Exercise – 2B

1. Verify that 3, -2, 1 are the zeros of the cubic polynomial p(x) = $(x^3 - 2x^2 - 5x + 6)$ and verify the relation between it zeros and coefficients.

Sol:

The given polynomial is p(x) = $(x^3 - 2x^2 - 5x + 6)$

$\therefore p(3) = (3^3 - 2 \times 3^2 - 5 \times 3 + 6) = (27 - 18 - 15 + 6) = 0$

$p(-2) = [\,(-2^3) - 2 \times (-2)^2 - 5 \times (-2) + 6] = (-8 - 8 + 10 + 6) = 0$

$p(1) = (1^3 - 2 \times 1^2 - 5 \times 1 + 6) = (1 - 2 - 5 + 6) = 0$

$\therefore$ 3, –2 and 1are the zeroes of p(x),

Let $\alpha = 3$, $\beta = -2$ and $\gamma = 1$. Then we have:

$(\alpha + \beta + \gamma) = (3 - 2 + 1) = 2 = \frac{-(coefficient\ of\ x^2)}{(coefficient\ of\ x^3)}$

$(\alpha\beta + \beta\gamma + \gamma\alpha) = (-6 - 2 + 3) = \frac{-5}{1} = \frac{coefficient\ of\ x}{coefficient\ of\ x^3}$

$\alpha\beta\gamma = \{\,3 \times (-2) \times 1\} = \frac{-6}{1} = \frac{-(constant\ term)}{(coefficient\ of\ x^3)}$

2. Verify that 5, -2 and $\frac{1}{3}$ are the zeroes of the cubic polynomial p(x) = $(3x^3 - 10x^2 - 27x + 10)$ and verify the relation between its zeroes and coefficients.

Sol:

$p(x) = (3x^3 - 10x^2 - 27x + 10)$

$p(5) = (3 \times 5^3 - 10 \times 5^2 - 27 \times 5 + 10) = (375 - 250 - 135 + 10) = 0$

$p(-2) = [3 \times (-2^3) - 10 \times (-2^2) - 27 \times (-2) + 10] = (-24 - 40 + 54 + 10) = 0$

$p\left(\frac{1}{3}\right) = \left\{3 \times \left(\frac{1}{3}\right)^3 - 10 \times \left(\frac{1}{3}\right)^2 - 27 \times \frac{1}{3} + 10\right\} = \left(3 \times \frac{1}{27} - 10 \times \frac{1}{9} - 9 + 10\right)$

$= \left(\frac{1}{9} - \frac{10}{9} + 1\right) = \left(\frac{1-10-9}{9}\right) = \left(\frac{0}{9}\right) = 0$

$\therefore$ 5, –2 and $\frac{1}{3}$ are the zeroes of p(x).

Let $\alpha = 5$, $\beta = -2$ and $\gamma = \frac{1}{3}$. Then we have:

$(\alpha + \beta + \gamma) = \left(5 - 2 + \frac{1}{3}\right) = \frac{10}{3} = \frac{-(coefficient\ of\ x^2)}{(coefficient\ of\ x^3)}$

$(\alpha\beta + \beta\gamma + \gamma\alpha) = \left(-10 - \frac{2}{3} + \frac{5}{3}\right) = \frac{-27}{3} = \frac{coefficient\ of\ x}{coefficient\ of\ x^3}$

$\alpha\beta\gamma = \left\{5 \times (-2) \times \frac{1}{3}\right\} = \frac{-10}{3} = \frac{-(constant\ term)}{(coefficient\ of\ x^3)}$

3. Find a cubic polynomial whose zeroes are 2, -3and 4.

Sol:

If the zeroes of the cubic polynomial are a, b and c then the cubic polynomial can be found as

$x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc$ ……(1)

Let a = 2, b = –3 and c = 4

Substituting the values in 1, we get

$x^3 - (2 - 3 + 4)x^2 + (-6 - 12 + 8)x - (-24)$

$\Rightarrow x^3 - 3x^2 - 10x + 24$

4. Find a cubic polynomial whose zeroes are $\frac{1}{2}$, 1 and –3.

Sol:

If the zeroes of the cubic polynomial are a, b and c then the cubic polynomial can be found as

$x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc$ ……(1)

Let a = $\frac{1}{2}$, b = 1 and c = –3

Substituting the values in (1), we get

$x^3 - \left(\frac{1}{2} + 1 - 3\right)x^2 + \left(\frac{1}{2} - 3 - \frac{3}{2}\right)x - \left(\frac{-3}{2}\right)$

$\Rightarrow x^3 - \left(\frac{-3}{2}\right)x^2 - 4x + \frac{3}{2}$

$\Rightarrow 2x^3 + 3x^2 - 8x + 3$

5. Find a cubic polynomial with the sum of its zeroes, sum of the products of its zeroes taken two at a time and the product of its zeroes as 5, -2 and -24 respectively.

Sol:

We know the sum, sum of the product of the zeroes taken two at a time and the product of the zeroes of a cubic polynomial then the cubic polynomial can be found as

x^3 – (sum of the zeroes)x^2 + (sum of the product of the zeroes taking two at a time)x – product of zeroes

Therefore, the required polynomial is

$x^3 - 5x^2 - 2x + 24$

6. If f(x) = $x^3 - 3x + 5x - 3$ is divided by g(x) = $x^2 - 2$

Sol:

$$
\begin{array}{r|l}
 & x - 3 \\
\hline
x - 2 & x^3 - 3x^2 + 5x - 3 \\
 & x^3 \qquad - 2x \\
 & - \qquad\quad + \\
\hline
 & -3x^2 + 7x - 3 \\
 & -3x^2 \qquad + 6 \\
 & + \qquad\quad - \\
\hline
 & 7x - 9 \\
\hline
\end{array}
$$

Quotient q(x) = x – 3

Remainder r(x) = 7x – 9

7. If $f(x) = x^4 - 3x^2 + 4x + 5$ is divided by $g(x) = x^2 - x + 1$

Sol:

$$x^2 + x - 3$$

$$x^2 - x + 1 \;\big)\; x^4 + 0x^3 - 3x^2 + 4x + 5$$

$$x^4 - x^3 + x^2$$

$$- \quad + \quad -$$

$$x^3 - 4x^2 + 4x + 5$$

$$x^3 - x^2 + x$$

$$- \quad + \quad -$$

$$-3x^2 + 3x + 5$$

$$-3x^2 + 3x - 3$$

$$+ \quad - \quad +$$

$$8$$

Quotient $q(x) = x^2 + x - 3$

Remainder $r(x) = 8$

8. If $f(x) = x^4 - 5x + 6$ is divided by $g(x) = 2 - x^2$.

Sol:

We can write

$f(x)$ as $x^4 + 0x^3 + 0x^2 - 5x + 6$ and $g(x)$ as $-x^2 + 2$

$$-x^2 - 2$$

$$-x^2 + 2 \;\big)\; x^4 + 0x^3 + 0x^2 - 5x + 6$$

$$x^4 \quad -2x^2$$

$$- \quad +$$

$$2x^2 - 5x + 6$$

$$2x^2 \quad - 4$$

$$- \quad +$$

$$-5x + 10$$

Quotient $q(x) = -x^2 - 2$

Remainder $r(x) = -5x + 10$

9. By actual division, show that $x^2 - 3$ is a factor of $2x^4 + 3x^3 - 2x^2 - 9x - 12$.

Sol:

Let $f(x) = 2x^4 + 3x^3 - 2x^2 - 9x - 12$ and g(x) as $x^2 - 3$

$$
\begin{array}{r|l}
 & 2x^2 + 3x + 4 \\
\hline
x^2 - 3 & 2x^4 + 3x^3 - 2x^2 - 9x - 12 \\
 & 2x^4 \quad\quad - 6x^2 \\
 & - \quad\quad\quad + \\
\hline
 & 3x^3 + 4x^2 - 9x - 12 \\
 & 3x^3 \quad\quad - 9x \\
 & - \quad\quad\quad + \\
\hline
 & 4x^2 - 12 \\
 & 4x^2 - 12 \\
 & - \quad + \\
\hline
 & x \\
\hline
\end{array}
$$

Quotient $q(x) = 2x^2 + 3x + 4$

Remainder r(x) = 0

Since, the remainder is 0.

