Exercise - 1B

1. Using prime factorization, find the HCF and LCM of

(i) 36, 84

(ii) 23, 31

(iii) 96, 404

(iv) 144, 198

(v) 396, 1080

(vi) 1152, 1664

In each case verify that HCF × LCM = product of given numbers.

Sol:

(i) Prime factorization:

$36 = 2^2 \times 3$

$84 = 2^2 \times 3 \times 7$

HCF = product of smallest power of each common prime factor in the numbers $= 2^2 \times 3 = 12$

LCM = product of greatest power of each prime factor involved in the numbers $= 2^2 \times 3^2 \times 7 = 252$

(ii) Prime factorization:

$23 = 23$

$31 = 31$

HCF = product of smallest power of each common prime factor in the numbers = 1

LCM = product of greatest power of each prime factor involved in the numbers $= 23 \times 31 = 713$

(iii) Prime factorization:

$96 = 2^5 \times 3$

$404 = 2^2 \times 101$

HCF = product of smallest power of each common prime factor in the numbers $= 2^2 = 4$

LCM = product of greatest power of each prime factor involved in the numbers $= 2^5 \times 3 \times 101 = 9696$

(iv) Prime factorization:

$144 = 2^4 \times 3^2$
$198 = 2 \times 3^2 \times 11$

HCF = product of smallest power of each common prime factor in the numbers $= 2 \times 3^2 = 18$

LCM = product of greatest power of each prime factor involved in the numbers $= 2^4 \times 3^2 \times 11 = 1584$

(v) Prime factorization:

$396 = 2^2 \times 3^2 \times 11$
$1080 = 2^3 \times 3^3 \times 5$

HCF = product of smallest power of each common prime factor in the numbers $= 2^2 \times 3^2 = 36$

LCM = product of greatest power of each prime factor involved in the numbers $= 2^3 \times 3^3 \times 5 \times 11 = 11880$

(vi) Prime factorization:

$1152 = 2^7 \times 3^2$
$1664 = 2^7 \times 13$

HCF = product of smallest power of each common prime factor in the numbers
$= 2^7 = 128$

LCM = product of greatest power of each prime factor involved in the numbers
$= 2^7 \times 3^2 \times 13 = 14976$

2. Using prime factorization, find the HCF and LCM of

(i) 8, 9, 25
(ii) 12,15, 21
(iii) 17,23,29
(iv) 24, 36, 40
(v) 30, 72, 432
(vi) 21, 28, 36, 45

Sol:

(i) $8 = 2 \times 2 \times 2 = 2^3$
$9 = 3 \times 3 = 3^2$
$25 = 5 \times 5 = 5^2$

HCF = product of smallest power of each common prime factor in the numbers = 1

LCM = product of greatest power of each prime factor involved in the numbers
$= 2^3 \times 3^2 \times 5^2 = 1800$

(ii) $12 = 2 \times 2 \times 3 = 2^2 \times 3$
$15 = 3 \times 5$
$21 = 3 \times 7$

HCF = product of smallest power of each common prime factor in the numbers = 3

LCM = product of greatest power of each prime factor involved in the numbers

$= 2^2 \times 3 \times 5 \times 7 = 420$

(iii) $17 = 17$
$23 = 23$
$29 = 29$
HCF = product of smallest power of each common prime factor in the numbers = 1
LCM = product of greatest power of each prime factor involved in the numbers
$= 17 \times 23 \times 29 = 11339$

(iv) $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$
$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$
$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$
HCF = product of smallest power of each common prime factor in the numbers
$= 2^2 = 4$
LCM = product of greatest power of each prime factor involved in the numbers
$= 2^3 \times 3^2 \times 5 = 360$

(v) $30 = 2 \times 3 \times 5$
$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$
$432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^4 \times 3^3$
HCF = product of smallest power of each common prime factor in the numbers
$= 2 \times 3 = 6$
LCM = product of greatest power of each prime factor involved in the numbers
$= 2^4 \times 3^3 \times 5 = 2160$

