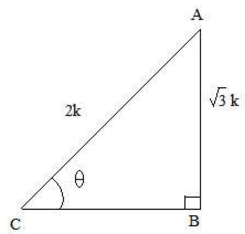
1. If $\sin\ \theta = \frac{\sqrt{3}}{2}$, find the value of all T- ratios of θ

Sol:

Let us first draw a right ΔABC, right angled at B and $\angle C = \theta$

Now, we know that $\sin\theta = \frac{Prependicular}{hypotenuse} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$



So, if AB = $\sqrt{3}k, then\ AC = 2k, where\ k\ is\ a\ positive\ number.$

Now, using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 - AB^2 = (2k)^2 - (\sqrt{3}k)^2$

$\Rightarrow BC^2 = 4k^2 - 3k^2 = k^2$

$\Rightarrow$ BC = k

Now, finding the other T-rations using their definitions, we get:

Cos $\theta = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

Tan $\theta = \frac{AB}{BC} = \frac{\sqrt{3}k}{k} = \sqrt{3}$

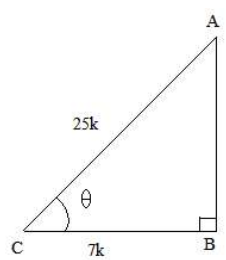
$\therefore$ cot $\theta = \frac{1}{\tan\theta} = \frac{1}{\sqrt{3}}, cosec\ \theta = \frac{1}{\sin\theta} = \frac{2}{\sqrt{3}}\ and\ \sec\theta = \frac{1}{\cos\theta} = 2$

2. If $\cos\ \theta = \frac{7}{25}$, find the value of all T-ratios of θ

Sol:

Let us first draw a right ΔABC, right angled at B and $\angle C = \theta$.

Now, we know that $\cos\theta = \frac{Base}{hypotenuse} = \frac{BC}{AC} = \frac{7}{25}$



So, if BC = 7k, then AC = 25k, were k is a positive number.

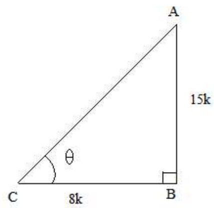
Now, using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = AC^2 - BC^2 = (25k)^2 - (7k)^2$
$\Rightarrow AB^2 = 625k^2 - 49k^2 = 576k^2$
$\Rightarrow$ AB = 24k
Now, finding the trigonometric ratios using their definitions, we get:
Sin $\theta = \frac{AB}{AC} = \frac{24k}{25k} = \frac{24}{25}$
Tan $\theta = \frac{AB}{BC} = \frac{24k}{7k} = \frac{24}{7}$
$\therefore$ cot $\theta = \frac{1}{\tan\theta} = \frac{7}{24}$, $cosec\ \theta = \frac{1}{\sin\theta} = \frac{25}{24}$ $and\ sec\theta = \frac{1}{\cos\theta} = \frac{25}{7}$

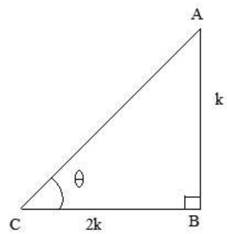
3. If $\tan\theta = \frac{15}{8}$, find the values of all T-ratios of θ

Sol:
Let us first draw a right ΔABC, right angled at B and $\angle C = \theta$
Now, we know that tan θ = $\frac{Perpendicular}{Base} = \frac{AB}{BC} = \frac{15}{8}$



So, if BC = 8k, then AB = 15k where k is positive number.
Now, using Pythagoras theorem, we have:
$AC^2 = AB^2 + BC^2 = (15k)^2 + (8k)^2$
$\Rightarrow AC^2 = 225k^2 + 64k^2 = 289k^2$
$\Rightarrow$ AC = 17k
Now, finding the other T-ratios using their definitions, we get:
Sin $\theta = \frac{AB}{AC} = \frac{15k}{17k} = \frac{15}{17}$
Cos $\theta = \frac{BC}{AC} = \frac{8k}{17k} = \frac{8}{17}$
$\therefore$ cot $\theta = \frac{1}{\tan\theta} = \frac{8}{15}$, $cosec\ \theta = \frac{1}{\sin\theta} = \frac{17}{15}$ $and\ sec\theta = \frac{1}{\cos\theta} = \frac{17}{8}$

4. If cot $\theta = 2$ find all the values of all T-ratios of θ
Sol:
Let us first draw a right ΔABC, right angled at B and $\angle C = \theta$
Now, we know that cot θ = $\frac{Base}{Perpendicular} = \frac{BC}{AB} = 2$



So, if BC = 2k, then AB = k, is a positive number.

Now, using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2 = (2k)^2 + (k)^2$

$\Rightarrow AC^2 = 4k^2 + k^2 = 5k^2$

$\Rightarrow AC = \sqrt{5}k$

Now, finding the other T-ratios using their definitions, we get:

Sin θ = $\frac{AB}{AC} = \frac{5}{\sqrt{5}k} = \frac{1}{\sqrt{2}}$

Cos θ = $\frac{BC}{AC} = \frac{2k}{\sqrt{5}k} = \frac{2}{\sqrt{5}}$

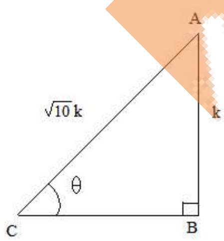
∴ tan θ = $\frac{1}{\cot\theta} = \frac{1}{2}, cosec\ \theta = \frac{1}{\sin\theta} = \sqrt{5}\ and\ sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{5}}{2}$

5. If cosec $\theta = \sqrt{10}$ find all the values of all T-ratios of θ

Sol:

Let us first draw a right ΔABC, right angled at B and $\angle C = \theta$

Now, we know that cosec θ = $\frac{Hypotenuse}{Perpendicular} = \frac{AC}{AB} = \frac{\sqrt{10}}{1}$



