

1. Without using trigonometric tables, evaluate:

$$(i) \frac{\sin 16^\circ}{\cos 74^\circ} \quad (ii) \frac{\sec 11^\circ}{\operatorname{cosec} 79^\circ} \quad (iii) \frac{\tan 27^\circ}{\cot 63^\circ}$$

$$(iv) \frac{\cos 35^\circ}{\sin 55^\circ} \quad (v) \frac{\operatorname{cosec} 42^\circ}{\sec 48^\circ} \quad (vi) \frac{\cot 38^\circ}{\tan 52^\circ}$$

Sol:

$$\begin{aligned} (i) \frac{\sin 16^\circ}{\cos 74^\circ} &= \frac{\sin (90^\circ - 74^\circ)}{\cos 74^\circ} \\ &= \frac{\cos 74^\circ}{\cos 74^\circ} \quad [\because \sin (90 - \theta) = \cos \theta] \\ &= 1 \end{aligned}$$

$$\begin{aligned} (ii) \frac{\sec 11^\circ}{\operatorname{cosec} 79^\circ} &= \frac{\sec (90^\circ - 79^\circ)}{\operatorname{cosec} 79^\circ} \\ &= \frac{\operatorname{cosec} 79^\circ}{\operatorname{cosec} 79^\circ} \quad [\because \sec (90 - \theta) = \operatorname{cosec} \theta] \\ &= 1 \end{aligned}$$

$$\begin{aligned} (iii) \frac{\tan 27^\circ}{\cot 63^\circ} &= \frac{\tan (90^\circ - 63^\circ)}{\cot 63^\circ} \\ &= \frac{\cos 63^\circ}{\cos 63^\circ} \quad [\because \tan (90 - \theta) = \cot \theta] \\ &= 1 \end{aligned}$$

$$\begin{aligned} (iv) \frac{\cos 35^\circ}{\sin 55^\circ} &= \frac{\cos (90^\circ - 55^\circ)}{\sin 55^\circ} \\ &= \frac{\sin 55^\circ}{\sin 55^\circ} \quad [\because \cos (90 - \theta) = \sin \theta] \\ &= 1 \end{aligned}$$

$$\begin{aligned} (v) \frac{\operatorname{cosec} 42^\circ}{\sec 48^\circ} &= \frac{\operatorname{cosec} (90^\circ - 48^\circ)}{\sec 48^\circ} \\ &= \frac{\sec 48^\circ}{\sec 48^\circ} \quad [\because \operatorname{cosec} (90 - \theta) = \sec \theta] \\ &= 1 \end{aligned}$$

$$\begin{aligned} (vi) \frac{\cot 38^\circ}{\tan 52^\circ} &= \frac{\cot (90^\circ - 52^\circ)}{\tan 52^\circ} \end{aligned}$$

$$= \frac{\tan 52^\circ}{\tan 52^\circ} \quad [\because \tan(90 - \theta) = \cot \theta]$$

$$= 1$$

2. Without using trigonometric tables, prove that:

- (i) $\cos 81^\circ - \sin 9^\circ = 0$ (ii) $\tan 71^\circ - \cot 19^\circ = 0$
 (iii) $\operatorname{cosec} 80^\circ - \sec 10^\circ = 0$ (iv) $\operatorname{cosec}^2 72^\circ - \tan^2 18^\circ = 1$
 (v) $\cos^2 75^\circ + \cos^2 15^\circ = 1$ (vi) $\tan^2 66^\circ - \cot^2 24^\circ = 0$
 (vii) $\sin^2 48^\circ + \sin^2 42^\circ = 1$ (viii) $\cos^2 57^\circ - \sin^2 33^\circ = 0$
 (ix) $(\sin 65^\circ + \cos 25^\circ)(\sin 65^\circ - \cos 25^\circ) = 0$

Sol:

(i) LHS = $\cos 81^\circ - \sin 9^\circ$
 $= \cos(90^\circ - 9^\circ) - \sin 9^\circ$
 $= \sin 9^\circ - \sin 9^\circ$
 $= 0$
 $= \text{RHS}$

(ii) LHS = $\tan 71^\circ - \cot 19^\circ$
 $= \tan(90^\circ - 19^\circ) - \cot 19^\circ$
 $= \cot 19^\circ - \cot 19^\circ$
 $= 0$
 $= \text{RHS}$