(vi) $21 = 3 \times 7$
$28 = 2 \times 2 \times 7 = 2^2 \times 7$
$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$
$45 = 5 \times 3 \times 3 = 5 \times 3^2$
HCF = product of smallest power of each common prime factor in the numbers = 1
LCM = product of greatest power of each prime factor involved in the numbers
$= 2^2 \times 3^2 \times 5 \times 7 = 1260$

3. The HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161, find the other.

Sol:
Let the two numbers be *a* and *b*.
Let the value of *a* be 161.
Given: HCF = 23 and LCM = 1449
We know, $a \times b = \text{HCF} \times \text{LCM}$
$\Rightarrow 161 \times b = 23 \times 1449$
$\Rightarrow \quad \therefore b = \frac{23 \times 1449}{161} = \frac{33327}{161} = 207$
Hence, the other number *b* is 207.

4. The HCF of two numbers is 145 and their LCM is 2175. If one of the numbers is 725, find the other.

Sol:

HCF of two numbers = 145

LCM of two numbers = 2175

Let one of the two numbers be 725 and other be x.

Using the formula, product of two numbers = HCF × LCM

we conclude that

$725 \times x = 145 \times 2175$

$x = \frac{145 \times 2175}{725}$

$= 435$

Hence, the other number is 435.

5. The HCF of two numbers is 18 and their product is 12960. Find their LCM.

Sol:

HCF of two numbers = 18

Product of two numbers = 12960

Let their LCM be x.

Using the formula, product of two numbers = HCF × LCM

we conclude that

$12960 = 18 \times x$

$x = \frac{12960}{18}$

$= 720$

Hence, their LCM is 720.

6. Is it possible to have two numbers whose HCF is 18 and LCM is 760? Give reason.

Sol:

No, it is not possible to have two numbers whose HCF is 18 and LCM is 760.

Since, HCF must be a factor of LCM, but 18 is not factor of 760.

7. Find the simplest form of

(i) $\frac{69}{92}$ (ii) $\frac{473}{645}$ (iii) $\frac{1095}{1168}$ (iv) $\frac{368}{496}$

Sol:

(i) Prime factorization of 69 and 92 is:

$69 = 3 \times 23$

$92 = 2^2 \times 23$

Therefore, $\frac{69}{92} = \frac{3 \times 23}{2^2 \times 23} = \frac{3}{2^2} = \frac{3}{4}$

Thus, simplest form of $\frac{69}{92}$ is $\frac{3}{4}$.

(ii) Prime factorization of 473 and 645 is:

$473 = 11 \times 43$

$645 = 3 \times 5 \times 43$

Therefore, $\frac{473}{645} = \frac{11 \times 43}{3 \times 5 \times 43} = \frac{11}{15}$

Thus, simplest form of $\frac{473}{645}$ is $\frac{11}{15}$.

(iii) Prime factorization of 1095 and 1168 is:

$1095 = 3 \times 5 \times 73$

$1168 = 2^4 \times 73$

Therefore, $\frac{1095}{1168} = \frac{3 \times 5 \times 73}{2^4 \times 73} = \frac{15}{16}$

Thus, simplest form of $\frac{1095}{1168}$ is $\frac{15}{16}$.

(iv) Prime factorization of 368 and 496 is:

$368 = 2^4 \times 23$

$496 = 2^4 \times 31$

Therefore, $\frac{368}{496} = \frac{2^4 \times 23}{2^4 \times 31} = \frac{23}{31}$

Thus, simplest form of $\frac{368}{496}$ is $\frac{23}{31}$.

8. Find the largest number which divides 438 and 606 leaving remainder 6 in each case.

Answer:

Largest number which divides 438 and 606, leaving remainder 6 is actually the largest number which divides 438 – 6 = 432 and 606 – 6 = 600, leaving remainder 0.

Therefore, HCF of 432 and 600 gives the largest number.

Now, prime factors of 432 and 600 are:

$432 = 2^4 \times 3^3$

$600 = 2^3 \times 3 \times 5^2$

HCF = product of smallest power of each common prime factor in the numbers = $2^3 \times 3$ = 24

Thus, the largest number which divides 438 and 606, leaving remainder 6 is 24.

9. Find the largest number which divides 320 and 457 leaving remainders 5 and 7 respectively.

Answer:

We know that the required number divides 315 (320 – 5) and 450 (457 – 7).

