Exercise – 4C

1. $\Delta ABC \sim \Delta DEF$ and their areas are respectively 64 cm^2 and 121cm^2. If EF = 15.4cm, find BC.

Sol:

It is given that Δ ABC ~ Δ DEF.

Therefore, ratio of the areas of these triangles will be equal to the ration of squares of their corresponding sides.

$$\frac{ar\ (\Delta ABC)}{ar\ (\Delta DEF)} = \frac{BC^2}{EF^2}$$

Let BC be X cm.

$$\Rightarrow \frac{64}{121} = \frac{x^2}{(15.4)^2}$$

$$\Rightarrow x^2 = \frac{64 \times 15.4 \times 15.4}{121}$$

$$\Rightarrow x = \sqrt{\frac{(64 \times 15.4 \times 15.4)}{121}}$$

$$= \frac{8 \times 15.4}{11}$$

= 11.2

Hence, BC = 11.2 cm

2. The areas of two similar triangles ABC and PQR are in the ratio 9:16. If BC = 4.5cm, find the length of QR.

Sol:

It is given that Δ ABC ~ Δ PQR

Therefore, the ration of the areas of triangles will be equal to the ratio of squares of their corresponding sides.

$$\frac{ar\ (\Delta ABC)}{ar\ (\Delta PQR)} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \frac{9}{16} = \frac{4^2}{QR^2}$$

$$\Rightarrow QR^2 = \frac{4.5 \times 4.5 \times 16}{9}$$

$$\Rightarrow QR = \sqrt{\frac{(4.5 \times 4.5 \times 16)}{9}}$$

$$= \frac{4.5 \times 4}{3}$$

= 6 cm

Hence, QR = 6 cm

3. $\Delta ABC \sim \Delta PQR$ and ar(ΔABC) = 4, ar(ΔPQR) . If BC = 12cm, find QR.

Sol:

Given : $ar\ (\Delta\ ABC\ ) = 4ar\ (\Delta\ PQR\ )$

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{4}{1}$

$\because$ ΔABC ~ ΔPQR

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$

$\therefore \frac{BC^2}{QR^2} = \frac{4}{1}$

$\Rightarrow QR^2 = \frac{12^2}{4}$

$\Rightarrow QR^2 = 36$

$\Rightarrow QR = 6\ cm$

Hence, QR = 6 cm

4. The areas of two similar triangles are 169cm^2 and 121cm^2 respectively. If the longest side of the larger triangle is 26cm, find the longest side of the smaller triangle.

Sol:

It is given that the triangles are similar.

Therefore, the ratio of the areas of these triangles will be equal to the ratio of squares of their corresponding sides.

Let the longest side of smaller triangle be X cm.

$\frac{ar\ (Larger\ triangle)}{ar\ (Smaller\ triangle)} = \frac{(Longest\ side\ of\ larger\ traingle)^2}{(Longest\ side\ of\ smaller\ traingle)^2}$

$\Rightarrow \frac{169}{121} = \frac{26^2}{x^2}$

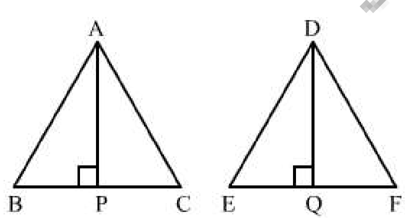
$\Rightarrow x = \sqrt{\frac{26 \times 26 \times 121}{169}}$

= 22

Hence, the longest side of the smaller triangle is 22 cm.

5. ΔABC ~ ΔDEF and their areas are respectively 100cm^2 and 49cm^2. If the altitude of ΔABC is 5cm, find the corresponding altitude of ΔDEF.

Sol:



It is given that ΔABC ~ ΔDEF.

Therefore, the ration of the areas of these triangles will be equal to the ratio of squares of their corresponding sides.

Also, the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

Let the altitude of ΔABC be AP, drawn from A to BC to meet BC at P and the altitude of ΔDEF be DQ, drawn from D to meet EF at Q.

