Exercise - 1C

1. Without actual division, show that each of the following rational numbers is a terminating decimal. Express each in decimal form.

(i) $\frac{23}{2^3 \times 5^2}$ (ii) $\frac{24}{125}$ (iii) $\frac{171}{800}$ (iv) $\frac{15}{1600}$ (v) $\frac{17}{320}$

(vi) $\frac{19}{3125}$

Answer:

(i) $\frac{23}{2^3 \times 5^2} = \frac{23 \times 5}{2^3 \times 5^3} = \frac{115}{1000} = 0.115$

We know either 2 or 5 is not a factor of 23, so it is in its simplest form

Moreover, it is in the form of $(2^m \times 5^n)$.

Hence, the given rational is terminating.

(ii) $\frac{24}{125} = \frac{24}{5^3} = \frac{24 \times 2^3}{5^3 \times 2^3} = \frac{192}{1000} = 0.192$

We know 5 is not a factor of 23, so it is in its simplest form.

Moreover, it is in the form of $(2^m \times 5^n)$.

Hence, the given rational is terminating.

(iii) $\frac{171}{800} = \frac{171}{2^5 \times 5^2} = \frac{171 \times 5^3}{2^5 \times 5^5} = \frac{21375}{100000} = 0.21375$

We know either 2 or 5 is not a factor of 171, so it is in its simplest form.

Moreover, it is in the form of $(2^m \times 5^n)$.
Hence, the given rational is terminating.

(iv) $\frac{15}{1600} = \frac{15}{2^6 \times 5^2} = \frac{15 \times 5^4}{2^6 \times 5^6} = \frac{9375}{1000000} = 0.009375$

We know either 2 or 5 is not a factor of 15, so it is in its simplest form.
Moreover, it is in the form of $(2^m \times 5^n)$.
Hence, the given rational is terminating.

(v) $\frac{17}{320} = \frac{17}{2^6 \times 5} = \frac{17 \times 5^5}{2^6 \times 5^6} = \frac{53125}{1000000} = 0.053125$

We know either 2 or 5 is not a factor of 17, so it is in its simplest form.
Moreover, it is in the form of $(2^m \times 5^n)$.
Hence, the given rational is terminating.

(vi) $\frac{19}{3125} = \frac{19}{5^5} = \frac{19 \times 2^5}{5^5 \times 2^5} = \frac{608}{100000} = 0.00608$

We know either 2 or 5 is not a factor of 19, so it is in its simplest form.
Moreover, it is in the form of $(2^m \times 5^n)$.
Hence, the given rational is terminating.

2. Without actual division show that each of the following rational numbers is a non-terminating repeating decimal.

(i) $\frac{11}{2^3 \times 3}$ (ii) $\frac{73}{2^3 \times 3^3 \times 5}$ (iii) $\frac{129}{2^2 \times 5^7 \times 7^5}$ (iv) $\frac{9}{35}$

(v) $\frac{77}{210}$ (vi) $\frac{32}{147}$ (vii) $\frac{29}{343}$ (viii) $\frac{64}{455}$

Answer:

(i) $\frac{11}{2^3 \times 3}$

We know either 2 or 3 is not a factor of 11, so it is in its simplest form.
Moreover, $(2^3 \times 3) \neq (2^m \times 5^n)$
Hence, the given rational is non – terminating repeating decimal.

(ii) $\frac{73}{2^3 \times 3^3 \times 5}$

We know 2, 3 or 5 is not a factor of 73, so it is in its simplest form.
Moreover, $(2^2 \times 3^3 \times 5) \neq (2^m \times 5^n)$
Hence, the given rational is non-terminating repeating decimal.

(iii) $\frac{129}{2^2 \times 5^7 \times 7^5}$

We know 2, 5 or 7 is not a factor of 129, so it is in its simplest form.
Moreover, $(2^2 \times 5^7 \times 7^5) \neq (2^m \times 5^n)$
Hence, the given rational is non-terminating repeating decimal.

(iv) $\frac{9}{35} = \frac{9}{5 \times 7}$

We know either 5 or 7 is not a factor of 9, so it is in its simplest form.
Moreover, $(5 \times 7) \neq (2^m \times 5^n)$
Hence, the given rational is non-terminating repeating decimal.

(v) $\frac{77}{210} = \frac{77 \div 7}{210 \div 7} = \frac{11}{30} = \frac{11}{2 \times 3 \times 5}$

We know 2, 3 or 5 is not a factor of 11, so $\frac{11}{30}$ is in its simplest form.

Moreover, $(2 \times 3 \times 7) \neq (2^m \times 5^n)$

Hence, the given rational is non-terminating repeating decimal.

(vi) $\frac{32}{147} = \frac{32}{3 \times 7^2}$

We know either 3 or 7 is not a factor of 32, so it is in its simplest form.

Moreover, $(3 \times 7^2) \neq (2^m \times 5^n)$

Hence, the given rational is non-terminating repeating decimal.

(vii) $\frac{29}{343} = \frac{29}{7^3}$

We know 7 is not a factor of 29, so it is in its simplest form.

Moreover, $7^3 \neq (2^m \times 5^n)$

Hence, the given rational is non-terminating repeating decimal.

(viii) $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

We know 5, 7 or 13 is not a factor of 64, so it is in its simplest form.

Moreover, $(5 \times 7 \times 13) \neq (2^m \times 5^n)$

Hence, the given rational is non-terminating repeating decimal.

3. Express each of the following as a rational number in its simplest form:

(i) $0.\overline{8}$ (ii) $2.\overline{4}$ (iii) $0.\overline{24}$ (iv) $0.\overline{12}$ (v) $2.\overline{24}$ (vi) $0.\overline{365}$

Answer:

(i) Let $x = \overline{0.8}$

$\therefore x = 0.888$ …(1)

$10x = 8.888$ …(2)

On subtracting equation (1) from (2), we get

$9x = 8 \Rightarrow x = \frac{8}{9}$

$\therefore 0.8 = \frac{8}{9}$

(ii) Let $x = \overline{2.4}$

$\therefore x = 2.444$ …(1)

$10x = 24.444$ …(2)

On subtracting equation (1) from (2), we get

$9x = 22 \Rightarrow x = \frac{22}{9}$

$\therefore 2.4 = \frac{22}{9}$

(iii) Let $x = \overline{0.24}$

$\therefore x = 0.2424$ …(1)

$100x = 24.2424$ …(2)

On subtracting equation (1) from (2), we get

$99x = 24 \Rightarrow x = \frac{8}{33}$

$\therefore \overline{0.24} = \frac{8}{33}$

(iv) Let x = $\overline{0.12}$

$\therefore x = 0.1212$ …(1)

$100x = 12.1212$ …(2)

On subtracting equation (1) from (2), we get

$99x = 12 \Rightarrow x = \frac{4}{33}$

$\therefore \overline{0.12} = \frac{4}{33}$

(v) Let x = $\overline{2.24}$

$\therefore x = 2.2444$ …(1)

$10x = 22.444$ …(2)

$100x = 224.444$ …(3)

On subtracting equation (2) from (3), we get

$90x = 202 \Rightarrow x = \frac{202}{90} = \frac{101}{45}$

$\therefore \overline{2.24} = \frac{101}{45}$

(vi) Let x = $\overline{0.365}$

$\therefore x = 0.3656565$ …(1)

$10x = 3.656565$ …(2)

$1000x = 365.656565$ …(3)

On subtracting equation (2) from (3), we get

$990x = 362 \Rightarrow x = \frac{362}{990} = \frac{181}{495}$

$\therefore \overline{0.365} = \frac{181}{495}$