∴ Required number = HCF (315, 450)

On applying Euclid's lemma, we get:

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315 ) 450 ( 1
    - 315
      135 ) 315 ( 2
          - 270
             45 ) 135 ( 3
                - 135
                    0
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Therefore, the HCF of 315 and 450 is 45.
Hence, the required number is 45.

10. Find the least number which when divides 35, 56 and 91 leaves the same remainder 7 in each case.

Answer:

Least number which can be divided by 35, 56 and 91 is LCM of 35, 56 and 91.
Prime factorization of 35, 56 and 91 is:
$35 = 5 \times 7$
$56 = 2^3 \times 7$
$91 = 7 \times 13$
LCM = product of greatest power of each prime factor involved in the numbers = $2^3 \times 5 \times 7 \times 13 = 3640$
Least number which can be divided by 35, 56 and 91 is 3640.
Least number which when divided by 35, 56 and 91 leaves the same remainder 7 is $3640 + 7 = 3647$.
Thus, the required number is 3647.

11. Find the smallest number which when divides 28 and 32, leaving remainders 8 and 12 respectively.

Answer:

Let the required number be x.
Using Euclid's lemma,
$x = 28p + 8$ and $x = 32q + 12$, where p and q are the quotients
$\Rightarrow 28p + 8 = 32q + 12$
$\Rightarrow 28p = 32q + 4$
$\Rightarrow 7p = 8q + 1$….. (1)
Here $p = 8n – 1$ and $q = 7n – 1$ satisfies (1), where n is a natural number
On putting $n = 1$, we get
$p = 8 – 1 = 7$ and $q = 7 – 1 = 6$
Thus, $x = 28p + 8$
$= 28 \times 7 + 8$

= 204

Hence, the smallest number which when divided by 28 and 32 leaves remainders 8 and 12 is 204.

12. Find the smallest number which when increased by 17 is exactly divisible by both 468 and 520.

Answer:

The smallest number which when increased by 17 is exactly divisible by both 468 and 520 is obtained by subtracting 17 from the LCM of 468 and 520.

Prime factorization of 468 and 520 is:

$468 = 2^2 \times 3^2 \times 13$

$520 = 2^3 \times 5 \times 13$

LCM = product of greatest power of each prime factor involved in the numbers = $2^2 \times 3^2 \times 5 \times 13 = 4680$

The required number is 4680 – 17 = 4663.

Hence, the smallest number which when increased by 17 is exactly divisible by both 468 and 520 is 4663.

13. Find the greatest number of four digits which is exactly divisible by 15, 24 and 36.

Answer:

Prime factorization:

$15 = 3 \times 5$

$24 = 2^3 \times 3$

$36 = 2^2 \times 3^2$

LCM = product of greatest power of each prime factor involved in the numbers = $2^3 \times 3^2 \times 5 = 360$

Now, the greatest four digit number is 9999.

On dividing 9999 by 360 we get 279 as remainder.

Thus, 9999 – 279 = 9720 is exactly divisible by 360.

Hence, the greatest number of four digits which is exactly divisible by 15, 24 and 36 is 9720.

14. In a seminar, the number of participants in Hindi, English and mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required, if in each room, the same number of participants are to be seated and all of them being in the same subject.

Answer:

Minimum number of rooms required = $\frac{Total\ number\ of\ participants}{HCF\ (60,84,108)}$

Prime factorization of 60, 84 and 108 is:

$60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

$108 = 2^2 \times 3^3$

HCF = product of smallest power of each common prime factor in the numbers = $2^2 \times 3$ = 12

Total number of participants = 60 + 84 + 108 = 252

Therefore, minimum number of rooms required = $\frac{252}{12}$ = 21

Thus, minimum number of rooms required is 21.

15. Three sets of English, Mathematics and Science books containing 336, 240 and 96 books respectively have to be stacked in such a way that all the books are stored subject wise and the height of each stack is the same. How many stacks will be there?

Answer:

Total number of English books = 336

Total number of mathematics books = 240

Total number of science books = 96

∴ Number of books stored in each stack = HCF (336, 240, 96)

Prime factorization:

$336 = 2^4 \times 3 \times 7$

$240 = 2^4 \times 3 \times 5$

$96 = 2^5 \times 3$

∴ HCF = Product of the smallest power of each common prime factor involved in the numbers = $2^4 \times 3 = 48$

Hence, we made stacks of 48 books each.