(iii) LHS = $\operatorname{cosec} 80^\circ - \sec 10^\circ$
 $= \operatorname{cosec}(90^\circ - 10^\circ) - \sec 10^\circ$
 $= \sec 10^\circ - \sec 10^\circ$
 $= 0$
 $= \text{RHS}$

(iv) LHS = $\operatorname{cosec}^2 72^\circ - \tan^2 18^\circ$
 $= \operatorname{cosec}^2(90^\circ - 18^\circ) - \tan^2 18^\circ$
 $= \sec^2 18^\circ - \tan^2 18^\circ$
 $= 1$
 $= \text{RHS}$

(v) LHS = $\cos^2 75^\circ + \cos^2 15^\circ$
 $= \cos^2(90^\circ - 15^\circ) + \cos^2 15^\circ$
 $= \sin^2 15^\circ + \cos^2 15^\circ$
 $= 1$
 $= \text{RHS}$

(vi) LHS = $\tan^2 66^\circ - \cot^2 24^\circ$
 $= \tan^2(90^\circ - 24^\circ) - \cot^2 24^\circ$
 $= \cot^2 24^\circ - \cot^2 24^\circ$
 $= 0$
 $= \text{RHS}$

$$\begin{aligned}
 \text{(vii) LHS} &= \sin^2 48^\circ + \sin^2 42^\circ \\
 &= \sin^2 (90^\circ - 42^\circ) + \sin^2 42^\circ \\
 &= \cos^2 42^\circ + \sin^2 42^\circ \\
 &= 1 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(viii) LHS} &= \cos^2 57^\circ - \sin^2 33^\circ \\
 &= \cos^2 (90^\circ - 33^\circ) - \sin^2 33^\circ \\
 &= \sin^2 33^\circ - \sin^2 33^\circ \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ix) LHS} &= (\sin 65^\circ + \cos 25^\circ) (\sin 65^\circ - \cos 25^\circ) \\
 &= \sin^2 65^\circ - \cos^2 25^\circ \\
 &= \sin^2 (90^\circ - 25^\circ) - \cos^2 25^\circ \\
 &= \cos^2 25^\circ - \cos^2 25^\circ \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

3. Without using trigonometric tables, prove that:

$$\text{(i) } \sin 53^\circ \cos 37^\circ + \cos 53^\circ \sin 37^\circ = 1$$

$$\text{(ii) } \cos 54^\circ \cos 36^\circ - \sin 54^\circ \sin 36^\circ = 0$$

$$\text{(iii) } \sec 70^\circ \sin 20^\circ + \cos 20^\circ \operatorname{cosec} 70^\circ = 2$$

$$\text{(iv) } \sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ = 0$$

$$\text{(v) } (\sin 72^\circ + \cos 18^\circ) (\sin 72^\circ - \cos 18^\circ) = 0$$

$$\text{(vi) } \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$$

Sol:

$$\begin{aligned}
 \text{(i) LHS} &= \sin 53^\circ \cos 37^\circ + \cos 53^\circ \sin 37^\circ \\
 &= \sin (90^\circ - 37^\circ) \cos 37^\circ + \cos (90^\circ - 37^\circ) \sin 37^\circ \\
 &= \cos 37^\circ \cos 37^\circ + \sin 37^\circ \sin 37^\circ \\
 &= \cos^2 37^\circ + \sin^2 37^\circ \\
 &= 1 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) LHS} &= \cos 54^\circ \cos 36^\circ - \sin 54^\circ \sin 36^\circ \\
 &= \cos (90^\circ - 36^\circ) \cos 36^\circ - \sin (90^\circ - 36^\circ) \sin 36^\circ \\
 &= \sin 36^\circ \cos 36^\circ - \cos 36^\circ \sin 36^\circ \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) LHS} &= \sec 70^\circ \sin 20^\circ + \cos 20^\circ \operatorname{cosec} 70^\circ \\
 &= \sec (90^\circ - 20^\circ) \sin 20^\circ + \cos 20^\circ \operatorname{cosec} (90^\circ - 20^\circ) \\
 &= \operatorname{cosec} 20^\circ \cdot \frac{1}{\operatorname{cosec} 20^\circ} + \frac{1}{\sec 20^\circ} \cdot \sec 20^\circ \\
 &= 1 + 1
 \end{aligned}$$

