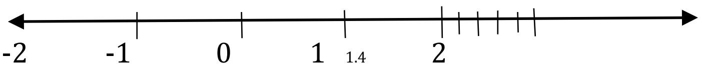
Chapter 1 :- Number System

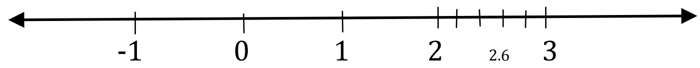
Exercise 1A

Answer1. Yes 0 is rational number because 0 can be written as $\frac{0}{1}$ which is in form of $\frac{p}{q}$, where p and q are integers and q ≠ 0

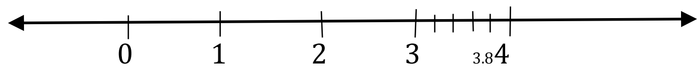
Answer 2.i)$\frac{5}{7}$=1.4



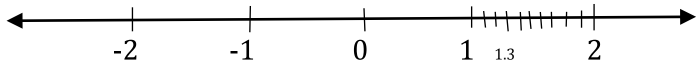
ii)$\frac{8}{3} = 2.6$



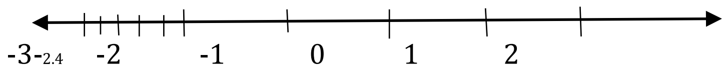
iii)$-\frac{23}{6} = 3.83$



iv)1.3



v)-2.4



Answer3. i) $\frac{3}{8}$ *and* $\frac{2}{5}$

Let, $\frac{3}{8} = x$ and $\frac{2}{5} = y$

Rational number lying between these two are

$\frac{1}{2}(x + y) = \frac{1}{2}(\frac{3}{8} + \frac{2}{5}) = \frac{1}{2} \times \frac{31}{40} = \frac{31}{80}$

ii)1.3 and 1.4

Let, x = 1.3 and y = 1.4

Rational number lying between these two are

$\frac{1}{2}(x+y) = \frac{1}{2}(1.3 + 1.4) = \frac{1}{2}(2.7) = 1.35$

iii) -1 and $\frac{1}{2}$

Let, x = -1 and y = $\frac{1}{2}$

Rational number lying between these two are

$$\frac{1}{2}(x+y) = \frac{1}{2}\left(-1+\frac{1}{2}\right) = \frac{1}{2}\left(\frac{-2+1}{2}\right) = \frac{1}{2} \times \frac{-1}{2} = -\frac{1}{4}$$

iv) $-\frac{3}{4}$ and $-\frac{2}{5}$

Let x = $-\frac{3}{4}$ and y = $-\frac{2}{5}$

Rational number lying between these two are

$$\frac{1}{2}(x+y) = \frac{1}{2}\left((-\frac{3}{4}) + (-\frac{2}{5})\right) = \frac{1}{2}\left(-\frac{23}{20}\right) = -\frac{23}{40}$$

v) $\frac{1}{9}$ *and* $\frac{2}{9}$

Let x = $\frac{1}{9}$ and y = $\frac{2}{9}$

Rational number lying between these two are

$\frac{1}{2}(x+y) = \frac{1}{2}\left(\frac{1}{9}+\frac{2}{9}\right) = \frac{1}{2} \times \frac{3}{9} = \frac{3}{18}$ or $\frac{1}{6}$

Answer4.

A rational number lying between $\frac{3}{5}$ *and* $\frac{7}{8}$ is $\frac{1}{2}\left(\frac{3}{5}+\frac{7}{8}\right)$,

That is, $\frac{59}{80}$

Now rational number between $\frac{59}{80}$ *and* $\frac{7}{8}$ is

$$\frac{1}{2}\left(\frac{59}{80}+\frac{7}{8}\right) = \frac{1}{2} \times \frac{129}{80} = \frac{129}{160}$$

And, a rational lying between $\frac{3}{5}$ *and* $\frac{59}{80}$

$$\frac{1}{2}\left(\frac{3}{5}+\frac{59}{80}\right)=\frac{1}{2}\times\frac{107}{80}=\frac{107}{160}$$

3 rational numbers are -$\frac{107}{160}, \frac{59}{80}, \frac{129}{160}$

Answer5.

Let n = 4

We convert $\frac{3}{7}$ and $\frac{5}{7}$ into equivalent rational number by multiplying the number by multiplying the numerator and denominator by (n+1), i.e., 5.

Thus, $\frac{3}{7}=\frac{3}{7}\times\frac{5}{5}=\frac{15}{35}$ *and* $\frac{5}{7}=\frac{5}{7}\times\frac{5}{5}=\frac{25}{35}$

We have $\frac{15}{35}<\frac{16}{35}<\frac{17}{35}<\frac{18}{35}<\frac{19}{35}<\frac{20}{35}<\frac{21}{35}<\frac{22}{35}<\frac{23}{35}<\frac{24}{35}<\frac{25}{35}$

OR

$$\frac{3}{7}<\frac{16}{35}<\frac{17}{35}<\frac{18}{35}<\frac{19}{35}<\frac{4}{7}<\frac{22}{35}<\frac{23}{35}<\frac{24}{35}<\frac{5}{7}$$

Hence, 4 rational numbers are $\frac{19}{35}, \frac{4}{7}, \frac{22}{35}, \frac{23}{35}$

Answer 6.

Let n = 6

We convert 2 and 3 into equivalent rational number by multiplying the number by multiplying the numerator and denominator by (n+1), i.e., 7.

Thus, $\frac{2}{1}=\frac{2}{1}\times\frac{7}{7}=\frac{14}{7}$ *and* $\frac{3}{1}=\frac{3}{1}\times\frac{7}{7}=\frac{21}{7}$

We have, $\frac{14}{7}<\frac{15}{7}<\frac{16}{7}<\frac{17}{7}<\frac{18}{7}<\frac{19}{7}<\frac{20}{7}<\frac{21}{7}$

OR

$$2<\frac{15}{7}<\frac{16}{7}<\frac{17}{7}<\frac{18}{7}<\frac{19}{7}<\frac{20}{7}<3$$

Hence 6 rational numbers are $\frac{15}{7}<\frac{16}{7}<\frac{17}{7}<\frac{18}{7}<\frac{19}{7}<\frac{20}{7}$

Answer7.

Let $x=\frac{3}{5}$ and $y=\frac{2}{3}$. clearly $x<y$. n = 6

Make denominator same $\frac{3}{5}\times\frac{3}{3}=\frac{9}{15}$ *and* $\frac{2}{3}\times\frac{5}{5}=\frac{10}{15}$

Let n = 5

We convert $\frac{9}{15}$ and $\frac{10}{15}$ into equivalent rational number by multiplying the number by multiplying the numerator and denominator by (n+1), i.e., 6.

Thus, $\frac{9}{15} = \frac{9}{15} \times \frac{6}{6} = \frac{54}{90}$ *and* $\frac{9}{15} = \frac{10}{15} \times \frac{6}{6} = \frac{60}{90}$

We have, $\frac{54}{90} < \frac{55}{90} < \frac{56}{90} < \frac{57}{90} < \frac{58}{90} < \frac{59}{90} < \frac{60}{90}$

OR

$$\frac{9}{15} < \frac{11}{18} < \frac{28}{25} < \frac{19}{30} < \frac{29}{30} < \frac{59}{90} < \frac{10}{15}$$

Hence 5 rational numbers are $\frac{11}{18} < \frac{28}{25} < \frac{19}{30} < \frac{29}{30} < \frac{59}{90}$

Answer8.

Here x = 2.1 and y = 2.2, clearly x < y, n= 16

Let d $= \frac{y-x}{(n+1)} = \frac{2.2-2.1}{17} = \frac{0.1}{17} = 0.005$

Hence, the required numbers between 2.1 and 2.2 are

(x + d), (x + 2d), (x + 3d), (x + 4d), (x + 5d), (x + 6d), (x + 7d), (x + 8d), (x + 9d), (x + 10d), (x + 11d),

(x + 11d), (x + 12d), (x + 13d), (x + 14d), (x + 15d), (x + 16d)

i.e., 2.105, 2.110, 2.115, 2.120, 2.125, 2.130, 2.135, 2.140, 2.145, 2.150, 2.155, 2.160, 2.165, 2.170

2.175, 2.180

Answer.9.

i) True,all natural numbers together with 0 form the collection W of all whole numbers, written as W = [0,1,2,3,4.......].
ii) False, 0 is not a natural number but it's a whole number.
iii) False, the least whole number is 0. Negative integers are not whole number.
iv) True, every integer in the $\frac{p}{q}$ can be written, where p and q are integers and $q \neq 0$.
v) False, fractional numbers are not integers.
vi) False, fractional numbers are not whole numbers.

Exercise 1B

Answer.1.

i) Denominator of $\frac{13}{80}$ is 80

And, $80 = 2^4 \times 5^1$

80 has no prime factor other than 2 and 5

$\therefore$ it is a terminating decimal

ii) Denominator of $\frac{7}{24}$ is 24

And, $24 = 2^3 \times 3^1$

24 has prime factor 3 which is other than 2 and 5

$\therefore$ it is not a terminating decimal

iii) Denominator of $\frac{5}{12}$ is 12

And, $12 = 2^2 \times 3^1$

12 has prime factor 3 which is other than 2 and 5

$\therefore$ it is not a terminating decimal

iv) Denominator of $\frac{31}{375}$ is 375

And, $375 = 5^3 \times 3^1$

375 has prime factor 3 which is other than 2 and 5

$\therefore$ it is not a terminating decimal

v) Denominator of $\frac{16}{125}$ is 125

And, $125 = 5^3$

125 has prime factor only 5 which is other than 2 and 5

$\therefore$ it is not a terminating decimal

Answer2.

i) $\frac{5}{8} = 0.625$,

It is terminating decimal because it ends after a finite number of digits.

ii) $\frac{7}{25} = 0.28$,

It is terminating decimal because it ends after a finite number of digits.

iii) $\frac{3}{11} = 0.\overline{27}$,

It is non terminating decimal because it doesn't end after a finite number of digits.

iv) $\frac{5}{13} = 0.\overline{384615}$

It is non terminating decimal because it doesn't end after a finite number of digits.

v) $\frac{11}{24} = 0.458\overline{3}$

It is non terminating decimal because it doesn't end after a finite number of digits.

vi) $\frac{261}{400} = 0.6525$

It is terminating decimal because it ends after a finite number of digits.

vii) $\frac{231}{625} = 0.3696$

It is terminating decimal because it ends after a finite number of digits.

viii) $2\frac{5}{12} = \frac{29}{12} = 2.41\overline{6}$

It is non terminating decimal because it doesn't end after a finite number of digits.

Answer.3.

i) Let $x = 0.\overline{2}$

Then, $x = 0.222$ .... (i)

Since repeating block has only one digit 2, we multiply its 10 which is

$10x = 2.222$ .... (ii)

Subtract (ii) – (i)

$9x = 2$

So, $x = \frac{2}{9}$

ii) Let $x = 0.\overline{53}$

Then, $x = 0.5353$ .... (i)

Since repeating block has only two digits, we multiply its 100 which is

$100x = 59.5353$ .... (ii)

Subtract (ii) – (i)

$99x = 53$

So, $x = \frac{53}{99}$

iii) Let $x = 2.\overline{93}$

Then, $x = 2.9393$ .... (i)

Since repeating block has only 2digit , we multiply its 100 which is

$100x = 293.9393$ .... (ii)

Subtract (ii) – (i)

$99x = 291$

So, $x = \frac{291}{99}$ *or* $\frac{97}{33}$

iv) Let $x = 18.\overline{48}$

Then, $x = 18.4848$ .... (i)

Since repeating block has only digit 2, we multiply its 100 which is

$100x = 1848.4848$ .... (ii)

Subtract (ii) – (i)

$99x = 1830$

So, $x = \frac{1830}{99}$ *or* $\frac{610}{33}$

v) Let $x = 0.\overline{235}$

Then, $x = 0.235235$ .... (i)

Since repeating block has only 3 digits, we multiply its 1000 which is

$1000x = 235.235235$ .... (ii)

Subtract (ii) – (i)

$909x = 235$

So, $x = \frac{235}{999}$

vi) Let $x = 0.00\overline{32}$

Then, $x = 0.003232$ .... (i)

Since repeating block has only 2 digits, we multiply its 100 which is

$10000x = 32.3232$ .... (ii)

Subtract (ii) – (i)

$9999x = 32.32$

So, $x = \frac{32.32}{9999} = \frac{3232}{999900} = \frac{8}{2475}$

vii) Let $x = 1.3\overline{23}$

Then, $x = 1.32323$ .... (i)

Since repeating block has only 2digit , we multiply its 100 which is

$100x = 132.32323$ .... (ii)

Subtract (ii) – (i)

$99x = 131$

So, $x = \frac{131}{99}$

viii) Let $x = 0.3\overline{178}$

Then, $x = 0.3178178$ .... (i)

Since repeating block has only 3digit , we multiply its 100 which is

$1000x = 317.8178$ .... (ii)

Subtract (ii) – (i)

$999x = 317.5$

So, $x = \frac{317.5}{9990} = \frac{3175}{9990} = \frac{635}{1998}$

ix) Let $x = 32.12\overline{35}$

Then, $x = 32.1235$ .... (i)

Since repeating block has only 2digit, we multiply its 100 which is

$100x = 3212.3535$ .... (ii)

Subtract (ii) – (i)

$99x = 3180.23$

So, $x = \frac{3180.23}{99}$ or $\frac{318023}{9900}$

x) Let $x = 0.40\overline{7}$

Then, $x = 0.407$ .... (i)

Since repeating block has only one digit , we multiply its 10 which is

$10x = 4.077$ .... (ii)

Subtract (ii) – (i)

$9x = 3.67$

So, $x = \frac{3.67}{9}$ or $\frac{367}{900}$

Answer4.

Let $x = 2.\overline{36}$

Then, $x = 2.3636$ .... (i)

Since repeating block has only 2 digits, we multiply its 100 which is

$100x = 236.3636$ .... (ii)

Subtract (ii) – (i)

$99x = 234$

So, $x = \frac{234}{99}$

Let $y = 0.\overline{23}$

Then, $y = 0.2323$ .... (i)

Since repeating block has only 2 digits, we multiply its 100 which is

$100y = 23.2323$ .... (ii)

Subtract (ii) – (i)

$99y = 23$

So, $y = \frac{23}{99}$

$x + y = \frac{234}{99} + \frac{23}{99} = \frac{257}{99}$

Answer.5.

Let $x = 0.\overline{38}$

Then, $x = 0.3838$ .... (i)

Since repeating block has only 2 digits, we multiply its 100 which is

$100x = 38.3838$ .... (ii)

Subtract (ii) – (i)

$99x = 38$

So, $x = \frac{38}{99}$

Let $y = 1.\overline{27}$

Then, $y = 1.2727$ .... (i)

Since repeating block has only 2 digits, we multiply its 100 which is

$100x = 127.2727$ .... (ii)

Subtract (ii) – (i)

$99x = 126$

So, $x = \frac{126}{99}$

$\frac{38}{99} + \frac{126}{99} = \frac{164}{99}$ which is in form $\frac{p}{q}$

Exercise 1C

Answer1. Irrational numbers are those numbers which can neither be expressed as a terminating decimal nor as a repeating decimal, is called an irrational number.Rational numbers can be possible in both fraction and decimal form. On the contrary, an irrational number can only be presented in decimal form but not in a fraction.

Eg. Rational number	Irrational numbers
1) $2 = \frac{2}{1}$	1) 0.200200020000
2) $-3.1 = \frac{-31}{10}$	2) $\sqrt{2}, \sqrt{3}$

Answer.2.

i) $\sqrt{\frac{3}{81}}$

it is irrational number because $\sqrt{3}$ is irrational number and $\sqrt{81} = 9$ which is rational number. Dividing a irrational number with rational is always Irrational number.

ii) $\sqrt{361} = 19$

it is a rational number, as it can be written in the form $\frac{p}{q}$and decimal form also.

iii) $\sqrt{21}$

it is a irrational number because it does not have a perfect square.

iv) $\sqrt{1.44} = 2.1$

it is rational number because it can be written in form $\frac{p}{q}$ and decimal form also.

v) $\frac{2}{3}\sqrt{6}$

It is airrational number as $\sqrt{6}$ is irrational number and $\frac{2}{3} = 0.\overline{66}$is rational

and rational X irrational is always Irrational number.

vi) $4.1276 = \frac{41276}{10000}$

it is rational number because it can be written in form$\frac{p}{q}$ and decimal form also.

vii) $\frac{22}{7} = 3.14$

it is rational number because it can be written in form $\frac{p}{q}$ and decimal form also.

viii)1.232332333...

it is a irrational number because it is nonterminating and nonrepeating decimal.

ix)3.040040004...

it is a irrational number because it is nonterminating and nonrepeating decimal.

x)$2.3565656... = 2.3\overline{56} = \frac{23\overline{56}}{1000}$

it is rational number because it can be written in form $\frac{p}{q}$ and decimal form also.

xi)$6.834834... = 6.\overline{834} = \frac{6\overline{834}}{1000}$

it is rational number because it can be written in form $\frac{p}{q}$ and decimal form also.

Answer.3. Yes,

Let take $\mathrm{x} = 2$ and $\mathrm{y} = \sqrt{2}$

When we add $\mathrm{x} + \mathrm{y} = 2 + \sqrt{2}$

Which is rational + irrational = Irrational, as it is irrational number because it is nonterminating and nonrepeating decimal.

Answer.4. Yes

If a is an rational number b is an irrational number then ab will always be irrational.

As rational X irrational is always Irrational

Expect $\mathrm{a} = 0$, other all are irrational number only.

Answer.5. No

$\sqrt{2} \times \sqrt{2} = 2$

Irrational ×Irrational is not always Irrational, it may be Rational

As 2 is a rational number whereas $\sqrt{2}$ is a rational number.