Then,

$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AP^2}{DQ^2}$

$\Rightarrow \frac{100}{49} = \frac{5^2}{DQ^2}$

$\Rightarrow \frac{100}{49} = \frac{25}{DQ^2}$

$\Rightarrow DQ^2 = \frac{49 \times 25}{100}$

$\Rightarrow DQ = \sqrt{\frac{49 \times 25}{100}}$

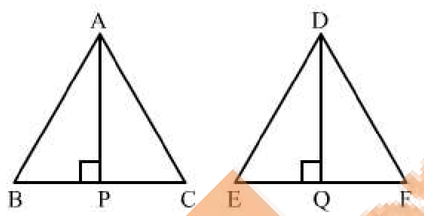
$\Rightarrow DQ = 3.5\ cm$

Hence, the altitude of ΔDEF is 3.5 cm

6. The corresponding altitudes of two similar triangles are 6cm and 9cm respectively. Find the ratio of their areas.

Sol:

Let the two triangles be ABC and DEF with altitudes AP and DQ, respectively.



It is given that Δ ABC ~ Δ DEF.

We know that the ration of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AP)^2}{(DQ)^2}$

$\Rightarrow \frac{ar(\Delta ABC)}{ar(DEF)} = \frac{6^2}{9^2}$

$= \frac{36}{81}$

$= \frac{4}{9}$

Hence, the ratio of their areas is 4 : 9

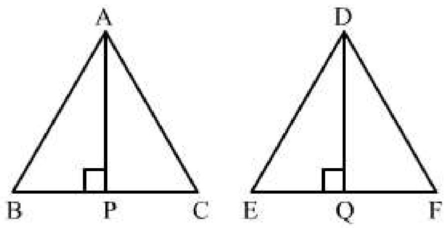
7. The areas of two similar triangles are $81cm^2$ and $49cm^2$ respectively. If the altitude of the first triangle is 6.3cm, find the corresponding altitude of the other.

Sol:

It is given that the triangles are similar.

Therefore, the areas of these triangles will be equal to the ratio of squares of their corresponding sides.

Also, the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.
Let the two triangles be ABC and DEF with altitudes AP and DQ, respectively.



$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AP^2}{DQ^2}$

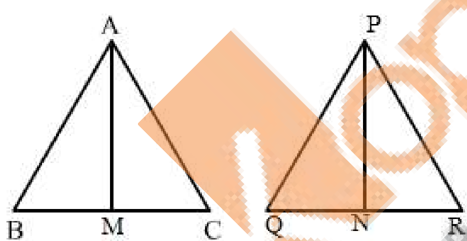
$\Rightarrow \frac{81}{49} = \frac{6.3^2}{DQ}$

$\Rightarrow DQ^2 = \frac{49}{81} \times 6.3^2$

$\Rightarrow DQ^2 = \sqrt{\frac{49}{81} \times 6.3 \times 6.3}$

Hence, the altitude of the other triangle is 4.9 cm.

8. The areas of two similar triangles are 64cm^2 and 100cm^2 respectively. If a median of the smaller triangle is 5.6cm, find the corresponding median of the other.
Sol:
Let the two triangles be ABC and PQR with medians AM and PN, respectively.



Therefore, the ratio of areas of two similar triangles will be equal to the ratio of squares of their corresponding medians.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AM^2}{PN^2}$

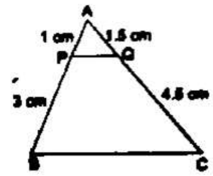
$\Rightarrow \frac{64}{100} = \frac{5.6^2}{PN^2}$

$\Rightarrow PN^2 = \frac{64}{100} \times 5.6^2$

$\Rightarrow PN^2 = \sqrt{\frac{100}{64} \times 5.6 \times 5.6}$

= 7 cm
Hence, the median of the larger triangle is 7 cm.

9. In the given figure, ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1cm, PB = 3cm, AQ = 1.5cm, QC = 4.5cm, prove that area of ΔAPQ is $\frac{1}{16}$ of the area of ΔABC.



Sol:

We have :

$\frac{AP}{AB} = \frac{1}{1+3} = \frac{1}{4}$ and $\frac{AQ}{AC} = \frac{1.5}{1.5+4.5} = \frac{1.5}{6} = \frac{1}{4}$

$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$

Also, $\angle A = \angle A$

By SAS similarity, we can conclude that ΔAPQ- ΔABC.