$$= 2$$

$$= \text{RHS}$$

$$\begin{aligned} \text{(iv) LHS} &= \sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ \\ &= \sin 35^\circ \cos(90^\circ - 55^\circ) - \cos 35^\circ \sin(90^\circ - 55^\circ) \\ &= \sin 35^\circ \cos 35^\circ - \cos 35^\circ \sin 35^\circ \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{(v) LHS} &= (\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ - \cos 18^\circ) \\ &= (\sin 72^\circ + \cos 18^\circ)[\cos(90^\circ - 72^\circ) - \cos 18^\circ] \\ &= (\sin 72^\circ + \cos 18^\circ)(\cos 18^\circ - \cos 18^\circ) \\ &= (\sin 72^\circ + \cos 18^\circ)(0) \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{(vi) LHS} &= \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ \\ &= \cot(90^\circ - 48^\circ) \cot(90^\circ - 23^\circ) \tan 42^\circ \tan 67^\circ \\ &= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ \\ &= \frac{1}{\tan 42^\circ} \times \frac{1}{\tan 67^\circ} \times \tan 42^\circ \times \tan 67^\circ \\ &= 1 \\ &= \text{RHS} \end{aligned}$$

4. Without using trigonometric tables, prove that:

$$\text{(i) } \frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2\cos 70^\circ \operatorname{cosec} 20^\circ = 0$$

$$\text{(ii) } \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ = 2$$

$$\text{(iii) } \frac{2 \sin 68^\circ}{\cos 22^\circ} + \frac{2 \cot 15^\circ}{5 \tan 75^\circ} - \frac{3 \tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan 70^\circ}{5} = 1$$

$$\text{(iv) } \frac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3} (\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ) = 2$$

$$\text{(v) } \frac{7 \cos 55^\circ}{3 \sin 35^\circ} - \frac{4[\cos 70^\circ \operatorname{cosec} 20^\circ]}{3(\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ)} = 1$$

Sol:

$$\begin{aligned} \text{(i) LHS} &= \frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2\cos 70^\circ \operatorname{cosec} 20^\circ \\ &= \frac{\sin 70^\circ}{\sin(90^\circ - 20^\circ)} + \frac{\sec(90^\circ - 20^\circ)}{\sec 70^\circ} - 2\cos 70^\circ \sec(90^\circ - 20^\circ) \\ &= \frac{\sin 70^\circ}{\sin 70^\circ} + \frac{\sec 70^\circ}{\sec 70^\circ} - 2\cos 70^\circ \sec 70^\circ \\ &= 1 + 1 - 2 \times \cos 70^\circ \times \frac{1}{\cos 70^\circ} \\ &= 2 - 2 \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned}
 \text{(ii) LHS} &= \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ \\
 &= \frac{\cos 80^\circ}{\sin(90^\circ - 10^\circ)} + \sin(90^\circ - 59^\circ) \operatorname{cosec} 31^\circ \\
 &= \frac{\cos 80^\circ}{\cos 80^\circ} + \sin 31^\circ \operatorname{cosec} 31^\circ \\
 &= 1 + \sin 31^\circ \times \frac{1}{\sin 31^\circ} \\
 &= 1 + 1 \\
 &= 2 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) LHS} &= \frac{2 \sin 68^\circ}{\cos 22^\circ} - \frac{2 \cot 15^\circ}{5 \tan 75^\circ} - \frac{3 \tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan 70^\circ}{5} \\
 &= \frac{2 \sin 68^\circ}{\sin(90^\circ - 22^\circ)} - \frac{2 \cot 15^\circ}{5 \tan(90^\circ - 75^\circ)} - \frac{3 \times 1 \times \cot(90^\circ - 20^\circ) \times \cot(90^\circ - 40^\circ) \times \tan 50^\circ \times \tan 70^\circ}{5} \\
 &= \frac{2 \sin 68^\circ}{\sin 68^\circ} - \frac{2 \cot 15^\circ}{5 \cot 15^\circ} - \frac{3 \times \cot 70^\circ \cot 50^\circ \tan 50^\circ \tan 70^\circ}{5} \\
 &= 2 - \frac{2}{5} - \frac{3 \times \frac{1}{\tan 70^\circ} \times \frac{1}{\tan 50^\circ} \times \tan 50^\circ \times \tan 70^\circ}{5} \\
 &= 2 - \frac{2}{5} - \frac{3}{5} \\
 &= \frac{10 - 2 - 3}{5} \\
 &= \frac{5}{5} \\
 &= 1 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv) LHS} &= \frac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3} (\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ) \\
 &= \frac{\sin 18^\circ}{\sin(90^\circ - 72^\circ)} + \sqrt{3} [\cot(90^\circ - 10^\circ) \times \frac{1}{\sqrt{3}} \times \cot(90^\circ - 40^\circ) \times \tan 50^\circ \times \tan 80^\circ] \\
 &= \frac{\sin 18^\circ}{\sin 18^\circ} + \sqrt{3} \left(\frac{\cot 80^\circ \times \cot 50^\circ \times \tan 50^\circ \times \tan 80^\circ}{\sqrt{3}} \right) \\
 &= 1 + \left(\frac{1}{\tan 80^\circ} \times \frac{1}{\tan 50^\circ} \times \tan 50^\circ \times \tan 80^\circ \right) \\
 &= 1 + 1 \\
 &= 2 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v) LHS} &= \frac{7 \cos 55^\circ}{3 \sin 35^\circ} - \frac{4 (\cos 70^\circ \operatorname{cosec} 20^\circ)}{3 (\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ)} \\
 &= \frac{7 \cos 55^\circ}{3 \cos(90^\circ - 35^\circ)} - \frac{4 (\sin(90^\circ - 70^\circ) \operatorname{cosec} 20^\circ)}{3 (\cot(90^\circ - 5^\circ) \times \cot(90^\circ - 25^\circ) \times 1 \times \tan 65^\circ \times \tan 85^\circ)} \\
 &= \frac{7 \cos 55^\circ}{3 \cos 55^\circ} - \frac{4 (\sin 20^\circ \operatorname{cosec} 20^\circ)}{3 (\cot 85^\circ \cot 65^\circ \tan 65^\circ \tan 85^\circ)} \\
 &= \frac{7}{3} - \frac{4 \left(\sin 20^\circ \times \frac{1}{\sin 20^\circ} \right)}{3 \left(\frac{1}{\tan 85^\circ} \times \frac{1}{\tan 65^\circ} \times \tan 65^\circ \times \tan 85^\circ \right)} \\
 &= \frac{7}{3} - \frac{4}{3} \\
 &= \frac{3}{3}
 \end{aligned}$$