Answer.6.

i) $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$
ii) $(2 + \sqrt{3})$ and $(7 + \sqrt{3})$
iii) $(5 + \sqrt{3})$ and $(\sqrt{3} - 6)$
iv) $(3 + \sqrt{2})$ and $(3 - \sqrt{2})$
v) $(3 + \sqrt{3})$ and $(4 - \sqrt{3})$

vi) $(2+\sqrt{3})$ and $(2 - \sqrt{3})$

vii) $\sqrt{3}$and $\sqrt{18}$

viii) $\sqrt{3}\ and\ \sqrt{27}$

Answer.7.

i) $3+\sqrt{3}$

As rational + irrational is always Irrational

ii) $\sqrt{7} - 2$

As irrational – rational is always Irrational

iii) $\sqrt[3]{5} \times \sqrt[3]{25}$

As irrational × irrational is Rational

iv) $\sqrt{7} \times \sqrt{343}$

As irrational × irrational is Rational.

v) $\sqrt{\frac{3}{117}}$

As irrational ÷ irrational is Rational

vi) $\sqrt{8} \times \sqrt{2}$

As irrational × irrational is Rational

Answer.8.

A rational number between 2 and 2.5 is $\frac{2+2.5}{2} = 2.25$

An irrational number between 2 and 2.5 is $\sqrt{2 \times 2.5} = \sqrt{5}$

Answer.9.

$\sqrt{2} = 1.414$ and $\sqrt{3} = 1.732$

Now, we can write n number of rational numbers between these. That is just greater than 1.414 and less than 1.732 and it should be terminating or not terminating but repeating.

Infinite numbers lie between $\sqrt{2}\ and\ \sqrt{3}$

Like, 1.4144144414444, 1.505005000, 1.606006000

Answer.10.

2 rational number between 0.5 and 0.55 can be anything

⇨ 0.50 and $0.5\overline{2}$

2 irrational number between 0.5 and 0.55

⇨ 0.511511151111 and 0.535335333

There are infinite number which lie between 0.5 and 0.55

Answer.11. $\frac{5}{7}$ *and* $\frac{9}{11}$

First, we have to change the number into decimal or rationaldecimal form of $\frac{5}{7}$ is 0.71428571 and the decimal form of $\frac{9}{11}$ is 0.81818181

Any rational number can lie between 0.71428571 and 0.81818181

3 Irrational number are

⇨ 0.737337333, 0.747447444, 0.797997999

Answer.12. A rational is always in the form $\frac{p}{q}$

So, we have $\frac{53}{250}$ *and* $\frac{105}{500}$.

Answer.13. Two irrational number that lie between 0.16 and 0.17 can be many

⇨ 0.16116111 and 0.1656656665

Answer.14.

i) True, rational + rational = Rational

Eg. $\frac{5}{9} + \frac{17}{9} = \frac{22}{9}$

ii) False, irrational + irrational ≠ irrational

Eg. Let a = $(1 + \sqrt{2})$ and b = $(1 - \sqrt{2})$

So, a + b = 2 which is rational number.

iii) True, rational × rational = rational

Eg. 4.5 X 2.3 = 10.35

iv) False, irrational $\times$ irrational $\neq$ irrational.

Eg.$\sqrt{8} \times \sqrt{2} = \sqrt{16} \rightarrow 4$

v) True, rational $\times$ irrational $=$ irrational

Eg.$2 \times \sqrt{2} = 2\sqrt{2}$

vi) False, rational $\times$ irrational $\neq$ rational number.

Eg. $3 \times \sqrt{2} = 3\sqrt{2}$, which is a irrational number.

vii) False,
As rational numbers can be written in form $\frac{p}{q}$. Rational numbers cannot be written on number line and all numbers written on number line are real.

viii) True,
Rational numbers are numbers which can be expressed as fractions. Real number is a set of all the numbers. So, each and every kind of number, rational or irrational will be considered as a real number.

ix) True,
π is a number whose exact value is not $\frac{22}{7}$.
π has a value which is nonterminating and nonrepeating.

Exercise 1D

Answer.1. ADD

i) $\left(2\sqrt{3}-5\sqrt{2}\right)+\left(\sqrt{3}+2\sqrt{2}\right)=\left(2\sqrt{2}\ -5\sqrt{2}\right)+\left(2\sqrt{3}+\ \sqrt{3}\right)$

$=\sqrt{2}(2-5)+\ \sqrt{3}(2+1)$

$=\ (-3\sqrt{2})+3\sqrt{3}$

$=\ 3(\sqrt{3}-\sqrt{2})$

ii) $\left(2\sqrt{2}+5\sqrt{3}-7\sqrt{5}\right)+\left(3\sqrt{3}-\ \sqrt{2}+\ \sqrt{5}\right)=(2\sqrt{2}-\sqrt{2}+5\sqrt{3}+3\sqrt{3}\ +\ \sqrt{5}-7\sqrt{5})$

$=(\sqrt{2}(2-1)+\sqrt{3}(5+3)+\sqrt{5}(1-7)$

$=\ \sqrt{2}+8\sqrt{3}-6\sqrt{5}$

iii) $\left(\frac{2}{3}\sqrt{7}-\ \frac{1}{2}\sqrt{2}+6\sqrt{11}\right)+\left(\frac{1}{3}\sqrt{7}+\ \frac{3}{2}\sqrt{2}-\sqrt{11}\right)=$

$\left(\frac{2}{3}\sqrt{7}+\frac{1}{3}\sqrt{7}+\frac{3}{2}\sqrt{2}-\frac{1}{2}\sqrt{2}+6\sqrt{11}-\sqrt{11}\right)$

$=\left(\sqrt{7}\left(\frac{2}{3}+\frac{1}{3}\right)+\ \sqrt{2}\left(\frac{3}{2}-\frac{1}{2}\right)+\sqrt{11}(6-1)\right)$

$=(\sqrt{7}+\ \sqrt{2}+5\sqrt{11})$

Answer.2. MULTIPLY

i) $\left(3\sqrt{5}\right)\times\left(2\sqrt{5}\right)=\left(3\times2\times\sqrt{5}\times\sqrt{5}\right)$

$=\left(6\times\sqrt{25}\right)$

$=(6\times5)=30 \qquad \left\{\sqrt{25}=5\right\}$

ii) $\left(6\sqrt{15}\right)\times4\left(\sqrt{3}\right)=\left(6\times4\times\sqrt{15}\times\sqrt{3}\right)$

$=\left(24\times\sqrt{5}\ \times\sqrt{3}\times\sqrt{3}\right)\left\{\sqrt{15}=\ \sqrt{5}\times\sqrt{3}\right\}$

$=\left(24\times3\times\sqrt{5}\right)\left\{\sqrt{3}\times\sqrt{3}=\ \sqrt{9}=3\right\}$

$=\ 72\sqrt{5}$

iii) $\left(2\sqrt{6}\right)\times\left(3\sqrt{3}\right)=\left(2\times3\times\sqrt{6}\times\sqrt{3}\right)$

$=\left(6\times\sqrt{2}\times\sqrt{3}\times\sqrt{3}\right)\left\{\sqrt{6}=\sqrt{3}\times\sqrt{2}\right\}$

$=\left(6\times3\times\sqrt{2}\right)\left\{\sqrt{3}\times\sqrt{3}=\sqrt{9}=3\right\}$

$=18\sqrt{2}$

iv) $\left(3\sqrt{8}\right)\times\left(3\sqrt{2}\right)=\left(3\times3\times\sqrt{8}\times\sqrt{2}\right)$

$=\left(9\times\sqrt{16}\right)$

$=(9\times4)\left\{\sqrt{16}=4\right\}$

$=36$

v) $\left(\sqrt{10}\right)\times\left(\sqrt{40}\right)=\left(\sqrt{2}\times\sqrt{5}\right)\times\left(\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{5}\right)$

$$= \left((\sqrt{2} \times \sqrt{2}) \times (\sqrt{2} \times \sqrt{2}) \times (\sqrt{5} \times \sqrt{5})\right)$$

$$= (2 \times 2 \times 5)\{\sqrt{2} \times \sqrt{2} = \sqrt{4} = 2, \sqrt{5} \times \sqrt{5} = \sqrt{25} = 5\}$$

$$= 20$$

vi) $(3\sqrt{28}) \times (2\sqrt{7}) = (3 \times 2 \times \sqrt{28} \times \sqrt{7})$

$$= (6 \times \sqrt{196})$$

$$= (6 \times 14)\{\sqrt{196} = 14\}$$

$$= 84$$

Answer.3. DIVIDE

i) $(16\sqrt{6}) \div (4\sqrt{2}) = \frac{16\sqrt{6}}{4\sqrt{2}} = \frac{16\times\sqrt{2}\times\sqrt{3}}{4\times\sqrt{2}}\{\sqrt{6} = \sqrt{2} \times \sqrt{3})$

$$= 4\sqrt{3}$$

ii) $(12\sqrt{15}) \div (4\sqrt{3}) = \frac{12\sqrt{15}}{4\sqrt{3}} = \frac{12\times\sqrt{3}\times\sqrt{5}}{4\times\sqrt{3}}\{\sqrt{15} = \sqrt{3} \times \sqrt{5}\}$

$$= 4\sqrt{3}$$

iii) $(18\sqrt{21}) \div (6\sqrt{7}) = \frac{18\sqrt{21}}{6\sqrt{7}} = \frac{18\times\sqrt{7}\times\sqrt{3}}{6\times\sqrt{7}}\{\sqrt{21} = \sqrt{7} \times \sqrt{3}\}$

$$= 3\sqrt{3}$$

Answer.4. Simplify

i) $(3 - \sqrt{11})(3 + \sqrt{11}) = 3 \times 3 + 3 \times \sqrt{11} - \sqrt{11} \times 3 - \sqrt{11} \times \sqrt{11}$

$$= 9 + 3\sqrt{11} - 3\sqrt{11} - 11\{\sqrt{11} \times \sqrt{11} = \sqrt{121} = 11\}$$

$$= -2$$

ii) $(-3 + \sqrt{5})(-3 - \sqrt{5}) = (-3 \times -3 + (-3) \times \sqrt{5} - \sqrt{5} \times -3 - \sqrt{5} \times \sqrt{5})$

$$= (9 - 3\sqrt{5} + 3\sqrt{5} - 5)\{\sqrt{5} \times \sqrt{5} = \sqrt{25} = 5\}$$

$$= 4$$

iii) $(3 - \sqrt{3})^2 = (a - b)^2 = a^2 + b^2 - 2ab$

Here, $a = 3$ and $b = \sqrt{3}$

$$= 3^2 + (\sqrt{3})^2 - 2(3 \times \sqrt{3})$$

$$= 9 + 3 - 6\sqrt{3}\{\sqrt{3} \times \sqrt{3} = \sqrt{9} = 3\}$$

$$= 12 - 6\sqrt{3}$$

iv) $\left(\sqrt{5}-\sqrt{3}\right)^2 = (a-b)^2 = a^2+b^2-2ab$

Here, $a=\sqrt{5}$ and $b=\sqrt{3}$

$$= \left(\sqrt{5}\right)^2 + (\sqrt{3})^2 - 2\left(\sqrt{5}\times\sqrt{3}\right)$$
$$= 5+3-2\sqrt{15}\left\{\sqrt{5}\times\sqrt{5}=\sqrt{25}=5, \sqrt{3}\times\sqrt{3}=\sqrt{3}=3\right\}$$
$$= 8-2\sqrt{15}$$

v) $\left(5+\sqrt{7}\right)\left(2+\sqrt{5}\right) = \left(5\times 2+5\times\sqrt{5}+\sqrt{7}\times 2+\sqrt{7}\times\sqrt{5}\right)$

$$= \left(10+5\sqrt{5}+2\sqrt{7}+\sqrt{35}\right)$$

vi) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{3}\right) = \left(\sqrt{5}\times\sqrt{2}-\sqrt{5}\times\sqrt{3}-(-\sqrt{2})\times\sqrt{2}-(-\sqrt{2})\times\sqrt{3}\right)$

$$= \left(\sqrt{10}-\sqrt{15}-2+\sqrt{6}\right)\left\{\sqrt{2}\times\sqrt{2}=\sqrt{4}=2\right\}$$

Answer.5. $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)^2 = \left(3+\sqrt{3}\right)\left(2^2+\sqrt{2}^2+2\left(2\times\sqrt{2}\right)\right)$

$$= \left(3+\sqrt{3}\right)\left(4+2+4\sqrt{2}\right)\left\{\sqrt{2}\times\sqrt{2}=\sqrt{4}=2\right\}$$

$$= \left(3+\sqrt{3}\right)\left(6+4\sqrt{2}\right)$$

$$= \left(3\times 6+3\times 4\sqrt{2}+\sqrt{3}\times 6+\sqrt{3}\times 4\sqrt{2}\right)$$

$$= \left(18+12\sqrt{2}+6\sqrt{3}+4\sqrt{6}\right)$$

Answer.6.

i) $\left(5-\sqrt{5}\right)\left(5+\sqrt{5}\right) = \left(5^2-\sqrt{5}^2\right)\{(a+b)(a-b)=(a^2-b^2)\}$

$$= (25-5) = 5\ \{\sqrt{5}\times\sqrt{5}=\sqrt{25}=5$$

It is Rational number.

ii) $\left(\sqrt{3}+2\right)^2 = (a+b)^2 = a^2+b^2+2ab$

$$= \sqrt{3}^2+2^2+2\left(\sqrt{3}\times 2\right)$$
$$= 3+2+4\sqrt{3}\left\{\sqrt{2}\times\sqrt{2}=\sqrt{4}=2\right\}$$
$$= 6+4\sqrt{3}$$

It is Irrational number.

iii) $\frac{2\sqrt{13}}{3\sqrt{52}-4\sqrt{117}} = \frac{2\sqrt{13}}{3\times\sqrt{4}\times\sqrt{13}-4\times\sqrt{9}\times\sqrt{13}}$

$$= \frac{2\sqrt{13}}{3\times 2\times\sqrt{13}-4\times 3\times\sqrt{13}} = \frac{2\sqrt{13}}{\sqrt{13}(6-12)} = \frac{2}{-6} = -0.\overline{3}$$

It is Rational number.

iv) $\sqrt{8}+4\sqrt{32}-6\sqrt{2} = \sqrt{2}\times\sqrt{4}+4\times\sqrt{4}\times\sqrt{4}\times\sqrt{2}-6\sqrt{2}$

$$= 2\sqrt{2}+16\sqrt{2}-6\sqrt{2}$$

$= \sqrt{2}(2 + 16 - 6) = 12\sqrt{2}$

It is irrational number.

Answer.7.

i) $(5 + \sqrt{11})(5 - \sqrt{11}) = (a^2 - b^2) = \left(5^2 - \sqrt{11}^2\right)$

$$= (25 - 11)\{\sqrt{11} \times \sqrt{11} = \sqrt{121} = 11\}$$

$= 14$ chocolates

ii) She wanted to make others children happy by distributing chocolates and she has caring nature.

Answer.8,

i) $(3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50}) = (3 \times \sqrt{9} \times \sqrt{5} - \sqrt{25} \times \sqrt{5} + \sqrt{2} \times \sqrt{25} \times \sqrt{4} - \sqrt{25} \times \sqrt{2})$

$$= (3 \times 3 \times \sqrt{5} - 5\sqrt{5} + 5 \times 2\sqrt{2} - 5\sqrt{2})$$
$$= (9\sqrt{5} - 5\sqrt{5} + 10\sqrt{2} - 5\sqrt{2})$$
$$= (4\sqrt{5} + 5\sqrt{2})$$

ii) $\left(\frac{2\sqrt{3}}{\sqrt{6}} - \frac{3\sqrt{140}}{\sqrt{28}} + \frac{\sqrt{55}}{\sqrt{99}}\right) = (\frac{2\times\sqrt{2}\times\sqrt{3}\times\sqrt{10}}{\sqrt{2}\times\sqrt{3}} - \frac{3\times\sqrt{4}\times\sqrt{7}\times\sqrt{5}}{\sqrt{4}\times\sqrt{7}} + \frac{\sqrt{5}\times\sqrt{11}}{\sqrt{9}\times\sqrt{11}}$

$$= \left(2\sqrt{5} - 3\sqrt{5} + \frac{\sqrt{5}}{3}\right) = \left(-\sqrt{5} + \frac{\sqrt{5}}{3}\right)$$

Make denominator same, multiply it by 3

$$= \left(\frac{-3\sqrt{5} + \sqrt{5}}{3}\right) = -\frac{2}{3}$$

iii) $\sqrt{72} + \sqrt{800} - \sqrt{18} = (\sqrt{2} \times \sqrt{4} \times \sqrt{9} + \sqrt{2} \times \sqrt{16} \times \sqrt{25} - \sqrt{2} \times \sqrt{9})$

$$= (2 \times 3 \times \sqrt{2} + 4 \times 5\sqrt{2} - 3\sqrt{2})$$
$$= (6\sqrt{2} + 20\sqrt{2} - 3\sqrt{2})$$
$$= 23\sqrt{2}$$

Exercise.1E

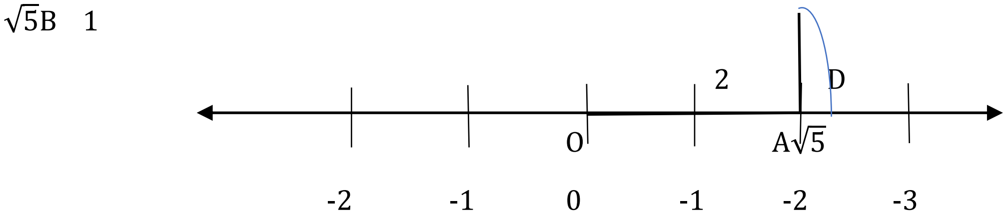
Answer.1. $\sqrt{5}$

Let M'OM be a horizontal line, taken as the x-axis and let O be the origin. Let O be 0.

