Exercise – 2A

1. Find the zeros of the polynomial $f(x) = x^2 + 7x + 12$ and verify the relation between its zeroes and coefficients.

Sol:

$x^2 + 7x + 12 = 0$

$\Rightarrow x^2 + 4x + 3x + 12 = 0$

$\Rightarrow x(x+4) + 3(x+4) = 0$

$\Rightarrow (x+4)(x+3) = 0$

$\Rightarrow (x + 4) = 0$ or $(x + 3) = 0$

$\Rightarrow x = -4$ or $x = -3$

Sum of zeroes $= -4 + (-3) = \frac{-7}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= (-4)(-3) = \frac{12}{1} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

2. Find the zeroes of the polynomial $f(x) = x^2 - 2x - 8$ and verify the relation between its zeroes and coefficients.

Sol:

$x^2 - 2x - 8 = 0$

$\Rightarrow x^2 - 4x + 2x - 8 = 0$

$\Rightarrow x(x - 4) + 2(x - 4) = 0$

$\Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow (x - 4) = 0$ or $(x+2) = 0$

$\Rightarrow x = 4$ or $x = -2$

Sum of zeroes $= 4 + (-2) = 2 = \frac{2}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= (4)(-2) = \frac{-8}{1} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

3. Find the zeroes of the quadratic polynomial $f(x) = x^2 + 3x - 10$ and verify the relation between its zeroes and coefficients.

Sol:

We have:

$f(x) = x^2 + 3x - 10$

$= x^2 + 5x - 2x - 10$

$= x(x + 5) - 2(x + 5)$

$= (x - 2)(x + 5)$

$\therefore f(x) = 0 \Rightarrow (x - 2)(x + 5) = 0$

$\Rightarrow x - 2 = 0$ or $x + 5 = 0$

$\Rightarrow x = 2$ or $x = -5$.

So, the zeroes of f(x) are 2 and -5.

Sum of zeroes = 2 + (−5) = −3 = $\frac{-3}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = 2 × (−5) = −10 = $\frac{-10}{1} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

4. Find the zeroes of the quadratic polynomial f(x) = $4x^2$ - 4x - 3 and verify the relation between its zeroes and coefficients.

Sol:

We have:

f(x) = $4x^2$ - 4x - 3

= $4x^2$ - (6x - 2x) - 3

= $4x^2$ - 6x + 2x - 3

= 2x (2x - 3) + 1(2x - 3)

= (2x + 1) (2x - 3)

∴ f(x) = 0 ⇒ (2x + 1) (2x - 3)= 0

⇒ 2x + 1= 0 or 2x - 3 = 0

⇒ x = $\frac{-1}{2}$ or x = $\frac{3}{2}$

So, the zeroes of f(x) are $\frac{-1}{2}$ and $\frac{3}{2}$.

Sum of zeroes = $\left(\frac{-1}{2}\right) + \left(\frac{3}{2}\right) = \frac{-1+3}{2} = \frac{2}{2} = 1 = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\left(\frac{-1}{2}\right) \times \left(\frac{3}{2}\right) = \frac{-3}{4} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

5. Find the zeroes of the quadratic polynomial f(x) = $5x^2$ - 4 - 8x and verify the relationship between the zeroes and coefficients of the given polynomial.

Sol:

We have:

f(x) = $5x^2$ - 4 - 8x

= $5x^2$ - 8x - 4

= $5x^2$ - (10x - 2x) - 4

= $5x^2$ - 10x + 2x - 4

= 5x (x - 2) + 2(x - 2)

= (5x + 2) (x - 2)

∴ f(x) = 0 ⇒ (5x + 2) (x - 2) = 0

⇒ 5x + 2= 0 or x - 2 = 0

⇒ x = $\frac{-2}{5}$ or x = 2

So, the zeroes of f(x) are $\frac{-2}{5}$ and 2.

Sum of zeroes = $\left(\frac{-2}{5}\right) + 2 = \frac{-2+10}{5} = \frac{8}{5} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\left(\frac{-2}{5}\right) \times 2 = \frac{-4}{5} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

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6. Find the zeroes of the polynomial f(x) = $2\sqrt{3}x^2 - 5x + \sqrt{3}$ and verify the relation between its zeroes and coefficients.

