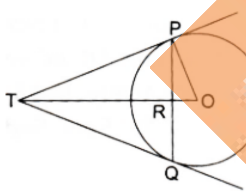
$$\Rightarrow 36 = 30 + 2x$$

$$\Rightarrow 2x = 6$$

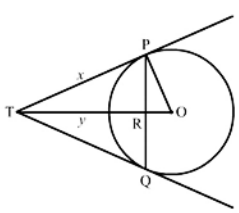
$$\Rightarrow x = 3cm$$

$$\therefore AB = 6 + 3 = 9cm \text{ and } AC = 9 + 3 = 12cm$$

13. PQ is a chord of length 4.8 cm of a circle of radius 3cm. The tangents at P and Q intersect at a point T as shown in the figure. Find the length of TP.



Sol:



Let *TR* = y and TP= x
We know that the perpendicular drawn from the center to me chord bisects It.

$$\therefore PR = RQ$$

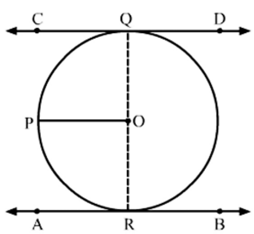
Now, $PR + RQ = 4.8$

$$\Rightarrow PR + PR = 4.8$$

$$\Rightarrow PR = 2.4$$

Now, in right triangle POR
By Using Pythagoras theorem, we have
$PO^2 = OR^2 + PR^2$
$\Rightarrow 3^2 = OR^2 + (2.4)^2$
$\Rightarrow OR^2 = 3.24$
$\Rightarrow OR = 1.8$
Now, in right triangle TPR
By Using Pythagoras theorem, we have
$TP^2 = TR^2 + PR^2$
$\Rightarrow x^2 = y^2 + (2.4)^2$
$\Rightarrow x^2 = y^2 + 5.76 \qquad ....(1)$
Again, In right triangle TPQ
By Using Pythagoras theorem, we have
$TO^2 = TP^2 + PO^2$
$\Rightarrow (y+1.8)^2 = x^2 + 3^2$
$\Rightarrow y^2 + 3.6y + 3.24 = x^2 + 9$
$\Rightarrow y^2 + 3.6y = x^2 + 5.76 \qquad ......(2)$
Solving (1) and (2), we get
$x = 4cm \text{ and } y = 3.2cm$
$\therefore TP = 4cm$

14. Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre.
Sol:



Suppose CD and AB are two parallel tangents of a circle with center O
Construction: Draw a line parallel to CD passing through O i.e. OP
We know that the radius and tangent are perpendicular at their point of contact.
$\angle OQC = \angle ORA = 90°$
Now, $\angle OQC + \angle POQ = 180°$ (co-interior angles)
$\Rightarrow \angle POQ = 180° - 90° = 90°$
Similarly, Now, $\angle ORA + \angle POR = 180°$ (co-interior angles)

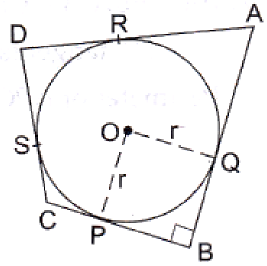
$\Rightarrow \angle POQ = 180° - 90° = 90°$

Now, $\angle POR + \angle POQ = 90° + 90° = 180°$

Since, $\angle POR \text{ and } \angle POQ$ are linear pair angles whose sum is 180°

Hence, QR is a straight line passing through center O.

15. In the given figure, a circle with center O, is inscribed in a quadrilateral ABCD such that it touches the side BC, AB, AD and CD at points P, Q, R and S respectively. If AB = 29cm, AD = 23cm, $\angle B = 90°$ and DS=5cm then find the radius of the circle.



Sol:

We know that tangent segments to a circle from the same external point are congruent

Now, we have

DS = DR, AR = AQ

Now AD = 23 cm

$\Rightarrow AR + RD = 23$

$\Rightarrow AR = 23 - RD$

$\Rightarrow AR = 23 - 5 \; [\because DS = DR = 5]$

$\Rightarrow AR = 18cm$

Again, AB = 29 cm

$\Rightarrow AQ + QB = 29$

$\Rightarrow QB = 29 - AQ$

$\Rightarrow QB = 29 - 18 \qquad [\because AR = AQ = 18]$

$\Rightarrow QB = 11cm$

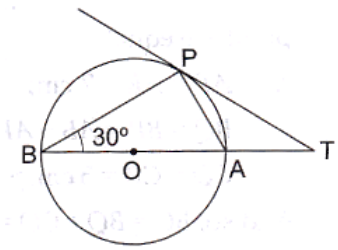
Since all the angles are in a quadrilateral BQOP are right angles and $OP = BQ$

Hence, BQOP is a square.

We know that all the sides of square are equal.

Therefore, BQ = PO = 11 cm

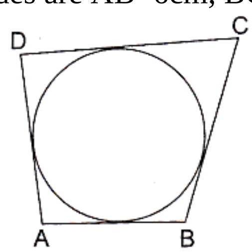
16. In the given figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If $\angle PBT = 30°$, prove that BA : AT = 2 : 1.



Sol:
AB is the chord passing through the center
So, AB is the diameter
Since, angle in a semicircle is a right angle
$\therefore \angle APB = 90°$
By using alternate segment theorem
We have $\angle APB = \angle PAT = 30°$
Now, in ΔAPB
$\angle BAP + \angle APB + \angle BAP = 180°$ (Angle sum property of triangle)
$\Rightarrow \angle BAP = 180° - 90° - 30° = 60°$
Now, $\angle BAP = \angle APT + \angle PTA$ (Exterior angle property)
$\Rightarrow 60° = 30° + \angle PTA$
$\Rightarrow \angle PTA = 60° - 30° = 30°$
We know that sides opposite to equal angles are equal
$\therefore AP = AT$
In right triangle ABP
$\sin \angle ABP = \dfrac{AP}{BA}$
$\Rightarrow \sin 30° = \dfrac{AT}{BA}$
$\Rightarrow \dfrac{1}{2} = \dfrac{AT}{BA}$
$\therefore BA : AT = 2 : 1$

Exercise – 12B

1. In the adjoining figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB=6cm, BC=9cm and CD=8 cm. Find the length of side AD.



Sol:

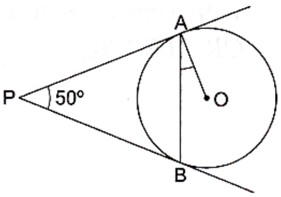
We know that when a quadrilateral circumscribes a circle then sum of opposites sides is equal to the sum of other opposite sides.

$\therefore AB + CD = AD + BC$

$\Rightarrow 6 + 8 = AD = 9$

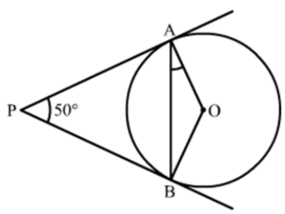
$\Rightarrow AD = 5cm$

2. In the given figure, PA and PB are two tangents to the circle with centre O. If $\angle APB = 50°$ then what is the measure of $\angle OAB$.



Sol:

Construction: Join OB



We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OBP = \angle OAP = 90°$

Now, In quadrilateral AOBP

$\angle AOB + \angle OBP + \angle APB + \angle OAP = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle AOB + 90° + 50° + 90° = 360°$

$\Rightarrow 230° + \angle BOC = 360°$

$\Rightarrow \angle AOB = 130°$

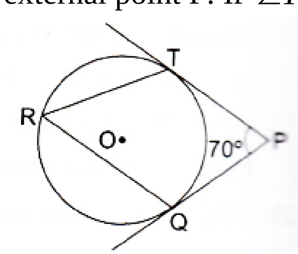
Now, In isosceles triangle AOB

$\angle AOB + \angle OAB + \angle OBA = 180°$ [Angle sum property of a triangle]

$\Rightarrow 130° + 2\angle OAB = 180°$ $[\because \angle OAB = \angle OBA]$

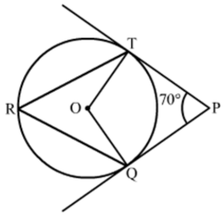
$\Rightarrow \angle OAB = 25°$

3. In the given figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ = 70°$, find the $\angle TRQ$.



Sol:

Construction: Join OQ and OT



We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OTP = \angle OQP = 90°$

Now, In quadrilateral OQPT

$\angle QOT + \angle OTP + \angle OQP + \angle TPO = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle QOT + 90° + 90° + 70° = 360°$

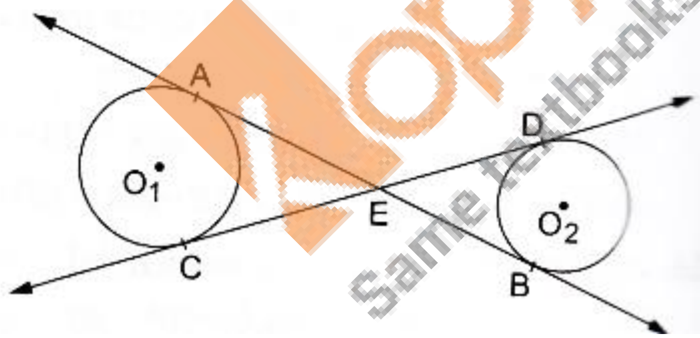
$\Rightarrow 250° + \angle QOT = 360°$

$\Rightarrow \angle QOT = 110°$

We know that the angle subtended by an arc at the center is double the angle subtended by the arc at any point on the remaining part of the circle.

$\therefore \angle TRQ = \frac{1}{2}(\angle QOT) = 55°$

4. In the given figure common tangents AB and CD to the two circles with centres O_1 *and* O_2 intersect at E. Prove that AB=CD.



Sol:

We know that tangent segments to a circle from the same external point are congruent.

So, we have

$EA = EC$ for the circle having center O_1

and

$ED = EB$ for the circle having center O_1

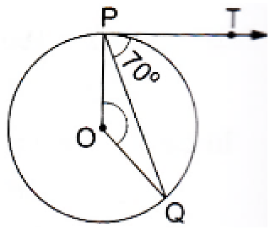
Now, Adding ED on both sides in EA = EC. we get

$EA + ED = EC + ED$

$\Rightarrow EA + EB = EC + ED$

$\Rightarrow AB = CD$

5. If PT is a tangent to a circle with center O and PQ is a chord of the circle such that $\angle QPT = 70°$, then find the measure of $\angle POQ$.



Sol:

We know that the radius and tangent are perpendicular at their point of contact.

$\therefore \angle OPT = 90°$

Now, $\angle OPQ = \angle OPT - \angle TPQ = 90° - 70° = 20°$

Since, OP = OQ as both are radius

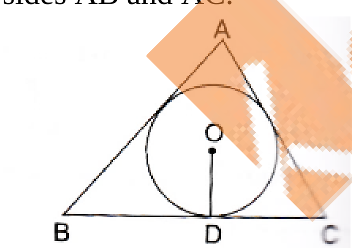
$\therefore \angle OPQ = \angle OQP = 20°$ (Angles opposite to equal sides are equal)

Now, In isosceles Δ POQ

$\angle POQ + \angle OPQ + \angle OQP = 180°$ (Angle sum property of a triangle)

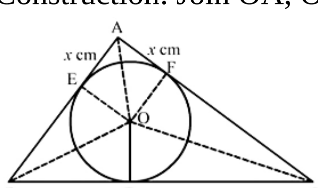
$\Rightarrow \angle POQ = 180° - 20° = 140°$

6. In the given figure, a triangle ABC is drawn to circumscribe a circle of radius 2 cm such that the segments BD and DC into which BC is divided by the point of contact D, are of lengths 4cm and 3cm respectively. If the area of $\Delta ABC = 21 cm^2$ then find the lengths of sides AB and AC.



Sol:

Construction: Join OA, OB, OC, OE $\perp$ AB at E and OF $\perp$ AC at F



We know that tangent segments to a circle from the same external point are congruent

Now, we have

$AE = AF, BD = BE = 4cm$ and $CD = CF = 3cm$

Now,

$Area(\Delta ABC) = Area(\Delta BOC) + Area(\Delta AOB) + Area(\Delta AOC)$

$\Rightarrow 21=\frac{1}{2}\times BC\times OD+\frac{1}{2}\times AB\times OE+\frac{1}{2}\times AC\times OF$

$\Rightarrow 42=7\times 2+(4+x)\times 2+(3+x)\times 2$

$\Rightarrow 21=7+4+x+3+x$

$\Rightarrow 21=14+2x$

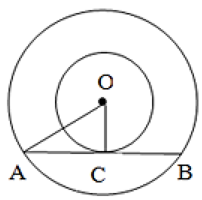
$\Rightarrow 2x=7$

$\Rightarrow x=3.5cm$

$\therefore AB=4+3.5=7.5cm \ and \ AC=3+3.5=6.5cm$

7. Two concentric circles are of radii 5cm and 3cm. Find the length of the chord of the larger circle (in cm) which touches the smaller circle.