$$= \frac{3}{3}$$

$$= 1$$

$$= \text{RHS}$$

5. Prove that:

$$(i) \sin \theta \cos(90^\circ - \theta) + \sin(90^\circ - \theta) \cos \theta = 1$$

$$(ii) \frac{\sin \theta}{\cos(90^\circ - \theta)} + \frac{\cos \theta}{\sin(90^\circ - \theta)} = 2$$

$$(iii) \frac{\sin \theta \cos(90^\circ - \theta) \cos \theta}{\sin(90^\circ - \theta)} + \frac{\cos \theta \sin(90^\circ - \theta) \sin \theta}{\cos(90^\circ - \theta)} = 1$$

$$(iv) \frac{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan \theta}{\operatorname{cosec}(90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot \theta} = 2$$

$$(v) \frac{\cos(90^\circ - \theta)}{1 + \sin(90^\circ - \theta)} + \frac{1 + \sin(90^\circ - \theta)}{\cos(90^\circ - \theta)} = 2 \operatorname{cosec} \theta$$

$$(vi) \frac{\sec(90^\circ - \theta) \operatorname{cosec} \theta - \tan(90^\circ - \theta) \cot \theta + \cos^2 25^\circ + \cos^2 65^\circ}{3 \tan 27^\circ \tan 63^\circ} = \frac{2}{3}$$

$$(vii) \cot \theta \tan(90^\circ - \theta) - \sec(90^\circ - \theta) \operatorname{cosec} \theta + \sqrt{3} \tan 12^\circ \tan 60^\circ \tan 78^\circ = 2$$

Sol:

$$(i) \text{LHS} = \sin \theta \cos(90^\circ - \theta) + \sin(90^\circ - \theta) \cos \theta$$

$$= \sin \theta \sin \theta + \cos \theta \cos \theta$$

$$= \sin^2 \theta + \cos^2 \theta$$

$$= 1$$

$$= \text{RHS}$$

Hence proved.

$$(ii) \text{LHS} = \frac{\sin \theta}{\cos(90^\circ - \theta)} + \frac{\cos \theta}{\sin(90^\circ - \theta)}$$

$$= \frac{\sin \theta}{\sin \theta} + \frac{\cos \theta}{\cos \theta}$$

$$= 1 + 1$$

$$= 2$$

$$= \text{RHS}$$

Hence proved.