Take OA = 2unit and draw AB⊥OA such that AB = 1unit. Join OB.

According to Pythagoras theorem, in △OAB

$$OB = \sqrt{OA^2 + AB^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$$



OD = OA = $\sqrt{5}$. Thus D, represents $\sqrt{5}$ on number line.

Answer.2.

Let M'OM be a horizontal line, taken as the x-axis and let O be the origin. Let O be 0.

Take OA = 2unit and draw AB⊥OA such that AB = 1unit. Join OB.

According to Pythagoras theorem,

$$OB = \sqrt{OA^2 + AB^2} = \sqrt{\left(\sqrt{2}\right)^2 + 1^2} = \sqrt{3}$$

Answer.3. Draw a line of 10 cm MN

Increase it 1 cm more from N to O

Draw a bisector of MO

Now draw a 90° angle from N and name the point E and taking NE as a radius draw an arc

Take the point D where the line cuts

Now ND= NE

Answer.4.

Construct a right angles triangle ABC such that AB=2cm and BC=2cm.

Apply Pythagoras theorem $AC = \sqrt{AB^2 + BC^2} = \sqrt{2^2 + 2^2} = \sqrt{4+4} = \sqrt{8}$

Taking A as centre and AC as radius, Draw a arc cutting the number line at D.

D represents AC=AD=$\sqrt{8}$.

Point D represents irrational number $\sqrt{8}$ on number line.

Answer.5.

Draw a line named XY with measure 4.7unit.

From Yadd 1cm and mark as Z

Take the half of XZ and mark it O

Take the measure of XO as radius and with O as centre draw a semi-circle.

And from Y draw a line perpendicular to XY, touching the semi-circle at D.

Take compass pointer on Y and the pencil point on D and then draw an arc on no. line.

Hence mark on no. line as root 4.7.

Answer.6.

Draw a line AB of 10.5 units

Extend the line BC = 1 units and AC = 11.5 units

Take the midpoint of AC as O = $\frac{11.5}{2}$ = 5.75

Draw the semicircle from A to C with O as center.

Draw a line Perpendicular to AC at point C until it bisects and intersects with the semi-circle

forming a new point D, ND, CD = $\sqrt{10.5}$

With point M as center draw an arc equal to length MD on the number AC intersecting at

Point E. Now OD = OE = $\sqrt{10.5}$

Answer.7

.Draw a line named XY with measure 7.28 unit.

From Yadd 1cm and mark as Z

Take the half of XZ and mark it O

Take the measure of XO as radius and with O as centre draw a semi-circle.

And from Y draw a line perpendicular to XY, touching the semi-circle at D.

Take compass pointer on Y and the pencil point on D and then draw an arc on no. line.

Hence mark on no. line as root 7.28.

Answer8.

Draw a line AB of 1+9.5 = 10.5 units

Extend the line BC = 1 units and AC = 11.5 units

Take the midpoint of AC as $O = \frac{11.5}{2} = 5.75$

Draw the semicircle from A to C with O as center.

Draw a line Perpendicular to AC at point C until it bisects and intersects with the semi-circle

forming a new point D, ND, $CD = \sqrt{10.5}$

With point M as center draw an arc equal to length MD on the number AC intersecting at

Point E. Now $OD = OE = \sqrt{10.5} = 1 + \sqrt{9.5}$

Answer9.

Draw a line AB is lie between 3-4 ,and given no. is 3.765 if we divide the no. 3 by 3.1,

3.2,3.3,…………,3.9

Here, 3.7 lies between 3.6 and 3.8

On further division we found that 3.76 lies between 3.71 and 3.79

On further division we get 3.765 lies between 3.761 and 3.769

Hence, we get the point 3.765 on the drawn line.

Answer10.

Draw a line AB is lie between 4 – 5 and given no is $4.\overline{67}$= 4.6777

Here 4.6 lies between 4.1 –4.9

On further divide we found that 4.67 lies between 4.61 – 4.69

On further divided we found that 4.677 lies between 4.671 – 4.679

On further divided we found that 4.6777 lies between 4.6771 – 4.6779

Hence, we get the point on no. line $4.\overline{67}$.

Exercise.1F

Answer.1. $\frac{1}{\sqrt{2}+\sqrt{3}} = \frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}^2-\sqrt{3}^2} = \frac{\sqrt{2}-\sqrt{3}}{2-3} = \frac{\sqrt{2}-\sqrt{3}}{-1} = \sqrt{3}-\sqrt{2}\{\sqrt{3}\times\sqrt{3}=\sqrt{9}=3\}$

$$\{\sqrt{2}\times\sqrt{2}=\sqrt{4}=2\}$$

Answer.2.

i) $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{\sqrt{7}^2} = \frac{\sqrt{7}}{7}$

ii) $\frac{\sqrt{5}}{2\sqrt{3}} = \frac{\sqrt{5}}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} = \frac{2\sqrt{15}}{4\times\sqrt{3}^2} = \frac{2\sqrt{15}}{12} = \frac{\sqrt{15}}{6}$

iii) $\frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-\sqrt{3}^2} = \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1} = 2-\sqrt{3}$

iv) $\frac{1}{\sqrt{5}-2} = \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{\sqrt{5}^2-2^2} = \frac{\sqrt{5}+2}{5-4} = \frac{\sqrt{5}+2}{1} = 5+\sqrt{2}$

v) $\frac{1}{5+3\sqrt{2}} = \frac{1}{5+3\sqrt{2}} \times \frac{5-3\sqrt{2}}{5-3\sqrt{2}} = \frac{5-3\sqrt{2}}{5^2-(3\sqrt{2})^2} = \frac{5-3\sqrt{2}}{25-9\times2} = \frac{5-3\sqrt{2}}{25-18} = \frac{5-3\sqrt{2}}{7}$

vi) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}^2-\sqrt{6}^2} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$

vii) $\frac{4}{\sqrt{11}-\sqrt{7}} = \frac{4}{\sqrt{11}-\sqrt{7}} \times \frac{\sqrt{11}+\sqrt{7}}{\sqrt{11}+\sqrt{7}} = \frac{4(\sqrt{11}+\sqrt{7})}{\sqrt{11}^2+\sqrt{7}^2} = \frac{4(\sqrt{11}+\sqrt{7})}{11-7} = \frac{4(\sqrt{11}+\sqrt{7})}{4} = \sqrt{11}+\sqrt{7}$

viii) $\frac{1+\sqrt{2}}{2-\sqrt{2}} = \frac{1+\sqrt{2}}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{1\times2+1\times\sqrt{2}+\sqrt{2}\times2+\sqrt{2}\times\sqrt{2}}{2^2-\sqrt{2}^2} = \frac{2+\sqrt{2}+2\sqrt{2}+2}{4-2}\{\sqrt{2}\times\sqrt{2}=\sqrt{4}=2\}$

$$= \frac{4+3\sqrt{2}}{4-2} = \frac{4+3\sqrt{2}}{2}$$

ix) $\frac{3-2\sqrt{2}}{3+2\sqrt{2}} = \frac{3-2\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{(3-2\sqrt{2})^2}{3^2-(2\sqrt{2})^2} = \frac{3^2+(2\sqrt{2})^2-2(3\times2\sqrt{2})}{9-4\times2} = \frac{9+4\times2-12\sqrt{2}}{9-8}$

$$= \frac{9+8-12\sqrt{2}}{1} = 17-12\sqrt{2}$$

Answer.3. Given, $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732, \sqrt{5} = 2.236$ *and* $\sqrt{10} = 3.162$

i) $\frac{2}{\sqrt{5}} = \frac{2}{2.236} = \frac{2}{2.236} \times \frac{1000}{1000} = \frac{2000}{2236} = 0.894$

ii) $\frac{2-\sqrt{3}}{\sqrt{3}} = \frac{2-\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}-\sqrt{3}^2}{\sqrt{3}^2} = \frac{2\sqrt{3}-3}{3} = \frac{2\times1.732-3}{3} = \frac{3.464-3}{3} = \frac{0.464}{3} = 0.155(approx)$

iii) $\frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}} = \frac{3.162-2.236}{1.414} = \frac{0.962}{1.414} = 0.655(approx.)$

Answer.4.

i) $\frac{\sqrt{2}-1}{\sqrt{2}+1} = a + b\sqrt{2}$

LHS,

$$\frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{(\sqrt{2}-1)^2}{\sqrt{2}^2 - 1^2} = \frac{\sqrt{2}^2 + 1^2 - 2(\sqrt{2}\times 1)}{2-1} \{(a-b)^2 = a^2 + b^2 - 2ab\}$$

$$= \frac{2+1-2\sqrt{2}}{1} = 3 - 2\sqrt{2}$$

As LHS=RHS

$$\therefore 3 - 2\sqrt{2} = a + b\sqrt{2}$$

So, $a = 3$ and $-2\sqrt{2} = b\sqrt{2}; b = -2$

ii) $\frac{2-\sqrt{5}}{2+\sqrt{5}} = a\sqrt{5} + b$

LHS,

$$\frac{2-\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{(2-\sqrt{5})^2}{2^2 - \sqrt{5}^2} = \frac{2^2 - \sqrt{5}^2 - 2(2\times\sqrt{5})}{4-5}$$

$$= \frac{4+5-4\sqrt{5}}{-1} \{(a-b)^2 = a^2 + b^2 - 2ab\}$$

$$= \frac{9-4\sqrt{5}}{-1} = 4\sqrt{5} - 9$$

As LHS=RHS

$$\therefore 4\sqrt{5} - 9 = a\sqrt{5} + b$$

So, $4\sqrt{5} = a\sqrt{5}; a = 4$ *and* $b = -9$

iii) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6}$

LHS,

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})^2}{\sqrt{3}^2 - \sqrt{2}^2} = \frac{\sqrt{3}^2 + \sqrt{2}^2 + 2(\sqrt{3}\times\sqrt{2})}{3-2} = \frac{3+2+2\sqrt{6}}{1} \{(a-b)^2 = a^2 + b^2 - 2ab\}$$

$$= 5 + 2\sqrt{6}$$

As LHS=RHS

$$\therefore 5 + 2\sqrt{6} = a + b\sqrt{6}$$

So, $a = 5$ $\quad$ *and* $\quad$ $2\sqrt{6} = b\sqrt{6}; b = 2$

iv) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$

LHS,

$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{(5+2\sqrt{3})(7-4\sqrt{3})}{7^2-(4\sqrt{3})^2} = \frac{5\times 7 + 5\times -4\sqrt{3} + 2\sqrt{3}\times 7 + 2\sqrt{3}\times -4\sqrt{3}}{49 - 16\times 3}$$

$$= \frac{35 - 20\sqrt{3} + 14\sqrt{3} - 8\times 3}{49-48} = \frac{35 - 24 - 6\sqrt{3}}{1} = 11 - 6\sqrt{3}$$

As LHS=RHS

$$\therefore 11 - 6\sqrt{3} = a + b\sqrt{3}$$

So, $a = 11$ $\quad$ *and* $\quad$ $-6\sqrt{3} = b\sqrt{3}; b = -6$

Answer.5. Given, $\sqrt{3} = 1.732, \sqrt{5} = 2.236, \sqrt{6} = 2.449$ *and* $\sqrt{10} = 3.162$

i) $\frac{1}{\sqrt{6}+\sqrt{5}} = \frac{1}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}^2-\sqrt{5}^2} = \frac{\sqrt{6}-\sqrt{5}}{6-5} = \frac{2.449-2.236}{1} = 0.213$

ii) $\frac{6}{\sqrt{5}+\sqrt{3}} = \frac{6}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{6(\sqrt{5}-\sqrt{3})}{\sqrt{5}^2-\sqrt{3}^2} = \frac{6(\sqrt{5}-\sqrt{3})}{5-3} = \frac{6(\sqrt{5}-\sqrt{3})}{2} = 3(2.236 - 1.732) = 3 \times 0.504 = 1.512$

iii) $\frac{1}{4\sqrt{3}-3\sqrt{5}} = \frac{1}{4\sqrt{3}-3\sqrt{5}} \times \frac{4\sqrt{3}+3\sqrt{5}}{4\sqrt{3}+3\sqrt{5}} = \frac{4\sqrt{3}+3\sqrt{5}}{(4\sqrt{3})^2-(3\sqrt{5})^2} = \frac{4\times 1.732+3\times 2.236}{16\times 3-9\times 5} = \frac{6.928+6.708}{48-45} = \frac{13.636}{3} = 4.545$

iv) $\frac{3+\sqrt{5}}{3-\sqrt{5}} = \frac{3+\sqrt{5}}{3-\sqrt{5}} = \frac{(3+\sqrt{5})^2}{3^2-\sqrt{5}^2} = \frac{3^2+\sqrt{5}^2+2(3\times\sqrt{5})}{9-5} = \frac{9+5+6\sqrt{5}}{4} = \frac{14+6\sqrt{5}}{4} = \frac{14+6\times 2.236}{4} = \frac{14+13.416}{4} = \frac{27.416}{4} = 6.854$

v) $\frac{1+2\sqrt{3}}{2-\sqrt{3}} = \frac{1+2\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{(1+2\sqrt{3})(2+\sqrt{3})}{2^2-\sqrt{3}^2} = \frac{1\times 2+1\times\sqrt{3}+2\sqrt{3}\times 2+2\sqrt{3}\times\sqrt{3}}{4-3} = \frac{2+\sqrt{3}+4\sqrt{3}+2\times\sqrt{3}^2}{1} =$
$2 + 5\sqrt{3} + 6 = 8 + 5\times 1.732 = 8 + 8.66 = 16.66$

vi) $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}\times\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}=\frac{(\sqrt{5}+\sqrt{2})^2}{\sqrt{5}^2-\sqrt{2}^2}=\frac{\sqrt{5}^2+\sqrt{2}^2+2(\sqrt{5}\times\sqrt{2})}{5-2}=\frac{5+2+2\sqrt{10}}{3}=\frac{7+2\sqrt{10}}{3}=$
$\frac{7+2\times3.162}{3}=\frac{13.324}{3}=4.441$

Answer.6.

i) $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}=\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{16}\times\sqrt{3}+\sqrt{9}\times\sqrt{2}}=\frac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}\times\frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}=$

$$\frac{7\sqrt{3}\times4\sqrt{3}+7\sqrt{3}\times-3\sqrt{2}-5\sqrt{2}\times4\sqrt{3}-5\sqrt{2}\times-3\sqrt{2}}{(4\sqrt{3})^2-(3\sqrt{2})^2}=\frac{28\times3-21\sqrt{6}-20\sqrt{6}-15\times2}{16\times3-9\times2}$$

$$=\frac{84-41\sqrt{6}+30}{48-18}=\frac{114-41\sqrt{6}}{30}$$

ii) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}=\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}\times\frac{3\sqrt{5}+2\sqrt{6}}{3\sqrt{5}+2\sqrt{6}}=\frac{2\sqrt{6}\times3\sqrt{5}+2\sqrt{6}\times2\sqrt{6}+(-\sqrt{5})\times3\sqrt{5}+(-\sqrt{5})\times2\sqrt{6}}{(3\sqrt{5})^2-(2\sqrt{6})^2}=$
$\frac{6\sqrt{30}+(2\sqrt{6})^2-3\times\sqrt{5}^2-2\sqrt{30}}{9\times5-4\times6}=\frac{4\sqrt{30}+4\times6-3\times5}{45-24}=\frac{4\sqrt{30}+24-15}{21}=\frac{4\sqrt{30}+9}{21}$

Answer.7.

i) $\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}=\frac{4+\sqrt{5}}{4-\sqrt{5}}\times\frac{4+\sqrt{5}}{4+\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}\times\frac{4-\sqrt{5}}{4-\sqrt{5}}=\frac{(4+\sqrt{5})^2}{4^2-\sqrt{5}^2}+\frac{(4-\sqrt{5})^2}{4^2-\sqrt{5}^2}=\frac{4^2+\sqrt{5}^2+2(4\times\sqrt{5})}{16-5}+$
$\frac{4^2+\sqrt{5}^2-2(4\times\sqrt{5})}{16-5}=\frac{16+5+8\sqrt{11}}{11}+\frac{16+5-8\sqrt{11}}{11}=\frac{21+8\sqrt{11}+21-8\sqrt{11}}{11}=\frac{42}{11}$

ii) $\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{2}{\sqrt{5}-\sqrt{3}}-\frac{3}{\sqrt{2}-\sqrt{5}}$

$$\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}^2-\sqrt{2}^2}=\frac{\sqrt{3}-\sqrt{2}}{3-2}=\frac{\sqrt{3}-\sqrt{2}}{1}=\sqrt{3}-\sqrt{2}$$

$$\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{2(\sqrt{5}+\sqrt{3})}{\sqrt{5}^2-\sqrt{3}^2}=\frac{2(\sqrt{5}+\sqrt{3})}{5-3}=\frac{2(\sqrt{5}+\sqrt{3})}{2}=\sqrt{5}+\sqrt{3}$$

$$\frac{3}{\sqrt{2}-\sqrt{5}}\times\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}+\sqrt{5}}=\frac{3(\sqrt{2}+\sqrt{5})}{\sqrt{2}^2-\sqrt{5}^2}=\frac{3(\sqrt{2}+\sqrt{5})}{2-5}=\frac{3(\sqrt{2}+\sqrt{5})}{-3}=-\sqrt{2}-\sqrt{5}$$

Equate to question

$$=\sqrt{3}-\sqrt{2}-(\sqrt{5}+\sqrt{3})—\sqrt{2}-\sqrt{5}=\sqrt{3}-\sqrt{2}-\sqrt{5}-\sqrt{3}+\sqrt{2}+\sqrt{5}=0$$

iii) $\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}+\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{(2+\sqrt{3})^2}{2^2-\sqrt{3}^2} + \frac{(2-\sqrt{3})^2}{2^2-\sqrt{3}^2} + \frac{(\sqrt{3}-1)^2}{\sqrt{3}^2-1^2}$