Sol:



Given Two circles have the same center O and AB is a chord of the larger circle touching the smaller circle at C; also. $OA=5$ cm and $OC=3$ cm

In $\Delta OAC, OA^2=OC^2+AC^2$

$\therefore AC^2=OA^2-OC^2$

$\Rightarrow AC^2=5^2-3^2$

$\Rightarrow AC^2=25-9$

$\Rightarrow AC^2=16$

$\Rightarrow AC=4cm$

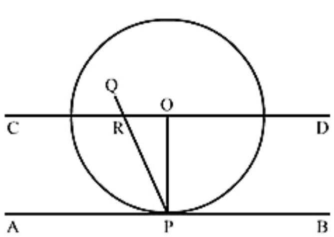
$\therefore AB=2AC$ (Since perpendicular drawn from the center of the circle bisects the chord)

$\therefore AB=2\times 4=8cm$

The length of the chord of the larger circle is 8 cm.

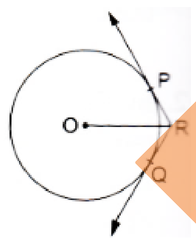
8. Prove that the perpendicular at the point of contact of the tangent to a circle passes through the centre.

Sol:

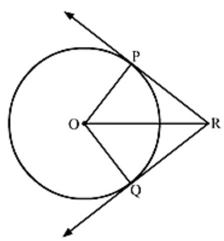


Let AB be the tangent to the circle at point P with center O.
To prove: PQ passes through the point O.
Construction: Join OP.
Through O, draw a straight line CD parallel to the tangent AB.
Proof: Suppose that PQ doesn't passes through point O.
PQ intersect CD at R and also intersect AB at P
AS, CD || AB. PQ is the line of intersection.
$\angle ORP = \angle RPA$ (Alternate interior angles)
but also.
$\angle RPA = 90° (OP \perp AB)$
$\Rightarrow \angle ORP = 90°$
$\angle ROP + \angle OPA = 180°$ (Co interior angles)
$\Rightarrow \angle ROP + 90° = 180°$
$\Rightarrow \angle ROP = 90°$
Thus, the ΔORP has 2 right angles i.e., $\angle ORP$ and $\angle ROP$ which is not possible
Hence, our supposition is wrong
$\therefore$ PQ passes through the point O.

9. In the given figure, two tangents RQ, and RP and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120°$, then prove that OR = PR + RQ.



Sol:



Construction Join PO and OQ
In $\Delta POR \text{ and } \Delta QOR$
$OP = OQ$ (Radii)
$RP = RQ$ (Tangents from the external point are congruent)
$OR = OR$ (Common)
By SSS congruency, $\Delta POR \cong \Delta QOR$
$\angle PRO = \angle QRO$ (C.P.C.T)

Now, $\angle PRO + \angle QRO = \angle PRQ$

$\Rightarrow 2\angle PRO = 120°$

$\Rightarrow \angle PRO = 60°$

Now. In ΔPOR

$\cos 60° = \frac{PR}{OR}$

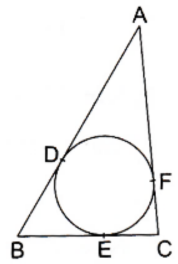
$\Rightarrow \frac{1}{2} = \frac{PR}{OR}$

$\Rightarrow OR = 2PR$

$\Rightarrow OR = PR + PR$

$\Rightarrow OR = PR + RQ$

10. In the given figure, a cradle inscribed in a triangle ABC touches the sides AB, BC and CA at points D, E and F respectively. If AB = 14cm, BC = 8cm and CA=12 cm. Find the length AD, BE and CF.



Sol:

We know that tangent segments to a circle from the same external point are congruent

Now, we nave

$AD = AF, BD = BE$ and $CE = CF$

Now $AD + BD = 14cm$ .....(1)

$AF + FC = 12cm$

$\Rightarrow AD + FC = 12cm$ ....(2)

$BE + EC = 8cm$

$\Rightarrow BD + FC = 8cm$ ....(3)

Adding all these we get

$AD + BD + AD + FC + BD + FC = 342$

$\Rightarrow 2(AD + BD + FC) = 34$

$\Rightarrow AD + BO + FC = 17cm$ ....(4)

Solving (1) and (4), we get

$FC = 3cm$

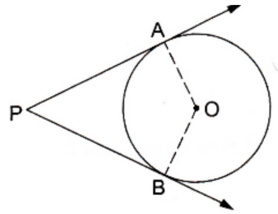
Solving (2) and (4), we get

$BD = 5cm = BE$

Solving (3) and (4), we get

and $AD = 9cm$

11. In the given figure, O is the centre of the circle. PA and PB are tangents. Show that AOBP is cyclic quadrilateral.



Sol:

We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OBP = \angle OAP = 90°$

Now, In quadrilateral AOBP

$\angle APB + \angle AOB + \angle OBP + \angle OAP = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle APB + \angle AOB + 90° + 90° = 360°$

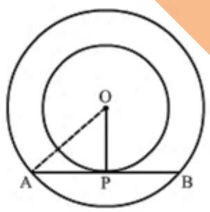
$\Rightarrow \angle APB + \angle AOB = 180°$

Since, the sum of the opposite angles of the quadrilateral is 180°

Hence, AOBP is a cyclic quadrilateral

12. In two concentric circles, a chord of length 8cm of the large circle touches he smaller circle. If the radius of the larger circle is 5cm then find the radius of the smaller circle.

Sol:



We know that the radius and tangent are perpendicular at their point of contact

Since, the perpendicular drawn from the centre bisect the chord

$\therefore AP = PB = \frac{AB}{2} = 4cm$

In right triangle AOP

$AO^2 = OP^2 + PA^2$

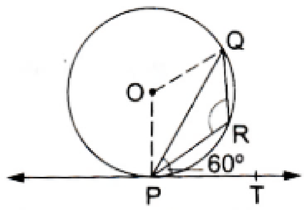
$\Rightarrow 5^2 = OP^2 + 4^2$

$\Rightarrow OP^2 = 9$

$\Rightarrow OP = 3cm$

Hence, the radius of the smaller circle is 3 cm.

13. In the given figure, PQ is chord of a circle with centre O an PT is a tangent. If $\angle QPT = 60°$, find the $\angle PRQ$.



Sol:
We know that the radius and tangent are perpendicular at their point of contact

$\therefore \angle OPT = 90°$

Now, $\angle OPQ = \angle OPT - \angle QPT = 90° - 60° = 30°$

Since, $OP = OQ$ as born is radius

$\therefore \angle OPQ = \angle OQP = 30°$ (Angles opposite to equal sides are equal)

Now, In isosceles, POQ

$\angle POQ + \angle OPQ + \angle OQP = 180°$ (Angle sum property of a triangle)

$\Rightarrow \angle POQ = 180° - 30° - 30° = 120°$

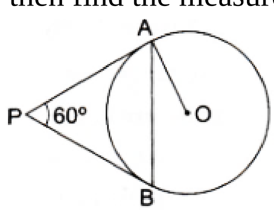
Now, $\angle POQ + \text{reflex} \angle POQ = 360°$ (Complete angle)

$\Rightarrow$ reflex $\angle POQ = 360° - 120° = 240°$

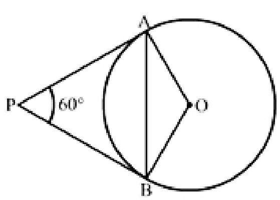
We know that the angle subtended by an arc at the centre double the angle subtended by the arc at any point on the remaining part of the circle

$\therefore \angle PRQ = \frac{1}{2}(reflex \angle POQ) = 120°$

14. In the given figure, PA and PB are two tangents to the circle with centre O. If $\angle APB = 60°$, then find the measure of $\angle OAB$.



Sol:
Construction: Join OB



We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OBP = \angle OAP = 90°$

Now, In quadrilateral AOBP

$\angle AOB + \angle OBP + \angle APB + \angle OAP = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle AOB + 90° + 60° + 90° = 360°$

$\Rightarrow 240° + \angle AOB = 360°$

$\Rightarrow \angle AOB = 120°$

Now, In isosceles triangle AOB

$\angle AOB + \angle OAB + \angle OBA = 180°$ [Angle sum property of a triangle]

$\Rightarrow 120° + 2\angle OAB = 180°$ $[\because \angle OAB = \angle OBA]$

$\Rightarrow \angle OAB = 30°$

Exercise – Multiple Choice Questions

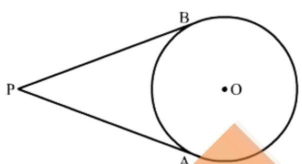
1. The number of tangents that can be drawn form an external point to a circle is

(a) 1 (b) 2 (3) (d) 4

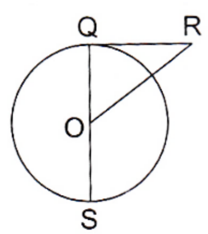
Answer: (b) 2

Sol:

We can draw only two tangents from an external point to a circle.



2. In the given figure, RQ is a tangent to the circle with centre O, If SQ = 6 cm and QR = 4 cm. then OR is equal to



(a) 2.5 cm (b) 3 cm (c) 5 cm (d) 8 cm

Answer: (c) 5 cm

Sol:

We know that the radius and tangent are perpendicular at their point of contact

$OQ = \frac{1}{2} QS = 3cm$ [$\because$ Radius is half of diameter]

Now, in right triangle OQR

By using Pythagoras theorem, we have

$OR^2 = RQ^2 + OQ^2$

$= 4^2 + 3^2$

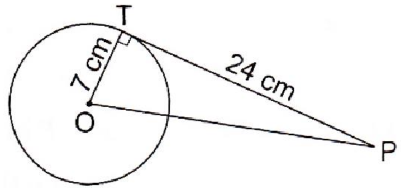
$= 16 + 9$

$= 25$

$\therefore OR^2 = 25$

$\Rightarrow OR = 5cm$

3. In a circle of radius 7 cm, tangent PT is drawn from a point P such that PT =24 cm. If O is the centre of the circle, then length OP = ?



(a) 30 cm (b) 28 cm (c) 25 cm (d) 18 cm

Answer: (c) 25 cm

Sol:

The tangent at any point of a circle is perpendicular to the radius at the point of contact

$\therefore OT \perp PT$

From right – angled triangle PTO,

$\therefore OP^2 = OT^2 + PT^2$ [Using Pythagoras' theorem]

$\Rightarrow OP^2 = (7)^2 + (24)^2$

$\Rightarrow OP^2 = 49 + 576$

$\Rightarrow OP^2 = 625$

$\Rightarrow OP = \sqrt{625}$

$\Rightarrow OP = 25cm$

4. Which of the following pairs of lines in a circle cannot be parallel?

(a) two chords (b) a chord and tangent (c) two tangents (d) two diameters

Answer: (d) two diameters

Sol:

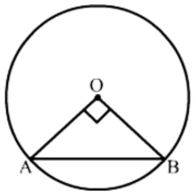
Two diameters cannot be parallel as they perpendicularly bisect each other.