So, if AC = $(\sqrt{10})k, then\ AB = k\ is\ a\ positive\ number.$

Now, by using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 + BC^2$

$\Rightarrow BC^2 = 9k^2$

$\Rightarrow$ BC = 3k

Now, finding the other T-ratios using their definitions, we get:

tan θ = $\frac{AB}{BC} = \frac{k}{3k} = \frac{1}{3}$

cos θ = $\frac{BC}{AC} = \frac{3k}{\sqrt{10}k} = \frac{3}{\sqrt{10}}$

∴ sin θ = $\frac{1}{\cos ec\ \theta} = \frac{1}{\sqrt{10}}, \cot\theta = \frac{1}{\tan\theta} = 3\ and\ sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{10}}{3}$

6. If $\sin\,\theta = \dfrac{a^2 - b^2}{a^2 + b^2}$ find all the values of all T-ratios of θ

Sol:

We have $\sin\theta = \frac{a^2-b^2}{a^2+b^2}$

As,

$\text{Cos}^2\,\theta = 1 - \sin^2\theta$

$= 1 - \left(\frac{a^2-b^2}{a^2+b^2}\right)^2$

$= \frac{1}{1} - \frac{\left(a^2-b^2\right)^2}{(a^2+b^2)^2}$

$= \frac{\left(a^2+b^2\right)^2 - \left(a^2-b^2\right)^2}{(a^2+b^2)}$

$= \frac{[(a^2+b^2)-(a^2-b^2)][(a^2+b^2)+(a^2-b^2)]}{(a^2+b^2)^2}$

$= \frac{[a^2+b^2-a^2+b^2][a^2+b^2+a^2-b^2]}{(a^2+b^2)^2}$

$= \frac{[2b^2][2a^2]}{(a^2+b^2)^2}$

$\Rightarrow \cos^2\theta = \frac{4a^2b^2}{(a^2+b^2)^2}$

$\Rightarrow cos\theta = \sqrt{\frac{4a^2b^2}{(a^2+b^2)^2}}$

$\Rightarrow cos\theta = \frac{2ab}{(a^2+b^2)}$

Also,

$tan\theta = \frac{sin\theta}{cos\theta}$

$= \frac{\left(\frac{a^2-b^2}{a^2+b^2}\right)}{\left(\frac{2ab}{a^2+b^2}\right)}$

$= \frac{a^2-b^2}{2ab}$

Now,

$cosec\theta = \frac{1}{sin\theta}$

$= \frac{1}{\left(\frac{a^2-b^2}{a^2-b^2}\right)}$

$= \frac{a^2+b^2}{a^2-b^2}$

Also,

$sec\theta = \frac{1}{cos\theta}$

$= \frac{1}{\left(\frac{2ab}{a^2+b^2}\right)}$

$= \frac{a^2+b^2}{2ab}$

And,

$cot\theta = \frac{1}{tan\theta}$

$= \frac{1}{\left(\frac{a^2-b^2}{2ab}\right)}$

$= \frac{2ab}{a^2-b^2}$

7. If 15 cot A = 8 find all the values of sin A and sec A

Sol:

We have,

$15 \cot A = 8$

$\Rightarrow \cot A = \frac{8}{15}$

As,

$cosec^2 A = 1 + cot^2 A$

$= 1 + \left(\frac{8}{15}\right)^2$

$= 1 + \frac{64}{225}$

$= \frac{225+64}{225}$

$\Rightarrow cosec^2 A = \frac{289}{225}$

$\Rightarrow cosec\ A = \sqrt{\frac{289}{225}}$

$\Rightarrow cosec\ A = \frac{17}{15}$

$\frac{1}{\sin A} = \frac{17}{15}$

Sin A = $\frac{15}{17}$

Also,

$\text{Cos}^2 A = 1 - \sin^2 A$

$= 1 - \left(\frac{15}{17}\right)^2$

$= 1 - \frac{225}{289}$

$= \frac{289-225}{289}$

$\Rightarrow cos^2 A = \frac{64}{289}$

$\Rightarrow \cos A = \sqrt{\frac{64}{289}}$

$\Rightarrow \cos A = \frac{8}{17}$

$\Rightarrow \frac{1}{\sec A} = \frac{8}{17}$

$\Rightarrow \sec A = \frac{17}{8}$

8. If $\sin A = \frac{9}{41}$ find all the values of cos A and tan A

Sol:

We have sin A $= \frac{9}{41}$

As,

$\text{Cos}^2 A = 1 - \sin^2 A$

$= 1 - \left(\frac{9}{41}\right)^2$

$= 1 - \frac{81}{1681}$

$= \frac{1681-81}{1681}$

$\Rightarrow \cos^2 A = \frac{1600}{1681}$

$\Rightarrow \cos A = \sqrt{\frac{1600}{1681}}$

$\Rightarrow \cos A = \frac{40}{41}$

Also,

$\text{Tan}\, A = \frac{sinA}{cosA}$

$= \frac{\left(\frac{9}{41}\right)}{\left(\frac{40}{41}\right)}$

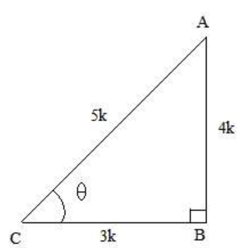
$= \frac{9}{40}$

9. If cos θ=0.6 show that (5sin θ -3tan θ) = 0

Sol:

Let us consider a right ΔABC right angled at B.

Now, we know that cos θ = 0.6 $= \frac{BC}{AC} = \frac{3}{5}$



So, if BC = 3k, then AC = 5k, where k is a positive number.

Using Pythagoras theorem, we have:

$Ac^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = AC^2 - BC^2$

$\Rightarrow AB^2 = (5k)^2 - (3k)^2 = 25k^2 - 9k^2$

$\Rightarrow AB^2 = 16k^2$

$\Rightarrow AB = 4k$

Finding out the other T-rations using their definitions, we get:

$\text{Sin}\,\theta = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

$\tan\theta = \frac{AB}{BC} = \frac{4k}{3k} = \frac{4}{3}$

Substituting the values in the given expression, we get:

$5\sin\theta - 3\tan\theta$

$\Rightarrow 5\left(\frac{4}{5}\right) - 3\left(\frac{4}{3}\right)$

$\Rightarrow 4 - 4 = 0 = RHS$

i.e., LHS = RHS

Hence, Proved.

