16. If A and B are acute angles such that tan A = $\frac{1}{3}$, tan B = $\frac{1}{2}$ and tan (A + B) = $\frac{\tan A+\tan B}{1-\tan A\tan B}$, show that A + B = 45^0.

Sol:

Given:

tan A = $\frac{1}{3}$ and tan B = $\frac{1}{2}$

tan (A + B) = $\frac{\tan A+\tan B}{1-\tan A\tan B}$

On substituting these values in RHS of the expression, we get:

$\frac{\tan A+\tan B}{1-\tan A\tan B} = \frac{\left(\frac{1}{3}+\frac{1}{2}\right)}{\left(1-\frac{1}{3}\times\frac{1}{3}\right)} = \frac{\left(\frac{5}{6}\right)}{\left(1-\frac{1}{6}\right)} = \frac{\left(\frac{5}{6}\right)}{\left(\frac{5}{6}\right)} = 1$

$\Rightarrow$ tan (A + B) = 1 = tan 45^0 [$\because$ tan 450 = 1]

$\therefore$ A + B = 45^0

17. Using the formula, tan 2A = $\frac{2\tan A}{1-tan^2A}$, find the value of tan 60^0, it being given that tan 30^0 = $\frac{1}{\sqrt{3}}$.

Sol:

A = 30^0

$\Rightarrow$ 2A = 2 $\times$ 30^0 = 60^0

By substituting the value of the given T-ratio, we get:

tan 2A = $\frac{2\tan A}{1-tan^2A}$

$\Rightarrow$ tan 60^0 = $\frac{2\tan 30^0}{1-tan^2 30^0} = \frac{2\times\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1-\frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{2}{3}} = \left(\frac{2}{\sqrt{3}}\right)\times\frac{3}{2} = \sqrt{3}$

$\therefore$ tan 60^0 = $\sqrt{3}$

18. Using the formula, cos A = $\sqrt{\frac{1+\cos 2A}{2}}$, find the value of cos 30^0, it being given that cos 60^0 = $\frac{1}{2}$.

Sol:

A = 30^0

$\Rightarrow$ 2A = 2 $\times$ 30^0 = 60^0

By substituting the value of the given T-ratio, we get:

Cos A = $\sqrt{\frac{1+\cos 2A}{2}}$

Cos 30^0 = $\sqrt{\frac{1+\cos 60^0}{2}} = \sqrt{\frac{1+\frac{1}{2}}{2}} = \sqrt{\frac{\frac{3}{2}}{2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\therefore$ cos A = $\frac{\sqrt{3}}{2}$

19. Using the formula, sin A = $\sqrt{\frac{1-\cos 2A}{2}}$, find the value of sin 30^0, it being given that cos 60^0 = $\frac{1}{2}$.

Sol:

A = 30^0

$\Rightarrow$ 2A = 2 $\times$ 30^0 = 60^0

By substituting the value of the given T-ratio, we get:

sin A = $\sqrt{\frac{1-\cos 2A}{2}}$

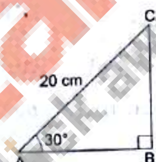
sin 30^0 = $\sqrt{\frac{1-\cos 60^0}{2}} = \sqrt{\frac{1-\frac{1}{2}}{2}} = \sqrt{\frac{\frac{1}{2}}{2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\therefore$ sin $30^0 = \frac{1}{2}$

20. In the adjoining figure, ΔABC is a right-angled triangle in which $\angle$B = 90^0, $\angle 30^0$ and AC = 20cm.

Find (i) BC, (ii) AB.

Sol:



From the given right-angled triangle, we have:

$\frac{BC}{AC}$ = sin 30^0

$\Rightarrow \frac{BC}{20} = \frac{1}{2}$

$\Rightarrow$ BC = $\frac{20}{2}$ = 10cm

Also, $\frac{AB}{AC}$ = cos 30^0

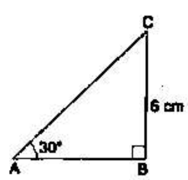
$\Rightarrow \frac{AB}{20} = \frac{\sqrt{3}}{2}$

$\Rightarrow$ AB = $\left(20 \times \frac{\sqrt{3}}{2}\right) = 10\sqrt{3}$ cm

$\therefore$ BC = 10cm and AB = $10\sqrt{3}$ cm

21. In the adjoining figure, ΔABC is right-angled at B and $\angle$A = 30^0. If BC = 6cm, find (i) AB, (ii) AC.

Sol:



From the given right-angled triangle, we have:

$\frac{BC}{AB}$ = tan 30^0

$\Rightarrow \frac{6}{AB} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ AB = $6\sqrt{3}$cm

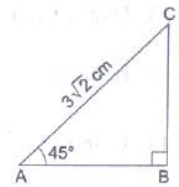
Also, $\frac{BC}{AC}$ = sin 30^0

$\Rightarrow \frac{6}{AC} = \frac{1}{2}$

$\Rightarrow$ AC = (2×6) = 12 cm

$\therefore$ AB = $6\sqrt{3}$ cm and AC = 12 cm

22. In the adjoining figure, ΔABC is right-angled at B and $\angle A = 45^0$. If $AC = 3\sqrt{2}$cm, find (i) BC, (ii) AB.