5. The chord of a circle of radius 10 cm subtends a right angle at its centre. The length of the chord (in cm) is

(a) $\frac{5}{\sqrt{2}}$ (b) $5\sqrt{2}$ (c) $10\sqrt{2}$ (d) $10\sqrt{3}$

Answer: (c) $10\sqrt{2}$

Sol:



In right triangle AOB

By using Pythagoras theorem, we have

$AB^2 = BO^2 + OA^2$

$= 10^2 + 10^2$

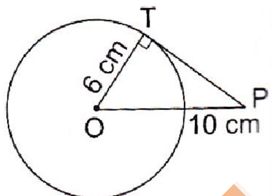
$= 100 + 100$

$= 200$

$\therefore OR^2 = 200$

$\Rightarrow OR = 10\sqrt{2}\,cm$

6. In the given figure, PT is tangent to the circle with centre O. If OT = 6 cm and OP = 10 cm then the length of tangent PT is



(a) 8 cm (b) 10 cm (c) 12 cm (d) 16 cm

Answer: (a) 8 cm

Sol:

In right triangle PTO

By using Pythagoras theorem, we have

$PO^2 = OT^2 + TP^2$

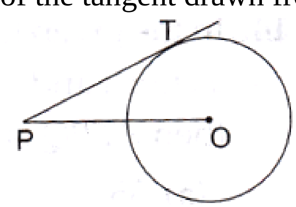
$\Rightarrow 10^2 = 6^2 + TP^2$

$\Rightarrow 100 = 36 + TP^2$

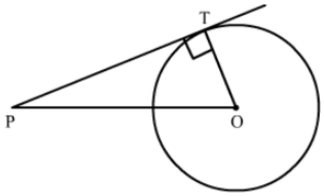
$\Rightarrow TP^2 = 64$

$\Rightarrow TP = 8cm$

7. In the given figure, point P is 26 cm away from the center O of a circle and the length PT of the tangent drawn from P to the circle is 24 cm. Then, the radius of the circle is



(a) 10 cm (b) 12 cm (c) 13 cm (d) 15 cm
Answer: (a) 10 cm
Sol:
Construction: Join OT.



We know that the radius and tangent are perpendicular at their point of contact
In right triangle PTO
By using Pythagoras theorem, we have

$PO^2 = OT^2 + TP^2$

$\Rightarrow 26^2 = OT^2 + 24^2$

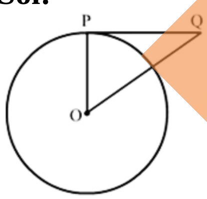
$\Rightarrow 676 = OT^2 + 576$

$\Rightarrow TP^2 = 100$

$\Rightarrow TP = 10\,cm$

8. PQ is a tangent to a circle with centre O at the point P. If ΔOPQ is an isosceles triangle, then $\angle OQP$ is equal to

(a) $30°$ (b) $45°$ (c) $60°$ (d) $90°$
Answer: (b) $45°$
Sol:



We know that the radius and tangent are perpendicular at their point of contact
Now, In isosceles right triangle POQ

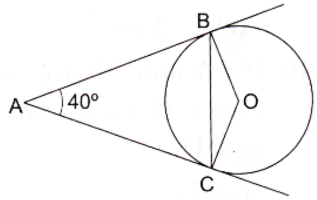
$\angle POQ + \angle OPQ + \angle OQP = 180°$ [Angle sum property of a triangle]

$\Rightarrow 2\angle OQP + 90° = 180°$

$\Rightarrow \angle OQP = 45°$

9. In the given figure, AB and AC are tangents to the circle with center O such that $\angle BAC = 40°$. Then , $\angle BOC =$ $40°$.

(d) 90°



(a) 80° (b) 100° (c) 120° (d) 140°

Answer: (d) 140°

Sol:

We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OBA = \angle OCA = 90°$

Now, In quadrilateral ABOC

$\angle BAC + \angle OCA + \angle OBA + \angle BOC = 360°$ [Angle sum property of quadrilateral]

$\Rightarrow 40° + 90° + 90° + \angle BOC = 360°$

$\Rightarrow 220° + \angle BOC = 360°$

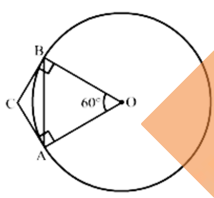
$\Rightarrow \angle BOC = 140°$

10. If a chord AB subtends an angle of $60°$ at the center of a circle, then he angle between the tangents to the circle drawn form A and B is

(a) $30°$ (b) $60°$ (c) $90°$ (d) $120°$

Answer: (d) 120°

Sol:



We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OBC = \angle OAC = 90°$

Now, In quadrilateral ABOC

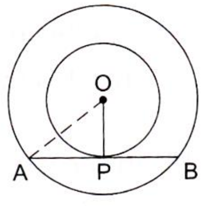
$\angle ACB + \angle OAC + \angle OBC + \angle AOB = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle ACB + 90° + 90° + 60° = 360°$

$\Rightarrow \angle ACB + 240° = 360°$

$\Rightarrow \angle ACB = 120°$

11. In the given figure, O is the centre of two concentric circles of radii 6 cm and 10 cm. AB is a chord of outer circle which touches the inner circle. The length of chord AB is



(a) 8cm (b) 14 cm (c) 16 cm (d) $\sqrt{136}$ cm

Answer: (c) 16 cm

Sol:

We know that the radius and tangent are perpendicular at their point of contact

In right triangle AOP

$AO^2 = OP^2 + PA^2$

$\Rightarrow 10^2 = 6^2 + PA^2$

$\Rightarrow PA^2 = 64$

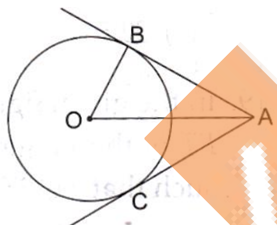
$\Rightarrow PA = 8cm$

Since, the perpendicular drawn from the center bisect the chord

$\therefore PA = PB = 8cm$

Now, $AB = AP + PB = 8 + 8 = 16cm$

12. In the given figure, AB and AC are tangents to a circle with centre O and radius 8 cm. If OA=17 cm, then the length of AC (in cm) is



(a) 9 (b) 15 (c) $\sqrt{353}$ (d) 25

Answer: (b) 15

Sol:

We know that the radius and tangent are perpendicular at their point of contact

In right triangle AOB

By using Pythagoras theorem, we have

$OA^2 = AB^2 + OB^2$

$\Rightarrow 17^2 = AB^2 + 8^2$

$\Rightarrow 289 = AB^2 + 64$

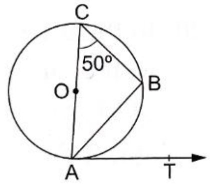
$\Rightarrow AB^2 = 225$

$\Rightarrow AB = 15cm$

The tangents drawn from the external point are equal

Therefore, the length of AC is 15 cm

13. In the given figure, 0 is the centre of a circle, AOC is its diameter such that $\angle ACB = 50°$. If AT is the tangent to the circle at the point A, then $\angle BAT = ?$



(a) 40° (b) 50° (c) 60° (d) 65°

Answer: (b) 50°

Sol:

$\angle ABC = 90°$ (Angle in a semicircle)

In ΔABC, we have: $\angle ACB + \angle CAB + \angle ABC = 180°$

$\Rightarrow 50° + \angle CAB + 90° = 180°$

$\Rightarrow \angle CAB = (180° - 140°)$

$\Rightarrow \angle CAB = 40°$

Now, $\angle CAT = 90°$ (Tangents drawn from an external point are perpendicular to the radius at the point of contact)

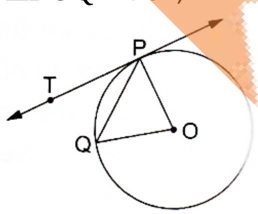
$\therefore \angle CAB + \angle BAT = 90°$

$\Rightarrow 40° + \angle BAT = 90°$

$\Rightarrow \angle BAT = (90° - 40°)$

$\Rightarrow \angle BAT = 50°$

14. In the given figure, O is the center of a circle, PQ is a chord and Pt is the tangent at P. If $\angle POQ = 70°$, then $\angle TPQ$ is equal to



(a) 35° (b) 45° (c) 55° (d) 70°

Answer: (a) 35°

Sol:

We know that the radius and tangent are perpendicular at their point of contact

Since, OP = OQ

$\because POQ$ is a isosceles right triangle

Now, In isosceles right triangle POQ

$\angle POQ + \angle OPQ + \angle OQP = 180°$ [Angle sum proper of a triangle]

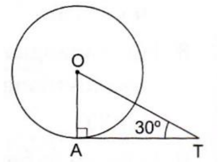
$\Rightarrow 70° + 2\angle OPQ = 180°$

$\Rightarrow \angle OPQ = 55°$

Now, $\angle TPQ + \angle OPQ = 90°$

$\Rightarrow \angle TPQ = 35°$

15. In the given figure, AT is a tangent to the circle with center O such that OT = 4 cm and $\angle OTA = 30°$, Then, $AT = ?$



(a) 4 cm (b) 2 cm (c) $2\sqrt{3}$ cm (d) $4\sqrt{3}$ cm

Answer: (c) $2\sqrt{3}$ cm

Sol:

$OA \perp AT$

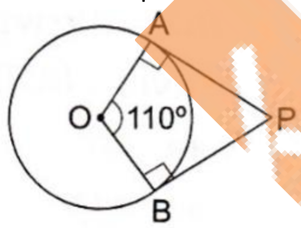
So, $\frac{AT}{OT} = \cos 30°$

$\Rightarrow \frac{AT}{4} = \frac{\sqrt{3}}{2}$

$\Rightarrow AT = \left(\frac{\sqrt{3}}{2} \times 4\right)$

$\Rightarrow AT = 2\sqrt{3}$

16. If PA and PB are two tangents to a circle with centre O such that $\angle AOB = 110°$ then $\angle APB$ is equal to



(a) $55°$ (b) $60°$ (c) $70°$ (d) $90°$

Answer: (c) $70°$

Sol:

Given, PA and PB are tangents to a circle with center O, with $\angle AOB = 110°$.

Now, we know that tangents drawn from an external point are perpendicular to the radius at the point of contact.

So, $\angle OAP = 90°$ and $\angle OBP = 90°$

$\Rightarrow \angle OAP + \angle OBP = 90° + 90° = 180°$, which shows that OABP is a cyclic quadrilateral.

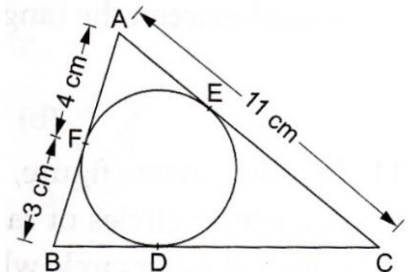
$\therefore \angle AOB + \angle APB = 180°$

$\Rightarrow 110° + \angle APB = 180°$

$\Rightarrow \angle APB = 180° - 110°$

$\Rightarrow \angle APB = 70°$

17. In the given figure, the length of BC is



(a) 7 cm (b) *10 cm* (c) 14 cm (d) 15 cm

Answer: (b) 10 cm

Sol:

We know that tangent segments to a circle from the same external point are congruent

Therefore, we have

$AF = AE = 4 cm$

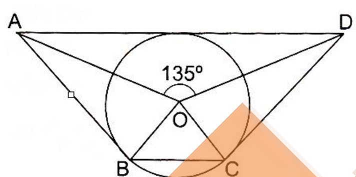
$BF = BD = 3 cm$

$EC = AC - AE = 11 - 4 = 7 cm$

$CD = CE = 7 cm$

$\therefore BC = BD + DC = 3 + 7 = 10 cm$

18. In the given figure, If $\angle AOD = 135°$ then $\angle BOC$ equal to



(a) $25°$ (b) $45°$ (c) $52.5°$ (d) $62.5°$

Answer: (b) $45°$

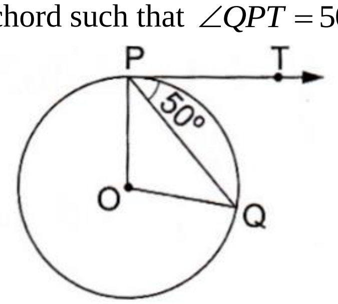
Sol:

We know that the sum of angles subtended by opposite sides of a quadrilateral having a circumscribed circle is 180 degrees

$\therefore \angle AOD + \angle BOC = 180°$

$\Rightarrow \angle BOC = 180° - 135° = 45°$

19. In the given figure, O is the centre of a circle and PT is the tangent to the circle. If PQ is a chord such that $\angle QPT = 50°$ then $\angle POQ = ?$



(a) $100°$ (b) $90°$ (c) $80°$ (d) $75°$

Answer: (a) $100°$

Sol:

Given, $\angle QPT = 50°$

And $\angle OPT = 90°$ (Tangents drawn from an external point are perpendicular to the radius at the point of contact)

$\therefore \angle OPQ = (\angle OPT - \angle QPT) = (90° - 50°) = 40°$

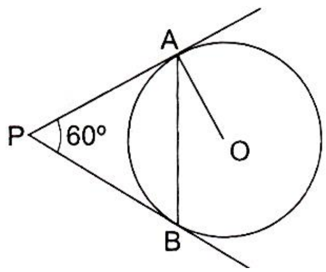
$OP = OQ$ (Radius of the same circle)

$\Rightarrow \angle OQP = \angle OPQ = 40°$

In $\Delta POQ, \angle POQ + \angle OQP + \angle OPQ = 180°$

$\therefore \angle POQ = 180° - (40° + 40°) = 100°$

20. In the given figure, PA and PB are two tangents to th4e circle with centre O. If $\angle APB = 60°$ then $\angle OAB$ is

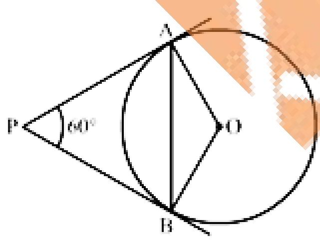


(a) $15°$ (b) $30°$ (c) $60°$ (d) $90°$

Answer: (b) $30°$

Sol:

Construction: Join OB



We know that the radius and tangent are perpendicular at the point of contact

$\therefore \angle OBP = \angle OAP = 90°$

Now, In quadrilateral AOBP

$\angle AOB + \angle OBP + \angle APB + \angle OAP = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle AOB + 90° + 60° + 90° = 360°$

$\Rightarrow 240° + \angle AOB = 360°$

$\Rightarrow \angle AOB = 120°$

Now, In isosceles triangles AOB

$\angle AOB + \angle OAB + \angle OBA = 180°$ [Angle sum property of a triangle]

$\Rightarrow 120° + 2\angle OAB = 180°$ $[\because \angle OAB = \angle OBA]$

$\Rightarrow \angle OAB = 30°$

21. If two tangents inclined at an angle of $60°$ are drawn to a circle of a radius 3 cm then the length of each tangent is

(a) 3 cm (b) $\frac{3\sqrt{3}}{2}$ cm (c) $3\sqrt{3}$ cm (d) 6 cm

Answer: (c) $3\sqrt{3}$ cm

Sol:

Given, PA and PB are tangents to circle with center O and radius 3 cm and $\angle APB = 60°$.