$$(iii) \text{LHS} = \frac{\sin \theta \cos(90^\circ - \theta) \cos \theta}{\sin(90^\circ - \theta)} + \frac{\cos \theta \sin(90^\circ - \theta) \sin \theta}{\cos(90^\circ - \theta)}$$

$$= \frac{\sin \theta \sin \theta \cos \theta}{\cos \theta} + \frac{\cos \theta \cos \theta \sin \theta}{\sin \theta}$$

$$= \sin^2 \theta + \cos^2 \theta$$

$$= 1$$

$$= \text{RHS}$$

Hence proved.

$$\begin{aligned}
 \text{(iv) LHS} &= \frac{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan \theta}{\operatorname{cosec}(90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot \theta} \\
 &= \frac{\sin \theta \operatorname{cosec} \theta \tan \theta}{\sec \theta \cos \theta \tan \theta} + \frac{\cot \theta}{\cot \theta} \\
 &= 1 + 1 \\
 &= 2 \\
 &= \text{RHS}
 \end{aligned}$$

Hence proved.

$$\begin{aligned}
 \text{(v) LHS} &= \frac{\cos(90^\circ - \theta)}{1 + \sin(90^\circ - \theta)} + \frac{1 + \sin(90^\circ - \theta)}{\cos(90^\circ - \theta)} \\
 &= \frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} \\
 &= \frac{\sin^2 \theta + (1 + \cos \theta)^2}{(1 + \cos \theta) \sin \theta} \\
 &= \frac{\sin^2 \theta + 1 + \cos^2 \theta + 2 \cos \theta}{(1 + \cos \theta) \sin \theta} \\
 &= \frac{1 + 1 + 2 \cos \theta}{(1 + \cos \theta) \sin \theta} \\
 &= \frac{2 + 2 \cos \theta}{(1 + \cos \theta) \sin \theta} \\
 &= \frac{2(1 + \cos \theta)}{(1 + \cos \theta) \sin \theta} \\
 &= 2 \frac{1}{\sin \theta} \\
 &= 2 \operatorname{cosec} \theta \\
 &= \text{RHS}
 \end{aligned}$$

Hence proved.

$$\begin{aligned}
 \text{(vi) LHS} &= \frac{\sec(90^\circ - \theta) \operatorname{cosec} \theta - \tan(90^\circ - \theta) \cot \theta + \cos^2 25^\circ + \cos^2 65^\circ}{3 \tan 27^\circ \tan 63^\circ} \\
 &= \frac{\operatorname{cosec} \theta \operatorname{cosec} \theta - \cot \theta \cot \theta + \sin^2(90^\circ - 25^\circ) + \cos^2 65^\circ}{3 \tan 27^\circ \cot(90^\circ - 63^\circ)} \\
 &= \frac{\operatorname{cosec}^2 \theta - \cot^2 \theta + \sin^2 65^\circ + \cos^2 65^\circ}{3 \tan 27^\circ \cot 27^\circ} \\
 &= \frac{1 + 1}{3 \times \tan 27^\circ \times \frac{1}{\tan 27^\circ}} \\
 &= \frac{2}{3} \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii) LHS} &= \cot \theta \tan(90^\circ - \theta) - \sec(90^\circ - \theta) \operatorname{cosec} \theta + \sqrt{3} \tan 12^\circ \tan 60^\circ \tan 78^\circ \\
 &= \cot \theta \cot \theta - \operatorname{cosec} \theta \operatorname{cosec} \theta + \sqrt{3} \tan 12^\circ \times \sqrt{3} \times \cot(90^\circ - 78^\circ) \\
 &= \cot^2 \theta - \operatorname{cosec}^2 \theta + 3 \tan 12^\circ \cot 12^\circ \\
 &= -1 + 3 \times \tan 12^\circ \times \frac{1}{\tan 12^\circ} \\
 &= -1 + 3 \\
 &= 2 \\
 &= \text{RHS}
 \end{aligned}$$