Sol:

$2\sqrt{3}x^2 - 5x + \sqrt{3}$

$\Rightarrow 2\sqrt{3}x^2 - 2x - 3x + \sqrt{3}$

$\Rightarrow 2x(\sqrt{3}x - 1) - \sqrt{3}(\sqrt{3}x - 1) = 0$

$\Rightarrow (\sqrt{3}x - 1)$ or $(2x - \sqrt{3}) = 0$

$\Rightarrow (\sqrt{3}x - 1) = 0$ or $(2x - \sqrt{3}) = 0$

$\Rightarrow x = \frac{1}{\sqrt{3}}$ or $x = \frac{\sqrt{3}}{2}$

$\Rightarrow x = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ or $x = \frac{\sqrt{3}}{2}$

Sum of zeroes = $\frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{6} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{6} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

7. Find the zeroes of the quadratic polynomial $2x^2 - 11x + 15$ and verify the relation between the zeroes and the coefficients.

Sol:

$f(x) = 2x^2 - 11x + 15$

$= 2x^2 - (6x + 5x) + 15$

$= 2x^2 - 6x - 5x + 15$

$= 2x(x - 3) - 5(x - 3)$

$= (2x - 5)(x - 3)$

$\therefore f(x) = 0 \Rightarrow (2x - 5)(x - 3) = 0$

$\Rightarrow 2x - 5 = 0$ or $x - 3 = 0$

$\Rightarrow x = \frac{5}{2}$ or $x = 3$

So, the zeroes of f(x) are $\frac{5}{2}$ and 3.

Sum of zeroes = $\frac{5}{2} + 3 = \frac{5+6}{2} = \frac{11}{2} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\frac{5}{2} \times 3 = \frac{-15}{2} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

8. Find the zeroes of the quadratic polynomial $4x^2 - 4x + 1$ and verify the relation between the zeroes and the coefficients.

Sol:

$4x^2 - 4x + 1 = 0$

$\Rightarrow (2x)^2 - 2(2x)(1) + (1)^2 = 0$

$\Rightarrow (2x - 1)^2 = 0$ $\quad [\because a^2 - 2ab + b^2 = (a-b)^2]$

$\Rightarrow (2x - 1)^2 = 0$

$\Rightarrow x = \frac{1}{2}$ or $x = \frac{1}{2}$

Sum of zeroes $= \frac{1}{2} + \frac{1}{2} = 1 = \frac{1}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

9. Find the zeroes of the quadratic polynomial $(x^2 - 5)$ and verify the relation between the zeroes and the coefficients.

Sol:

We have:

$f(x) = x^2 - 5$

It can be written as $x^2 + 0x - 5$.

$= \left(x^2 - (\sqrt{5})^2\right)$

$= (x + \sqrt{5})(x - \sqrt{5})$

$\therefore f(x) = 0 \Rightarrow (x + \sqrt{5})(x - \sqrt{5}) = 0$

$\Rightarrow x + \sqrt{5} = 0$ or $x - \sqrt{5} = 0$

$\Rightarrow x = -\sqrt{5}$ or $x = \sqrt{5}$

So, the zeroes of f(x) are $-\sqrt{5}$ and $\sqrt{5}$.

Here, the coefficient of x is 0 and the coefficient of x^2 is 1.

Sum of zeroes $= -\sqrt{5} + \sqrt{5} = \frac{0}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= -\sqrt{5} \times \sqrt{5} = \frac{-5}{1} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

10. Find the zeroes of the quadratic polynomial $(8x^2 - 4)$ and verify the relation between the zeroes and the coefficients.

Sol:

We have:

$f(x) = 8x^2 - 4$

It can be written as $8x^2 + 0x - 4$

$= 4\ \{(\sqrt{2}x)^2 - (1)^2\}$

$= 4(\sqrt{2}x + 1)(\sqrt{2}x - 1)$

$\therefore f(x) = 0 \Rightarrow (\sqrt{2}x + 1)(\sqrt{2}x - 1) = 0$

$\Rightarrow (\sqrt{2}x + 1) = 0$ or $\sqrt{2}x - 1 = 0$

$\Rightarrow x = \frac{-1}{\sqrt{2}}$ or $x = \frac{1}{\sqrt{2}}$

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So, the zeroes of f(x) are $\frac{-1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$

Here the coefficient of x is 0 and the coefficient of x^2 is $\sqrt{2}$

Sum of zeroes = $\frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{-1+1}{\sqrt{2}} = \frac{0}{\sqrt{2}} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\frac{-1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{-1 \times 4}{2 \times 4} = \frac{-4}{8} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

11. Find the zeroes of the quadratic polynomial ($5y^2 + 10y$) and verify the relation between the zeroes and the coefficients.