$= \frac{2^2+\sqrt{3}^2+2(2\times\sqrt{3})}{4-3} + \frac{2^2+\sqrt{3}^2-2(2\times\sqrt{3})}{4-3} + \frac{\sqrt{3}^2+1^2-2(\sqrt{3}\times1)}{3-1}$

$= \frac{4+3+4\sqrt{3}}{1} + \frac{4+3-4\sqrt{3}}{1} + \frac{3+1-2\sqrt{3}}{2}$

$= \frac{7+4\sqrt{3}}{1} + \frac{7-4\sqrt{3}}{1} + \frac{2(2-\sqrt{3})}{2} = 7 + 4\sqrt{3} + 7 - 4\sqrt{3} + 2 - \sqrt{3} = 16 - \sqrt{3}$

iv) $\frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

$= \frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} + \frac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}} \times \frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}} - \frac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$

$= \frac{2\sqrt{12}-2\sqrt{18}}{\sqrt{2}^2-\sqrt{3}^2} + \frac{6\sqrt{12}-6\sqrt{6}}{\sqrt{6}^2-\sqrt{3}^2} - \frac{8\sqrt{18}-8\sqrt{6}}{\sqrt{6}^2-\sqrt{2}^2}$

$= \frac{2\sqrt{12}-2\sqrt{18}}{2-3} + \frac{6\sqrt{12}-6\sqrt{6}}{6-3} - \frac{8\sqrt{18}-8\sqrt{6}}{6-2}$

$= \frac{2\sqrt{12}-2\sqrt{18}}{-1} + \frac{6\sqrt{12}-6\sqrt{6}}{3} - \frac{8\sqrt{18}-8\sqrt{6}}{4}$

$= \frac{-2\sqrt{12}+2\sqrt{18}}{1} + \frac{3(2\sqrt{12}-2\sqrt{6})}{3} - \frac{4(2\sqrt{18}-2\sqrt{6})}{4} = 2\sqrt{18} - 2\sqrt{12} + 2\sqrt{12} - 2\sqrt{6} - 2\sqrt{18} + 2\sqrt{6} = 0$

Answer.8.

i) $\frac{1}{3+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+1} = 1$

LHS,

$= \frac{1}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{3-\sqrt{7}}{3^2-\sqrt{7}^2} + \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}^2-\sqrt{5}^2} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}^2-\sqrt{3}^2} + \frac{\sqrt{3}-1}{\sqrt{3}^2-1^2}$

$= \frac{3-\sqrt{7}}{9-7} + \frac{\sqrt{7}-\sqrt{5}}{7-5} + \frac{\sqrt{5}-\sqrt{3}}{5-3} + \frac{\sqrt{3}-1}{3-1}$

$= \frac{3-\sqrt{7}}{2} + \frac{\sqrt{7}-\sqrt{5}}{2} + \frac{\sqrt{5}-\sqrt{3}}{2} + \frac{\sqrt{3}-1}{2} = \frac{3-\sqrt{7}+\sqrt{7}-\sqrt{5}+\sqrt{5}-\sqrt{3}+\sqrt{3}-1}{2}$

$= \frac{3-1}{2} = \frac{2}{2} = 1$

As LHS=RHS, hence Proved

ii) $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}} = 2$

LHS

$$=\frac{1}{1+\sqrt{2}}\times\frac{1-\sqrt{2}}{1-\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}\times\frac{\sqrt{3}-\sqrt{4}}{\sqrt{3}-\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}\times\frac{\sqrt{4}-\sqrt{5}}{\sqrt{4}-\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}\times\frac{\sqrt{5}-\sqrt{6}}{\sqrt{5}-\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}\times\frac{\sqrt{6}-\sqrt{7}}{\sqrt{6}-\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}\times\frac{\sqrt{7}-\sqrt{8}}{\sqrt{7}-\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}\times\frac{\sqrt{8}-\sqrt{9}}{\sqrt{8}-\sqrt{9}}$$

$$=\frac{1-\sqrt{2}}{1^2-\sqrt{2}^2}+\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}^2-\sqrt{3}^2}+\frac{\sqrt{3}-\sqrt{4}}{\sqrt{3}^2-\sqrt{4}^2}+\frac{\sqrt{4}-\sqrt{5}}{\sqrt{4}^2-\sqrt{5}^2}+\frac{\sqrt{5}-\sqrt{6}}{\sqrt{5}^2-\sqrt{6}^2}+\frac{\sqrt{6}-\sqrt{7}}{\sqrt{6}^2-\sqrt{7}^2}+\frac{\sqrt{7}-\sqrt{8}}{\sqrt{7}^2-\sqrt{8}^2}+\frac{\sqrt{8}-\sqrt{9}}{\sqrt{8}^2-\sqrt{9}^2}$$

$$=\frac{1-\sqrt{2}}{1-2}+\frac{\sqrt{2}-\sqrt{3}}{2-3}+\frac{\sqrt{3}-\sqrt{4}}{3-4}+\frac{\sqrt{4}-\sqrt{5}}{4-5}+\frac{\sqrt{5}-\sqrt{6}}{5-6}+\frac{\sqrt{6}-\sqrt{7}}{6-7}+\frac{\sqrt{7}-\sqrt{8}}{7-8}+\frac{\sqrt{8}-\sqrt{9}}{8-9}$$

$$=\frac{1-\sqrt{2}}{-1}+\frac{\sqrt{2}-\sqrt{3}}{-1}+\frac{\sqrt{3}-\sqrt{4}}{-1}+\frac{\sqrt{4}-\sqrt{5}}{-1}+\frac{\sqrt{5}-\sqrt{6}}{-1}+\frac{\sqrt{6}-\sqrt{7}}{-1}+\frac{\sqrt{7}-\sqrt{8}}{-1}+\frac{\sqrt{8}-\sqrt{9}}{-1}$$

$$=\frac{1-\sqrt{2}+\sqrt{2}-\sqrt{3}+\sqrt{3}-\sqrt{4}+\sqrt{4}-\sqrt{5}+\sqrt{5}-\sqrt{6}+\sqrt{6}-\sqrt{7}+\sqrt{7}-\sqrt{8}+\sqrt{8}-\sqrt{9}}{-1}=\frac{1-3}{-1}=\frac{-2}{-1}=2$$

As LHS=RHS, hence Proved

Answer.9.

$$\frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}=a+b\sqrt{5}$$

Solve LHS

$$\frac{7+3\sqrt{5}}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$=\frac{7\times3+7\times\left(-\sqrt{5}\right)+3\sqrt{5}\times3+3\sqrt{5}\times\left(-\sqrt{5}\right)}{3^2-\sqrt{5}^2}-\frac{7\times3+7\times\sqrt{5}+\left(-3\sqrt{5}\right)\times3+\left(-3\sqrt{5}\right)\times\sqrt{5}}{3^2-\sqrt{5}^2}$$

$$=\frac{21-7\sqrt{5}+9\sqrt{5}-3\times\sqrt{5}^2}{9-5}-\frac{21+7\sqrt{5}-9\sqrt{5}-3\times\sqrt{5}^2}{9-5}$$

$$=\frac{21+2\sqrt{5}-3\times5}{4}-\frac{21-2\sqrt{5}-3\times5}{4}=\frac{21+2\sqrt{5}-15-21+2\sqrt{5}+15}{4}$$

$$=\frac{4\sqrt{5}}{4}$$

Equate LHS to RHS

$$\frac{4\sqrt{5}}{4}=a+b\sqrt{5}\ \therefore a=0;\quad b\sqrt{5}=\sqrt{5},b=1$$

Answer.10.

$$\frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}+\sqrt{11}}+\frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}=\frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}+\sqrt{11}}\times\frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}-\sqrt{11}}+\frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}\times\frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}+\sqrt{11}}$$

$$=\frac{(\sqrt{13}-\sqrt{11})^2}{\sqrt{13}^2-\sqrt{11}^2}+\frac{(\sqrt{13}+\sqrt{11})^2}{\sqrt{13}^2-\sqrt{11}^2}$$

$$=\frac{\sqrt{13}^2+\sqrt{11}^2-2(\sqrt{13}\times\sqrt{11})}{13-11}+\frac{\sqrt{13}^2+\sqrt{11}^2+2(\sqrt{13}\times\sqrt{11})}{13-11}$$

$$=\frac{13+11-2\sqrt{143}}{2}+\frac{13+11+2\sqrt{143}}{2}=\frac{24-2\sqrt{143}+24+2\sqrt{143}}{2}=\frac{48}{2}=24$$

Answer.11. Given $x=3+2\sqrt{2}$

Put x value in $x+\frac{1}{x}$

$$x+\frac{1}{x}=3+2\sqrt{2}+\frac{1}{3+2\sqrt{2}}=3+2\sqrt{2}+\frac{1}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}=3+2\sqrt{2}+\frac{3-2\sqrt{3}}{3^2-2\sqrt{2}^2}$$

$$3+2\sqrt{2}+\frac{3-2\sqrt{3}}{9-4\times 2}=3+2\sqrt{2}+\frac{3-2\sqrt{2}}{9-8}=3+2\sqrt{2}+\frac{3-2\sqrt{2}}{1}=3+2\sqrt{2}+3-2\sqrt{2}=3+3$$
$$=6$$

It is Rational number.

Answer.12. Given$x=2-\sqrt{3}$

Put x value in $\left(x-\frac{1}{x}\right)^3$

$$(a-b)^3=a^3-b^3-3ab(a-b)$$

Here $a/x=2-\sqrt{3}$ *and* $b/\frac{1}{x}=\frac{1}{2-\sqrt{3}}$ *or* $\left(\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{2+\sqrt{3}}{2^2-\sqrt{3}^2}=\frac{2+\sqrt{3}}{4-3}=\frac{2+\sqrt{3}}{1}\right)=2+\sqrt{3}$

$$\left(x-\frac{1}{x}\right)^3=\left(2-\sqrt{3}-2-\sqrt{3}\right)^3=\left(-2\sqrt{3}\right)^3=-8\times\sqrt{27}=-8\times\sqrt{9}\times\sqrt{3}$$
$$=\quad -8\times 3\times\sqrt{3}=-24\sqrt{3}$$

Answer.13. Given$x=9-4\sqrt{5}$

Put x value in $x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2\left(x\times\frac{1}{x}\right)=\left(x+\frac{1}{x}\right)^2-2$.....(i)

$$\frac{1}{x}=\frac{1}{9-4\sqrt{5}}\times\frac{9+4\sqrt{5}}{9+4\sqrt{5}}=\frac{9+4\sqrt{5}}{9^2-\left(4\sqrt{5}\right)^2}=\frac{9+4\sqrt{5}}{81-16\times\sqrt{5}^2}=\frac{9+4\sqrt{5}}{81-80}=\frac{9+4\sqrt{5}}{1}=9+4\sqrt{5}$$

Put in equation (i)

$$\left(9-4\sqrt{5}+9+4\sqrt{5}\right)^2-2=18^2-2=324-2=322$$

Answer.14. Given$x = \frac{5-\sqrt{21}}{2}$

$$\frac{1}{x} = \frac{1}{\frac{5-\sqrt{21}}{2}} = \frac{2}{5-\sqrt{21}} = \frac{2}{5-\sqrt{21}} \times \frac{5+\sqrt{21}}{5+\sqrt{21}} = \frac{2(5+\sqrt{21})}{5^2-\sqrt{21}^2} = \frac{2(5+\sqrt{21})}{25-21} = \frac{2(5+\sqrt{21})}{4}$$
$$= \frac{(5+\sqrt{21})}{2}$$

$$x + \frac{1}{x} = \frac{5-\sqrt{21}}{2} + \frac{5+\sqrt{21}}{2} = \frac{5-\sqrt{21}+\sqrt{21}+5}{2} = \frac{10}{2} = 5$$

Answer.15. Given$a = 3 - 2\sqrt{2}$

$$a^2 = (3-2\sqrt{2})^2 = 3^2 + (2\sqrt{2})^2 - 2(3\times 2\sqrt{2}) = 9 + 4\times 2 - 12\sqrt{2} = 17 - 12\sqrt{2}$$

$$\frac{1}{a^2} = \frac{1}{(3-2\sqrt{2})^2} = \frac{1}{3^2+(2\sqrt{2})^2-2(3\times 2\sqrt{2})} = \frac{1}{9+4\times 2-12\sqrt{2}} = \frac{1}{9+8-12\sqrt{2}}$$
$$= \frac{1}{17-12\sqrt{2}}$$

Rationalise

$$\frac{1}{17-12\sqrt{2}} \times \frac{17+12\sqrt{2}}{17+12\sqrt{2}} = \frac{17+12\sqrt{2}}{17^2-(12\sqrt{2})^2} = \frac{17+12\sqrt{2}}{289-144\times 2} = \frac{17+12\sqrt{2}}{289-288} = \frac{17+12\sqrt{2}}{1}$$
$$= 17 + 12\sqrt{2}$$

$$a^2 - \frac{1}{a^2} = 17 - 12\sqrt{2} - (17+12\sqrt{2}) = 17 - 12\sqrt{2} - 17 - 12\sqrt{2} = -24\sqrt{2}$$

Answer.16. Given$x = \sqrt{13} + 2\sqrt{3}$

$$\frac{1}{x} = \frac{1}{\sqrt{13}+2\sqrt{3}} \times \frac{\sqrt{13}-2\sqrt{3}}{\sqrt{13}-2\sqrt{3}} = \frac{\sqrt{13}-2\sqrt{3}}{\sqrt{13}^2-(2\sqrt{3})^2} = \frac{13-2\sqrt{3}}{13-4\times 3} = \frac{13-2\sqrt{3}}{13-12} = 13 - 2\sqrt{3}$$

$$x - \frac{1}{x} = \sqrt{13} + 2\sqrt{3} - (\sqrt{13} - 2\sqrt{3}) = \sqrt{13} + 2\sqrt{3} - \sqrt{13} + 2\sqrt{3} = 4\sqrt{3}$$

Answer.17. Given$x = 2 + \sqrt{3}$

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-\sqrt{3}^2} = \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$$

$$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\left(x\times\frac{1}{x}\right)\left(x+\frac{1}{x}\right)=\left(x+\frac{1}{x}\right)^3-3(1)\left(x+\frac{1}{x}\right)$$

$$=\left(2+\sqrt{3}+2-\sqrt{3}\right)^3-3\left(2+\sqrt{3}+2-\sqrt{3}\right)$$

$$=(4)^3-3(4)=64-12=52$$

Answer.18. Given $x=\frac{5-\sqrt{3}}{5+\sqrt{3}}$ *and* $y=\frac{5+\sqrt{3}}{5-\sqrt{3}}$

$$x=\frac{5-\sqrt{3}}{5+\sqrt{3}}\times\frac{5-\sqrt{3}}{5-\sqrt{3}}=\frac{\left(5-\sqrt{3}\right)^2}{5^2-\sqrt{3}^2}=\frac{5^2+\sqrt{3}^2-2\left(5\times\sqrt{3}\right)}{25-3}=\frac{25+3-10\sqrt{3}}{22}=\frac{28-10\sqrt{3}}{22}$$

$$=\frac{2\left(14-5\sqrt{3}\right)}{22}=\frac{14-5\sqrt{3}}{11}$$

$$y=\frac{5+\sqrt{3}}{5-\sqrt{3}}\times\frac{5+\sqrt{3}}{5+\sqrt{3}}=\frac{\left(5+\sqrt{3}\right)^2}{5^2-\sqrt{3}^2}=\frac{5^2+\sqrt{3}^2+2\left(5\times\sqrt{3}\right)}{25-3}=\frac{25+3+10\sqrt{3}}{22}=\frac{28+10\sqrt{3}}{22}$$

$$=\frac{2\left(14+5\sqrt{3}\right)}{22}=\frac{14+5\sqrt{3}}{11}$$

$$x-y=\frac{14-5\sqrt{3}}{11}-\frac{14+5\sqrt{3}}{11}=\frac{14-5\sqrt{3}-14-5\sqrt{3}}{11}=\frac{-10\sqrt{3}}{11}$$

Answer.19. Given $a=\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ *and* $b=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

Equation $3a^2+4ab-3b^2$

$$3\left(\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}\right)^2+4\left(\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}\right)-3\left(\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}\right)^2$$

$$3\left(\left(\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}\right)^2-\left(\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}\right)^2\right)+4=3\left(\frac{\left(7+2\sqrt{10}\right)^2-\left(7-2\sqrt{10}\right)^2}{7^2-\left(2\sqrt{10}\right)^2}\right)+4$$

$$=3\left(\frac{49+40+28\sqrt{10}-\left(49+40-28\sqrt{10}\right)}{49-40}\right)+4$$

$$=3\left(\frac{49+40+28\sqrt{10}-49-40+28\sqrt{10}}{9}\right)+4=3\left(\frac{56\sqrt{10}}{9}\right)+4$$

$$=\frac{56\sqrt{10}}{3}+4$$

Answer.20.