6. Without using trigonometric tables, prove that:

$$(i) \tan 5^\circ \tan 25^\circ \tan 30^\circ \tan 65^\circ \tan 85^\circ = 1$$

$$(ii) \cot 12^\circ \cot 38^\circ \cot 52^\circ \cot 60^\circ \cot 78^\circ = \frac{1}{\sqrt{3}}$$

$$(iii) \cos 15^\circ \cos 35^\circ \operatorname{cosec} 55^\circ \cos 60^\circ \operatorname{cosec} 75^\circ = \frac{1}{2}$$

$$(iv) \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 180^\circ = 0$$

$$(v) \left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2 = 2$$

Sol:

$$\begin{aligned} (i) \text{ LHS} &= \tan 5^\circ \tan 25^\circ \tan 30^\circ \tan 65^\circ \tan 85^\circ \\ &= \tan (90^\circ - 85^\circ) \tan (90^\circ - 65^\circ) \times \frac{1}{\sqrt{3}} \times \frac{1}{\cos 60^\circ} \frac{1}{\cos 85^\circ} \\ &= \cot 85^\circ \cot 65^\circ \frac{1}{\sqrt{3}} \frac{1}{\cos 60^\circ} \frac{1}{\cos 85^\circ} \\ &= \frac{1}{\sqrt{3}} = \text{RHS} \end{aligned}$$

$$\begin{aligned} (ii) \text{ LHS} &= \cot 12^\circ \cot 38^\circ \cot 52^\circ \cot 60^\circ \cot 78^\circ \\ &= \tan (90^\circ - 12^\circ) \times \tan (90^\circ - 38^\circ) \times \cot 52^\circ \times \frac{1}{\sqrt{3}} \times \cot 78^\circ \\ &= \frac{1}{\sqrt{3}} \times \tan 78^\circ \times \tan 52^\circ \times \cot 52^\circ \times \cot 78^\circ \\ &= \frac{1}{\sqrt{3}} \times \tan 78^\circ \times \tan 52^\circ \times \frac{1}{\tan 52^\circ} \times \frac{1}{\tan 78^\circ} \\ &= \frac{1}{\sqrt{3}} \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} (iii) \text{ LHS} &= \cos 15^\circ \cos 35^\circ \operatorname{cosec} 55^\circ \cos 60^\circ \operatorname{cosec} 75^\circ \\ &= \cos (90^\circ - 75^\circ) \cos (90^\circ - 55^\circ) \frac{1}{\sin 55^\circ} \times \frac{1}{2} \times \frac{1}{\sin 75^\circ} \\ &= \sin 75^\circ \sin 55^\circ \frac{1}{\sin 55^\circ} \times \frac{1}{2} \times \frac{1}{\sin 75^\circ} \\ &= \frac{1}{2} = \text{RHS} \end{aligned}$$

$$\begin{aligned} (iv) \text{ LHS} &= \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 180^\circ \\ &= \cos 1^\circ \times \cos 2^\circ \times \cos 3^\circ \times \dots \times \cos 90^\circ \times \dots \times \cos 180^\circ \\ &= \cos 1^\circ \times \cos 2^\circ \times \cos 3^\circ \times \dots \times 0 \times \dots \times \cos 180^\circ \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} (v) \text{ LHS} &= \left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2 \\ &= \left(\frac{\cos (90^\circ - 49^\circ)}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\cos (90^\circ - 49^\circ)}\right)^2 \\ &= \left(\frac{\cos 41^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\cos 41^\circ}\right)^2 \\ &= 1^2 + 1^2 \end{aligned}$$

$$= 1 + 1$$

$$= 2$$

$$= \text{RHS}$$

Disclaimer: The RHS of (v) given in textbook is incorrect. There should be 2 instead 1. The same has been corrected in the solution here.

7. Prove that:

$$(i) \sin(70^\circ + \theta) - \cos(20^\circ - \theta) = 0$$

$$(ii) \tan(55^\circ - \theta) - \cot(35^\circ + \theta) = 0$$

$$(iii) \operatorname{cosec}(67^\circ + \theta) - \sec(23^\circ - \theta) = 0$$

$$(iv) \operatorname{cosec}(65^\circ + \theta) - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \cot(35^\circ + \theta) = 0$$

$$(v) \sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 80^\circ \tan 89^\circ = 1$$