Sol:

We have,

$f(u) = 5u^2 + 10u$

It can be written as 5u (u+2)

$\therefore f(u) = 0 \Rightarrow 5u = 0$ or $u + 2 = 0$

$\Rightarrow u = 0$ or $u = -2$

So, the zeroes of f (u) are −2 and 0.

Sum of the zeroes = $-2 + 0 = -2 = \frac{-2 \times 5}{1 \times 5} = \frac{-10}{5} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ u^2)}$

Product of zeroes = $-2 \times 0 = 0 = \frac{0 \times 5}{1 \times 5} = \frac{-0}{5} = \frac{constant\ term}{(coefficient\ of\ u^2)}$

12. Find the zeroes of the quadratic polynomial ($3x^2 - x - 4$) and verify the relation between the zeroes and the coefficients.

Sol:

$3x^2 - x - 4 = 0$

$\Rightarrow 3x^2 - 4x + 3x - 4 = 0$

$\Rightarrow x(3x - 4) + 1(3x - 4) = 0$

$\Rightarrow (3x - 4)(x + 1) = 0$

$\Rightarrow (3x - 4)$ or $(x + 1) = 0$

$\Rightarrow x = \frac{4}{3}$ or $x = -1$

Sum of zeroes = $\frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\frac{4}{3} \times (-1) = \frac{-4}{3} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

13. Find the quadratic polynomial whose zeroes are 2 and -6. Verify the relation between the coefficients and the zeroes of the polynomial.

Sol:

Let $\alpha = 2$ and $\beta = -6$

Sum of the zeroes, $(\alpha + \beta) = 2 + (-6) = -4$

Product of the zeroes, $\alpha\beta = 2 \times (-6) = -12$

$\therefore$ Required polynomial $= x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (-4)x - 12$

$= x^2 + 4x - 12$

Sum of the zeroes $= -4 = \frac{-4}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= -12 = \frac{-12}{1} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

14. Find the quadratic polynomial whose zeroes are $\frac{2}{3}$ and $\frac{-1}{4}$. Verify the relation between the coefficients and the zeroes of the polynomial.

Sol:

Let $\alpha = \frac{2}{3}$ and $\beta = \frac{-1}{4}$.

Sum of the zeroes $= (\alpha + \beta) = \frac{2}{3} + \left(\frac{-1}{4}\right) = \frac{8-3}{12} = \frac{5}{12}$

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Product of the zeroes, $\alpha\beta = \frac{2}{3} \times \left(\frac{-1}{4}\right) = \frac{-2}{12} = \frac{-1}{6}$

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$\therefore$ Required polynomial $= x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - \frac{5}{12}x + \left(\frac{-1}{6}\right)$

$= x^2 - \frac{5}{12}x - \frac{1}{6}$

Sum of the zeroes $= \frac{5}{12} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= \frac{-1}{6} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

15. Find the quadratic polynomial, sum of whose zeroes is 8 and their product is 12. Hence, find the zeroes of the polynomial.

Sol:

Let α and β be the zeroes of the required polynomial f(x).

Then $(\alpha + \beta) = 8$ and $\alpha\beta = 12$

$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$\Rightarrow f(x) = x^2 - 8x + 12$

Hence, required polynomial $f(x) = x^2 - 8x + 12$

$\therefore f(x) = 0 \Rightarrow x^2 - 8x + 12 = 0$

$\Rightarrow x^2 - (6x + 2x) + 12 = 0$

$\Rightarrow x^2 - 6x - 2x + 12 = 0$

$\Rightarrow x(x - 6) - 2(x - 6) = 0$

$\Rightarrow (x - 2)(x - 6) = 0$

$\Rightarrow (x - 2) = 0$ or $(x - 6) = 0$

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$\Rightarrow$ x = 2 or x = 6

So, the zeroes of f(x) are 2 and 6.

16. Find the quadratic polynomial, sum of whose zeroes is 0 and their product is -1. Hence, find the zeroes of the polynomial.

Sol:

Let α and β be the zeroes of the required polynomial f(x).

Then $(\alpha + \beta) = 0$ and $\alpha\beta = -1$

$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$\Rightarrow f(x) = x^2 - 0x + (-1)$

$\Rightarrow f(x) = x^2 - 1$

Hence, required polynomial $f(x) = x^2 - 1$.

$\therefore f(x) = 0 \Rightarrow x^2 - 1 = 0$

$\Rightarrow (x + 1)(x - 1) = 0$

$\Rightarrow (x + 1) = 0$ or $(x - 1) = 0$

$\Rightarrow x = -1$ or $x = 1$

So, the zeroes of f(x) are -1 and 1.