$a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}, b = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Rationalise

$$a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}-\sqrt{2})^2}{\sqrt{3}^2-\sqrt{2}^2} = \frac{\sqrt{3}^2+\sqrt{2}^2-2(\sqrt{3}\times\sqrt{2})}{3-2} = \frac{3+2-2\sqrt{6}}{1} = 5-2\sqrt{6}$$

$$b = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})^2}{\sqrt{3}^2-\sqrt{2}^2} = \frac{\sqrt{3}^2+\sqrt{2}^2+2(\sqrt{3}\times\sqrt{2})}{3-2} = \frac{3+2+2\sqrt{6}}{1} = 5+2\sqrt{6}$$

$$a^2+b^2-5ab = (5-2\sqrt{6})^2+(5+2\sqrt{6})^2-5[(5-2\sqrt{6})(5+2\sqrt{6})]$$

$$= 5^2+(2\sqrt{6})^2-2(5\times 2\sqrt{6})+5^2+(2\sqrt{6})^2+2(5\times 2\sqrt{6})-5\left(5^2-(2\sqrt{6})^2\right)$$

$$= 25+4\times 6-20\sqrt{6}+25+4\times 6+20\sqrt{6}-5(25-4\times 6)$$

$$= 25+24-20\sqrt{6}+25+24-20\sqrt{6}-5(25-24)$$

$$= 50+48-5 = 98-5 = 93$$

Answer.21 $p = \frac{3-\sqrt{5}}{3+\sqrt{5}}, q = \frac{3+\sqrt{5}}{3-\sqrt{5}}$

Rationalise

$$p = \frac{3-\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{(3-\sqrt{5})^2}{3^2-\sqrt{5}^2} = \frac{3^2+\sqrt{5}^2-2(3\times\sqrt{5})}{9-5} = \frac{9+5-6\sqrt{5}}{4} = \frac{14-6\sqrt{5}}{4}$$
$$= \frac{7-3\sqrt{5}}{2}$$

$$q = \frac{3+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{(3+\sqrt{5})^2}{3^2-\sqrt{5}^2} = \frac{3^2+\sqrt{5}^2+2(3\times\sqrt{5})}{9-5} = \frac{9+5+6\sqrt{5}}{4} = \frac{14+6\sqrt{5}}{4}$$
$$= \frac{7+3\sqrt{5}}{2}$$

$$p.q = \frac{7-3\sqrt{5}}{2} \times \frac{7+3\sqrt{5}}{2} = \frac{7^2-(3\sqrt{5})^2}{4} = \frac{49-45}{4} = \frac{4}{4} = 1$$

$$p^2+q^2 = (p+q)^2-2.p.q$$

$$= \left(\frac{7-3\sqrt{5}}{2}+\frac{7+3\sqrt{5}}{2}\right)^2-2\times 1 = \left(\frac{7-3\sqrt{5}+7+3\sqrt{5}}{2}\right)^2-2 = \left(\frac{14}{2}\right)^2-2 = 7^2-2 = 49-2$$
$$= 47$$

Answer.22.

i) $$\frac{1}{\sqrt{7}+\sqrt{6}-\sqrt{13}}=\frac{1}{(\sqrt{7}+\sqrt{6})-\sqrt{13}}\times\frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{(\sqrt{7}+\sqrt{6})+\sqrt{13}}=\frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{(\sqrt{7}+\sqrt{6})^2-\sqrt{13}^2}=\frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{\left(\sqrt{7}^2+\sqrt{6}^2+2(\sqrt{7}\times\sqrt{6})\right)-13}$$

$$=\frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{(7+6+2\sqrt{42})-13}=\frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{13+2\sqrt{42}-13}=\frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{2\sqrt{42}}=\frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{2\sqrt{42}}\times\frac{\sqrt{42}}{\sqrt{42}}$$

$$\frac{\sqrt{7}\times\sqrt{42}+\sqrt{6}\times\sqrt{42}+\sqrt{13}\times\sqrt{42}}{2\times\sqrt{42}^2}=\frac{\sqrt{294}+\sqrt{252}+\sqrt{546}}{2\times 42}$$

$$=\frac{\sqrt{49}\times\sqrt{6}+\sqrt{36}\times\sqrt{7}+\sqrt{546}}{84}=\frac{7\sqrt{6}+6\sqrt{7}+\sqrt{546}}{84}$$

ii) $$\frac{3}{\sqrt{3}+\sqrt{5}-\sqrt{2}}=\frac{3}{(\sqrt{3}+\sqrt{5})-\sqrt{2}}\times\frac{(\sqrt{3}+\sqrt{5})+\sqrt{2}}{(\sqrt{3}+\sqrt{5})+\sqrt{2}}=\frac{3\left((\sqrt{3}+\sqrt{5})+\sqrt{2}\right)}{(\sqrt{3}+\sqrt{5})^2-\sqrt{2}^2}=\frac{3\left((\sqrt{3}+\sqrt{5})+\sqrt{2}\right)}{\left(\sqrt{3}^2+\sqrt{5}^2+2(\sqrt{3}+\sqrt{5})\right)-2}$$

$$\Rightarrow\frac{3\left((\sqrt{3}+\sqrt{5})+\sqrt{2}\right)}{6+2\sqrt{3}\sqrt{5}}=\frac{3}{2}\times\frac{(\sqrt{3}+\sqrt{5})+\sqrt{2}}{3+\sqrt{3}\sqrt{5}}=\frac{3}{2}\times\frac{(\sqrt{3}+\sqrt{5})+\sqrt{2}}{3+\sqrt{15}}\times\frac{3-\sqrt{15}}{3-\sqrt{15}}$$

$$\Rightarrow\frac{3}{2}\times\frac{(\sqrt{3}+\sqrt{5}+\sqrt{2})(3-\sqrt{15})}{9-15}=\frac{3}{2}\times\frac{(\sqrt{3}+\sqrt{5}+\sqrt{2})(3-\sqrt{15})}{-6}=-\frac{1}{4}(3\sqrt{3}+3\sqrt{5}+3\sqrt{2}-3\sqrt{5}-5\sqrt{3}-\sqrt{30})$$

$$\Rightarrow-\frac{1}{4}(-2\sqrt{3}+3\sqrt{2}-\sqrt{30})=\frac{2\sqrt{3}-3\sqrt{2}+\sqrt{30}}{4}$$

iii) $$\frac{4}{2+\sqrt{3}+\sqrt{7}}=\frac{4}{(2+\sqrt{3})+\sqrt{7}}\times\frac{(2+\sqrt{3})-\sqrt{7}}{(2+\sqrt{3})-\sqrt{7}}=\frac{4\left((2+\sqrt{3})-\sqrt{7}\right)}{(2+\sqrt{3})^2-\sqrt{7}^2}=\frac{4\left((2+\sqrt{3})-\sqrt{7}\right)}{\left(2^2+\sqrt{3}^2+2(2\times\sqrt{3})\right)-7}=\frac{4\left((2+\sqrt{3})-\sqrt{7}\right)}{(4+3+4\sqrt{3})-7}$$

$$=\frac{4\left((2+\sqrt{3})-\sqrt{7}\right)}{7+4\sqrt{3}-7}=\frac{4\left((2+\sqrt{3})-\sqrt{7}\right)}{4\sqrt{3}}=\frac{\left((2+\sqrt{3})-\sqrt{7}\right)}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{2\times\sqrt{3}+\sqrt{3}^2-\sqrt{7}\times\sqrt{3}}{\sqrt{3}^2}=\frac{2\sqrt{3}+3-\sqrt{21}}{3}$$

Answer.23. Given $\sqrt{2}=1.414\ \ and\ \sqrt{6}=2.449$,

$$\frac{1}{\sqrt{3}-\sqrt{2}-1}=\frac{1}{\sqrt{3}-(\sqrt{2}-1)}\times\frac{\sqrt{3}+(\sqrt{2}-1)}{\sqrt{3}+(\sqrt{2}-1)}=\frac{\sqrt{3}+(\sqrt{2}-1)}{\sqrt{3}^2-\left(\sqrt{2}^2+1^2-2.\sqrt{2}.1\right)}$$

$$=\frac{\sqrt{3}+(\sqrt{2}-1)}{3-(2+1-2\sqrt{2})}=\frac{\sqrt{3}+(\sqrt{2}-1)}{3-3+2\sqrt{2}}$$

$$\frac{\sqrt{3}+(\sqrt{2}-1)}{2\sqrt{2}}\times\frac{2\sqrt{2}}{2\sqrt{2}}=\frac{2\left(\sqrt{6}+\sqrt{2}^2-\sqrt{2}\right)}{4\times\sqrt{2}^2}=\frac{\sqrt{6}+2-\sqrt{2}}{2\times 2}=\frac{\sqrt{6}+2-\sqrt{2}}{4}$$
$$=\frac{2.449+2-1.414}{4}=\frac{3.035}{4}=0.758$$

Answer.24. Given$x=\frac{1}{2-\sqrt{3}}$

Rationalise

$$x=\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{2+\sqrt{3}}{2^2-\sqrt{3}^2}=\frac{2+\sqrt{3}}{4-3}=2+\sqrt{3}$$

⇨ $x^3-2x^2-7x+5=(2+\sqrt{3})^3-2(2+\sqrt{3})^2-7(2+\sqrt{3})+5$

$$=2^3+\sqrt{3}^3+3[(2\times\sqrt{3})(2+\sqrt{3})]-2\left(2^2+\sqrt{3}^2+2(2\times\sqrt{3})\right)-14-7\sqrt{3}+5$$

$$=8+3\sqrt{3}+6\sqrt{3}(2+\sqrt{3})-2(4+3+4\sqrt{3})-9-7\sqrt{3}$$

$$=8+3\sqrt{3}+12\sqrt{3}+6.\sqrt{3}^2-2(7+4\sqrt{3})-9-7\sqrt{3}$$

$$=8+15\sqrt{3}+18-14-8\sqrt{3}-9-7\sqrt{3}=26-14-9+15\sqrt{3}-15\sqrt{3}=26-23=3$$

Answer.25.Given$\sqrt{5}=2.236\ and\ \sqrt{10}=3.162$

$$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}=\frac{15}{\sqrt{10}+\sqrt{4}\times\sqrt{5}+\sqrt{4}+\sqrt{10}-\sqrt{5}-\sqrt{16}\times\sqrt{5}}$$
$$=\frac{15}{\sqrt{10}+2\sqrt{5}+2\sqrt{10}-\sqrt{5}-4\sqrt{5}}$$

$$=\frac{15}{3\sqrt{10}-3\sqrt{5}}=\frac{15}{3(\sqrt{10}-\sqrt{5})}=\frac{5}{\sqrt{10}-\sqrt{5}}=\frac{5}{3.162-2.236}=\frac{5}{0.926}=5.399(approx.)$$

EXERCISE 1G

Answer 1 –:

(i) –: $2^{\frac{2}{3}} \times 2^{\frac{1}{3}} = 2^{\frac{2}{3}+\frac{1}{3}} = 2^1 = 2$ $\quad \because a^m + a^n = a^{m+n}$

(ii) –: $2^{\frac{2}{3}} \times 2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} = 2^{\frac{13}{15}}$ $\quad \because a^m + a^n = a^{m+n}$

(iii) –: $7^{\frac{5}{6}} \times 7^{\frac{2}{3}} = 7^{\frac{5}{6}+\frac{2}{3}} = 7^{\frac{9}{6}} = 7^{\frac{3}{2}}$ $\quad \because a^m + a^n = a^{m+n}$

(iv) –: $(1296)^{\frac{1}{4}} \times (1296)^{\frac{1}{2}} = (1296)^{\frac{1}{4}+\frac{1}{2}} = (1296)^{\frac{3}{4}} = (6^4)^{\frac{3}{4}} = 6^{4\times\frac{3}{4}} = 6^3$
$= 216$ $\quad \because a^m + a^n = a^{(m+n)} \& (a^m)^n = a^{mn}$

Answer2 –:

(i) –: $\dfrac{6^{\frac{1}{4}}}{6^{\frac{1}{5}}} = 6^{\left(\frac{1}{4}-\frac{1}{5}\right)} = 6^{\frac{1}{20}}$ $\quad \because \dfrac{a^m}{a^n} = a^{(m-n)}$

(ii) –: $\dfrac{8^{\frac{1}{2}}}{8^{\frac{2}{3}}} = 8^{\frac{1}{2}-\frac{2}{3}} = 8^{\frac{3-2}{8}} = 8^{\left(-\frac{1}{6}\right)}$ $\quad \because \dfrac{a^m}{a^n} = a^{(m-n)}$

(iii) –: $\dfrac{5^{\frac{6}{7}}}{5^{\frac{2}{3}}} = 5^{\left(\frac{6}{7}-\frac{2}{3}\right)} = 5^{\frac{4}{21}}$ $\quad \because \dfrac{a^m}{a^n} = a^{(m-n)}$

Answer 3 –: (i) –: $3^{\frac{1}{4}} \times 5^{\frac{1}{4}} = (3 \times 5)^{\frac{1}{4}} = 15^{\frac{1}{4}}$ $\quad \because a^m \times b^m = (ab)^m$

(ii) –: $2^{\frac{5}{6}} \times 3^{\frac{5}{8}} = (2 \times 3)^{\frac{5}{8}} = 6^{\frac{5}{8}}$ $\quad \because a^m \times b^m = (ab)^m$

(iii) –: $6^{\frac{1}{2}} \times 7^{\frac{1}{2}} = (6 \times 7)^{\frac{1}{2}} = 42^{\frac{1}{2}}$ $\quad \because a^m \times b^m = (ab)^m$

Answer4 –: (i) –: $(3^4)^{\frac{1}{4}} = 3^{4\times\frac{1}{4}} = 3^1 = 3$ $\quad \because (a^m)^n = a^{(mn)}$

(ii) –: $(3^{\frac{1}{3}})^4 = 3^{\frac{1}{3}\times 4} = 3^{\frac{4}{3}}$ $\quad \because (a^m)^n = a^{(mn)}$

(iii) –: $\left(\dfrac{1}{3^4}\right)^{\frac{1}{2}} = \left(\dfrac{1}{3}\right)^{4\times\frac{1}{2}} = \left(\dfrac{1}{3}\right)^2 = \dfrac{1}{9}$ $\quad \because (a^m)^n = a^{(mn)}$

Answer 5 –: (i) –: $(125)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5^{\left(3\times\frac{1}{3}\right)} = 5^1 = 5$ $\quad \because (a^m)^n = a^{(mn)}$

(ii) –: $(64)^{\frac{1}{6}} = (2^6)^{\frac{1}{6}} = 2^{\left(6\times\frac{1}{6}\right)} = 2^1 = 2$ $\quad \because (a^m)^n = a^{(mn)}$

(iii) –: $(25)^{\frac{3}{2}} = (5^2)^{\frac{3}{2}} = 5^{\left(2\times\frac{3}{2}\right)} = 5^3 = 125$ $\quad \because (a^m)^n = a^{(mn)}$

(iv) –: $(81)^{\frac{3}{4}} = (3^4)^{\frac{3}{4}} = 3^{\left(4\times\frac{3}{4}\right)} = 3^3 = 27$ $\quad \because (a^m)^n = a^{(mn)}$

(v) –: $(64)^{-\frac{1}{2}} = (8^2)^{-\frac{1}{2}} = 8^{-\left(2\times\frac{1}{2}\right)} = 8^{-1} = \dfrac{1}{8}$ $\quad \because (a^m)^n = a^{(mn)} \& a^{-1} = \dfrac{1}{a}$

(vi) −: $(8)^{-\frac{1}{3}} = (2^3)^{-\frac{1}{3}} = 2^{-\left(3\times\frac{1}{3}\right)} = 2^{-1} = \frac{1}{2}$ $\because (a^m)^n = a^{(mn)} \& a^{-1} = \frac{1}{a}$

Answer6 −: Given $a = 2, b = 3$

(i) −: $\left(a^b + b^a\right)^{-1} = \frac{1}{2^3 + 3^2} = \frac{1}{8+9} = \frac{1}{17}$ $\because a^{-1} = \frac{1}{a}$

(ii) −: $\left(a^a + b^b\right)^{-1} = \frac{1}{2^2 + 3^3} = \frac{1}{4+27} = \frac{1}{31}$ $\because a^{-1} = \frac{1}{a}$

Answer7

$$\text{(i)} -: \left(\frac{81}{49}\right)^{-\frac{3}{2}} = \left(\left(\frac{9}{7}\right)^2\right)^{-\frac{3}{2}} = \left(\frac{9}{7}\right)^{-\frac{3}{2}\times 2} = \left(\frac{9}{7}\right)^{-3} = \left(\frac{7}{9}\right)^3 = \frac{343}{729}$$

$$\because (a^m)^n = a^{(mn)} \& a^{-1} = \frac{1}{a}$$

(ii) −: $(14641)^{0.25} = (14641)^{\frac{1}{4}} = (11^4)^{\frac{1}{4}} = 11^{4\times\frac{1}{4}} = 11^1 = 11$

$$\because (a^m)^n = a^{(mn)}$$

(iii) −: $\left(\frac{32}{243}\right)^{-\frac{4}{5}} = \left(\left(\frac{2}{3}\right)^5\right)^{-\frac{4}{5}} = \left(\frac{2}{3}\right)^{-\left(5\times\frac{4}{5}\right)} = \left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4 = \frac{81}{16}$

$$\because (a^m)^n = a^{(mn)} \& a^{-1} = \frac{1}{a}$$

(iv) −: $\left(\frac{7776}{243}\right)^{-\frac{3}{5}} = \left(\left(\frac{6}{3}\right)^5\right)^{-\frac{3}{5}} = \left(\frac{6}{3}\right)^{-\frac{3}{5}\times 5} = \left(\frac{3}{6}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$$\because (a^m)^n = a^{(mn)} \& a^{-1} = \frac{1}{a}$$