Tangents drawn from an external point are equal; so, PA = PB.

And OP is the bisector of $\angle APB$, which gives $\angle OPB = \angle OPA = 30°$.

$OA \perp PA$. So, from right – angled ΔOPA, we have:

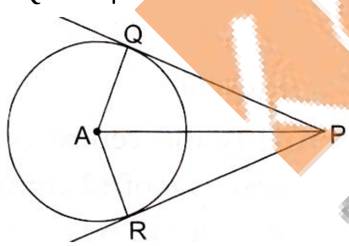
$$\frac{OA}{AP} = \tan 30°$$

$$\Rightarrow \frac{OA}{AP} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{3}{AP} = \frac{1}{\sqrt{3}}$$

$$= AP = 3\sqrt{3}\,cm$$

22. In the given figure, PQ and PR are tangents to a circle with centre A. If $\angle QPA = 27°$ then $\angle QAR$ equals



(a) $63°$ (b) $117°$ (c) $126°$ (d) $153°$

Answer: (c) $126°$

Sol:

We know that the radius and tangent are perpendicular at the point of contact

Now, In ΔPQA

$\angle PQA + \angle QAP + \angle APQ = 180°$ [Angle sum property of a triangle]

$\Rightarrow 90° + \angle QAP + 27° = 180°$ $[\because \angle OAB = \angle OBA]$

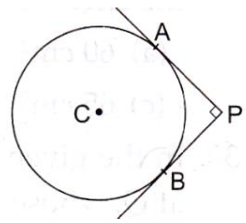
$\Rightarrow \angle QAP = 63°$

In ΔPQA and ΔPRA

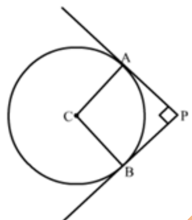
$PQ = PR$ (Tangents draw from same external point are equal)

$QA = RA$ (Radio of the circle)
$AP = AP$ (common)
By SSS congruency
$\Delta PQA \cong \Delta PRA$
$\angle QAP = \angle RAP = 63°$
$\therefore \angle QAR = \angle QAP + \angle RAP = 63° + 63° = 126°$

23. In the given figure, PQ and PR are tangents to a circle with centre A. If $\angle QPA = 27°$ then $\angle QAR$ equals

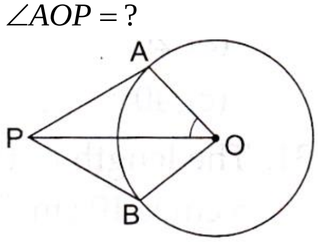


(a) $63°$ (b) $117°$ (c) $126°$ (d) $153°$
Answer: (b) $117°$
Sol:
Construction: Join CA and CB



We know that the radius and tangent are perpendicular at their point of contact
$\because \angle CAP = \angle CBP = 90°$
Since, in quadrilateral ACBP all the angles are right angles
$\therefore$ $ACPB$ is a rectangle
Now, we know that the pair of opposite sides are equal in rectangle
$\therefore CB = AP \text{ and } CA = BP$
Therefore, $CB = AP = 4cm$ and $CA = BP = 4cm$

24. If PA and PB are two tangents to a circle with centre O such that $\angle APB = 80°$. Then, $\angle AOP = ?$



(a) $40°$ (b) $50°$ (c) $60°$ (d) $70°$
Answer: (b) $50°$

Sol:
Given, PA and PB are two tangents to a circle with center O and $\angle APB = 80°$

$\therefore \angle APO = \frac{1}{2}\angle APB = 40°$

[Since they are equally inclined to the line segment joining the center to that point and $\angle OAP = 90°$]
[Since tangents drawn from an external point are perpendicular to the radius at the point of contact]
Now, in triangle *AOP*:

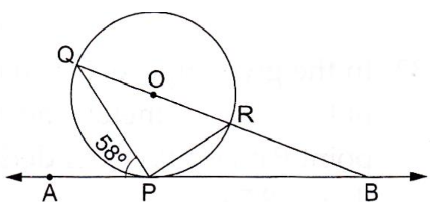
$\angle AOP + \angle OAP + \angle APO = 180°$

$\Rightarrow \angle AOP + 90° + 40° = 180°$

$\Rightarrow \angle AOP = 180° - 130°$

$\Rightarrow \angle AOP = 50°$

25. In the given figure, O is the centre of the circle. AB is the tangent to the circle at the point P. If $\angle APQ = 58°$ then the measure of $\angle PQB$ is



(a) $32°$ (b) $58°$ (c) $122°$ (d) $132°$

Answer: (a) $32°$

Sol:
We know that a chord passing through the center is the diameter of the circle.

$\because \angle QPR = 90°$ (Angle in a semi circle is 90°)

By using alternate segment theorem
We have $\angle APQ = \angle PRQ = 58°$

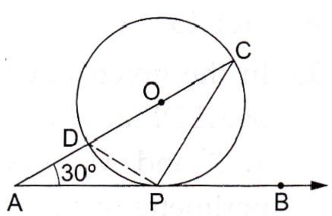
Now, In Δ PQR

$\angle PQR + \angle PRQ + \angle QPR = 180°$ [Angle sum properly of a triangle]

$\Rightarrow \angle PQR + 58° + 90° = 180°$

$\Rightarrow \angle PQR = 32°$

26. In the given figure, O is the centre of the circle. AB is the tangent to the circle at the point P. If $\angle PAO = 30°$ then $\angle CPB + \angle ACP$ is equal to



(a) $60°$ (b) $90°$ (c) $120°$ (d) $150°$

Answer: (b) $90°$

Sol:

We know that a chord passing through the center is the diameter of the circle.

$\because \angle DPC = 90°$ (Angle in a semicircle is 90°)

Now, In ΔCDP

$\angle CDP + \angle DCP + \angle DPC = 180°$ [Angle sum property of a triangle]

$\Rightarrow \angle CDP + \angle DCP + 90° = 180°$

$\Rightarrow \angle CDP + \angle DCP = 90°$

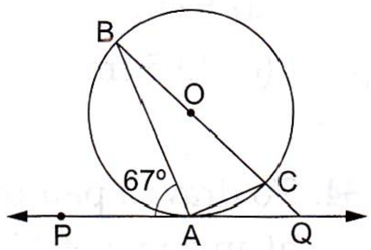
By using alternate segment theorem

We have $\angle CDP = \angle CPB$

$\therefore \angle CPB + \angle ACP = 90°$

27. In the given figure, PQ is a tangent to a circle with centre O, A is the point of contact. If $\angle PAB = 67°$, then the measure of $\angle AQB$ is

(a) $73°$ (b) $64°$ (c) $53°$ (d) $44°$



Answer: (d) $44°$

Sol:

We know that a chord passing through the center is the diameter of the circle.

$\because BAC = 90°$ (Angle in a semicircle is $90°$)

By using alternate segment theorem

We have $\angle PAB = \angle ACB = 67°$

Now, In ABC

$\angle ABC + \angle ACB + \angle BAC = 180°$ [Angle sum property of a triangle]

$\Rightarrow \angle ABC + 67° + 90° = 180°$

$\Rightarrow \angle ABC = 23°$

Now, $\angle BAQ = 180° - \angle PAB$ [Linear pair angles]

$= 180° - 67°$

$= 113°$

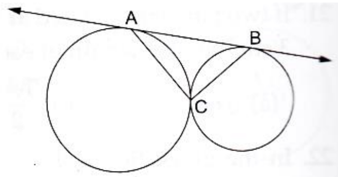
Now, In ΔABQ

$\angle ABQ + \angle AQB + \angle BAQ = 180°$ [Angle sum property of a triangle]

$\Rightarrow 23° + \angle AQB + 113° = 180°$

$\Rightarrow \angle AQB = 44°$

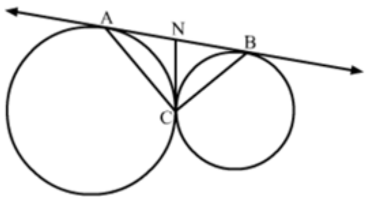
28. In the given figure, two circles touch each other at C and AB is a tangent to both the circles. The measure of $\angle ACB$ is



(a) $45°$ (b) $60°$ (c) $90°$ (d) $120°$

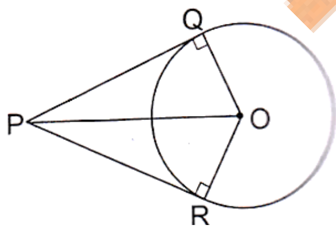
Answer: (c) $90°$

Sol:



We know that tangent segments to a circle from the same external point are congruent
Therefore, we have
$NA = NC$ and $NC = NB$
We also know that angle opposite to equal sides is equal
$\therefore \angle NAC = \angle NCA$ and $\angle NBC = \angle NCB$
Now, $\angle ANC + \angle BNC = 180°$ [Linear pair angles]
$\Rightarrow \angle NBC + \angle NCB + \angle NAC + \angle NCA = 180°$ [Exterior angle property]
$\Rightarrow 2\angle NCB + 2\angle NCA = 180°$
$\Rightarrow 2(\angle NCA + \angle NCA) = 180°$
$\Rightarrow \angle ACB = 90°$

29. O is the centre of a circle of radius 5 cm. At a distance of 13 cm form O, a point P is taken. From this point, two tangents PQ and PR are drawn to the circle. Then, the area of quad. PQOR is



(a) $60\,cm^2$ (b) $32.5\,cm^2$ (c) $65\,cm^2$ (d) $30\,cm^2$

Answer: (a) $60\,cm^2$

Sol:

Given,
$OQ = OR = 5cm, OP = 13cm.$
$\angle OQP = \angle ORP = 90°$ (Tangents drawn from an external point are perpendicular to the radius at the point of contract)

From right – angled ΔPOQ:

$PQ^2 = \left(OP^2 - OQ^2\right)$

$\Rightarrow PQ^2 = \left(OP^2 - OQ^2\right)$

$\Rightarrow PQ^2 = 13^2 - 5^2$

$\Rightarrow PQ^2 = 169 - 25$

$\Rightarrow PQ = 144$

$\Rightarrow PQ = \sqrt{144}$

$\Rightarrow PQ = 12\,cm$

$\therefore ar\left(\Delta OQP\right) = \frac{1}{2} \times PQ \times OQ$

$\Rightarrow ar\left(\Delta OQP\right) = \left(\frac{1}{2} \times 12 \times 5\right) cm^2$

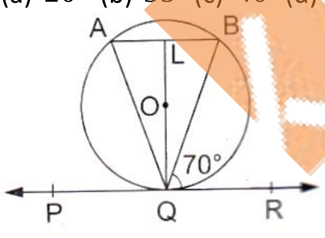
$\Rightarrow ar\left(\Delta OQP\right) = 30 cm^2$

Similarly, $ar\left(\Delta ORP\right) = 30\,cm^2$

$\therefore ar\left(quad.PQOR\right) = \left(30 + 30\right) cm^2 = 60\,cm^2$

30. In the given figure, PQR is a tangent to the circle at Q, whose centre is O and AB is a chord parallel to PR such that $\angle BQR = 70°$. Then, $\angle AQB = ?$

(a) $20°$ (b) $35°$ (c) $40°$ (d) $45°$



Answer: (c) $40°$

Sol:

Since, $AB \parallel PR, BQ$ is transversal

$\angle BQR = \angle ABQ = 70°$ [Alternative angles]

$OQ \perp PQR$ (Tangents drawn from an external point are perpendicular to the radius at the point of contract)

and $AB \parallel PQR$

$\therefore QL \perp AB$; so, $OL \perp AB$

$\therefore OL$ bisects chord AB [Perpendicular drawn from the center bisects the chord]

From ΔQLA *and* QLB:

$\angle QLA = \angle QLB = 90°$

$LA = LB$ (OL bisects chord AB)

QL is the common side.