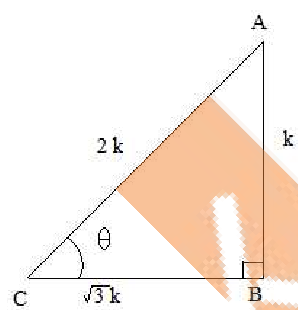
10. If cosec θ= 2 show that $\left(\cot\theta + \frac{\sin\theta}{1+\cos\theta}\right) = 2$

Sol:

Let us consider a right ΔABC, right angled at B and $\angle C = \theta$.

Now, it is given that cosec θ = 2.

Also, $\sin\theta = \frac{1}{\cos ec\theta} = \frac{1}{2} = \frac{AB}{AC}$



So, if AB =k, then AC =2k, where k is a positive number.

Using Pythagoras theorem, we have:

$\Rightarrow AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 - AB^2$

$\Rightarrow BC^2\ (2k)^2 - (k)^2$

$\Rightarrow BC^2 = 3k^2$

$\Rightarrow BC = \sqrt{3}k$

Finding out the other T-ratios using their definitions, we get:

$\cos\theta = \frac{BC}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\text{Tan}\,\theta = \frac{AB}{BC} = \frac{k}{\sqrt{3}k} = \frac{1}{\sqrt{3}}$

$\text{Cot}\,\theta = \frac{1}{\tan\theta} = \sqrt{3}$

Substituting these values in the given expression, we get:

$\text{Cot}\,\theta + \frac{\sin\theta}{1+\cos\theta}$

$= \sqrt{3} + \frac{\left(\frac{1}{2}\right)}{1+\frac{\sqrt{3}}{2}}$

$= \sqrt{3} + \frac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}}$

$= \sqrt{3} + \frac{1}{2+\sqrt{3}}$

$= \frac{\sqrt{3}(2+\sqrt{3})+1}{2+\sqrt{3}}$

$= \frac{2\sqrt{3}+3+1}{2+\sqrt{3}}$

$= \frac{2(2+\sqrt{3})}{2+\sqrt{3}} = 2$

i.e., LHS = RHS

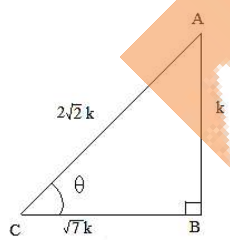
Hence proved.

11. If $\tan\theta = \frac{1}{\sqrt{7}}$ show that $\frac{\left(\cos ec^2\theta - \sec^2\theta\right)}{\cos ec^2\theta + \sec^2\theta} = \frac{3}{4}$

Sol:

Let us consider a right ΔABC, right angled at B and $\angle C = \theta$.

Now it is given that $\tan\theta = \frac{AB}{BC} = \frac{1}{\sqrt{7}}$



So, if AB = k, then BC = $\sqrt{7}k$, *wher k is a positive number*.

Using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (k)^2 + \left(\sqrt{7}k\right)^2$

$\Rightarrow AC^2 = k^2 + 7k^2$

$\Rightarrow AC = 2\sqrt{2}k$

Now, finding out the values of the other trigonometric ratios, we have:

$\text{Sin}\,\theta = \frac{AB}{AC} = \frac{k}{2\sqrt{2}k} = \frac{1}{2\sqrt{2}}$

$\text{Cos}\,\theta = \frac{BC}{AC} = \frac{\sqrt{7}k}{2\sqrt{2}k} = \frac{\sqrt{7}}{2\sqrt{2}}$

$\therefore cosec\ \theta = \frac{1}{\sin\theta} = 2\sqrt{2}\ and\ \sec\theta = \frac{1}{\cos\theta} = \frac{2\sqrt{2}}{\sqrt{7}}$

Substituting the values of cosec θ and sec θ in the give expression, we get:

$\frac{cosec^2\theta - \sec^2\theta}{cosec^2\theta + \sec^2\theta}$

$= \frac{(2\sqrt{2})^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{(2\sqrt{2})^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$

$= \frac{8 - \left(\frac{8}{7}\right)}{8 + \left(\frac{8}{7}\right)}$

$= \frac{\frac{56-8}{7}}{\frac{56+8}{7}}$

$= \frac{48}{64} = \frac{3}{4} = RHS$

i.e., LHS = RHS

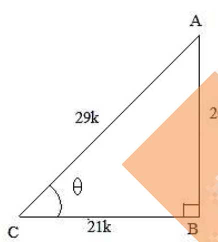
Hence proved.

12. If $\tan\theta = \frac{20}{21}$, show that $\frac{(1 - \sin\theta + \cos\theta)}{(1 + \sin\theta + \cos\theta)} = \frac{3}{7}$

Sol:

Let us consider a right ΔABC right angled at B and $\angle C = \theta$

Now, we know that $\tan\theta = \frac{AB}{BC} = \frac{2\theta}{21}$



So, if AB = 20k, then BC = 21k, where k is a positive number.

Using Pythagoras theorem, we get:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (20k)^2 + (21k)^2$

$\Rightarrow AC^2 = 841k^2$

$\Rightarrow AC = 29k$

Now. $\text{Sin}\,\theta = \frac{AB}{AC} = \frac{20}{29}$ *and* $\cos\theta = \frac{BC}{AC} = \frac{21}{29}$

Substituting these values in the give expression, we get:

$LHS = \frac{1 - \sin\theta + cos\theta}{1 + \sin\theta + \cos\theta}$

$= \frac{1 - \frac{20}{29} + \frac{21}{29}}{1 + \frac{20}{29} + \frac{21}{29}}$

$= \frac{\frac{29-20+21}{29}}{\frac{29+20+21}{29}} = \frac{30}{70} = \frac{3}{7} = RHS$

∴ LHS = RHS

Hence proved.