Sol:

$$(i) \text{LHS} = \sin(70^\circ + \theta) - \cos(20^\circ - \theta)$$

$$= \sin\{90^\circ - (20^\circ - \theta)\} - \cos(20^\circ - \theta)$$

$$= \cos(20^\circ - \theta) - \cos(20^\circ - \theta)$$

$$= 0$$

$$= \text{RHS}$$

$$(ii) \text{LHS} = \tan(55^\circ - \theta) - \cot(35^\circ + \theta)$$

$$= \tan\{90^\circ - (35^\circ + \theta)\} - \cot(35^\circ + \theta)$$

$$= \cot(35^\circ + \theta) - \cot(35^\circ + \theta)$$

$$= 0$$

$$= \text{RHS}$$

$$(iii) \text{LHS} = \operatorname{cosec}(67^\circ + \theta) - \sec(23^\circ - \theta)$$

$$= \operatorname{cosec}\{90^\circ - (23^\circ - \theta)\} - \sec(23^\circ - \theta)$$

$$= \sec(23^\circ - \theta) - \sec(23^\circ - \theta)$$

$$= 0$$

$$= \text{RHS}$$

$$(iv) \text{LHS} = \operatorname{cosec}(65^\circ + \theta) - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \cot(35^\circ + \theta)$$

$$= \operatorname{cosec}\{90^\circ - (25^\circ - \theta)\} - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \cot\{90^\circ - (55^\circ - \theta)\}$$

$$= \sec(25^\circ - \theta) - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \tan(55^\circ - \theta)$$

$$= 0$$

$$= \text{RHS}$$

$$(v) \text{LHS} = \sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 80^\circ \tan 89^\circ$$

$$= \sin\{90^\circ - (40^\circ - \theta)\} - \cos(40^\circ - \theta) + \{\tan 1^\circ \tan(90^\circ - 1^\circ)\} \{\tan 10^\circ \tan(90^\circ - 10^\circ)\}$$

$$= \cos(40^\circ - \theta) - \cos(40^\circ - \theta) + (\tan 1^\circ \cot 1^\circ) (\tan 10^\circ \cot 10^\circ)$$

$$\begin{aligned}
 &= \left(\frac{1}{\cot 1^\circ} \times \cot 1^\circ \right) \left(\tan 10^\circ \times \frac{1}{\tan 10^\circ} \right) \\
 &= 1 \times 1 \\
 &= 1 \\
 &= \text{RHS}
 \end{aligned}$$

8. Express each of the following in terms of trigonometric ratios of angles lying between 0° and 45° .

(i) $\sin 67^\circ + \cos 75^\circ$

(ii) $\cot 65^\circ + \tan 49^\circ$

(iii) $\sec 78^\circ + \operatorname{cosec} 56^\circ$

(iv) $\operatorname{cosec} 54^\circ + \sin 72^\circ$

Sol:

(i) $\sin 67^\circ + \cos 75^\circ$

$$= \cos (90^\circ - 67^\circ) + \sin (90^\circ - 75^\circ)$$

$$= \cos 23^\circ + \sin 15^\circ$$

(ii) $\cot 65^\circ + \tan 49^\circ$

$$= \cos (90^\circ - 65^\circ) + \cot (90^\circ - 49^\circ)$$

$$= \cos 25^\circ + \cot 41^\circ$$

(iii) $\sec 78^\circ + \operatorname{cosec} 56^\circ$

$$= \sec (90^\circ - 12^\circ) + \operatorname{cosec} (90^\circ - 34^\circ)$$

$$= \operatorname{cosec} 12^\circ + \sec 34^\circ$$

(iv) $\operatorname{cosec} 54^\circ + \sin 72^\circ$

$$= \sec (90^\circ - 54^\circ) + \cos (90^\circ - 72^\circ)$$

$$= \sec 36^\circ + \cos 18^\circ$$

9. If A, B, C are the angles of a $\triangle ABC$, prove that $\tan \left(\frac{C+A}{2} \right) = \cot \frac{B}{2}$.

Sol:

In $\triangle ABC$,

$$A + B + C = 180^\circ$$

$$\Rightarrow A + C = 180^\circ - B \quad \dots\dots(i)$$

Now,

$$\text{LHS} = \tan \left(\frac{C+A}{2} \right)$$

$$= \tan \left(\frac{180^\circ - B}{2} \right) \quad [\text{Using (i)}]$$

$$= \tan \left(90^\circ - \frac{B}{2} \right)$$

$$= \cot \frac{B}{2}$$

$$= \text{RHS}$$

10. If $\cos 2\theta = \sin 4\theta$ and 2θ is acute, then find the value of θ .

Sol:

We have,

$$\cos 2\theta = \sin 4\theta$$

$$\Rightarrow \sin (90^\circ - 2\theta) = \sin 4\theta$$

Comparing both sides, we get

$$90^\circ - 2\theta = 4\theta$$

$$\Rightarrow 2\theta + 4\theta = 90^\circ$$

$$\Rightarrow 6\theta = 90^\circ$$

$$\Rightarrow \theta = \frac{90^\circ}{6}$$

$$\therefore \theta = 15^\circ$$

Hence, the value of θ is 15° .