Sol:

From the right-angled ΔABC, we have:

$\frac{BC}{AC} = \sin 45^0$

$\Rightarrow \frac{BC}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow BC = 3cm$

Also, $\frac{AB}{AC} = \cos 45^0$

$\Rightarrow \frac{AB}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow AB = 3$ cm

$\therefore$ BC = 3 cm and AB = 3 cm

23. If $\sin (A + B) = 1$ and $\cos (A – B) = 1$, $0^0 \leq (A + B) \leq 90^0$ and $A > B$, then find A and B.

Sol:

Here, $\sin (A + B) = 1$

$\Rightarrow \sin (A + B) = 90^0$ $[\because \sin 90^0 = 1]$

$\Rightarrow (A + B) = 90^0$ …….(i)

Also, $\cos (A – B) = 1$

$\Rightarrow \cos (A – B) = 0^0$ $[\because \cos 0^0 = 1]$

$\Rightarrow A – B = 0^0$ ….(ii)

Solving (i) and (ii), we get:

$A = 45^0$ and $B = 45^0$

24. If $\sin (A – B) = \frac{1}{2}$ and $\cos (A + B) = \frac{1}{2}$, $0^0 \leq (A + B) \leq 90^0$ and $A > B$, then find A and B.

Sol:

Here, $\sin (A – B) = \frac{1}{2}$

$\Rightarrow \sin (A – B) = 30^0$ $[\because \sin 30^0 = \frac{1}{2}]$

$\Rightarrow (A – B) = 30^0$ …….(i)

Also, $\cos (A + B) = \frac{1}{2}$

$\Rightarrow \cos (A + B) = \cos 60^0$ $[\because \cos 60^0 = \frac{1}{2}]$

$\Rightarrow A + B = 60^0$ ….(ii)

Solving (i) and (ii), we get:

$A = 45^0$ and $B = 15^0$

25. If $\tan (A – B) = \frac{1}{\sqrt{3}}$ and $\tan (A + B) = \sqrt{3}$, $0^0 \leq (A + B) \leq 90^0$ and $A > B$, then find A and B.

Sol:

Here, $\tan (A – B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan (A – B) = \tan 30^0$ $[\because \tan 30^0 = \frac{1}{\sqrt{3}}]$

$\Rightarrow (A – B) = 30^0$ …….(i)

Also, $\tan (A + B) = \sqrt{3}$

$\Rightarrow$ tan (A + B) = tan 60^0 [$\because$ tan $60^0 = \sqrt{3}$]

$\Rightarrow$ A + B = 60^0 …….(ii)

Solving (i) and (ii), we get:

A = 45^0 and B = 15^0

26. If 3x = cosec θ = and $\frac{3}{x}$ = cot θ find the value of $3\left(x^2 - \frac{1}{x^2}\right)$

Sol:

$3\left(x^2 - \frac{1}{x^2}\right)$

$= \frac{9}{3}\left(x^2 - \frac{1}{x^2}\right)$

$= \frac{1}{3}\left(9x^2 - \frac{9}{x^2}\right)$

$= \frac{1}{3}\left[(3x^2) - \left(\frac{3}{x}\right)^2\right]$

$= \frac{1}{3}[(cosec\ \theta)^2 - (cot\ \theta)^2]$

$= \frac{1}{3}(\text{cosec}^2\theta - \cot^2\theta)$

$= \frac{1}{3}(1) = \frac{1}{3}$

27. If sin (A+B) = sin A cos B + cos A sin B and cos (A-B) = cos A cos B + sin A sin B

(i) sin (75^0)

(ii) cos (15^0)

Sol:

Let A = 45^0 and B = 30^0

(i) As, sin(A + B) = sin A cos B + cos A sin B

$\Rightarrow$ sin ($45^0 + 30^0$) = sin 45^0 cos 30^0 + cos 45^0 sin 30^0

$\Rightarrow$ sin (75^0) = $\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$\Rightarrow$ sin (75^0) = $\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$\therefore$ sin (75^0) = $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) As, cos (A – B) = cos A cos B + sin A sin B

$\Rightarrow$ cos ($45^0 - 30^0$) = cos 45^0 cos 30^0 + sin 45^0 sin 30^0

$\Rightarrow$ cos (15^0) = $\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$\Rightarrow$ cos (15^0) = $\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$\therefore$ cos (15^0) = $\frac{\sqrt{3}+1}{2\sqrt{2}}$

Disclaimer: cos 15^0 can also be written by taking A = 60^0 and B = 45^0.