$\therefore \Delta QLA \cong \Delta QLB$ [By SAS congruency]

$\therefore \angle QAL = \angle QBL$

$\Rightarrow \angle QAB = \angle QBA$

$\therefore \Delta AQB$ is isosceles

$\therefore \angle LQA = \angle LQR$

$\angle LQP = \angle LQR = 90°$

$\angle LQB = (90° - 70°) = 20°$

$\therefore \angle LQA = \angle LQB = 20°$

$\Rightarrow \angle LQA = \angle LQB = 20°$

$\Rightarrow \angle AQB = \angle LQA + \angle LQB$

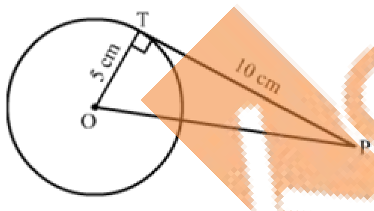
$= 40°$

31. The length of the tangent form an external point P to a circle of radius 5 cm is 10 cm. The distance of the point from the centre of the circle is

(a) 8 cm (b) $\sqrt{104}\ cm$ (c) 12 cm (d) $\sqrt{125}\ cm$

Answer: (b) $\sqrt{104}\ cm$

Sol:



We know that the radius and tangent are perpendicular at their point of contact

In right triangle PTO

By using Pythagoras theorem, we have

$PO^2 = OT^2 + TP^2$

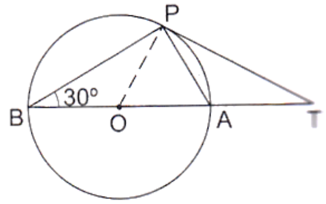
$\Rightarrow PO^2 = 5^2 + 10^2$

$\Rightarrow PO^2 = 25 + 100$

$\Rightarrow PO^2 = 125$

$\Rightarrow PO = \sqrt{125 cm}$

32. In the given figure, O is the centre of a circle, BOA is its diameter and the tangent at the point P meets BA extended at T. If $\angle PBO = 30°$ then $\angle PTA = ?$



(a) $60°$ (b) $30°$ (c) $15°$ (d) $45°$

Answer: (b) $30°$

Sol:

We know that a chord passing through the center is the diameter of the circle

$\because \angle BPA = 90°$ (Angle in a semicircle is 90°)

By using alternate segment theorem

We have $\angle APT = \angle ABP = 30°$

Now, In ΔABP

$\angle PBA + \angle BPA + \angle BAP = 180°$ [Angle sum property of a triangle]

$\Rightarrow 30° + 90° + \angle BAP = 180°$

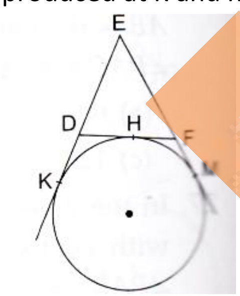
$\Rightarrow \angle BAP = 60°$

Now, $\angle BAP = \angle APT + \angle PTA$

$\Rightarrow 60° = 30° + \angle PTA$

$\Rightarrow \angle PTA = 30°$

33. In the given figure, a circle touches the side DF of ΔEDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm then the perimeter of ΔEDF is



(a) 9 cm (b) 12 cm (c) 13.5 cm (d) 18 cm

Answer: (d) 18 cm

Sol:

We know that tangent segments to a circle from the same external point are congruent.

Therefore, we have

$EK = EM = 9cm$

Now, $EK + EM = 18cm$

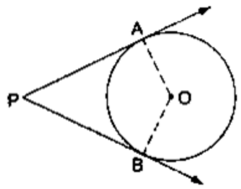
$\Rightarrow ED + DK + EF + FM = 18cm$

$\Rightarrow ED + DH + EF + HF = 18cm$ $(\because DK = DH \text{ and } FM = FH)$

$\Rightarrow ED + DF + EF = 18cm$

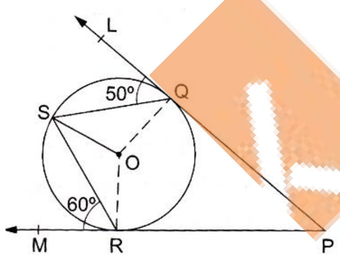
$\Rightarrow$ Perimeter of $\Delta EDF = 18cm$

34. To draw a pair of tangents to a circle, which are inclined to each other at an angle of $45°$, we have to draw tangents at the end points of those two radii, the angle between which is
(a) $105°$ (b) $135°$ (c) $140°$ (d) $145°$
Answer: (b) $135°$
Sol:



Suppose PA and PB are two tangents we want to draw which inclined at an angle of $45°$
We know that the radius and tangent are perpendicular at their point of contact
$\because \angle OBP = \angle OAP = 90°$
Now, in quadrilateral AOBP
$\angle AOB + \angle OBP + \angle OAP + \angle APB = 360°$ [Angle sum property of a quadrilateral]
$\Rightarrow \angle AOB + 90° + 90° + 45° = 360°$
$\Rightarrow \angle AOB + 225° = 360°$
$\Rightarrow \angle AOB = 135°$

35. In the given figure, O is the centre of a circle; PQL and PRM are the tangents at the points Q and R respectively and S is a point on the circle such that $\angle SQL = 50°$ and
$DE \perp DF\ OQ \perp BC\ and\ OR \perp AC.$
Then, $\angle QSR = ?$
$5\sqrt{3}\,cm$



(a) $40°$ (b) $50°$ (c) $60°$ (d) $70°$
Answer: (d) $70°$
Sol:
PQL is a tangent OQ is the radius; so, $\angle OQL = 90°$
$\therefore \angle OQS = (90° - 50°) = 40°$
Now, $OQ = OS$ (Radius of the same circle)
$\Rightarrow \angle OSQ = \angle OQS = 40°$
Similarly, $\angle ORS = (90° - 60°) = 30°$,
And, $OR = OS$ (Radius of the same circle)

$\Rightarrow \angle OSR = \angle ORS = 30°$

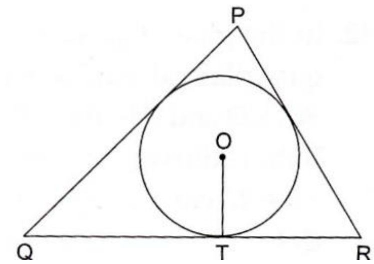
$\therefore \angle QSR = \angle OSQ + \angle OSR$

$\Rightarrow \angle QSR = (40° + 30°)$

$\Rightarrow \angle QSR = 70°$

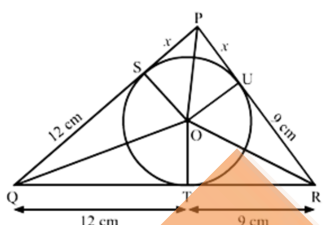
36. In the given figure, a triangle PQR is drawn to circumscribe a circle of radius 6 cm such that the segments QT and TR into which QR is divided by the point of contact T, are of lengths 12 cm and 9 cm respectively. If the area of $\Delta PQR = 189\, cm^2$ then the length of side of PQ is

(a) 17.5 cm (b) 20 cm (c) 22.5 cm (d) 25 cm



Answer: (c) 22.5 cm

Sol:



We know that tangent segments to a circle from the same external point are congruent.
Therefore, we have

$PS = PU = x$

$QT = QS = 12\, cm$

$RT = RU = 9\, cm$

Now,

$Ar(\Delta PQR) = Ar(\Delta POR) + Ar(\Delta QOR) + Ar(\Delta POQ)$

$\Rightarrow 189 = \frac{1}{2} \times OU \times PR + \frac{1}{2} \times OT \times QR + \frac{1}{2} \times OS \times PQ$

$\Rightarrow 378 = 6 \times (x + 9) + 6 \times (21) + 6 \times (12 + x)$

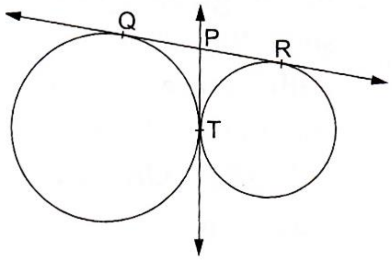
$\Rightarrow 63 = x + 9 + 21 + x + 12$

$\Rightarrow 2x = 21$

$\Rightarrow x = 10.5\, cm$

Now, $PQ = QS + SP = 12 + 10.5 + 10.5 = 22.5\, cm$

37. In the given figure, QR is a common tangent to the given circles, touching externally at the point T. The tangent at T meets QR at P. If PT= 3.8 cm then the length of QR is



(a) 1.9 cm (b) 3.8 cm (c) 5.7 cm (d) 7.6 cm

Answer: (d) 7.6 cm

Sol:

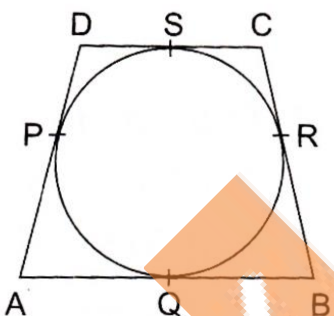
We know that tangent segments to a circle from the same external point are congruent.

Therefore, we have

$PT = PO$ = 3.8 cm and $PT = PR$ 3.8 cm

$\therefore QR = QP + PR = 3.8 + 3.8 = 7.6 cm$

38. In the given figure, quad. ABCD is circumscribed touching the circle at P, Q, R and S. If AP = 5 cm, BC= 7 c m and CS = 3 cm. Then, the length of AB = ?



(a) 9 cm (b) 10 cm (c) 12 cm (d) 8 cm

Answer: (a) 9 cm

Sol:

Tangents drawn from an external point to a circle are equal.

So, $AQ = AP = 5 cm$

$CR = CS = 3 cm$

And $BR = (BC - CR)$

$\Rightarrow BR = (7 - 3) cm$

$\Rightarrow BR = 4 cm$

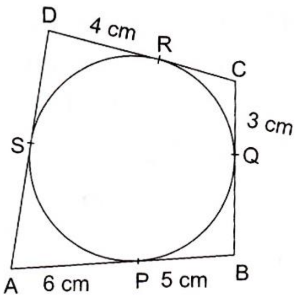
$BQ = BR = 4 cm$

$\therefore AB = (AQ + BQ)$

$\Rightarrow AB = (5 + 4) cm$

$\Rightarrow AB = 9 cm$

39. In the given figure, quad. ABCD is circumscribed, touching the circle at P, Q, R and S. If AP = 6 cm, BP = 5 cm, CQ = 3 cm and DR = 4 cm then perimeter of quad. ABCD is



(a) 18 cm (b) 27 cm (c) 36 cm (d) 32 cm

Answer: (c) 36 cm

Sol:

Given, $AP = 6cm, BP = 5cm, CQ = 3cm$ and $DR = 4cm$

Tangents drawn from an external point to a circle are equal

So, $AP = AS = 6cm, BP = BQ = 5cm, CQ = CR = 3cm, DR = DS = 4cm.$

$\therefore AB = AP + BP = 6 + 5 = 11cm$

$BC = BQ + CQ = 5 + 3 = 8cm$

$CD = CR + DR = 3 + 4 = 7cm$

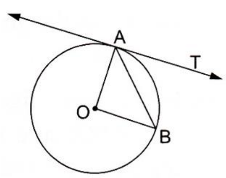
$AD = AS + DS = 6 + 4 = 10cm$

$\therefore$ Perimeter of quadrilateral $ABCD = AB + BC + CD + DA$

$= (11 + 8 + 7 + 10) cm$

$= 36 cm$

40. In the given figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If $\angle AOB = 100°$ then $\angle BAT$ is equal to