Hence, $x^2 - 3$ is a factor of $2x^4 + 3x^3 - 2x^2 - 9x - 12$

10. On dividing $3x^3 + x^2 + 2x + 5$ is divided by a polynomial g(x), the quotient and remainder are $(3x - 5)$ and $(9x + 10)$ respectively. Find g(x).

Sol:

By using division rule, we have

Dividend = Quotient $\times$ Divisor + Remainder

$\therefore 3x^3 + x^2 + 2x + 5 = (3x - 5)g(x) + 9x + 10$

$\Rightarrow 3x^3 + x^2 + 2x + 5 - 9x - 10 = (3x - 5)g(x)$

$\Rightarrow 3x^3 + x^2 - 7x - 5 = (3x - 5)g(x)$

$\Rightarrow g(x) = \frac{3x^3 + x^2 - 7x - 5}{3x - 5}$

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
3x - 5 & 3x^3 + x^2 - 7x - 5 \\
 & 3x^3 - 5x^2 \\
 & - \quad + \\
\hline
 & 6x^2 - 7x - 5 \\
 & 6x^2 - 10x \\
 & - \quad + \\
\hline
 & 3x - 5 \\
 & 3x - 5 \\
 & - \quad + \\
\hline
 & X \\
\hline
\end{array}
$$

$\therefore g(x) = x^2 + 2x + 1$

11. Verify division algorithm for the polynomial $f(x) = (8 + 20x + x^2 - 6x^3)$ by $g(x) = (2 + 5x - 3x^2)$.

Sol:

We can write f(x) as $-6x^3 + x^2 + 20x + 8$ and g(x) as $-3x^2 + 5x + 2$

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
-3x^2 + 5x + 2 & -6x^3 + x^2 + 20x + 8 \\
 & -6x^3 + 10x^2 + 4x \\
 & + \quad - \quad - \\
\hline
 & -9x^2 + 16x + 8 \\
 & -9x^2 + 15x + 6 \\
 & + \quad - \quad - \\
\hline
 & x + 2 \\
\hline
\end{array}
$$

Quotient = 2x + 3

Remainder = x + 2

By using division rule, we have

Dividend = Quotient × Divisor + Remainder

$\therefore -6x^3 + x^2 + 20x + 8 = (-3x^2 + 5x + 2)(2x + 3) + x + 2$

$\Rightarrow -6x^3 + x^2 + 20x + 8 = -6x^3 + 10x^2 + 4x - 9x^2 + 15x + 6 + x + 2$

$\Rightarrow -6x^3 + x^2 + 20x + 8 = -6x^3 + x^2 + 20x + 8$

12. It is given that –1 is one of the zeroes of the polynomial $x^3 + 2x^2 - 11x - 12$. Find all the zeroes of the given polynomial.

Sol:

Let $f(x) = x^3 + 2x^2 - 11x - 12$

Since – 1 is a zero of f(x), (x+1) is a factor of f(x).

On dividing f(x) by (x+1), we get

$$
\begin{array}{r|l|l}
x + 1 & x^3 + 2x^2 - 11x - 12 & x^2 + x + 12 \\
 & x^3 + x^2 & \\
 & - \quad - & \\
\hline
 & x^2 - 11x - 12 & \\
 & x^2 + x & \\
 & - \quad - & \\
\hline
 & -12x - 12 & \\
 & -12x - 12 & \\
 & + \quad + & \\
\hline
 & X & \\
\hline
\end{array}
$$

$f(x) = x^3 + 2x^2 - 11x - 12$
$= (x + 1)(x^2 + x - 12)$
$= (x + 1)\{x^2 + 4x - 3x - 12\}$
$= (x + 1)\{x(x+4) - 3(x+4)\}$
$= (x + 1)(x - 3)(x + 4)$

$\therefore f(x) = 0 \Rightarrow (x + 1)(x - 3)(x + 4) = 0$

$\Rightarrow (x + 1) = 0$ or $(x - 3) = 0$ or $(x + 4) = 0$

$\Rightarrow x = -1$ or $x = 3$ or $x = -4$

Thus, all the zeroes are – 1, 3 and – 4.