13. If sec $\theta = \frac{5}{4}$ show that $\frac{(\sin\theta - 2\cos\theta)}{(\tan\theta - \cot\theta)} = \frac{12}{7}$

Sol:

We have,

$\text{Sec}\,\theta = \frac{5}{4}$

$\Rightarrow \frac{1}{\cos\theta} = \frac{5}{4}$

$\Rightarrow \cos\theta = \frac{4}{5}$

Also,

$\text{Sin}^2\theta = 1 - \cos^2\theta$

$= 1 - \left(\frac{4}{5}\right)^2$

$= 1 - \frac{16}{25}$

$= \frac{9}{25}$

$\Rightarrow \sin\theta = \frac{3}{5}$

Now,

$LHS = \frac{(\sin\theta - 2\cos\theta)}{(\tan\theta - \cot\theta)}$

$= \frac{(\sin\theta - 2\cos\theta)}{\left(\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}\right)}$

$= \frac{(\sin\theta - 2\cos\theta)}{\left(\frac{\sin^2\theta - \cos^2\theta}{\sin\theta\ \cos\theta}\right)}$

$\frac{\sin\theta\cos\theta(\sin\theta - 2\cos\theta)}{(\sin^2\theta - \cos^2\theta)}$

$= \frac{\frac{3}{5}\times\frac{4}{5}\left(\frac{3}{5} - 2\times\frac{4}{5}\right)}{\left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2}$

$= \frac{\frac{12}{25}\left(\frac{3}{5} - \frac{8}{5}\right)}{\left(\frac{9}{25} - \frac{16}{25}\right)}$

$= \frac{\frac{12}{25}\times\left(\frac{-5}{5}\right)}{\left(\frac{-7}{25}\right)}$

$= \frac{12}{7}$

$=$ RHS

14. If cot $\theta = \frac{3}{4}$, show that $\sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \frac{1}{\sqrt{7}}$

Sol:

$LHS = \sqrt{\frac{\sec\theta - cosec\,\theta}{\sec\theta + cosec\,\theta}}$

$= \sqrt{\frac{\left(\frac{1}{\cos\theta} - \frac{1}{\sin\theta}\right)}{\left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)}}$

$= \sqrt{\frac{\left(\frac{\sin\theta - \cos\theta}{\sin\theta\cos\theta}\right)}{\left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right)}}$

$= \sqrt{\frac{\left(\frac{\sin\theta - \cos\theta}{\sin\theta}\right)}{\left(\frac{\sin\theta + \cos\theta}{\sin\theta}\right)}}$

$= \sqrt{\frac{\left(\frac{\sin\theta}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)}{\left(\frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)}}$

$= \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$

$= \sqrt{\frac{\left(1 - \frac{3}{4}\right)}{\left(1 + \frac{3}{4}\right)}}$

$= \sqrt{\frac{\left(\frac{1}{4}\right)}{\left(\frac{7}{4}\right)}}$

$= \sqrt{\frac{1}{7}}$

$= \frac{1}{\sqrt{7}}$

= RHS

15. If $\sin\theta = \frac{3}{4}$, show that $\sqrt{\frac{\cos ec^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \frac{\sqrt{7}}{3}$

Sol:

$LHS = \sqrt{\frac{cosec^2\theta - \cot^2\theta}{\sec^2 2 - 1}}$

$= \sqrt{\frac{1}{\tan^2\theta}}$

$= \sqrt{\cot^2\theta}$

$= \cot\theta$

$= \sqrt{cosec^2\theta - 1}$

$= \sqrt{\left(\frac{1}{\left(\frac{3}{4}\right)}\right)^2 - 1}$

$= \sqrt{\left(\frac{4}{3}\right)^2 - 1}$

$= \sqrt{\frac{16}{9} - 1}$

$= \sqrt{\frac{16-9}{9}}$

$= \sqrt{\frac{7}{9}}$

$= \frac{\sqrt{7}}{3}$

= RHS

16. If $\sin\ \theta = \frac{a}{b}$, show that $\left(\sec\theta + \tan\theta\right) = \sqrt{\frac{b+a}{b-a}}$

Sol:

$LHS = (\sec\theta + \tan\theta)$

$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= \frac{1+\sin\theta}{\cos\theta}$

$= \frac{1+\sin\theta}{\sqrt{1-\sin^2\theta}}$

$= \frac{(1+\frac{a}{b})}{\sqrt{1-\left(\frac{a}{b}\right)^2}}$

$= \frac{\left(\frac{1}{1}+\frac{a}{b}\right)}{\sqrt{\frac{1}{1}-\frac{a^2}{b^2}}}$

$= \frac{\left(\frac{b+a}{b}\right)}{\sqrt{\frac{b^2-a^2}{b^2}}}$

$= \frac{(b+a)}{\sqrt{(b+a)}\sqrt{(b-a)}}$

$= \frac{\sqrt{(b+a)}}{\sqrt{(b-a)}}$

$= \sqrt{\frac{b+a}{b-a}}$

= RHS

17. If $\cos\ \theta = \frac{3}{5}$, show that $\frac{(\sin\theta - \cot\theta)}{2\tan\theta} = \frac{3}{160}$

Sol:

$$LHS = \frac{(\sin\theta - \cot\theta)}{2\tan\theta}$$

$$= \frac{\sin\theta - \frac{\cos\theta}{\sin\theta}}{2(\frac{\sin\theta}{\cos\theta})}$$

$$= \frac{\frac{\sin^2\theta - \cos\theta}{\sin\theta}}{(\frac{2\sin\theta}{\cos\theta})}$$