(a) $40°$ (b) $50°$ (c) $90°$ (d) $100°$

Answer: (b) $50°$

Sol:

Given: AO and BC are the radius of the circle

Since, AO = BO

$\therefore \Delta AOB$ is an isosceles triangle

Now, in ΔAOB

$\angle AOB + \angle OBA + \angle OAB = 180°$ (Angle sum property of triangle)

$\Rightarrow 100° + \angle OAB + \angle OAB = 180°$ $(\angle OBA = \angle OAB)$

$\Rightarrow 2\angle OAB = 80°$

$\Rightarrow \angle OAB = 40°$

We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OAT = 90°$

$\Rightarrow \angle OAB + \angle BAT = 90°$

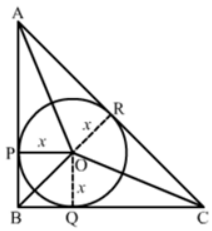
$\Rightarrow \angle BAT = 90° - 40° = 50°$

41. In a right triangle ABC, right angled at B, BC = 12 cm and AB = 5 cm. The radius of the circle inscribed in the triangle is

(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm

Answer: (b) 2 cm

Sol:



In right triangle ABC

By using Pythagoras theorem we have

$AC^2 = AB^2 + BC^2$

$= 5^2 + 12^2$

$= 25 + 144$

$= 169$

$\therefore AC^2 = 169$

$\Rightarrow AC = 13 cm$

Now,

$Ar(\Delta ABC) = Ar(\Delta AOB) + Ar(\Delta BOC) + Ar(\Delta AOC)$

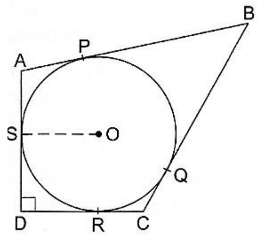
$\Rightarrow \frac{1}{2} \times AB \times BC = \frac{1}{2} \times OP \times AB + \frac{1}{2} \times OQ \times BC + \frac{1}{2} \times OR \times AC$

$\Rightarrow 5 \times 12 = x \times 5 + x \times 12 + x \times 13$

$\Rightarrow 60 = 30x$

$\Rightarrow x = 2 cm$

42. In the given figure, a circle is inscribed in a quadrilateral ABCD touching its sides AB, BC, CD and AD at P, Q, R and S respectively. If the radius of the circle is 10 cm, BC = 38 cm, PB = 27 cm and $AD \perp CD$ then the length of CD is

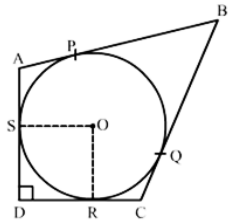


(a) 11 cm (b) 15 cm (c) 20 cm (d) 21 cm

Answer: (d) 21 cm

Sol:

Construction: Join OR



We know that tangent segments to a circle from the same external point are congruent.
Therefore, we have

$BP = BQ = 27\,cm$

$CQ = CR$

Now, $BC = 38\,cm$

$\Rightarrow BQ + QC = 38$

$\Rightarrow QC = 38 - 27 = 11\,cm$

Since, all the angles in quadrilateral DROS are right angles.
Hence, DROS is a rectangle.
We know that opposite sides of rectangle are equal

$\therefore OS = RD = 10\,cm$

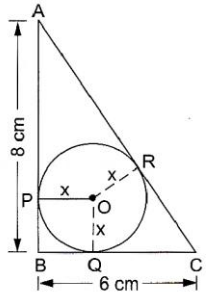
Now, $CD = CR + RD$

$= CQ + RD$

$= 11 + 10$

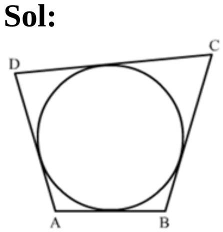
$= 21\,cm$

43. In the given figure, ΔABC is right-angled at B such that BC = 6 cm and AB = 8 cm. A circle with centre O has been inscribed the triangle. $OP \perp AB, OQ \perp BC$ and $OR \perp AC$. If OP = OQ = OR = x cm then x = ?



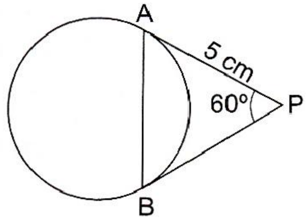
(a) 2 cm (b) 2.5 cm (c) 3 cm (d) 3.5 cm
Answer: (a) 2 cm
Sol:
Given, $AB = 8cm, BC = 6cm$
Now, in ΔABC :
$AC^2 = AB^2 + BC^2$
$\Rightarrow AC^2 = (8^2 + 6^2)$
$\Rightarrow AC^2 = (64 + 36)$
$\Rightarrow AC^2 = 100$
$\Rightarrow AC = \sqrt{100}$
$\Rightarrow AC = 10\,cm$
$PBQO$ is a square
$CR = CQ$ (Since the lengths of tangents drawn from an external point are equal)
$\therefore CQ = (BC - BQ) = (6 - x)cm$
Similarly, $AR = AP = (AB = BP) = (8 - x)cm$
$\therefore AC = (AR + CR) = [(8 - x) + (6 - x)]cm$
$\Rightarrow 10 = (14 - 2x)cm$
$\Rightarrow 2x = 4$
$\Rightarrow x = 2\,cm$
$\therefore$ The radius of the circle is 2 cm.

44. Quadrilateral ABCD is circumscribed to a circle. If AB= 6 cm, BC = 7cm and CD = 4cm then the length of AD is
(a) 3 cm (b) 4 cm (c) 6 cm (d) 7 cm
Answer: (a) 3 cm
Sol:



We know that when a quadrilateral circumscribes a circle then sum of opposes sides is equal to the sum of other opposite sides
$\therefore AB + DC = AD + BC$
$\Rightarrow 6 + 4 = AD + 7$
$\Rightarrow AD = 3cm$

45. In the given figure, PA and PB are tangents to the given circle such that PA = 5 cm and $\angle APB = 60°$. The length of chord AB is



(a) $5\sqrt{2}\,cm$ (b) 5 cm (c) $5\sqrt{3}\,cm$ (d) 7.5 cm

Answer: (b) 5 cm

Sol:

The lengths of tangents drawn from a point to a circle are equal

So, $PA = PB$ and therefore, $\angle PAB = \angle PBA = x$ (say).

Then, in ΔPAB:

$\angle PAB + \angle PBA + \angle APB = 180°$

$\Rightarrow x + x + 60° = 180°$

$\Rightarrow 2x = 180° - 60°$

$\Rightarrow 2x = 120°$

$\Rightarrow x = 60°$

$\therefore$ Each angle of ΔPAB is $60°$ and therefore, it is an equilateral triangle.

$\therefore AB = PA = PB = 5cm$

$\therefore$ The length of the chord AB is $5cm$.

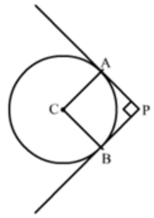
46. In the given figure, DE and DF are tangents from an external point D to a circle with centre A. If DE = 5 cm and $DE \perp DF$ then the radius of the circle is

(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

Answer: (c) 5 cm

Sol:

Construction: Join AF and AE



We know that the radius and tangent are perpendicular at their point of contact

$\because \angle AED = \angle AFD = 90°$

Since, in quadrilateral AEDF all the angles are right angles

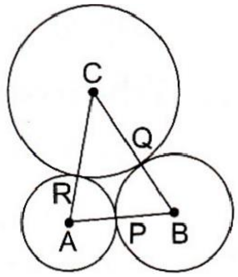
$\therefore AEDF$ is a rectangle

Now, we know that the pair of opposite sides is equal in rectangle

$\therefore AF = DE = 5cm$

Therefore, the radius of the circle is 5 cm

47. In the given figure, three circles with centres A, B, C respectively touch each other externally. If AB = 5 cm, BC = 7 cm and CA = 6 cm then the radius of the circle with centre A is



(a) 1.5 cm (b) 2 cm (c) 2.5 cm (d) 3 cm

Answer: (b) 2 cm

Sol:

Given, $AB = 5$ cm, $BC = 7$ cm and $CA = 6$ cm.

Let, $AR = AP = x$ cm.

$BQ = BP = y$ cm

$CR = CQ = z$ cm

(Since the length of tangents drawn from an external point arc equal)

Then, $AB = 5$ cm

$\Rightarrow AP + PB = 5cm$

$\Rightarrow x + y = 5$ ....(i)

Similarly, $y + z = 7$ ......(ii)

and $z + x = 6$ ....(iii)

Adding (i), (ii) and (iii), we get:

$(x + y) + (y + z) + (z + x) = 18$

$\Rightarrow 2(x + y + z) = 18$

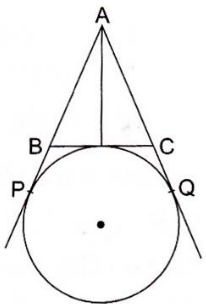
$\Rightarrow (x + y + z) = 9$ ....(iv)

Now, (iv) – (ii):

$\Rightarrow x = 2$

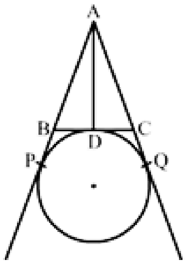
$\therefore$ The radius of the circle with center A is 2 cm.

48. In the given figure, AP, AQ and BC are tangents to the circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm then the length of AP is



(a) 15 cm (b) 10 cm (c) 9 cm (d) 7.5 cm

Answer: (d) 7.5 cm
Sol:



We know that tangent segments to a circle from the same external point are congruent
Therefore, we have

$AP = AQ$

$BP = BD$

$CQ = CD$

Now, $AB + BC + AC = 5 + 4 + 6 = 15$

$\Rightarrow AB + BD + DC + AC = 15 cm$

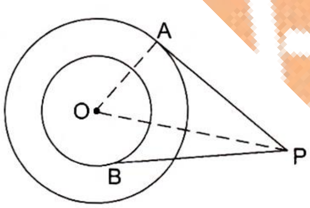
$\Rightarrow AB + BP + CQ + AC = 15 cm$

$\Rightarrow AP + AQ = 15 cm$

$\Rightarrow 2AP = 15 cm$

$\Rightarrow AP = 7.5 cm$

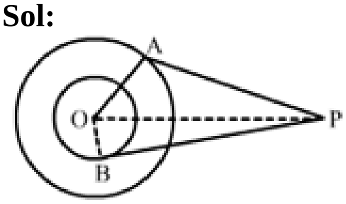
49. In the given figure, O is the centre of two concentric circles of radii 5 cm and 3 cm. From an external point p tangents PA and PB are drawn to these circles. If PA = 12 cm then PB is equal to



(a) $5\sqrt{2}\ cm$ (b) $3\sqrt{5}\ cm$ (c) $4\sqrt{10}\ cm$ (d) $5\sqrt{10}\ cm$

Answer: (c) $4\sqrt{10}\ cm$

Sol:



Given, $OP = 5 cm, PA = 12 cm$

Now, join O and B

Then, $OB = 3cm.$

Now, $\angle OAP = 90°$ (Tangents drawn from an external point are perpendicular to the radius at the point of contract)

Now, in ΔOAP:

$OP^2 = OA^2 + PA^2$

$\Rightarrow OP^2 = 5^2 + 12^2$

$\Rightarrow OP^2 = 25 + 144$

$\Rightarrow OP^2 = 169$

$\Rightarrow OP = \sqrt{169}$

$\Rightarrow OP = 13$

Now, in ΔOBP:

$PB^2 = OP^2 - OB^2$

$\Rightarrow PB^2 = 13^2 - 3^2$

$\Rightarrow PB^2 = 169 - 9$

$\Rightarrow PB^2 = 160$

$\Rightarrow PB = \sqrt{160}$

$\Rightarrow PB = 4\sqrt{10}cm$

50. Which of the following statements in not true?

(a) If a point P lies inside a circle, not tangent can be drawn to the circle, passing through p.

(b) If a point P lies on the circle, then one and only one tangent can be drawn to the circle at P.

(c) If a point P lies outside the circle, then only two tangents can be drawn to the circle form P.

(d) A circle can have more than two parallel tangents. parallel to a given line.

Answer: (d) A circle can have more than two parallel tangents. parallel to a given line.

Sol:

A circle can have more than two parallel tangents. parallel to a given line.

This statement is false because there can only be two parallel tangents to the given line in a circle.

51. Which of the following statements is not true?

(a) A tangent to a circle intersects the circle exactly at one point.