Answer8

(i) −: $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}} = 4 \times (216)^{\frac{2}{3}} + 1 \times (256)^{\frac{3}{4}} + 2 \times (243)^{\frac{1}{5}}$

$$\because (a^m)^n = a^{(mn)} \& a^{-1} = \frac{1}{a}$$

$$= 4 \times (6^3)^{\frac{2}{3}} + 1 \times (4^4)^{\frac{3}{4}} + 2 \times (3^5)^{\frac{1}{5}} = 4 \times (6)^{3\times\frac{2}{3}} + 1 \times (4)^{4\times\frac{3}{4}} + 2 \times (3)^{5\times\frac{1}{5}}$$

$$= 4 \times 6^2 + 1 \times +2 \times 3 = 4 \times 36 + 1 \times 64 + 6 = 214$$

(ii) −: $\left(\frac{64}{125}\right)^{-\frac{2}{3}} + \left(\frac{256}{625}\right)^{-\frac{1}{4}} + \left(\frac{3}{7}\right)^0 = \left(\left(\frac{4}{5}\right)^3\right)^{-\frac{2}{3}} + \left(\left(\frac{4}{5}\right)^4\right)^{-\frac{1}{4}} + 1$ $\{\because a^0 = 1\}$

$$= \left(\frac{4}{5}\right)^{3\times\left(-\frac{2}{3}\right)} + \left(\frac{4}{5}\right)^{4\times\left(-\frac{1}{4}\right)} + 1 = \left(\frac{4}{5}\right)^{-2} + \left(\frac{4}{5}\right)^{-1} + 1 = \frac{25}{16} + \frac{5}{4} + 1 = \frac{61}{16}$$

$$\because (a^m)^n = a^{(mn)} \& a^{-1} = \frac{1}{a}$$

(iii) −: $\left(\frac{81}{16}\right)^{-\frac{3}{4}}\left[\frac{\left(\frac{25}{9}\right)^{-\frac{3}{2}}}{\left(\frac{5}{2}\right)^{-3}}\right] = \left(\left(\frac{3}{2}\right)^{4}\right)^{\left(-\frac{3}{4}\right)}\left[\frac{\left(\frac{5}{3}\right)^{2\left(-\frac{3}{2}\right)}}{\left(\frac{5}{2}\right)^{-3}}\right] = \left(\frac{3}{2}\right)^{-4\times\frac{3}{4}}\left[\frac{\left(\frac{5}{3}\right)^{-2\times\frac{3}{2}}}{\left(\frac{5}{2}\right)^{-3}}\right] = \left(\frac{3}{2}\right)^{-3}\left[\frac{\left(\frac{5}{3}\right)^{-3}}{\left(\frac{5}{2}\right)^{-3}}\right]$

$= \frac{8}{27}\times\frac{27}{125}\times\frac{125}{8} = 1 \qquad \because (a^m)^n = a^{(mn)} \& a^{-1} = \frac{1}{a}$

(iv) −: $\frac{(25)^{\frac{5}{2}}\times(729)^{\frac{1}{3}}}{(125)^{\frac{2}{3}}\times(27)^{\frac{2}{3}}\times(8)^{\frac{4}{3}}} = \frac{(5^2)^{\frac{5}{2}}\times(9^3)^{\frac{1}{3}}}{(5^3)^{\frac{2}{3}}\times(3^3)^{\frac{2}{3}}\times(2^3)^{\frac{4}{3}}} = \frac{5^5\times 9^1}{5^2\times 3^2\times 2^4} = \frac{125}{16}$

$\because (a^m)^n = a^{(mn)}$

Answer 9 −: (i) −: $(1^3+2^3+3^3)^{\frac{1}{2}} = 36^{\frac{1}{2}} = 6$

(ii) −: $\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}} = \left[5\left((2^3)^{\frac{1}{3}}+(3^3)^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}} = \left[5\left(2^{\left(3\times\frac{1}{3}\right)}+3^{\left(3\times\frac{1}{3}\right)}\right)^3\right]^{\frac{1}{4}}$

$= [5(2^1+3^1)^3]^{\frac{1}{4}} = [5(5)^3]^{\frac{1}{4}} = [5\times 125]^{\frac{1}{4}} = \left[5^{4\times\frac{1}{4}}\right] = 5$

(iiii) −: $\frac{2^0+7^0}{5^0} = \frac{1+1}{1} = 2 \qquad \because \ a^0 = 1$

(iv) −: $\left[(16)^{\frac{1}{2}}\right]^{\frac{1}{2}} = 4^{\frac{1}{2}} = 2$

Answer10

(i) −: LHS $= \left[8^{-\frac{2}{3}}\times 2^{\frac{1}{2}}\times 25^{-\frac{5}{4}}\right]\div\left[32^{-\frac{2}{5}}\times 125^{-\frac{5}{6}}\right]$

$= \left[(2^3)^{-\frac{2}{3}}\times 2^{\frac{1}{2}}\times(5^2)^{-\frac{5}{4}}\right]\div\left[\left((2^5)^{-\frac{2}{5}}\right)\times(5^3)^{-\frac{5}{6}}\right]$

$= \frac{\left[2^{-2}\times 2^{\frac{1}{2}}\times 5^{-\frac{5}{2}}\right]}{\left[2^{-2}\times 5^{-\frac{5}{2}}\right]} = 2^{\frac{1}{2}} = \sqrt{2} = \text{RHS}$

(ii) −: LHS $= \left(\frac{64}{125}\right)^{-\frac{2}{3}} + \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \frac{\sqrt{25}}{\sqrt[3]{64}} = \left(\frac{125}{64}\right)^{\frac{2}{3}} + \frac{1}{\left(\left(\frac{4}{5}\right)^4\right)^{\frac{1}{4}}} + \frac{5}{4} = \left(\frac{5}{4}\right)^{3\times\frac{2}{3}} + \frac{1}{\frac{4}{5}^{4\times\frac{1}{4}}} + \frac{5}{4}$

$= \frac{25}{16}+\frac{5}{4}+\frac{5}{4} = \frac{65}{16} = \text{RHS}$

(iiii) −: LHS $= \left[7\left\{(81)^{\frac{1}{4}}+(256)^{\frac{1}{4}}\right\}^{\frac{1}{4}}\right]^4 = \left[7\left\{3^{4\times\frac{1}{4}}+4^{4\times\frac{1}{4}}\right\}^{\frac{1}{4}}\right]^4$

$= \left[7\{3+4\}^{\frac{1}{4}}\right]^4 = \left[7^{1+\frac{1}{4}}\right]^4 = 7^{\frac{5}{4}\times 4} = 7^5 = 16807 = \text{RHS}$

Answer 11 −: $\sqrt[4]{\sqrt[3]{x^2}} = \left((x^2)^{\frac{1}{3}}\right)^{\frac{1}{4}} = x^{2\times\frac{1}{3}\times\frac{1}{4}} = x^{\frac{1}{6}}$ $\qquad \because ((a^m)^n)^p = a^{mnp}$

Answer12 −: $\sqrt[3]{2}.\sqrt[4]{2}.\sqrt[12]{32} = 2^{\frac{1}{3}}.2^{\frac{1}{4}}.(32)^{\frac{1}{12}} = 2^{\frac{1}{3}}.2^{\frac{1}{4}}.(2)^{\frac{5}{12}} = 2^{\frac{1}{3}+\frac{1}{4}+\frac{5}{12}} = 2^{\frac{12}{12}} = 2$

Answer13 −: (i) −: $\left(\frac{(15)^{\frac{1}{3}}}{9^{\frac{1}{4}}}\right)^{-6} = \frac{\left((9)^{\frac{1}{4}}\right)^6}{\left((15)^{\frac{1}{3}}\right)^6} = \frac{3^{2\times\frac{1}{4}\times 6}}{15^{\frac{1}{3}\times 6}} = \frac{3^3}{15^2} = \frac{27}{225} = \frac{3}{25}$

(ii) −: $\left(\frac{(12)^{\frac{1}{5}}}{27^{\frac{1}{5}}}\right)^{\frac{5}{2}} = \frac{\left((12)^{\frac{1}{5}}\right)^{\frac{5}{2}}}{\left((27)^{\frac{1}{5}}\right)^{\frac{5}{2}}} = \frac{12^{\frac{1}{5}\times\frac{5}{2}}}{27^{\frac{1}{5}\times\frac{5}{2}}} = \frac{12^{\frac{1}{2}}}{27^{\frac{1}{2}}} = \left(\frac{12}{27}\right)^{\frac{1}{2}} = \frac{4^{\frac{1}{2}}}{9} = \frac{2}{3}$

(iii) −: $\left(\frac{(15)^{\frac{1}{4}}}{3^{\frac{1}{2}}}\right)^{-2} = \frac{\left((3)^{\frac{1}{2}}\right)^2}{\left((15)^{\frac{1}{4}}\right)^2} = \frac{3^{2\times\frac{1}{2}}}{15^{\frac{1}{4}\times 2}} = \frac{3^1}{15^{\frac{1}{2}}} = \frac{3}{15^{\frac{1}{2}}}$

Answer 14 −: (i) −: $\sqrt[5]{5x+2} = 2$

$$5x + 2 = 2^5$$

$$5x = 32 - 2 = 30$$

$$x = \frac{30}{5} = 6$$

(ii) −: $\sqrt[3]{3x-2} = 4$

$$3x - 2 = 4^3$$

$$3x = 64 + 2 = 66$$

$$x = \frac{66}{3} = 22$$

(iii) −: $\left(\frac{3}{4}\right)^3\left(\frac{4}{3}\right)^{-7} = \left(\frac{3}{4}\right)^{2x}$

$\left(\frac{3}{4}\right)^3\left(\frac{3}{4}\right)^7 = \left(\frac{3}{4}\right)^{2x}$

$$\left(\frac{3}{4}\right)^{3+7} = \left(\frac{3}{4}\right)^{2x}$$

$$2x = 10$$

$$x = 5$$

(iv) −: $5^{x-3} \times 3^{2x-8} = 225$

$5^{x-3} \times 3^{2x-8} = 25 \times 9 = 5^2 \times 3^2$

$$x - 3 = 2 \quad \& \; 2x - 8 = 2$$

$$x = 5 \quad \& \quad x = \frac{10}{2} = 5$$

(v) −: $\dfrac{3^{3x}.3^{2x}}{3^{x}} = \sqrt[4]{3^{20}}$

$3^{3x+2x-x} = 3^{\frac{20}{4}} = 3^{5}$

$3^{4x} = 3^{5}$

$$4x = 5$$

$$x = \frac{5}{4}$$

Answer15 −: (i) −: $\text{LHS} = \sqrt{x^{-1}y}.\sqrt{y^{-1}z}.\sqrt{z^{-1}x} = \sqrt{\frac{y}{x}}.\sqrt{\frac{z}{y}}.\sqrt{\frac{x}{z}} = \sqrt{\frac{xyz}{xyz}} = 1 = \text{RHS}$

$$\because a^{-1} = \frac{1}{a}$$

(ii) −: $\text{LHS} = \left(x^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}}.\left(x^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}}.\left(x^{\frac{1}{c-a}}\right)^{\frac{1}{c-b}}$

$$= x^{\frac{1}{a-b}\times\frac{1}{a-c}}.x^{\frac{1}{b-c}\times\frac{1}{b-a}}.x^{\frac{1}{c-a}\times\frac{1}{c-b}} = (x)^{\left(\frac{1}{(a-b)(a-c)}+\frac{1}{(b-c)(b-a)}+\frac{1}{(c-a)(c-b)}\right)}$$

$$= (x)^{\left(-\frac{1}{(a-b)(c-a)}-\frac{1}{(b-c)(a-b)}-\frac{1}{(c-a)(b-c)}\right)} = (x)^{\left(\frac{-(b-c)-(c-a)-(a-b)}{(a-b)(b-c)(c-a)}\right)} = x^{\frac{-b+c-c+a-a+b}{(a-b)(b-c)(c-a)}}$$

$$= x^{\frac{0}{(a-b)(b-c)(c-a)}} = x^{0} = 1$$

(iii) −: $\text{LHS} = \dfrac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\dfrac{x^{b}}{x^{a}}\right)^{c} = \dfrac{x^{ab-ac}}{x^{ab-bc}} \div \left(x^{b-a}\right)^{c} = \dfrac{x^{(ab-ac-ab+bc)}}{x^{bc-ac}}$

$$= x^{(ab-ab+bc-bc+ac-ac)} = x^{0} = 1 = \text{RHS}$$

(iv) −: $\text{LHS} = \dfrac{\left(x^{a+b}\right)^{2}\left(x^{b+c}\right)^{2}\left(x^{c+a}\right)^{2}}{(x^{a}x^{b}x^{c})^{4}} = \dfrac{x^{(2a+2b)}x^{(2b+2c)}x^{(2c+2a)}}{x^{4a}x^{4b}x^{4c}}$

$$= x^{(2a+2a+2b+2b+2c+2c-4a-4b-4c)} = x^{(4a-4a+4b-4b+4c-4c)} = x^{0} = 1 = \text{RHS}$$

16 −: $\left(\dfrac{x^{b}}{x^{c}}\right)^{b+c-a}.\left(\dfrac{x^{c}}{x^{a}}\right)^{c+a-b}.\left(\dfrac{x^{a}}{x^{b}}\right)^{a+b-c} = \left(x^{b-c}\right)^{b+c-a}.(x^{c-a})^{c+a-b}.\left(x^{a-b}\right)^{a+b-c}$

$$= x^{(b-c)(b+c-a)}.x^{(c-a)(c+a-b)}.x^{(a-b)(a+b-c)}$$

$$= x^{b^2+bc-ab-bc-c^2+ac}.x^{c^2+ac-bc-ac-a^2+ab}.x^{a^2+ab-ac-ab-b^2+bc}$$

$$= x^{a^2-a^2+b^2-b^2+c^2-c^2+ab-ab+ac-ac+bc-bc} = x^{0} = 1$$

Answer17 −: $\dfrac{\left(9^{n}\times3^{2}\times\left(3^{-\frac{n}{2}}\right)^{-2}-(27)^{n}\right)}{3^{3m}\times2^{3}} = \dfrac{1}{27}$

$$= \frac{\left(3^{2n}\times3^{2}\times(3)^{-\frac{n}{2}\times-2}-(3^{3})^{n}\right)}{3^{3m}\times2^{3}} = \frac{1}{27}$$

$$= \frac{(3^{2n+2} \times 3^{n} - 3^{3n})}{3^{3m} \times 2^{3}} = \frac{1}{27}$$

$$= \frac{3^{3n+2} - 3^{3n}}{3^{3m} \times 2^{3}} = \frac{1}{27}$$

$$= \frac{\left(3^{3n}(3^{2} - 1)\right)}{3^{3m} \times 8} = \frac{1}{27}$$

$$= \frac{(3^{3n} \times 8)}{(3^{3m} \times 8)} = \frac{1}{3^{3}}$$

$$= 3^{3n-3m} = 3^{-3}$$

$$3n - 3m = -3$$

$$n - m = -1$$

$$m - n = 1$$

Answer 18 $-$: $\sqrt[6]{6} = 1.348$

$\sqrt[3]{7} = 1.913$ $\sqrt[6]{6} < 1.682 < 1.913$

$\sqrt[4]{8} = 1.682$

MULTPLE CHOICE QUESTIONS

Answer1-(d)

∵ 0 can be written as $\frac{0}{1}$ and ration number can be written as $\frac{P}{Q}$where P & Q are integers and P $\neq$ 0.

Answer 2-(a)

∵ 0 lies between -3 and 3. 0 can be written as $\frac{0}{1}$ and rational number can be written as $\frac{P}{Q}$where P & Q are integers and P $\neq$ 0.

Answer3-(c)

$$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6} \quad and \ \frac{5}{3} \times \frac{2}{2} = \frac{10}{6}$$

Since, we take option c because

$$\frac{4}{6} < \frac{5}{6} < \frac{6}{6} < \frac{7}{6} < \cdots < \frac{10}{6}$$

So two rational numbers between $\frac{2}{3}$ and $\frac{5}{3}$ are $\frac{5}{6}$and$\frac{7}{6}$.

Answer4 -(d)

Every point on a number line represents a unique number.

Answer5 -(c)

$\sqrt{2} = 1.1.414213..$ Irrational number

$\sqrt{23} = 4.795831..$ Irrational number

$\sqrt{225} = 15$Rational number {∵ written as $\frac{P}{Q}$where P & Q are integers and P $\neq$ 0.}

0.1010010001…. Irrational number

Answer6-(d)

Every rational number is real number because natural numbers, whole numbers, and integers all of them are real number.

Answer7 -(c)

Between any two rational numbers there are infinitely many rational numbers.

Answer 8-(b)

The decimal representation of a rational number is either terminating or repeating.

Because rational number is in the form of $\frac{P}{Q}$where P & Q are integers and P $\neq$ 0.

Answer9-(d)

The decimal representation of an irrational number is neither terminating nor repeating.

Answer10-(d)

$0.25, 0.25\overleftarrow{28}, 0.\overleftarrow{2528}$ are rational numbers because rational number is either terminating or repeating.

0.5030030003.... is not rational number

Answer11-(d)

3.141141114... is irrational because an irrational number is neither terminating nor repeating.

Answer12-(d)

Rational number equivalent to $\frac{7}{19}$be$\frac{7}{19} \times \frac{2}{2} = \frac{14}{38}, \frac{7}{19} \times \frac{3}{3} = \frac{21}{57}$

Ans=$\frac{21}{57}$

Answer13-(b)

$-\frac{2}{3}$ *and* $-\frac{1}{5}$Is negative rational numbers and a positive numbers cannot lie between two negative numbers

Since $\frac{3}{10}$ not lie between$-\frac{2}{3}$ *and* $-\frac{1}{5}$.

Answer14-(c)

π= 3.1415926535..., which is non-terminating non-repeating

So π is irrational number.

Answer15-(d)

$\sqrt{2} = 1.41421356237...$ This is non-terminating, non-recurring.

Answer16-(a)

$\sqrt{23} = 4.79583152331271954....$

This is non-terminating, non-recurring. So $\sqrt{23}$ is an irrational number.

Answer17-(b)

$$\frac{17}{7} = 2.\mathbf{428571}428571 .... = 2.\overline{428571}$$

Repeating block of digits in the decimal expansion of $\frac{17}{7}$ is 6.

Answer18-(c)

$\sqrt{8} = 2.82842712.....$ This is non-terminating, non-recurring. So $\sqrt{8}$ is an irrational number.

Answer19-(d)

Sometimes rational and sometimes irrational.