$$= \frac{\cos\theta(\sin^2\theta - \cos\theta)}{2\sin^2\theta}$$

$$= \frac{\cos\theta(1 - \cos^2\theta - \cos\theta}{2(1 - \cos^2\theta)}$$

$$= \frac{\frac{3}{5}[1 - \left(\frac{3}{5}\right)^2 - \frac{3}{5}]}{2[1 - \left(\frac{3}{5}\right)^2]}$$

$$= \frac{\frac{3}{5}(\frac{1}{1} - \frac{9}{25} - \frac{3}{5})}{2(1 - \frac{9}{25})}$$

$$= \frac{\frac{3}{5}(\frac{25-9-15}{25})}{2(\frac{25-9}{25})}$$

$$= \frac{\frac{3}{5}(\frac{1}{25})}{2(\frac{16}{25})}$$

$$= \frac{3}{5\times2\times16}$$

$$= \frac{3}{160}$$

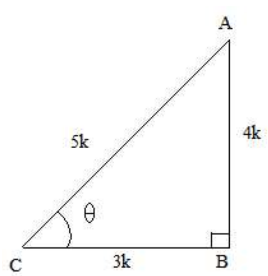
= RHS

18. If $\tan\ \theta = \frac{4}{3}$, show that $(\sin\theta + \cos\theta) = \frac{7}{5}$

Sol:

Let us consider a right ΔABC, right angled at B and $\angle C = \theta$

Now, we know that $\tan\theta = \frac{AB}{BC} = \frac{4}{3}$



So, if BC = 3k, then AB = 4k, where k is a positive number.

Using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2 = (4k)^2 + (3k)^2$

$\Rightarrow AC^2 = 16k^2 + 9k^2 = 25k^2$

$\Rightarrow$ AC = 5k

Finding out the values of $\sin\theta$ *and* $\cos\theta$ *using their definitions, we have*:

$\text{Sin}\,\theta = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

$\text{Cos}\,\theta = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

Substituting these values in the given expression, we get:

$(\sin\theta + \cos\theta) = \left(\frac{4}{5} + \frac{3}{5}\right) = \left(\frac{7}{5}\right) = RHS$

i.e., LHS = RHS

Hence proved.

19. If $\tan\theta = \frac{a}{b}$, show that $\frac{(a\sin\theta - b\cos\theta)}{(a\sin\theta + b\cos\theta)} = \frac{(a^2 - b^2)}{a^2 + b^2}$

Sol:

It is given that $\tan\theta = \frac{a}{b}$

$LHS = \frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$

Dividing the numerator and denominator by $\cos\theta$, *we get*:

$\frac{a\tan\theta - b}{a\tan\theta + b} \quad \left(\therefore \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

Now, substituting the value of $\tan\theta$ *in the above expression, we get*:

$\frac{a\left(\frac{a}{b}\right) - b}{a\left(\frac{a}{b}\right) + b}$

$= \frac{\frac{a^2}{b} - b}{\frac{a^2}{b} + b}$

$= \frac{a^2 - b^2}{a^2 + b^2} = RHS$

i.e., LHS = RHS

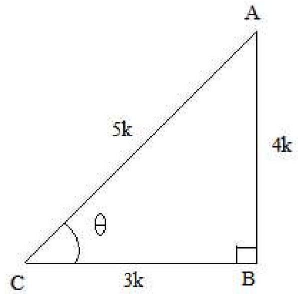
Hence proved.

20. If $3\tan\theta = 4$, show that $\frac{(4\cos\theta - \sin\theta)}{(4\cos\theta + \sin\theta)} = \frac{4}{5}$

Sol:

Let us consider a right ΔABC right angled at B and $\angle C = \theta$.

We know that $\tan\theta = \frac{AB}{BC} = \frac{4}{3}$



So, if BC = 3k, then AB = 4k, where k is a positive number.
Using Pythagoras theorem, we have:
$AC^2 = AB^2 + BC^2$
$\Longrightarrow AC^2 = 16k^2 + 9k^2$
$\Longrightarrow AC^2 = 25k^2$
$\Longrightarrow$ AC = 5k
Now, we have:
$\text{Sin}\,\theta = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$
$\text{Cos}\,\theta = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$
Substituting these values in the given expression, we get:
$\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$
$= \frac{4\left(\frac{3}{5}\right) - \frac{4}{5}}{2\left(\frac{3}{5}\right) + \frac{4}{5}}$
$= \frac{\frac{12}{5} - \frac{4}{5}}{\frac{6}{5} + \frac{4}{5}}$
$= \frac{\frac{12-4}{5}}{\frac{6+4}{5}}$
$= \frac{8}{10} = \frac{4}{5} = RHS$
i.e., LHS = RHS
Hence proved.

21. If $3\cot\theta = 2$, show that $\frac{(4\sin\theta - 4\cos\theta)}{(2\sin\theta + 6\cos\theta)} = \frac{1}{3}$

Sol:
It is given that $\cos\theta = \frac{2}{3}$
$LHS = \frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$
Dividing the above expression by $\sin\theta, we\ get$:
$\frac{4 - 3\cot\theta}{2 + 6\cot\theta}$ $\quad [\because \cot\theta = \frac{\cos\theta}{\sin\theta}]$
Now, substituting the values of $\cot\theta$ $in\ the\ above\ expression, we\ get$:

$\frac{4-3(\frac{2}{3})}{2+6(\frac{2}{3})}$

$= \frac{4-2}{2+4} = \frac{2}{6} = \frac{1}{3}$

i.e., LHS = RHS

Hence proved.