13. If 1 and –2 are two zeroes of the polynomial $(x^3 - 4x^2 - 7x + 10)$, find its third zero.

Sol:

Let $f(x) = x^3 - 4x^2 - 7x + 10$

Since 1 and –2 are the zeroes of f(x), it follows that each one of (x–1) and (x+2) is a factor of f(x).

Consequently, $(x-1)(x+2) = (x^2 + x - 2)$ is a factor of f(x).

On dividing f(x) by $(x^2 + x - 2)$, we get:

$x^2 + x - 2 \,)\, x^3 - 4x^2 - 7x + 10 \,(\, x - 5$
$x^3 + x^2 - 2x$
$- \quad - \quad +$
$-5x^2 - 5x + 10$
$-5x^2 - 5x + 10$
$+ \quad + \quad -$
X

$f(x) = 0 \Rightarrow (x^2 + x - 2)(x - 5) = 0$

$\Rightarrow (x - 1)(x + 2)(x - 5) = 0$

$\Rightarrow x = 1$ or $x = -2$ or $x = 5$

Hence, the third zero is 5.

14. If 3 and –3 are two zeroes of the polynomial $(x^4 + x^3 - 11x^2 - 9x + 18)$, find all the zeroes of the given polynomial.

Sol:

Let $x^4 + x^3 - 11x^2 - 9x + 18$

Since 3 and – 3 are the zeroes of f(x), it follows that each one of (x + 3) and (x – 3) is a factor of f(x).

Consequently, $(x - 3)(x + 3) = (x^2 - 9)$ is a factor of f(x).

On dividing f(x) by $(x^2 - 9)$, we get:

$$x^2 - 9 \overline{\smash{)}\, x^4 + x^3 - 11x^2 - 9x + 18 \,}\left(x^2 + x - 2 \right.$$
$x^4 \quad - 9x^2$
$- \quad +$
$x^3 - 2x^2 - 9x + 18$
$x^3 \quad - 9x$
$- \quad +$
$-2x^2 + 18$
$-2x^2 + 18$
$+ \quad -$
x

$f(x) = 0 \Rightarrow (x^2 + x - 2)(x^2 - 9) = 0$

$\Rightarrow (x^2 + 2x - x - 2)(x - 3)(x + 3)$

$\Rightarrow (x - 1)(x + 2)(x - 3)(x + 3) = 0$

$\Rightarrow x = 1$ or $x = -2$ or $x = 3$ or $x = -3$

Hence, all the zeroes are 1, -2, 3 and -3.

15. If 2 and -2 are two zeroes of the polynomial $(x^4 + x^3 - 34x^2 - 4x + 120)$, find all the zeroes of the given polynomial.

Sol:

Let $f(x) = x^4 + x^3 - 34x^2 - 4x + 120$

Since 2 and -2 are the zeroes of f(x), it follows that each one of $(x - 2)$ and $(x + 2)$ is a factor of f(x).

Consequently, $(x - 2)(x + 2) = (x^2 - 4)$ is a factor of f(x).

On dividing f(x) by $(x^2 - 4)$, we get:

$$x^2 - 4 \overline{\smash{)}\, x^4 + x^3 - 34x^2 - 4x + 120 \,}\left(x^2 + x - 2 \right.$$
$x^4 \quad - 4x^2$
$- \quad +$
$x^3 - 30x^2 - 4x + 120$
$x^3 \quad -4x$
$- \quad +$
$-30x^2 + 120$
$-30x^2 + 120$
$+ \quad -$
x

$f(x) = 0$

$\Rightarrow (x^2 + x - 30)(x^2 - 4) = 0$

⇒ $(x^2 + 6x - 5x - 30)(x - 2)(x + 2)$

⇒ $[x(x + 6) - 5(x + 6)](x - 2)(x + 2)$

⇒ $(x - 5)(x + 6)(x - 2)(x + 2) = 0$

⇒ $x = 5$ or $x = -6$ or $x = 2$ or $x = -2$

Hence, all the zeroes are 2, -2, 5 and -6.