17. Find the quadratic polynomial, sum of whose zeroes is $(\frac{5}{2})$ and their product is 1. Hence, find the zeroes of the polynomial.

Sol:

Let α and β be the zeroes of the required polynomial f(x).

Then $(\alpha + \beta) = \frac{5}{2}$ and $\alpha\beta = 1$

$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$\Rightarrow f(x) = x^2 - \frac{5}{2}x + 1$

$\Rightarrow f(x) = 2x^2 - 5x + 2$

Hence, the required polynomial is $f(x) = 2x^2 - 5x + 2$

$\therefore f(x) = 0 \Rightarrow 2x^2 - 5x + 2 = 0$

$\Rightarrow 2x^2 - (4x + x) + 2 = 0$

$\Rightarrow 2x^2 - 4x - x + 2 = 0$

$\Rightarrow 2x(x - 2) - 1(x - 2) = 0$

$\Rightarrow (2x - 1)(x - 2) = 0$

$\Rightarrow (2x - 1) = 0$ or $(x - 2) = 0$

$\Rightarrow x = \frac{1}{2}$ or $x = 2$

So, the zeros of f(x) are $\frac{1}{2}$ and 2.

18. Find the quadratic polynomial, sum of whose zeroes is $\sqrt{2}$ and their product is $(\frac{1}{3})$.

Sol:

We can find the quadratic equation if we know the sum of the roots and product of the roots by using the formula

x^2 – (Sum of the roots)x + Product of roots = 0

$\Rightarrow x^2 - \sqrt{2}x + \frac{1}{3} = 0$

$\Rightarrow 3x^2 - 3\sqrt{2}x + 1 = 0$

19. If $x = \dfrac{2}{3}$ and x = -3 are the roots of the quadratic equation $ax^2 + 2ax + 5x + 10$ then find the value of a and b.

Sol:

Given: $ax^2 + 7x + b = 0$

Since, $x = \frac{2}{3}$ is the root of the above quadratic equation

Hence, it will satisfy the above equation.

Therefore, we will get

$a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$

$\Rightarrow \frac{4}{9}a + \frac{14}{3} + b = 0$

$\Rightarrow 4a + 42 + 9b = 0$

$\Rightarrow 4a + 9b = -42$ …(1)

Since, x = –3 is the root of the above quadratic equation

Hence, It will satisfy the above equation.

Therefore, we will get

$a(-3)^2 + 7(-3) + b = 0$

$\Rightarrow 9a - 21 + b = 0$

$\Rightarrow 9a + b = 21$ …..(2)

From (1) and (2), we get

a = 3, b = –6

20. If $(x+a)$ is a factor of the polynomial $2x^2 + 2ax + 5x + 10$, find the value of a.

Sol:

Given: (x + a) is a factor of $2x^2 + 2ax + 5x + 10$

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So, we have

$x + a = 0$

$\Rightarrow x = -a$

Now, it will satisfy the above polynomial.

Therefore, we will get

$2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$

$\Rightarrow 2a^2 - 2a^2 - 5a + 10 = 0$

$\Rightarrow -5a = -10$

$\Rightarrow a = 2$

21. One zero of the polynomial $3x^3 + 16x^2 + 15x - 18$ is $\frac{2}{3}$. Find the other zeros of the polynomial.

Sol:

Given: $x = \frac{2}{3}$ is one of the zero of $3x^3 + 16x^2 + 15x - 18$

Now, we have

$x = \frac{2}{3}$

$\Rightarrow x - \frac{2}{3} = 0$

Now, we divide $3x^3 + 16x^2 + 15x - 18$ by $x - \frac{2}{3}$ to find the quotient

Quotient: $3x^2 + 18x + 27$

$x - \frac{2}{3}$) $3x^3 + 16x^2 + 15x - 18$

$3x^3 - 2x^2$

$-\quad +$

$18x^2 + 15x$

$18x^2 - 12x$

$-\quad +$

$27x - 18$

$27x - 18$

$-\quad +$

X

So, the quotient is $3x^2 + 18x + 27$

Now,

$3x^2 + 18x + 27 = 0$

$\Rightarrow 3x^2 + 9x + 9x + 27 = 0$

$\Rightarrow 3x(x + 3) + 9(x + 3) = 0$

$\Rightarrow (x + 3)(3x + 9) = 0$

$\Rightarrow (x + 3) = 0$ or $(3x + 9) = 0$

$\Rightarrow x = -3$ or $x = -3$

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