Ex- two irrational numbers = $\sqrt{2}$&$\sqrt{3}$

Product = $\sqrt{2} \times \sqrt{3} = \sqrt{6}$this is irrational.

But we take another two irrational numbers $\sqrt{2}$&$\sqrt{2}$

Product = $\sqrt{2} \times \sqrt{2} = 2$ this is rational.

Answer20-(d)

Every real number is either rational or irrational.

Answer21-(d)

$\pi = 3.14159265$...which is non-terminating non-repeatingso π is irrational number.

$\frac{22}{7}$is in the form of $\frac{P}{Q}$ where Qis not equal to 0 hence is not equal to 0 hence $\frac{22}{7}$is rational.

Answer22-(c)

Rational number between $\sqrt{2}$&$\sqrt{3}$ is 1.6

$\sqrt{2} = 1.41421...$

$\sqrt{3} = 1.73205...$

Answer23-(d)

The decimal representation of rational numbers is either terminating or repeating.

So 0.853853853... here 853 is repeating hence 0.853853853 is rational number.

Answer24-(a)

The product of a nonzero rational number with an irrational number is always irrational.

Answer25-(b)

Let $x = 0.\overline{2}$

Then $x = 0.22222...$ ..... (I)

Since the repeating block 2 has one digit, we multiply x by 10 to get

$10x = 2.22222....$ ...... (ii)

Subtract (i) from (ii)

$$9x = 2$$

$$x = \frac{2}{9}$$

Answer26-(c)

Let $x = 1.\overline{6}$

Then $x = 1.66666...$ ..... (I)

Since the repeating block 6 has one digit, we multiply x by 10 to get

$10x = 16.66666.$ ..... (ii)

Subtract (i) from (ii)

$$9x = 15$$

$$x = \frac{15}{9} = \frac{5}{3}$$

Answer27-(b)

Let $x = 0.\overline{54}$

Then $x = 0.5454545...$ ..... (I)

Since the repeating block 54 has two digits, we multiply x by 100 to get

$100x = 54.545454....$ ...... (ii)

Subtract (i) from (ii)

$$99x = 54$$

$$x = \frac{54}{99} = \frac{6}{11}$$

Answer28-(c)

Let $x = 0.3\overline{2}$

Then $x = 0.322222...$ ..... (I)

Since the repeating block 2 has one digit, we multiply x by 10 to get

$10x = 3.22222....$ ...... (ii)

$100x=32.22222...$ ...... (iii)

Subtract (ii) from (iii)

$$90x = 29$$

$$x = \frac{29}{90}$$

Answer29-(d)

Let $x = 0.12\overline{3}$

Then $x = 0.1233333...$ ..... (I)

Since the repeating block 3 has one digit, we multiply x by 10 to get

10x = 1.233333.... ...... (ii)

100x=12.33333... ...... (iii)

1000x=123.33333... .... (iv)

Subtract (iii) from (IV)

$$900x = 111$$

$$x = \frac{111}{900} = \frac{37}{300}$$

Answer30-(c)

Let a and b two rational numbers then irrational number between a and b = $\sqrt{ab}$

Irrational number between 5 and 6 = $\sqrt{5 \times 6} = \sqrt{30}$

Answer31-(d)

Let a and b two rational numbers then irrational number between a and b = $\sqrt{ab}$

Irrational number between $\sqrt{2}$ and $\sqrt{3} = \sqrt{\sqrt{2} \times \sqrt{3}} = \sqrt{\sqrt{6}} = \sqrt{6^{\frac{1}{2}}} = 6^{\frac{1}{2}^{\frac{1}{2}}} = 6^{\frac{1}{2} \times \frac{1}{2}} = 6^{\frac{1}{4}}$

Answer32-(c)

Let a and b two rational numbers then irrational number between a and b = $\sqrt{ab}$

Irrational number between $\frac{1}{7}$ and $\frac{2}{7} = \sqrt{\frac{1}{7} \times \frac{2}{7}}$

Answer33-(b)

Let x = $0.\overline{3}$

Then x = 0.3333.... .... (I)

10x = 3.3333.... .... (ii)

On subtracting (i) from (ii), we get

9x = 3

x = $\frac{3}{9}$

Let y = $0.\overline{4}$

Then y = 0.4444.... .... (I)

10y = 4.4444.... .... (ii)

On subtracting (i) from (ii), we get

$9y = 4$

$y = \frac{4}{9}$

$0.\overline{3} + 0.\overline{4} = x + y = \frac{3}{9} + \frac{4}{9} = \frac{7}{9}$

Answer34-(c)

Let $x = 2.\overline{45}$

Then $x = 2.4545....$ .... (I)

$100x = 245.4545.......$ .... (ii)

On subtracting (i) from (ii), we get

$99x = 243$

$x = \frac{243}{99}$

Let $y = 0.\overline{36}$

Then $y = 0.3636....$ .... (iii)

$100y = 36.3636....$ .... (iV)

On subtracting (iii) from (IV), we get

$99y = 36$

$y = \frac{36}{99}$

$2.\overline{45} + 0.\overline{36} = x + y = \frac{243}{99} + \frac{36}{99} = \frac{279}{99} = \frac{31}{11}$

Answer35-(b)

$(\sqrt{11} - \sqrt{7})(\sqrt{11} + \sqrt{7}) = (\sqrt{11})^2 - (\sqrt{7})^2 \qquad [(a-b)(a+b) = a^2 - b^2] = 11 - 7 = 4$

Answer36-(b)

$(-2 - \sqrt{3})(-2 + \sqrt{3}) = (-2)^2 - (\sqrt{3})^2 = 4 - 3 = 1 \quad [(a-b)(a+b) = a^2 - b^2]$

1 is positive and rational.

Answer37-(b)

$(6 + \sqrt{27}) - (3 + \sqrt{3}) + (1 - 2\sqrt{3}) = 6 + 3\sqrt{3} - 3 - \sqrt{3} + 1 - 2\sqrt{3} = 4$

4 is positive and rational.

Answer38-(c)

$$\frac{15\sqrt{15}}{3\sqrt{3}} = 5\sqrt{\frac{15}{3}} = 5\sqrt{5} \qquad \because \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Answer39-(a)

$$\sqrt{20} \times \sqrt{5} = \sqrt{20 \times 5} = \sqrt{100} = 10 \qquad \because \sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

Answer40-(b)

$$\frac{4\sqrt{12}}{12\sqrt{27}} = \frac{\sqrt{12}}{3\sqrt{27}} = \sqrt{\frac{12}{9 \times 27}} = \sqrt{\frac{3 \times 4}{9 \times 3 \times 9}} = \sqrt{\frac{4}{81}} = \frac{2}{9} \qquad \because \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Answer41-(b)

$$\sqrt{10} \times \sqrt{15} = \sqrt{150} = \sqrt{5 \times 5 \times 6} = 5\sqrt{6} \qquad \because \sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

Answer42 -(b)

$$\frac{\sqrt{32} + \sqrt{48}}{\sqrt{8} + \sqrt{12}} = \frac{\left(\sqrt{4 \times 8} + \sqrt{4 \times 12}\right)}{\sqrt{8} + \sqrt{12}} = \frac{\left(2\sqrt{8} + 2\sqrt{12}\right)}{\sqrt{8} + \sqrt{12}} = \frac{2\left(\sqrt{8} + \sqrt{12}\right)}{\sqrt{8} + \sqrt{12}} = 2$$

Answer43-(c)

$$(125)^{-\frac{1}{3}} = \left(\frac{1}{125}\right)^{\frac{1}{3}} = \left(\left(\frac{1}{5}\right)^{3}\right)^{\frac{1}{3}} = \frac{1}{5} \qquad \because a^{-1} = \frac{1}{a} \& a^{m^n} = a^{mn}$$

Answer44-(b)

$$7^{\frac{1}{2}}.8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}} = (56)^{\frac{1}{2}} \qquad \because a^n.b^n = (a \times b)^n$$

Answer45-(d)

$$\frac{13^{\frac{1}{5}}}{13^{\frac{1}{3}}} = 13^{\left(\frac{1}{5}-\frac{1}{3}\right)} = 13^{\left(\frac{3-5}{15}\right)} = 13^{-\frac{2}{15}} \qquad \because \frac{a^m}{a^n} = a^{m-n}$$

Answer46-(a)

$$\sqrt[4]{(64)^{-2}} = \sqrt[4]{(8^2)^{-2}} = \sqrt[4]{(8)^{-2*2}} = \sqrt[4]{(8)^{-4}} = (8)^{-4\times\frac{1}{4}} = (8)^{-1} = \frac{1}{8} \qquad \because a^{-1} = \frac{1}{a}$$

Answer47-(b)

$$\frac{(2^0 + 7^0)}{5^0} = \frac{1+1}{1} = 2 \qquad \because a^0 = 1$$

Answer48-(a)

$$(243)^{\frac{1}{5}} = (3^5)^{\frac{1}{5}} = 3^{5\times\frac{1}{5}} = 3 \qquad \because (a^m)^n = a^{mn}$$

Answer49-(c)

$9^3 + (-3)^3 - 6^3 = 729 - 27 - 216 = 729 - 243 = 486$

Answer50-(b)

$$(16)^{-\frac{1}{4}} \times \sqrt[4]{16} = (2^4)^{-\frac{1}{4}} \times (2^4)^{\frac{1}{4}} = 2^{-1} \times 2^1 = \frac{1}{2} \times 2 = 1$$

Answer 51 (c).

$\sqrt[4]{\sqrt[3]{2^2}} = \sqrt[4]{(2)^{\left(2\times\left(\frac{1}{3}\right)\right)}} = \sqrt[4]{(2)^{\left(\frac{2}{3}\right)}} = 2^{\left(\frac{2}{3}\times\frac{1}{4}\right)} = 2^{\frac{1}{6}}$

Answer 52. (d)

$(25)^{\frac{1}{3}} \times 5^{\frac{1}{3}} = (5)^{2\times\frac{1}{3}} \times 5^{\frac{1}{3}} = (5)^{\frac{2}{3}} \times 5^{\frac{1}{3}} = (5)^{\frac{2}{3}+\frac{1}{3}} = (5)^{\frac{3}{3}} = 5$

Answer 53. (a)

$\left[(81)^{\frac{1}{2}}\right]^{\frac{1}{2}} = \left[(9^2)^{\frac{1}{2}}\right]^{\frac{1}{2}} = (9)^{2\times\frac{1}{2}\times\frac{1}{2}} = (9)^{\frac{1}{2}} = (3^2)^{\frac{1}{2}} = 3$

Answer 54.(d)

lets check the option

(a) $\sqrt{5} = \left(\sqrt{5}\right)^2 = 5$ which is rational no.

So . This option is wrong .

(b) $\sqrt{2} = \left(\sqrt{2}\right)^2 = 2$ which is rational no.

So . This option is wrong .

(c) $\sqrt[3]{2} = \left(2^{\frac{1}{3}}\right)^2 = 2^{\frac{2}{3}}$ which is irrational no.

$\sqrt[3]{2} = \left(2^{\frac{1}{3}}\right)^4 = 2^{\frac{4}{3}}$ which is irrational no.

So . This option is also wrong .

(d)

$\sqrt[4]{2}= \left(2^{\frac{1}{4}}\right)^2 = 2^{\frac{2}{4}} = 2^{\frac{1}{2}} = \sqrt{2}$ which is irrational no.

$\sqrt[4]{2}= \left(2^{\frac{1}{4}}\right)^4 = 2^{\frac{4}{4}} = 2^1 = 2$ which is rational no.

So, this option is correct.

Answer 55. (b)

Given: $x = \frac{\sqrt{7}}{5}$ and $\frac{5}{x} = p\sqrt{7}$

so, $\frac{5}{x} = p\sqrt{7}$

putting the value of x in above eq.

So, $\frac{5}{\frac{\sqrt{7}}{5}} = p\sqrt{7} \Rightarrow \frac{25}{\sqrt{7}} = p\sqrt{7}$

$\Rightarrow 25 = p\sqrt{7} \times \sqrt{7}$

$\Rightarrow 25 = 7p$

$\Rightarrow p = \frac{25}{7}$

Answer 56 (b).

$\left(\frac{256x^{16}}{81y^4}\right)^{-\frac{1}{4}} = \left(\frac{81y^4}{256x^{16}}\right)^{\frac{1}{4}}$ (because $a^{-1} = \frac{1}{a}$)

$\Rightarrow \left(\frac{3^4y^4}{4^4(x^4)^4}\right)^{\frac{1}{4}} = \left(\left(\frac{3y}{4(x^4)}\right)^4\right)^{\frac{1}{4}} = \left(\frac{3y}{4x^4}\right)^{\left(4\times\frac{1}{4}\right)} = \left(\frac{3y}{4x^4}\right)^{(1)} = \frac{3y}{4x^4}$

Answer 57. (b)

$\Rightarrow \left(\frac{3y}{4x^4}\right)^{(1)}$

$= x^{p-q+q-r+r-p}$ (because $a^b.a^c = a^{b+c}$)

$= x^0$ ($a^0 = 1$, where a any no or variable)

$= 1$

Answer 58. (c)

$\Rightarrow\sqrt{p^{-1}q}.\sqrt{q^{-1}r}.\sqrt{r^{-1}p}$

$= \sqrt{\frac{q}{p}}.\sqrt{\frac{r}{q}}.\sqrt{\frac{p}{r}}$ (because$a^{-1} = \frac{1}{a}$)

$= \sqrt{\frac{q}{p}\times\frac{r}{q}\times\frac{p}{r}} = \sqrt{1} = 1$

Answer 59. (a)

$\Rightarrow\sqrt[3]{2}\times\sqrt[4]{2}\times\sqrt[12]{32}$

$= 2^{\frac{1}{3}}\times 2^{\frac{1}{4}}\times(32)^{\frac{1}{12}}$

$= 2^{\frac{1}{3}}\times 2^{\frac{1}{4}}\times(2^5)^{\frac{1}{12}} = 2^{\frac{1}{3}}\times 2^{\frac{1}{4}}\times(2)^{5\times\frac{1}{12}}$

$= 2^{\frac{1}{3}}\times 2^{\frac{1}{4}}\times(2)^{\frac{5}{12}} = 2^{\frac{1}{3}+\frac{1}{4}+\frac{5}{12}} = 2^{\frac{4+3+5}{12}}$

$= 2^{\frac{12}{12}} = 2^1$

$= 2$

Answer 60. (d)

$\left(\frac{2}{3}\right)^x\left(\frac{3}{2}\right)^{2x} = \frac{81}{16}$(Given)

$\Rightarrow\left(\frac{3}{2}\right)^{-x}\left(\frac{3}{2}\right)^{(2x)} = \frac{3^4}{2^4}$ .... ($\left(\frac{2}{3}\right)^x = \left(\frac{3}{2}\right)^{-x}$)

$\Rightarrow\left(\frac{3}{2}\right)^{(-x+2x)} = \frac{3^4}{2^4}$

$\Rightarrow\left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^4$

Comparing both side , we get $x = 4$

Answer 61.(d)

$(3^3)^2 = 9^x$(Given)

$\Rightarrow(3^3)^2 = (3^2)^x$ (9=3)

$\Rightarrow \quad x = 3$

so , $5^x = 5^3 = 5\times5\times5 = 125$

Answer 62. (b)

$\Rightarrow \frac{5^{(n+2)}-6\times5^{(n+1)}}{13\times5^{n}-2\times5^{(n+1)}}$

$= \frac{5^{(n+1)}.5^{1}-6\times5^{(n+1)}}{13\times5^{n}-2\times(5^{n}.5^{1})}$

$= \frac{5^{(n+1)}(5-6)}{5^{n}(13-2\times5)}$

$= \frac{5^{n}\times5\times(-1)}{5^{n}(13-10)}$

$= -\frac{5}{3}$

Answer 63. (d)

$\sqrt[3]{500} = 500^{\frac{1}{3}}$

To make numerator perfect cube , multiply and divide by 2.

$= \left(\frac{500\times2}{2}\right)^{\frac{1}{3}} = \left(\frac{1000}{2}\right)^{\frac{1}{3}} = \left(\frac{10^{3}}{2}\right)^{\frac{1}{3}} = \frac{(10^{3})^{\frac{1}{3}}}{2^{\frac{1}{3}}} = \frac{(10)^{3\times\frac{1}{3}}}{2^{\frac{1}{3}}} = \frac{10}{2^{\frac{1}{3}}} = \frac{10}{\sqrt[3]{2}}$

so the simplest factor is $\sqrt[3]{2}$

Answer 64.(b)

$\Rightarrow 2\sqrt{2}-\sqrt{3}$

$= (2\sqrt{2}-\sqrt{3})\times(2\sqrt{2}+\sqrt{3}) = (2\sqrt{2})^{2}-(\sqrt{3})^{2} = 8-3 = 5$

$$\Rightarrow so 2\sqrt{2}+\sqrt{3} is simplest factor of 2\sqrt{2}-\sqrt{3}$$

Answer 65. (d)

Rationalisation factor of $\frac{1}{2\sqrt{3}-\sqrt{5}}$ is $2\sqrt{3}+\sqrt{5}$

it can be written as $\sqrt{3\times4}+\sqrt{5} = \sqrt{12}+\sqrt{5}$.