(b) The point common to the circle and its tangent is called the point of contact.

(c) The tangent at any point of a circle is perpendicular to the radius of the circle through the point of contact.

(d) A straight line can meet a circle at one point only.

Answer: (d) A straight line can meet a circle at one point only.

Sol:
A straight be can meet a circle at one point only
This statement is not true because a straight line that is not a tangent but a secant cuts the circle at two points.

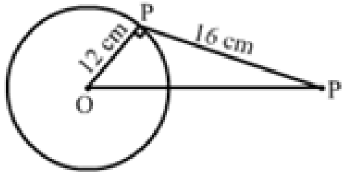
52. Which of the following statement is not true?
(a) A line which intersect a circle in tow points, is called secant of the circle.
(b) A line intersecting a circle at one point only, is called a tangent to the circle.
(c) The point at which a line touches the circle, is called the point of contact.
(d) A tangent to the circle can be drawn form a point inside the circle.
Answer: (d) A tangent to the circle can be drawn form a point inside the circle.
Sol:
A tangent to the circle can be drawn from a point Inside the circle.
This statement is false because tangents are the lines drawn from an external point to the circle that touch the circle at one point.

Assertion-and-Reason Type
Each question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer, use the following code:
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
(c) Assertion (A) is true and Reason (R) is false.
(d) Assertion (A) is false and Reason (R) is true.

53.

Assertion (A)	Reason (R)
At a point P of a circle with center O and radius 12 cm, a tangent PQ of length 16 cm is drawn., Then, OQ = 20 cm.	The tangent at any point of a circle is perpendicular to the radius through the point of contact.

The correct answer is (a) / (b) / (c) / (d).
Answer: (a) Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
Sol:



(a) Both Assertion (A) and Reason (R) are true and Reason (R) s a correct explanation of Assertion (A)
In $\Delta OPQ, \angle OPQ = 90°$

$\therefore OQ^2 = OP^2 + PQ^2$

$\Rightarrow OQ = \sqrt{OP^2 + PQ^2}$

$= \sqrt{12^2 + 16^2}$

$= \sqrt{144 + 256}$

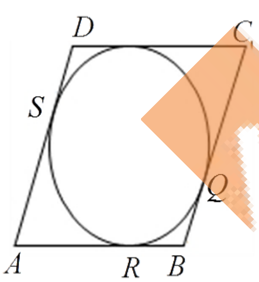
$= \sqrt{400}$

$= 20\,cm$

54.

Assertion (A)	Reason (R)
If two tangents are drawn to a circle from an external point then they subtend equal angles at the centre.	A parallelogram circumscribing a circle is rhombus.

The correct answer is (a) / (b) / (c) / (d).
Answer: (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
Sol:
Assertion -
We know that It two tangents are drawn to a circle from an external pout, they subtend equal angles at the center
Reason:



Given, a parallelogram *ABCD* circumscribes a circle with center O
AB = *BC* = *CD* = *AD*
We know that the tangents drawn from an external point to circle are equal

$\therefore AP = AS$ ....(i) [tangents from *A*]

$BP = BQ$ ....(ii) [tangents from *B*]

$CR = CQ$ ....(iii) [tangents from *C*]

$DR = DS$ ....(iv) [tangents from *D*]

$\therefore AB + CD = AP + BP + CR + DR$

$= AS + BQ + CQ + DS$ [from (i), (ii), (iii) and (iv)]

$= (AS + DS) + (BQ + CQ)$

$= AD + BC$

Thus, $(AB+CD)=(AD+BC)$

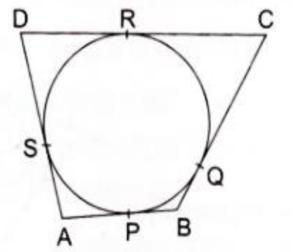
$\Rightarrow 2AB=2AD$ [$\because$ opposite sides of a parallelogram are equal]

$\Rightarrow AB=AD$

$\therefore CD=AB=AD=BC$

Hence, *ABCD* is a rhombus.

55.

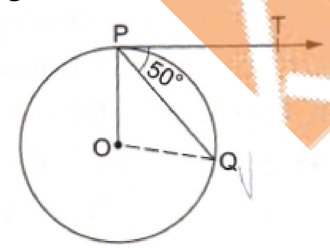
Assertion (A)	Reason (R)
In the given figure a quad. ABCD is drawn to circumscribe a given circle, as shown Then, AB + BC = AD + DC. 	In two concentric circles , the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

The correct answer is (a) / (b) / (c) / (d).

Answer: (d) Assertion (A) is false and Reason (R) is true.

Exercise - Formative Assessment

1. In the given figure, O is the center of a circle, PQ is a chord and the tangent PT at P makes an angle of $50°$ with PQ. Then, $\angle POQ=?$



(a) 130° (b) 100°

(c) 90° (d) 75°

Answer: (b) 100°

Sol:

Given, $\angle QPT=50°$

Now, $\angle OPT=90°$ (Since tangents drawn from an external point are perpendicular to the radius at point of contact)

$\therefore \angle OPQ=(\angle OPT-\angle QPT)=(90°-50°)=40°$

$OP=OQ$ (Radii of the same circle)

$\Rightarrow \angle OPQ = \angle OQP = 40°$

In ΔPOQ

$\angle POQ + \angle OPQ + \angle OQP = 180°$

$\Rightarrow \angle POQ + 40° + 40° = 180°$

$\Rightarrow \angle POQ = 180° - (40° + 40°)$

$\Rightarrow \angle POQ = 180° - 80°$

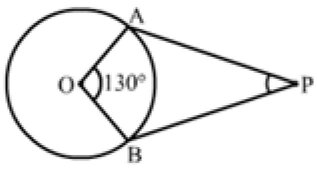
$\Rightarrow \angle POQ = 100°$

2. If the angles between two radii of a circle is $130°$, then the angle between the tangents at the ends of the radii is

(a) $65°$ (b) $40°$ (c) $50°$ (d) $90°$

Answer: (c) $50°$

Sol:



OA and *OB* are the two radii of a circle with center *O*.

Also, *AP* and *BP* are the tangents to the circle.

Given, $\angle AOB = 130°$

Now, $\angle OAB = \angle OBA = 90°$ (Since tangents drawn from an external point are perpendicular to the radius at point of contact)

In quadrilateral *OAPB*,

$\angle AOB + \angle OAB + \angle OBA + \angle APB = 360°$

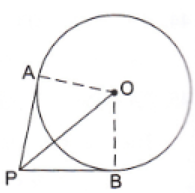
$\Rightarrow 130° + 90° + 90° + \angle APB = 360°$

$\Rightarrow \angle APB = 360° - (130° + 90° + 90°)$

$\Rightarrow \angle APB = 360° - 310°$

$\Rightarrow \angle APB = 50°$

3. If tangents PA and PB from a point P to a circle with center O are drawn so that $\angle APB = 80°$, then, $\angle POA$?



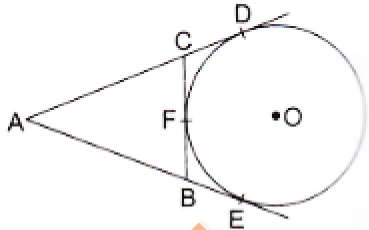
(a) $40°$ (b) $50°$

(c) $80°$ (d) $60°$

Answer: (b) $50°$

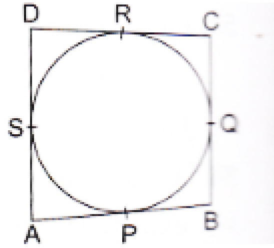
Sol:
From ΔOPA and ΔOPB
$OA = OB$ (Radii of the same circle)
OP (Common side)
$PA = PB$ (Since tangents drawn from an external point to a circle are equal)
$\therefore \Delta OPA \cong \Delta OPB$ (SSS rule)
$\therefore \angle APO = \angle BPO$
$\therefore \angle APO = \frac{1}{2}\angle APB = 40°$
And $\angle OAP = 90°$ (Since tangents drawn from an external point are perpendicular to the radius at point of contact)
Now, in $\Delta OAP, \angle AOP + \angle OAP + \angle APO = 180°$
$\Rightarrow \angle AOP + 90° + 40° = 180°$
$\Rightarrow \angle AOP = 180° - 130° = 50°$

4. In the given figure, AD and AE are the tangents to a circle with centre O and BC touches the circle at F. If AE = 5 cm, then perimeter of ΔABC is



(a) l5cm (b) 10cm
(c) 22.5 cm (d) 20cm
Answer: (b) 10cm
Sol:
Since the tangents from an external point are equal, we have
$AD = AE, CD = CF, BE = BF$
Perimeter of $\Delta ABC = AC + AB + CB$
$= (AD - CD) + (CF + BF) + (AE - BE)$
$= (AD - CF) + (CF + BF) + (AE - BF)$
$= AD + AE$
$= 2AE$
$= 2 \times 5$
$= 10\,cm$

5. In the given figure, a quadrilateral ABCD is drawn to circumscribe a circle such that its sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively. If AB = x cm, BC = 7cm, CR = 3cm and AS=5cm, find x.



Sol:

We know that tangent segments to a circle from the same external point are congruent

Now, we have

$CR = CQ, AS = AP$ and $BQ = BP$

Now, $BC = 7\,cm$

$\Rightarrow CQ + BQ = 7$

$\Rightarrow BQ = 7 - CQ$

$\Rightarrow BQ = 7 - 3$ $\quad [\because CQ = CR = 3]$

$\Rightarrow BQ = 4\,cm$

Again, $AB = AP + PB$

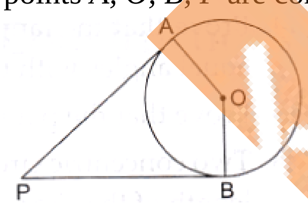
$= AP = BQ$

$= 5 + 4$ $\quad [\because AS = AP = 5]$

$= 9\,cm$

Hence, the value of x 9cm

6. In the given figure, PA and PB are the tangents to a circle with centre O. Show that the points A, O, B, P are concyclic.



Sol:

Here, OA = OB

And $OA \perp AP, OA \perp BP,$ (Since tangents drawn from an external point arc perpendicular to the radius at the point of contact)

$\therefore \angle OAP = 90°, \angle OBP = 90°$

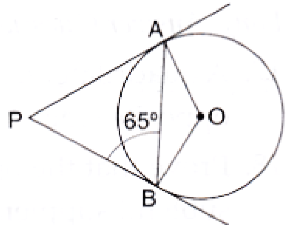
$\therefore \angle OAP + \angle OBP = 90° + 90° = 180°$

$\therefore \angle AOB + \angle APB = 180°$ $\quad$ (Since, $\angle OAP + \angle OBP + \angle AOB + \angle APB = 360°$)

Sum of opposite angle of a quadrilateral is 180°.

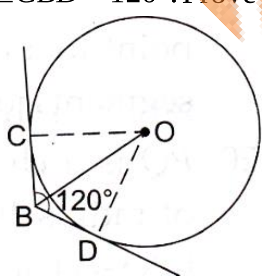
Hence A, O, B and P are concyclic.

7. In the given figure, PA and PB are two tangents form an externa point P to a circle with centre O. If $\angle PBA = 65°$, find the $\angle OAB$ and $\angle APB$.



Sol:
We know that tangents drawn from the external port are congruent
$\therefore PA = PB$
Now, In isosceles triangle APB
$\angle APB + \angle PBA = \angle PAB = 180°$ [Angle sum property of a triangle]
$\Rightarrow \angle APB + 65° + 65° = 180°$ $[\because \angle PBA = \angle PAB = 65°]$
$\Rightarrow \angle APB = 50°$
We know that the radius and tangent are perpendicular at their port of contact
$\therefore \angle OBP = \angle OAP = 90°$
Now, In quadrilateral $AOBP$
$\angle AOB + \angle OBP + \angle APB + \angle OAP = 360°$ [Angle sum property of a quadrilateral]
$\Rightarrow \angle AOB + 90° + 50° + 90° = 360°$
$\Rightarrow 230° + \angle BOC = 360°$
$\Rightarrow \angle AOB = 130°$
Now, In isosceles triangle AOB
$\angle AOB + \angle OAB + \angle OBA = 180°$ [Angle sum property of a triangle]
$\Rightarrow 130° + 2\angle OAB = 180°$ $[\because \angle OAB = \angle OBA]$
$\Rightarrow \angle OAB = 25°$

8. Two tangents segments BC and BD are drawn to a circle with center O such that $\angle CBD = 120°$. Prove that $OB = 2BC$



Ans:
Sol:
Here, OB is the bisector of $\angle CBD$.
(Two tangents are equally inclined to the line segment joining the center to that point)
$\therefore \angle CBO = \angle DBO = \frac{1}{2}\angle CBD = 60°$
$\therefore From\ \Delta BOD, \angle BOD = 30°$
Now, from right – angled ΔBOD,

$\Rightarrow \frac{BD}{OB} = \sin 30°$

$\Rightarrow OB = 2BD$

$\Rightarrow OB = 2BC$ (Since tangents from an external point are equal. i.e., $BC = BD$)

$\therefore OB = 2BC$

9. Fill in the blanks.

(i) A line intersecting a circle in two distinct points is called a ………

(ii) A circle can have parallel tangents at the most ...