22. If $3\cot\theta = 4$, show that $\frac{(1-\tan^2\theta)}{(1+\tan^2\theta)} = (\cos^2\theta - \sin^2\theta)$

Sol:

$LHS = \frac{(1-\tan^2\theta)}{(1+\tan^2\theta)}$

$= \frac{(1-\frac{1}{\cot^2\theta})}{(1+\frac{1}{\cot^2\theta})}$

$= \frac{\frac{\cot^2\theta-1}{\cot^2\theta}}{(\frac{\cot^2\theta+1}{\cot^2\theta})}$

$= \frac{\cot^2\theta-1}{\cot^2\theta+1}$

$= \frac{\left(\frac{4}{3}\right)^2-1}{\left(\frac{4}{3}\right)^2+1}$ $\left(As, 3\cot\theta = 4 \text{ or } \cot\theta = \frac{4}{3}\right)$

$= \frac{\frac{16}{9}-1}{\frac{16}{9}+1}$

$= \frac{(\frac{16-9}{9})}{(\frac{16+9}{9})}$

$= \frac{(\frac{7}{9})}{(\frac{25}{9})}$

$= \frac{7}{25}$

$RHS = (\cos^2\theta - \sin^2\theta)$

$= \frac{(\cos^2\theta-\sin^2\theta)}{1}$

$= \frac{(\frac{\cos^2\theta-\sin^2\theta}{\sin^2\theta})}{(\frac{1}{\sin^2\theta})}$

$= \frac{\frac{\cos^2\theta}{\sin^2\theta}-\frac{\sin^2\theta}{\sin^2\theta}}{cosec^2\theta}$

$= \frac{(\cot^2\theta-1)}{(\cot^2\theta+1)}$

$= \frac{[\left(\frac{4}{3}\right)^2-1]}{[\left(\frac{4}{3}\right)^2+1]}$

$= \frac{(\frac{16}{9} - \frac{1}{1})}{(\frac{16}{9} + \frac{1}{1})}$

$= \frac{(\frac{16-9}{9})}{(\frac{16+9}{9})}$

$= \frac{(\frac{7}{9})}{(\frac{25}{9})}$

$= \frac{7}{25}$

Since, LHS = RHS

Hence, verified.

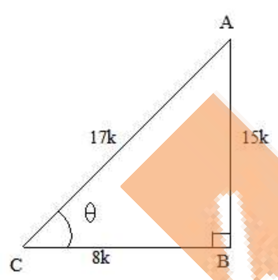
23. If $\sec\theta = \frac{17}{8}$ verify that $\frac{(3-4\sin^2\theta)}{(4\cos^2\theta-3)} = \frac{(3-\tan^2\theta)}{(1-\tan^3\theta)}$

Sol:

It is given that $\sec\theta = \frac{17}{8}$

Let us consider a right ΔABC right angled at B and $\angle C = \theta$

We know that $\cos\theta = \frac{1}{\sec\theta} = \frac{8}{17} = \frac{BC}{AC}$



So, if BC = 8k, then AC = 17k, where k is a positive number.

Using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = AC^2 - BC^2 = (17k)^2 - (8k)^2$

$\Rightarrow AB^2 = 289k^2 - 64k^2 = 225k^2$

$\Rightarrow$AB = 15k.

Now, $\tan\theta = \frac{AB}{BC} = \frac{15}{8}$ and $\sin\theta = \frac{AB}{AC} = \frac{15k}{17k} = \frac{15}{17}$

The given expression is $\frac{3-4\sin^2\theta}{4\cos^2\theta-3} = \frac{3-\tan^2\theta}{1-3\tan^2\theta}$

Substituting the values in the above expression, we get:

$LHS = \frac{3-4\left(\frac{15}{17}\right)^2}{4\left(\frac{8}{17}\right)^2-3}$

$= \frac{3-\frac{900}{289}}{\frac{256}{289}-3}$

$= \frac{867-900}{256-867} = -\frac{33}{-611} = \frac{33}{611}$

$RHS = \frac{3-\left(\frac{15}{8}\right)^2}{1-3\left(\frac{15}{8}\right)^2}$

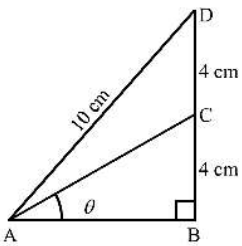
$= \frac{3-\frac{225}{64}}{1-\frac{675}{64}}$

$= \frac{192-255}{64-675} = -\frac{33}{-611} = \frac{33}{611}$

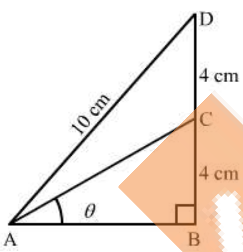
∴ LHS = RHS

Hence proved.

24. In the adjoining figure, $\angle B = 90^\circ$, $\angle BAC = \theta°$, $BC = CD = 4cm$ and AD = 10 cm. find (i) $\sin\theta$ and (ii) $\cos\theta$



Sol:



In ΔABD,

Using Pythagoras theorem, we get

$AB = \sqrt{AD^2 - BD^2}$

$= \sqrt{10^2 - 8^2}$

$= \sqrt{100 - 64}$

$= \sqrt{36}$

= 6 cm

Again,

In ΔABC,

Using Pythagoras therem, we get

$AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{6^2 + 4^2}$

$= \sqrt{36 + 16}$

$= \sqrt{52}$

$= 2\sqrt{13}\ cm$

Now,

(i) $\sin\theta = \frac{BC}{AC}$

$= \frac{4}{2\sqrt{13}}$

$= \frac{2}{\sqrt{13}}$

$= \frac{2\sqrt{13}}{13}$

(ii) $\cos\theta = \frac{AB}{AC}$

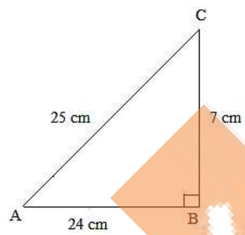
$= \frac{6}{2\sqrt{13}}$

$= \frac{3}{\sqrt{13}}$

$= \frac{3\sqrt{13}}{13}$

25. In a ΔABC, $\angle B = 90°$, AB= 24 cm and BC = 7 cm find (i) sin A (ii) cos A (iii) sin C (iv) cos C

Sol:



Using Pythagoras theorem, we get:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (24)^2 + (7)^2$