16. Find all the zeroes of $(x^4 + x^3 - 23x^2 - 3x + 60)$, if it is given that two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.

Sol:

Let $f(x) = x^4 + x^3 - 23x^2 - 3x + 60$

Since $\sqrt{3}$ and $-\sqrt{3}$ are the zeroes of f(x), it follows that each one of $(x - \sqrt{3})$ and $(x + \sqrt{3})$ is a factor of f(x).

Consequently, $(x - \sqrt{3})(x + \sqrt{3}) = (x^2 - 3)$ is a factor of f(x).

On dividing f(x) by $(x^2 - 3)$, we get:

$x^2 - 3 \;\big)\; x^4 + x^3 - 23x^2 - 3x + 60 \;\big(\; x^2 + x - 20$

$x^4 \quad - 3x^2$

$- \quad +$

$x^3 - 20x^2 - 3x + 60$

$x^3 \quad - 3x$

$- \quad +$

$-20x^2 + 60$

$-20x^2 + 60$

$+ \quad -$

---

x

$f(x) = 0$

⇒ $(x^2 + x - 20)(x^2 - 3) = 0$

⇒ $(x^2 + 5x - 4x - 20)(x^2 - 3)$

⇒ $[x(x + 5) - 4(x + 5)](x^2 - 3)$

⇒ $(x - 4)(x + 5)(x - \sqrt{3})(x + \sqrt{3}) = 0$

⇒ $x = 4$ or $x = -5$ or $x = \sqrt{3}$ or $x = -\sqrt{3}$

Hence, all the zeroes are $\sqrt{3}$, $-\sqrt{3}$, 4 and -5.

17. Find all the zeroes of $(2x^4 - 3x^3 - 5x^2 + 9x - 3)$, it is being given that two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.

Sol:

The given polynomial is $f(x) = 2x^4 - 3x^3 - 5x^2 + 9x - 3$

Since $\sqrt{3}$ and $-\sqrt{3}$ are the zeroes of f(x), it follows that each one of $(x - \sqrt{3})$ and $(x + \sqrt{3})$ is a factor of f(x).

Consequently, $(x - \sqrt{3})(x + \sqrt{3}) = (x^2 - 3)$ is a factor of f(x).

On dividing f(x) by $(x^2 - 3)$, we get:

$$
\begin{array}{r|l|l}
x^2 - 3 & 2x^4 - 3x^3 - 5x^2 + 9x - 3 & 2x^2 - 3x + 1 \\
 & 2x^4 \qquad - 6x^2 & \\
 & - \qquad\quad + & \\
\hline
 & -3x^3 + x^2 + 9x - 3 & \\
 & -3x^3 \qquad + 9x & \\
 & + \qquad\quad - & \\
\hline
 & x^2 - 3 & \\
 & x^2 - 3 & \\
 & - \quad + & \\
\hline
 & x & \\
\hline
\end{array}
$$

$f(x) = 0$

$\Rightarrow 2x^4 - 3x^3 - 5x^2 + 9x - 3 = 0$

$\Rightarrow (x^2 - 3)(2x^2 - 3x + 1) = 0$

$\Rightarrow (x^2 - 3)(2x^2 - 2x - x + 1) = 0$

$\Rightarrow (x - \sqrt{3})(x + \sqrt{3})(2x - 1)(x - 1) = 0$

$\Rightarrow x = \sqrt{3}$ or $x = -\sqrt{3}$ or $x = \frac{1}{2}$ or $x = 1$

Hence, all the zeroes are $\sqrt{3}$, $-\sqrt{3}$, $\frac{1}{2}$ and 1.