Answer 66. (d)

$\Rightarrow \frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^{2}-(\sqrt{2})^{2}} = \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$

Answer 67. (c)

if $x = 2 + \sqrt{3}$

then

$$x + \frac{1}{x} = 2 + \sqrt{3} + \frac{1}{2 + \sqrt{3}} = \frac{\left(2 + \sqrt{3}\right)^2 + 1}{2 + \sqrt{3}} = \frac{4 + 3 + 4\sqrt{3} + 1}{2 + \sqrt{3}} = \frac{8 + 4\sqrt{3}}{2 + \sqrt{3}} = \frac{4\left(2 + \sqrt{3}\right)}{2 + \sqrt{3}} = 4$$

Answer 68. (c)

$\Rightarrow \frac{1}{3+2\sqrt{2}} = ?$

$\Rightarrow \frac{1}{3+2\sqrt{2}} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{(3)^2-\left(2\sqrt{2}\right)^2} = \frac{3-2\sqrt{2}}{9-8} = 3 - 2\sqrt{2}$

Answer 69. (b)

if $x = 7 + 4\sqrt{3}$

then

$$x + \frac{1}{x} = 7 + 4\sqrt{3} + \frac{1}{7 + 4\sqrt{3}} = \frac{\left(7 + 4\sqrt{3}\right)^2 + 1}{7 + 4\sqrt{3}} = \frac{49 + 48 + 56\sqrt{3} + 1}{2 + \sqrt{3}} = \frac{98 + 56\sqrt{3}}{7 + 4\sqrt{3}}$$
$$= \frac{14\left(7 + 4\sqrt{3}\right)}{7 + 4\sqrt{3}} = 14$$

Answer 70.(c)

if $\sqrt{2} = 1.41$

then $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\left(\sqrt{2}\right)^2} = \frac{\sqrt{2}}{2} = \frac{1.41}{2} = 0.705$

Answer 71. ()

if $\sqrt{7} = 2.646$

then $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{\left(\sqrt{7}\right)^2} = \frac{\sqrt{7}}{7} = \frac{2.646}{7} = 0.378$

Answer 72.(d)

$\sqrt{3 - 2\sqrt{2}} = \sqrt{2 + 1 - 2\sqrt{2}}$

$= \sqrt{(\sqrt{2})^2 + (1)^2 - 2 \times \sqrt{2} \times 1} = \sqrt{\left((\sqrt{2}-1)^2\right)} = \sqrt{2} - 1$ [$(a-b)^2 = a^2 + b^2 - 2ab$]

Answer 73. (c)

$\sqrt{5+2\sqrt{6}} = \sqrt{2+3+2\sqrt{6}}$

$= \sqrt{(\sqrt{2})^2 + (\sqrt{3})^2 + 2 \times \sqrt{2} \times \sqrt{3}} = \sqrt{(\sqrt{2}+\sqrt{3})^2} = \sqrt{2} + \sqrt{3}$ [$(a+b)^2 = a^2 + b^2 + 2ab$]

Answer 74.(c)

if $\sqrt{2} = 1.414$

then $\sqrt{\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)}} = \sqrt{\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}} = \sqrt{\frac{(\sqrt{2}-1)^2}{\left(\sqrt{2}^2-1^2\right)}} = \sqrt{\frac{(\sqrt{2}-1)^2}{(2-1)}} = (\sqrt{2}-1) = (1.414-1) = 0.414$

Answer 75. (a)

If $x = 3 + \sqrt{8}$

then $x^2 + \frac{1}{x^2}$

so let's find the value of x^2 and $\frac{1}{x^2}$

$\Rightarrow x^2 = (3+\sqrt{8})^2 = \left(3^2 + \sqrt{8}^2 + 2 \times 3 \times \sqrt{8}\right) = (9 + 8 + 6\sqrt{8}) = (17 + 6\sqrt{8})$

and $\frac{1}{x} = \frac{1}{3+\sqrt{8}}$

$= \frac{1}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}} = \frac{3-\sqrt{8}}{3^2-(\sqrt{8})^2} = \frac{3-\sqrt{8}}{9-8} = 3 - \sqrt{8}$

so $\frac{1}{x^2} = \left(\frac{1}{x}\right)^2 = (3-\sqrt{8})^2 = \left(3^2 + (\sqrt{8})^2 - 2 \times 3 \times \sqrt{8}\right) = (9 + 8 - 6\sqrt{8}) = (17 - 6\sqrt{8})$

then $x^2 + \frac{1}{x^2} = (17 + 6\sqrt{8}) + (17 - 6\sqrt{8}) = 34$

Answer 76.(a)

As we know that if p and q are two rational no. Then a rational no between p and q is given by $\frac{1}{2}(p+q)$.

so, Reason (R) is true.

A rational no between $\frac{2}{5}$ and $\frac{2}{5} = \frac{1}{2}\left(\frac{2}{5} + \frac{3}{5}\right) = \frac{5}{10} = \frac{10}{20}$

A rational no between $\frac{2}{5}$ and $\frac{10}{20} = \frac{1}{2}\left(\frac{2}{5} + \frac{10}{20}\right) = \frac{9}{20}$

A rational no between $\frac{10}{20}$and $\frac{2}{5} = \frac{1}{2}\left(\frac{10}{20} + \frac{3}{5}\right) = \frac{11}{20}$

so, three rational no between $\frac{2}{5}$ and $\frac{2}{5}$is $\frac{10}{20}, \frac{09}{20}, \frac{11}{20}$

⇒ Assertion (A) is True and Reason (R) is correct explanation of of A

thus option a is correct.

Answer 77. (a)

As we know square root of a positive integer which is not a perfect square is an irrational no.

e.g $\sqrt{5}$ is an irrational no. i.e 5 is not perfect square .

⇒ Reason (R) is true.

So , Assertion (A) is true .

Since R gives A .

⇒ option a is correct

Answer 78. (b)

as we know that e and π are both irrational no.

Because e = 2.71828... which is not terminating or recurring.

Similarly π = 3.14159...which is not terminating or recurring.

So Assertion (A) and Reason (R) are true.

But R is not correct explanation of A.

So, Option b is correct.

Answer 79. (b)

As we know square root of a positive integer which is not a perfect square is an irrational no.

So,$\sqrt{3}$ is an irrational no. because 5 is not perfect square .

So Assertion (A) is True.

The sum of a rational and an irrational no. is an irrational no.

e.g $2 + \sqrt{5}$ is irrational no. Where 2 is rational no and $\sqrt{5}$is an irrational no.

So, Reason (R) is true .

But R is not correct explanation of A.

Thus option b is correct.

Answer 80.

(a) $6.\overline{54} = 6.5444444...$ that is a non terminating and repeating decimal so it is a rational no.

(b) $\pi = 3.14159.......$ that is a non terminating and no repeating decimal so it is an irrational no.

(c) $\frac{1}{7} = 0.\overline{142857}$ so it is repeating with 6 decimal digit so the length of period is 6

(d) If $x = 2 - \sqrt{3}$

then $x^2 + \frac{1}{x^2}$

so let's find the value of x^2 and $\frac{1}{x^2}$

$\Rightarrow x^2 = (2 - \sqrt{3})^2 = \left(2^2 + \sqrt{3}^2 - 2 \times 2 \times \sqrt{3}\right) = (4 + 3 - 4\sqrt{3}) = (7 + 4\sqrt{3})$

and $\frac{1}{x} = \frac{1}{2-\sqrt{3}}$

$= \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2} = \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$

so $\frac{1}{x^2} = \left(\frac{1}{x}\right)^2 = (2 - \sqrt{3})^2 = \left(2^2 + (\sqrt{3})^2 - 2 \times 2 \times \sqrt{3}\right) = (4 + 3 - 4\sqrt{3}) = (7 - 4\sqrt{3})$

then $x^2 + \frac{1}{x^2} = (7 + 4\sqrt{3}) + (7 - 4\sqrt{3}) = 14$

Answer 81.

(a) $\sqrt[4]{(81)^{-2}} = \sqrt[4]{(9^2)^{-2}} = \sqrt[4]{(9)^{2\times-2}} = \sqrt[4]{(9)^{-4}} = ((9)^{-4})^{\frac{1}{4}} = (9)^{-4\times\frac{1}{4}} = 9^{-1} = \frac{1}{9}$

(b) if $\left(\frac{a}{b}\right)^{x-2} = \left(\frac{b}{a}\right)^{x-4}$

then x = ?

$\Rightarrow \left(\frac{a}{b}\right)^{x-2} = \left(\frac{b}{a}\right)^{x-4}$

$\Rightarrow \left(\frac{a}{b}\right)^{x-2} = \left(\frac{a}{b}\right)^{-1\times(x-4)}$

$\Rightarrow\left(\frac{a}{b}\right)^{x-2} = \left(\frac{a}{b}\right)^{(-x+4)}$

$\Rightarrow x - 2 = -x + 4$

$\Rightarrow 2x = 6$

$\Rightarrow x = 3$

(c) . if $x = 9 + 4\sqrt{5}$

then

$\Rightarrow \frac{1}{x} = \frac{1}{9+4\sqrt{5}} = \frac{1}{9+4\sqrt{5}} \times \frac{9-4\sqrt{5}}{9-4\sqrt{5}} = \frac{9-4\sqrt{5}}{9^2-\left(4\sqrt{5}\right)^2} = \frac{9-4\sqrt{5}}{81-80} = 9 - 4\sqrt{5}$

$\Rightarrow \sqrt{x} - \frac{1}{\sqrt{x}}$

squaring the equation.

$\Rightarrow\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = \left(\sqrt{x}^2 + \left(\frac{1}{\sqrt{x}}\right)^2 - 2 \times \sqrt{x} \times \frac{1}{\sqrt{x}}\right) = \left(x + \frac{1}{x} - 2\right)$

$= \quad \left(9 + 4\sqrt{5}\right) + \left(9 - 4\sqrt{5}\right) - 2 = 9 + 9 - 2 = 16$

$\Rightarrow\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = 16$

$\Rightarrow\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = 4$

(d) $\left(\frac{81}{16}\right)^{\frac{-3}{4}} \times \left(\frac{64}{27}\right)^{\frac{-1}{3}} = \left(\frac{3^4}{2^4}\right)^{\frac{-3}{4}} \times \left(\frac{4^3}{3^3}\right)^{\frac{-1}{3}} = \left(\frac{3}{2}\right)^{4\times\frac{-3}{4}} \times \left(\frac{4}{3}\right)^{3\times\frac{-1}{3}} = \left(\frac{3}{2}\right)^{-3} \times \left(\frac{4}{3}\right)^{-1}$

$\Rightarrow\left(\frac{2}{3}\right)^3 \times \left(\frac{3}{4}\right)^1 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{3}{4} = \frac{2}{9}$

Very Short Answer Question

Answer 1.

(i) The sum of rational and irrational no. is always an irrational no.

(ii) Subtraction of rational and irrational no. is always an irrational no.

And same for Multiplication and Division.

e.g $2+\sqrt{5}$ is irrational no. Where 2 is rational no and $\sqrt{5}$is an irrational no.

Answer 2.

$\Rightarrow(3-\sqrt{11})(3+\sqrt{11})$

$=(3)^2-(\sqrt{11})^2$ ....[(a-b)(a+b) = a^2-b^2]

$=$ 9 -11

$=$ -2

Answer 3. $\Rightarrow\frac{665}{625}$

Answer 4.

$\Rightarrow(1296)^{0.17}\times(1296)^{0.08}$

$\Rightarrow(1296)^{0.17+0.08}=(1296)^{0.25}$ $(a^x \,.\, a^y = a^{x+y})$

$=(1296)^{\frac{25}{100}}=(6^4)^{\frac{1}{4}}=(6)^{4\times\frac{1}{4}}$

$=6^1=6$

Answer 5.

$6\sqrt{36}+5\sqrt{12}$..........................$(\sqrt{36}=6)$

$=6\times6+5\sqrt{3\times4}$................$(\sqrt{4}=2)$

$=36+5\times2\sqrt{3}$

$=36+10\sqrt{3}$

Answer 6.

if a and b are two distinct positive rational no. Then

$\sqrt{ab}$is an irrational no lying between a and b.

So the irrational no between 5 and 6 is $\sqrt{5 \times 6} = \sqrt{30}$

Answer 7.

$$\Rightarrow \frac{21\sqrt{12}}{10\sqrt{27}} = \frac{21\sqrt{3\times4}}{10\sqrt{3\times9}} = \frac{21\times2\sqrt{3}}{10\times3\sqrt{3}} = \frac{42\sqrt{3}}{30\sqrt{3}} = \frac{42}{30} = \frac{7}{5}$$

Answer 8.

$$\Rightarrow \frac{1}{\sqrt{3}+\sqrt{2}} = \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3} - \sqrt{2}$$

Answer 9.

$$\Rightarrow \left(\frac{2}{5}\right)^{2x-2} = \frac{32}{3125}$$

$$\Rightarrow \left(\frac{2}{5}\right)^{2x-2} = \frac{2^5}{5^5}$$

$$\Rightarrow \left(\frac{2}{5}\right)^{2x-2} = \left(\frac{2}{5}\right)^5$$

Comparing both side, we get

$\Rightarrow 2x - 2 = 5$

$\Rightarrow 2x = 5 + 2$

$\Rightarrow 2x = 7$

$\Rightarrow x = \frac{7}{2}$

Answer 10

$\Rightarrow (32)^{\frac{1}{5}} + (-7)^0 + (64)^{\frac{1}{2}}$

$\Rightarrow (2^5)^{\frac{1}{5}} + (-7)^0 + (8^2)^{\frac{1}{2}}$

$\Rightarrow (2)^{5\times\frac{1}{5}} + 1 + (8)^{2\times\frac{1}{2}}$ $\{(-7)^0 = 1\}$

$\Rightarrow 2 + 1 + 8 = 11$

Answer 11.

$$\left(\frac{81}{49}\right)^{\frac{-3}{2}} = \left(\frac{9^2}{7^2}\right)^{\frac{-3}{2}} = \left(\frac{9}{7}\right)^{2\times\left(\frac{-3}{2}\right)} = \left(\frac{9}{7}\right)^{-3} = \left(\frac{7}{9}\right)^{3} = \frac{7^3}{9^3} = \frac{343}{729}\left(\left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^{1}\right)$$

Answer 12.

$$\sqrt[4]{81x^8y^4z^{16}} = \sqrt[4]{3^4(x^2)^4(y^4)(z^4)^4} = \sqrt[4]{(3x^2yz^4)^4} = (3x^2yz^4)^{\left(4\times\frac{1}{4}\right)} = 3x^2yz^4$$

Answer 13. if a = 1, b = 2

then , $\left(a^b + b^a\right)^{-1} = \frac{1}{a^b+b^a}$

$\Rightarrow \quad \frac{1}{1^2+2^1} = \frac{1}{1+2} = \frac{1}{3}$

Answer 14.

$$\left(\frac{3125}{243}\right)^{\frac{4}{5}} = \left(\frac{5^5}{3^5}\right)^{\frac{4}{5}} = \left(\frac{5}{3}\right)^{5\times\frac{4}{5}} = \left(\frac{5}{3}\right)^{4} = \frac{625}{81}$$

Answer 15.

let take two irrational numbers as $2+\sqrt{3}$ and $2-\sqrt{3}$

$(2+\sqrt{3}) + (2-\sqrt{3}) = 2+2 = 4$, which is a rational no.

$(2+\sqrt{3}) \times (2-\sqrt{3}) = 2^2 - \sqrt{3}^2 = 4-3 = 1$, which is a rational no.

Answer 16.

Yes, Product of a rational and irrational no always irrational .

e.g let's take a rational no 4 and an irrational no $\sqrt{3}$,

product of two no $= 4\times\sqrt{3} = 4\sqrt{3}$, which is an irrational no.

Answer 17. let $x = \sqrt[3]{4}$

then , $x^2 = \left(\sqrt[3]{4}\right)^2 = (4)^{2\times\frac{1}{3}} = 4^{\frac{2}{3}}$, which is a irrational no.

$X^3 = \left(\sqrt[3]{4}\right)^3 = (4)^{3\times\frac{1}{3}} = 4$, which is a rational no.

Answer 18 reciprocal of $2+\sqrt{3} = \frac{1}{2+\sqrt{3}}$

$\Rightarrow \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-\sqrt{3}^2} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

Answer 19 If $\sqrt{10} = 3.162$

then $\frac{1}{\sqrt{10}} = \frac{1}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{\sqrt{10}^2} = \frac{\sqrt{10}}{10} = \frac{3.162}{10} = 0.3162$

Answer 20. $\left(2\sqrt{5}+3\sqrt{2}\right)^2$

$= \left(2\sqrt{5}\right)^2 + \left(3\sqrt{2}\right)^2 + 2 \times 2\sqrt{5} \times 3\sqrt{2}$

$= 20 + 18 + 12\sqrt{10} = 38 + 12\sqrt{10}$

Answer 21. if $(10)^x = 64$

then , $10^{\left(\frac{x}{2}+1\right)}$

$= 10^{\left(\frac{x}{2}\right)} \times 10^1 = \sqrt{10^x} \times 10 = \sqrt{64} \times 10 = \sqrt{8^2} \times 10 = (8^2)^{\frac{1}{2}} \times 10 = 8 \times 10 = 80$

Answer 22. $\frac{2^n+2^{n-1}}{2^{n+1}-2^n}$

$= \frac{2^{n-1}(2+1)}{2^n(2-1)} = \frac{2^{n-1}\times 3}{2^n} = \frac{2^n \times 2^{-1} \times 3}{2^n} = 3 \times \frac{1}{2} = \frac{3}{2}$ $\qquad (2^{-1} = \frac{1}{2})$

Answer 23. $\left[\left((256)^{\frac{-1}{2}}\right)^{\frac{-1}{4}}\right]^2$

$$= \left[\left((16^2)^{\frac{-1}{2}} \right)^{\frac{-1}{4}} \right]^2 = \left[\left((16)^{2 \times \frac{-1}{2}} \right)^{\frac{-1}{4}} \right]^2 = \left[(16^{-1})^{\frac{-1}{4}} \right]^2$$

$$= \left[(16^{-1})^{\frac{-1}{4}} \right]^2 = \left[(16)^{-1 \times \frac{-1}{4}} \right]^2 = \left[(16)^{\frac{1}{4}} \right]^2 = \left[(2^4)^{\frac{1}{4}} \right]^2 = \left[2^{4 \times \frac{1}{4}} \right]^2 = [2]^2 = 4$$