(iii) The common point of a tangent to a circle and the circle is called the ………

(iv) A circle can have ….. tangents

Sol:

(i) A line intersecting a circle at two district points is called a secant

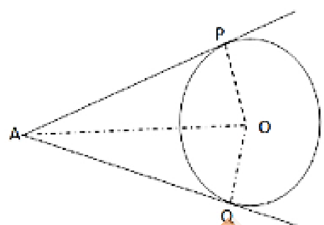
(ii) A circle can have two parallel tangents at the most

(iii) The common point of a tangent to a circle and the circle is called the point of contact.

(iv) A circle can have infinite tangents

10. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

Sol:



Given two tangents AP and AQ are drawn from a point A to a circle with center O.

To prove: $AP = AQ$

Join OP, OQ and OA.

AP is tangent at P and OP is the radius.

$\therefore OP \perp AP$ (Since tangents drawn from an external point are perpendicular to the radius at the point of contact)

Similarly, $OQ \perp AQ$

In the right ΔOPA and ΔOQA, we have:

$OP = OQ$ [radii of the same circle]

$\angle OPA = \angle OQA (= 90°)$

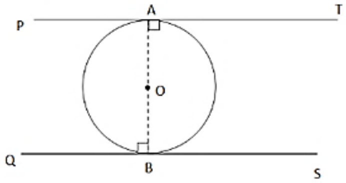
$OA = OA$ [Common side]

$\therefore \Delta OPA \cong \Delta OQA$ [By R.H.S – Congruence]

Hence, $AP = AQ$

11. Prove that the tangents drawn at the ends of the diameter of a circle are parallel.

Sol:



Here, PT and QS are the tangents to the circle with center O and AB is the diameter

Now, radius of a circle is perpendicular to the tangent at the point of contact

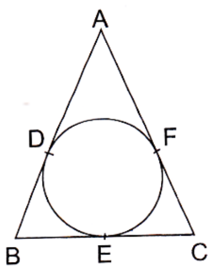
$\therefore OA \perp AT$ and $OB \perp BS$ (Since tangents drawn from an external point are perpendicular to the radius at point of contact)

$\therefore \angle OAT = \angle OBQ = 90°$

But $\angle OAT$ and $\angle OBQ$ are alternate angles.

$\therefore AT$ is parallel to BS.

12. In the given figure, if $AB = AC$, prove that $BE = CE$.



Sol:

Given, $AB = AC$

We know that the tangents from an external point are equal

$\therefore AD = AF, BD = BE \text{ and } CF = CE$ ....(i)

Now, $AB = AC$

$\Rightarrow AD + DB = AF + FC$

$\Rightarrow AF + DB = AF + FC$ $[from(i)]$

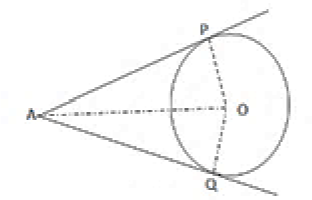
$\Rightarrow DB = FC$

$\Rightarrow BE = CE$ $[from(i)]$

Hence proved.

13. If two tangents are drawn to a circle from an external point, show that they subtend equal angles at the centre.

Sol:



Given: A circle with center O and a point A outside it. Also, AP and AQ are the two tangents to the circle

To prove: $\angle AOP = \angle AOQ$.

Proof : In ΔAOP and ΔAOQ, we have

$AP = AQ$ [tangents from an external point are equal]

$OP = OQ$ [radii of the same circle]

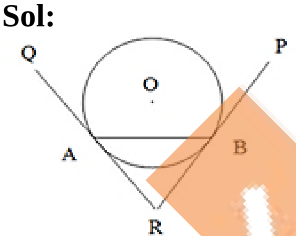
$OA = OA$ [common side]

$\therefore \Delta AOP \cong \Delta AOQ$ [by SSS – congruence]

Hence, $\angle AOP = \angle AOQ$ (c.p.c.t).

14. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

Sol:



Let RA and RB be two tangents to the circle with center O and let AB be a chord of the circle.

We have to prove that $\angle RAB = \angle RDA$.

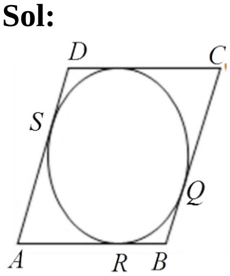
$\therefore$ Now, RA

$= RB$ (Since tangents drawn from an external point to a circle are equal)

In. $\Delta RAB, \angle RAB = \angle RDA$ (Since opposite sides are equal, their base angles are also equal)

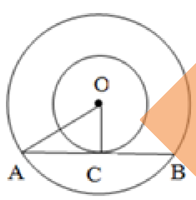
15. Prove that the parallelogram circumscribing a circle, is a rhombus.

Sol:



Given, a parallelogram *ABCD* circumscribes a circle with center O
$AB = BC = CD = AD$
We know that the tangents drawn from an external point to circle are equal
$\therefore AP = AS$ ....(i) [tangents from *A*]
$BP = BQ$ ....(ii) [tangents from *B*]
$CR = CQ$ ....(iii) [tangents from *C*]
$DR = DS$ ....(iv) [tangents from *D*]
$\therefore AB + CD = AP + BP + CR + DR$
$= AS + BQ + CQ + DS$ [from (i), (ii), (iii) and (iv)]
$= (AS + DS) + (BQ + CQ)$
$= AD + BC$
Thus, $(AB + CD) = (AD + BC)$
$\Rightarrow 2AB = 2AD$ [$\because$ opposite sides of a parallelogram are equal]
$\Rightarrow AB = AD$
$\therefore CD = AB = AD = BC$
Hence, *ABCD* is a rhombus.

16. Two concentric circles are of radii 5 cm and 3 cm respectively. Find the length of the chord of the larger circle which touches the smaller circle.
Sol:



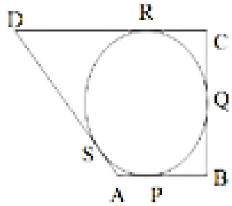
Given: Two circles have the same center *O* and *AB* is a chord of the larger circle touching the smaller circle at *C*. also, *OA* = *5* cm ad *OC 3* cm
In $\Delta OAC, OA^2 = OC^2 + AC^2$
$\therefore AC^2 = OA^2 - OC^2$
$\Rightarrow AC^2 = 5^2 - 3^2$
$\Rightarrow AC^2 = 25 - 9$
$\Rightarrow AC^2 = 16$
$\Rightarrow AC = 4cm$
$\therefore AB = 2AC$ (Since perpendicular drawn from the center of the circle bisects the chord)
$\therefore AB = 2 \times 4 = 8cm$
The length of the chord of the larger circle is 8cm.

17. A quadrilateral *ABCD* is drawn to circumscribe a circle. Prove that *AB* + *CD* = *AD* + *BC*.

Or

A quadrilateral is drawn to circumscribe a circle. Prove that the sum of opposite sides are equal.

Sol:



We know that the tangents drawn from an external point to circle are equal.

$\therefore AP = AS$ ....(i) [tangents from *A*]

$BP = BQ$ ....(ii) [tangents from *B*]

$CR = CQ$ ....(iii) [tangents from *C*]

$DR = DS$ ....(iv) [tangents from *D*]

$\therefore AB + CD = (AP + BP) + (CR + DR)$

$= (AS + BQ) + (CQ + DS)$ [using (i), (ii), (iii) and (iv)]

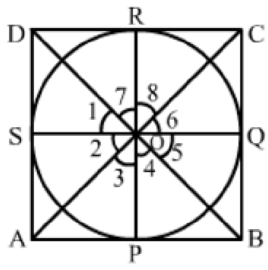
$= (AS + DS) + (BQ + CQ)$

$= AD + BC$

Hence, (*AB* + *CD*) = (*AD* + *BC*)

18. Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Sol:



Given, a quadrilateral *ABCD* circumference a circle with center *O*.

To prove: $\angle AOB + \angle COD = 180°$

And $\angle AOD + \angle BOC = 180°$

Join: OP, OQ, OR and OS.

We know that the tangents drawn from an external point of a circle subtend equal angles at the center.

$\therefore \angle 1 = \angle 7, \angle 2 = \angle 3, \angle 4 = \angle 5$ and $\angle 6 = \angle 8$

And $\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6+\angle 7+\angle 8=360^\circ$ [angles at a point]

$\Rightarrow (\angle 1+\angle 7)+(\angle 3+\angle 2)+(\angle 4+\angle 5)+(\angle 6+\angle 8)=360^\circ$

$2\angle 1+2\angle 2+2\angle 6+2\angle 5=360^\circ$

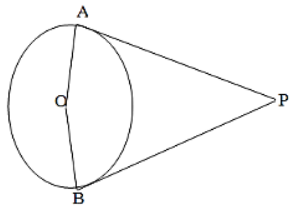
$\Rightarrow \angle 1+\angle 2+\angle 5+\angle 6=180^\circ$

$\Rightarrow \angle AOB+\angle COD=180^\circ$ and $\angle AOD+\angle BOC=180^\circ$

19. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact at the centre.

Ans:

Sol:



Given, PA and PB are the tangents drawn from a point P to a circle with center O. Also, the line segments OA and OB are drawn.

To prove: $\angle APB+\angle AOB=180^\circ$

We know that the tangent to a circle is perpendicular to the radius through the point of contact

$\therefore PA\perp OA$

$\Rightarrow \angle OAP=90^\circ$

$PB\perp OB$

$\Rightarrow \angle OBP=90^\circ$

$\therefore \angle OAP+\angle OBP=(90^\circ+90^\circ)=180^\circ$ ....(i)

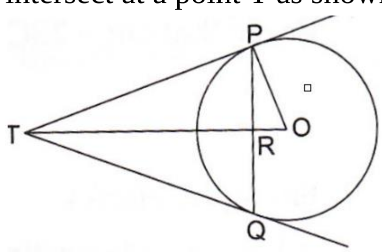
But we know that the sum of all the angles of a quadrilateral is 360°.

$\therefore \angle OAP+\angle OBP+\angle APB+\angle AOB=360^\circ$ ....(ii)

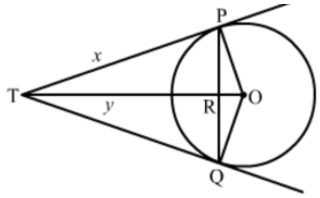
From (i) and (ii), we get:

$\angle APB+\angle AOB=180^\circ$

20. PQ is a chord of length 16 cm of a circle of radius 10 cm. The tangents at P and Q intersect at a point T as shown in the figure. Find the length of TP.



Sol:



Let TR = y and TP = x

We know that the perpendicular drawn from the center to the chord bisects it.

$\therefore PR = RQ$

Now, PR + RQ = 16

PR + PR = 16

$\Rightarrow PR = 8$

Now, in right triangle POR

By Using Pythagoras theorem, we have

$PO^2 = OR^2 + PR^2$

$\Rightarrow 10^2 = OR^2 + (8)^2$

$\Rightarrow OR^2 = 36$

$\Rightarrow OR = 6$

Now, in right triangle TPR

By Using Pythagoras theorem, we have

$TP^2 = TR^2 + PR^2$

$\Rightarrow x^2 = y^2 + (8)^2$

$\Rightarrow x^2 = y^2 + 64 \qquad ....(1)$

Again, in right triangle TPQ

By Using Pythagoras theorem, we have

$TO^2 = TP^2 + PO^2$

$\Rightarrow (y+6)^2 = x^2 + 10^2$

$\Rightarrow y^2 + 12y + 36 = x^2 + 100$

$\Rightarrow y^2 + 12y = x^2 + 64 \qquad ....(2)$

Solving (1) and (2), we get

$x = 10.67$

$\therefore TP = 10.67\,cm$