18. Obtain all other zeroes of $(x^4 + 4x^3 - 2x^2 - 20x - 15)$ if two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

Sol:

The given polynomial is $f(x) = x^4 + 4x^3 - 2x^2 - 20x - 15$.

Since $(x - \sqrt{5})$ and $(x + \sqrt{5})$ are the zeroes of f(x) it follows that each one of $(x - \sqrt{5})$ and $(x + \sqrt{5})$ is a factor of f(x).

Consequently, $(x - \sqrt{5})(x + \sqrt{5}) = (x^2 - 5)$ is a factor of f(x).

On dividing f(x) by $(x^2 - 5)$, we get:

$$
\begin{array}{r|l}
x^2 - 5 & x^4 + 4x^3 - 2x^2 - 20x - 15 \quad \big(\; 2x^2 - 3x + 1 \\
 & x^4 \qquad - 5x^2 \\
 & - \qquad\quad + \\
\hline
 & 4x^3 + 3x^2 - 20x - 15 \\
 & 4x^3 \qquad - 20x \\
 & - \qquad\quad + \\
\hline
 & 3x^2 - 15 \\
 & 3x^2 - 15 \\
 & - \quad + \\
\hline
 & x \\
\hline
\end{array}
$$

$f(x) = 0$

$\Rightarrow x^4 + 4x^3 - 7x^2 - 20x - 15 = 0$

$\Rightarrow (x^2 - 5)(x^2 + 4x + 3) = 0$

$\Rightarrow (x - \sqrt{5})(x + \sqrt{5})(x + 1)(x + 3) = 0$

$\Rightarrow x = \sqrt{5}$ or $x = -\sqrt{5}$ or $x = -1$ or $x = -3$

Hence, all the zeroes are $\sqrt{5}$, $-\sqrt{5}$, -1 and -3.

19. Find all the zeroes of polynomial $(2x^4 - 11x^3 + 7x^2 + 13x - 7)$, it being given that two of its zeroes are $(3 + \sqrt{2})$ and $(3 - \sqrt{2})$.

Sol:

The given polynomial is $f(x) = 2x^4 - 11x^3 + 7x^2 + 13x - 7$.

Since $(3 + \sqrt{2})$ and $(3 - \sqrt{2})$ are the zeroes of f(x) it follows that each one of $(x + 3 + \sqrt{2})$ and $(x + 3 - \sqrt{2})$ is a factor of f(x).

Consequently, $[(x - (3 + \sqrt{2})][(x - (3 - \sqrt{2})] = [(x - 3) - \sqrt{2}][(x - 3) + \sqrt{2}]$

$= [(x - 3)^2 - 2] = x^2 - 6x + 7$, which is a factor of f(x).

On dividing f(x) by $(x^2 - 6x + 7)$, we get:

$$
\begin{array}{r|l}
x^2 - 6x + 7 & 2x^4 - 11x^3 + 7x^2 + 13x - 7 \quad \big(\; 2x^2 + x - 1 \\
 & 2x^4 - 12x^3 + 14x^2 \\
 & - \qquad + \qquad - \\
\hline
 & x^3 - 7x^2 + 13x - 7 \\
 & x^3 - 6x^2 + 7x \\
 & - \qquad + \qquad - \\
\hline
 & -x^2 + 6x - 7 \\
 & -x^2 + 6x - 7 \\
\end{array}
$$

$+ \quad - \quad +$

x

f(x) = 0

$\Rightarrow 2x^4 - 11x^3 + 7x^2 + 13x - 7 = 0$

$\Rightarrow (x^2 - 6x + 7)(2x^2 + x - 7) = 0$

$\Rightarrow (x + 3 + \sqrt{2})(x + 3 - \sqrt{2})(2x - 1)(x + 1) = 0$

$\Rightarrow x = -3 - \sqrt{2}$ or $x = -3 + \sqrt{2}$ or $x = \frac{1}{2}$ or $x = -1$

Hence, all the zeroes are $(-3 - \sqrt{2})$, $(-3 + \sqrt{2})$, $\frac{1}{2}$ and -1.