$\Rightarrow AC^2 = 576 + 49 = 625$

$\Rightarrow AC = 25\ cm$

Now, for T-Ratios of $\angle A, base = AB\ and\ perpendicular = BC$

(i) $\sin A = \frac{BC}{AC} = \frac{7}{25}$

(ii) $\cos A = \frac{AB}{AC} = \frac{24}{25}$

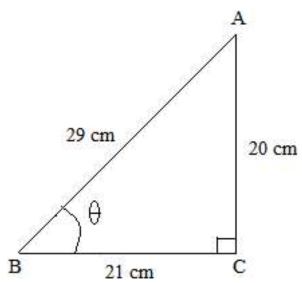
Similarly, for T-Ratios of $\angle C, base = BC\ and\ perpendicular = AB$

(iii) $\sin C = \frac{AB}{AC} = \frac{24}{25}$

(iv) $\cos C = \frac{BC}{AC} = \frac{7}{25}$

26. In ΔABC , $\angle C = 90°$ $\angle ABC = \theta°$ BC = 21 units . and AB= 29 units. Show that $\left(\cos^2\theta - \sin^2\theta\right) = \frac{41}{841}$

Sol:



Using Pythagoras theorem, we get:

$AB^2 = AC^2 + BC^2$

$\Rightarrow AC^2 = AB^2 - BC^2$

$\Rightarrow AC^2 = (29)^2 - (21)^2$

$\Rightarrow AC^2 = 841 - 441$

$\Rightarrow AC^2 = 400$

$\Rightarrow AC = \sqrt{400} = 20\ units$

Now, $\sin\theta = \frac{AC}{AB} = \frac{2\theta}{29}$ $and \cos\theta = \frac{BC}{AB} = \frac{21}{29}$

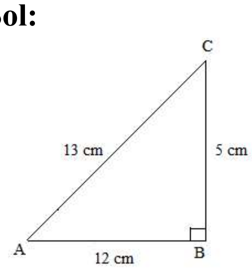
$\text{Cos}^2\theta - \sin^2\theta = \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2 = \frac{441}{841} - \frac{400}{841} = \frac{41}{841}$

Hence proved.

27. In a ΔABC, $\angle B = 90°$, AB = 12 cm and BC = 5 cm Find

(i) cos A (ii) cosec A (iii) cos C (iv) cosec C

Sol:



Using Pythagoras theorem, we get:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 12^2 + 5^2 = 144 + 25$

$\Rightarrow AC^2 = 169$

$\Rightarrow AC = 13\ cm$

Now, for T-Ratios of $\angle A, base = AB\ and\ perpendicular = BC$

(i)$\cos A = \frac{AB}{AC} = \frac{12}{13}$

(ii)$cosec\ A = \frac{1}{\sin A} = \frac{AC}{BC} = \frac{13}{5}$

Similarly, for T-Ratios of $\angle C, base = BC\ and\ perpendicular = AB$

(iii)$\cos C = \frac{BC}{AC} = \frac{5}{13}$

(iv)$cosec\ C = \frac{1}{\sin C} = \frac{AC}{AB} = \frac{13}{12}$

28. If $\sin\ \alpha = \frac{1}{2}$ prove that $\left(3\cos\alpha - 4\cos^2\alpha\right) = 0$

Sol:

$LHS = (3cosa - 4\cos^3 a)$

$= cos\ a(3 - 4\cos^2 a)$

$= \sqrt{1 - \sin^2 a}\,[3 - 4(1 - \sin^2 a)]$

$= \sqrt{1 - \left(\frac{1}{2}\right)^2}\left[3 - 4\left(1 - \left(\frac{1}{2}\right)^2\right)\right]$

$= \sqrt{\frac{1}{1} - \frac{1}{4}}\left[3 - 4\left(\frac{1}{1} - \frac{1}{4}\right)\right]$

$= \sqrt{\frac{3}{4}}\left[3 - 4\left(\frac{3}{4}\right)\right]$

$= \sqrt{\frac{3}{4}}[3 - 3]$

$= \sqrt{\frac{3}{4}}[0]$

= 0

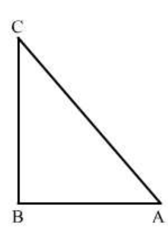
= RHS

29. IF $\Delta ABC, \angle B = 90°$ AND Tan $A = \frac{1}{\sqrt{3}}$. Prove that

(i) Sin A. cos C+ cos A. Sin c = 1

(ii) cos A. cos C -sin A. sin C = 0

Sol:



In ΔABC, $\angle B = 90^0$,

As, $\tan A = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

$Let\ BC = x\ and\ AB = x\sqrt{3}$

Using Pythagoras the get

$AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{(x\sqrt{3})^2 + x^2}$

$= \sqrt{3x^2 + x^2}$

$= \sqrt{4x^2}$

= 2x

Now,

(i)LHS = $\sin A . \cos C + \cos A . \sin C$

$= \frac{BC}{AC} . \frac{BC}{AC} + \frac{AB}{AC} . \frac{AB}{AC}$

$= \left(\frac{BC}{AC}\right)^2 + \left(\frac{AB}{AC}\right)^2$

$= \left(\frac{x}{2x}\right)^2 + \left(\frac{x\sqrt{3}}{2x}\right)^2$

$= \frac{1}{4} + \frac{3}{4}$

= 1

= RHS

(ii)LHS = $\cos A . \cos C - \sin A . sinC$

$= \frac{AB}{AC} . \frac{BC}{AC} - \frac{BC}{AC} . \frac{AB}{AC}$

$= \frac{x\sqrt{3}}{2x} . \frac{x}{2x} - \frac{x}{2x} . \frac{x\sqrt{3}}{2x}$

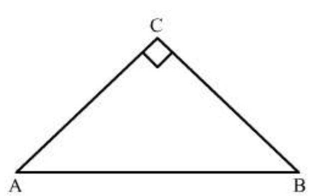
$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$

= 0

= RHS

30. If $\angle A$ and $\angle B$ are acute angles such that sin A = Sin B prove that $\angle A = \angle B$

Sol:



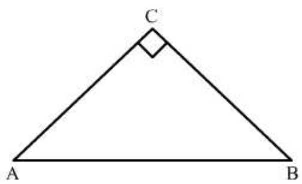
In ΔABC, $\angle C = 90^0$

$\text{Sin } A = \frac{BC}{AB}\ and$

$\text{Sin } B = \frac{AC}{AB}$

As, $\sin A = \sin B$
$\Rightarrow \frac{BC}{AB} = \frac{AC}{AB}$
$\Rightarrow BC = AC$
So, $\angle A = \angle B$ $(Angles\ opposite\ to\ equal\ sides\ are\ equal)$

31. If $\angle A$ and $\angle B$ are acute angles such that tan A= Tan B then prove that $\angle A = \angle B$
Sol:

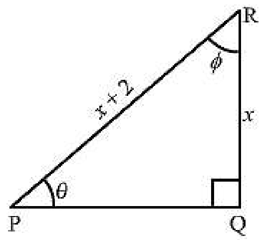


In ΔABC, $\angle C = 90^0$
Tan $A = \frac{BC}{AC}$ and
Tan $B = \frac{AC}{BC}$
As, $\tan A = \tan B$
$\Rightarrow \frac{BC}{AC} = \frac{AC}{BC}$
$\Rightarrow BC^2 = AC^2$
$\Rightarrow BC = AC$
So, $\angle A = \angle B$ $(Angles\ opposite\ to\ equal\ sides\ are\ equal)$

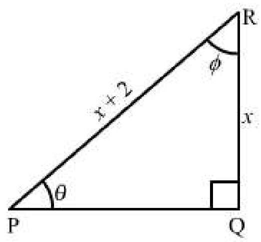
32. If a right ΔABC, right-angled at B, if tan A=1 then verify that 2sin A . cos A = 1
Sol:
We have,
Tan $A = 1$
$\Rightarrow \frac{sinA}{cosA} = 1$
$\Rightarrow \sin A = \cos A$
$\Rightarrow \sin A - \cos A = 0$
Squaring both sides, we get
$(sinA - cosA)^2 = 0$
$\Rightarrow \sin^2 A + \cos^2 A - 2 \sin A \,.\cos A = 0$
$\Rightarrow 1 - 2 \sin A \,.\cos A = 0$
$\therefore 2 \sin A \,. cosA = 1$

33. In the figure of ΔPQR, $\angle P = \theta°$ and $\angle R = \phi°$ find
(i) $\sqrt{X+1} \cot \phi$
(ii) $\sqrt{x^3 + x^2} \tan \theta$

(iii) cos θ



Sol:



In $\Delta PQR, \angle Q = 90^0$,

Using Pythagoras theorem, we get

$PQ = \sqrt{PR^2 - QR^2}$

$= \sqrt{(x+2)^2 - x^2}$

$= \sqrt{x^2 + 4x + 4 - x^2}$

$= \sqrt{4\,(x+1)}$

$= 2\sqrt{x+1}$

Now,

(i) $\left(\sqrt{x+1}\right) \cot \emptyset$

$= \left(\sqrt{x+1}\right) \times \frac{QR}{PQ}$

$= \left(\sqrt{x+1}\right) \times \frac{x}{2\sqrt{x+1}}$

$= \frac{x}{2}$

(ii) $\left(\sqrt{x^3 + x^2}\right) \tan \theta$

$= \left(\sqrt{x^2(x+1)}\right) \times \frac{QR}{PQ}$

$= x\,\sqrt{(x+1)} \times \frac{x}{2\sqrt{x+1}}$

$= \frac{x^2}{2}$

(iii) $cos\ \theta$

$= \frac{PQ}{PR}$ $\quad \theta = \frac{2\sqrt{x+1}}{x+2}$

34. If x = cosec A +cos A and y = cosec A – cos A then prove that $\left(\frac{2}{x+y}\right)^2+\left(\frac{x-y}{2}\right)^2=1$

Sol:

$LHS = \left(\frac{2}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2 - 1$

$= \left[\frac{2}{(cosec\ A+\cos A)+(cosecA-\cos A)}\right]^2 + \left[\frac{(cosecA+\cos A)-(cosecA-\cos A)}{2}\right]^2 - 1$

$= \left[\frac{2}{cosecA+\cos A+cosecA-\cos A}\right]^2 + \left[\frac{cosecA+\cos A-cosecA+\cos A}{2}\right]^2 - 1$

$= \left[\frac{2}{2\ cosecA}\right]^2 + \left[\frac{2\cos A}{2}\right]^2 - 1$

$= \left[\frac{1}{cosecA}\right]^2 + [cosA]^2 - 1$

$= [sinA]^2 + [cosA]^2 - 1$

$= \sin^2 A + \cos^2 A - 1$

= 1-1

= 0

= RHS

35. If x = cot A + cos A and y = cot A – cos A then prove that $\left(\frac{x-y}{x+y}\right)^2+\left(\frac{x-y}{2}\right)^2=1$

Sol:

$LHS = \left(\frac{x-y}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2$

$= \left[\frac{(cotA+cosA)-(cotA-cosA)}{(cotA+cosA)+(cotA-cosA)}\right]^2 + \left[\frac{(cotA+cosA)-(cotA-cosA)}{2}\right]^2$

$= \left[\frac{cotA+cosA-cotA+cosA}{cotA+cosA+cotA-cosA}\right]^2 + \left[\frac{cotA+cosA-cotA+cosA}{2}\right]^2$

$= \left[\frac{2cosA}{2cotA}\right]^2 + \left[\frac{2cosA}{2}\right]^2$

$= \left[\frac{cosA}{\left(\frac{cosA}{sinA}\right)}\right]^2 + [cosA]^2$

$= \left[\frac{sinA\ cosA}{cosA}\right]^2 + [cosA]^2$

$= [sinA]^2 + [cosA]^2$

$= \sin^2 A + \cos^2 A$

= 1

= RHS