11. If $\sec 2A = \operatorname{cosec} (A - 42^\circ)$, where $2A$ is an acute angle, find the value of A .

Sol:

We have,

$$\sec 2A = \operatorname{cosec} (A - 42^\circ)$$

$$\Rightarrow \operatorname{cosec} (90^\circ - 2A) = \operatorname{cosec} (A - 42^\circ)$$

Comparing both sides, we get

$$90^\circ - 2A = A - 42^\circ$$

$$\Rightarrow 2A + A = 90^\circ + 42^\circ$$

$$\Rightarrow 3A = 132^\circ$$

$$\Rightarrow A = \frac{132^\circ}{3}$$

$$\therefore A = 44^\circ$$

Hence, the value of A is 44° .

12. If $\sin 3A = \cos (A - 26^\circ)$, where $3A$ is an acute angle, find the value of A .

Sol:

$$\sin 3A = \cos (A - 26^\circ)$$

$$\Rightarrow \cos (90^\circ - 3A) = \cos (A - 26^\circ) \quad [\because \sin \theta = \cos (90^\circ - \theta)]$$

$$\Rightarrow 90^\circ - 3A = A - 26^\circ$$

$$\Rightarrow 116^\circ = 4A$$

$$\Rightarrow A = \frac{116^\circ}{4} = 29^\circ$$

13. If $\tan 2A = \cot (A - 12^\circ)$, where $2A$ is an acute angle, find the value of A .

Sol:

$$\tan 2A = \cot (A - 12^\circ)$$

$$\Rightarrow \cot (90^\circ - 2A) = \cot (A - 12^\circ) \quad [\because \tan \theta = \cot (90^\circ - \theta)]$$

$$\Rightarrow (90^\circ - 2A) = (A - 12^\circ)$$

$$\Rightarrow 102^\circ = 3A$$

$$\Rightarrow A = \frac{102^\circ}{3} = 34^\circ$$

14. If $\sec 4A = \operatorname{cosec} (A - 15^\circ)$, where $4A$ is an acute angle, find the value of A .

Sol:

$$\sec 4A = \operatorname{cosec} (A - 15^\circ)$$

$$\Rightarrow \operatorname{cosec} (90^\circ - 4A) = \operatorname{cosec} (A - 15^\circ) \quad [\because \sec \theta = \operatorname{cosec} (90^\circ - \theta)]$$

$$\Rightarrow (90^\circ - 4A) = (A - 15^\circ)$$

$$\Rightarrow 105^\circ = 5A$$

$$\Rightarrow A = \frac{105^\circ}{5} = 21^\circ$$

15. Without using trigonometric tables, evaluate the following.

Sol:

$$\begin{aligned} & \frac{2}{3} \operatorname{cosec}^2 58^\circ - \frac{2}{3} \cot 58^\circ \tan 32^\circ - \frac{5}{3} \tan 13^\circ \tan 37^\circ \tan 45^\circ \tan 53^\circ \tan 77^\circ \\ &= \frac{2}{3} (\operatorname{cosec}^2 58^\circ - \cot 58^\circ \tan 32^\circ) - \frac{5}{3} \tan 13^\circ \tan (90^\circ - 13^\circ) \tan 37^\circ \tan (90^\circ - 37^\circ) \\ & \quad (\tan 45^\circ) \\ &= \frac{2}{3} \{ \operatorname{cosec}^2 58^\circ - \cot 58^\circ \tan (90^\circ - 58^\circ) \} - \frac{5}{3} \tan 13^\circ \cot 13^\circ \tan 37^\circ \cot 37^\circ \quad (1) \\ &= \frac{2}{3} (\operatorname{cosec}^2 58^\circ - \cot 58^\circ \tan 58^\circ) - \frac{5}{3} \tan 13^\circ \frac{1}{\tan 13^\circ} \tan 37^\circ \frac{1}{\tan 37^\circ} \\ &= \frac{2}{3} (\operatorname{cosec}^2 58^\circ - \cot^2 58^\circ) - \frac{5}{3} \\ &= \frac{2}{3} - \frac{5}{3} \\ &= -1 \end{aligned}$$

Hence proved.