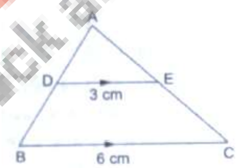
$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2} = \frac{1^2}{4^2} = \frac{1}{16}$

$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{1}{16}$

$\Rightarrow ar(\Delta APQ) = \frac{1}{16} \times ar\ (\Delta ABC)$

Hence proved.

10. In the given figure, DE ‖ BC. If DE = 3cm, BC = 6cm and ar(ΔADE) = 15cm^2, find the area of ΔABC.



Sol:

It is given that DE || BC

$\therefore \angle ADE = \angle ABC\ (Corresponding\ angles)$

$\angle AED = \angle ACB\ (Corresponding\ angles)$

By AA similarity, we can conclude that Δ ADE ~ Δ ABC

$\therefore \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2}$

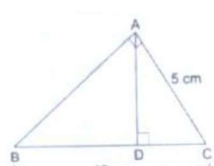
$\Rightarrow \frac{15}{ar(\Delta ABC)} = \frac{3^2}{6^2}$

$\Rightarrow ar\ (\Delta ABC) = \frac{15 \times 36}{9}$

= 60 cm^2

Hence, area of triangle ABC is 60 cm^2

11. ΔABC is right angled at A and AD⊥BC. If BC = 13cm and AC = 5cm, find the ratio of the areas of ΔABC and ΔADC.



Sol:

In ΔABC and ΔADC, we have:

$\angle BAC = \angle ADC = 90^0$

$\angle ACB = \angle ACD\ (common)$

By AA similarity, we can conclude that Δ BAC~ Δ ADC.

Hence, the ratio of the areas of these triangles is equal to the ratio of squares of their corresponding sides.

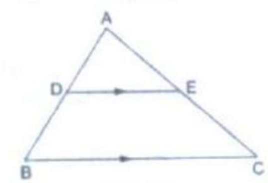
$\therefore \frac{ar(\Delta BAC)}{ar(\Delta ADC)} = \frac{BC^2}{AC^2}$

$\Rightarrow \frac{ar(\Delta BAC)}{ar(\Delta ADC)} = \frac{13^2}{5^2}$

$= \frac{169}{25}$

Hence, the ratio of areas of both the triangles is 169:25

12. In the given figure, DE ‖ BC and DE: BC = 3:5. Calculate the ratio of the areas of ΔADE and the trapezium BCED.



Sol:

It is given that DE || BC.

$\therefore \angle ADE = \angle ABC$ $(Corresponding\ angles)$

$\angle AED = \angle ACB$ $(Corresponding\ angles)$

Applying AA similarity theorem, we can conclude that Δ ADE ~ ΔABC.

$\therefore \frac{ar(\Delta ABC)}{ar(ADE)} = \frac{BC^2}{DE^2}$

Subtracting 1 from both sides, we get:

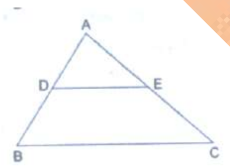
$\frac{ar(\Delta ABC)}{ar(\Delta ADE)} - 1 = \frac{5^2}{3^2} - 1$

$\Rightarrow \frac{ar(\Delta ABC) - ar(\Delta ADE)}{ar(\Delta ADE)} = \frac{25-9}{9}$

$\Rightarrow \frac{ar(BCED)}{ar(\Delta ADE)} = \frac{16}{9}$

Or, $\frac{ar(\Delta ADE)}{ar(BCED)} = \frac{9}{16}$

13. In ΔABC, D and E are the midpoints of AB and AC respectively. Find the ratio of the areas of ΔADE and ΔABC.



Sol:

It is given that D and E are midpoints of AB and AC.

Applying midpoint theorem, we can conclude that DE ∥ BC.

Hence, by B.P.T., we get :

$\frac{AD}{AB} = \frac{AE}{AC}$

Also, $\angle A = \angle A$

Applying SAS similarity theorem, we can conclude that Δ ADE~ Δ ABC.

Therefore, the ration of areas of these triangles will be equal to the ratio of squares of their corresponding sides.

$\therefore \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2}$