∴ Number of stacks = $\frac{336}{48} + \frac{240}{48} + \frac{96}{48}$ = (7+5+2) = 14

16. Three pieces of timber 42m, 49m and 63m long have to be divided into planks of the same length. What is the greatest possible length of each plank? How many planks are formed?

Answer:

The lengths of three pieces of timber are 42m, 49m and 63m respectively.

We have to divide the timber into equal length of planks.

∴ Greatest possible length of each plank = HCF (42, 49, 63)

Prime factorization:

$42 = 2 \times 3 \times 7$

$49 = 7 \times 7$

$63 = 3 \times 3 \times 7$

∴HCF = Product of the smallest power of each common prime factor involved in the numbers = 7

Hence, the greatest possible length of each plank is 7m.

17. Find the greatest possible length which can be used to measure exactly the lengths 7m, 3m 85cm and 12m 95cm.

Answer:

The three given lengths are 7m (700cm), 3m 85cm (385cm) and 12m 95m (1295cm). (∵1m = 100cm).

∴ Required length = HCF (700, 385, 1295)

Prime factorization:

$700 = 2 \times 2 \times 5 \times 5 \times 7 = 2^2 \times 5^2 \times 7$

$385 = 5 \times 7 \times 11$

$1295 = 5 \times 7 \times 37$

∴HCF = $5 \times 7 = 35$

Hence, the greatest possible length is 35cm.

18. Find the maximum number of students among whom 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and the same number of pencils.

Answer:

Total number of pens = 1001

Total number pencils = 910

∴ Maximum number of students who get the same number of pens and pencils = HCF (1001, 910)

Prime factorization:

$1001 = 11 \times 91$

$910 = 10 \times 91$

∴ HCF = 91

Hence, 91 students receive same number of pens and pencils.

19. Find the least number of square tiles required to pave the ceiling of a room 15m 17cm long and 9m 2cm broad.

Answer:

It is given that:

Length of a tile = 15m 17m = 1517cm [∵ 1m = 100cm]

Breadth of a tile = 9m 2m = 902cm

∴ Side of each square tile = HCF (1517, 902)

Prime factorization:

$1517 = 37 \times 41$

$902 = 22 \times 41$

∴ HCF = product of smallest power of each common prime factor in the numbers = 41

∴ Required number of tiles = $\frac{Area\ of\ ceiling}{Area\ of\ one\ tile} = \frac{1517 \times 902}{41 \times 41} = 37 \times 22 = 814$

20. Three measuring rods are 64 cm, 80 cm and 96 cm in length. Find the least length of cloth that can be measured an exact number of times, using any of the rods.

Answer:

Length of the three measuring rods are 64cm, 80cm and 96cm, respectively.

∴ Length of cloth that can be measured an exact number of times = LCM (64, 80, 96)

Prime factorization:

$64 = 2^6$

$80 = 2^4 \times 5$

$96 = 2^5 \times 3$

∴ LCM = product of greatest power of each prime factor involved in the numbers = $2^6 \times 3 \times 5$ = 960cm = 9.6m

Hence, the required length of cloth is 9.6m.

21. An electronic device makes a beep after every 60 seconds. Another device makes a beep after every 62 seconds. They beeped together at 10 a.m. At what time will they beep together at the earliest?

Answer:

Beep duration of first device = 60 seconds

Beep duration of second device = 62 seconds

∴ Interval of beeping together = LCM (60, 62)

Prime factorization:

$60 = 2^2 \times 3 \times 5$

$62 = 2 \times 31$

∴ LCM = $2^2 \times 3 \times 5 \times 31$ = 1860 seconds = $\frac{1860}{60}$ = 31min

Hence, they will beep together again at 10 : 31 a.m.

22. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8, 10, 12 minutes respectively. In 30 hours, how many times do they toll together?

Answer:

Six bells toll together at intervals of 2,4, 6, 8, 10 and 12 minutes, respectively.

Prime factorization:

$2 = 2$

$4 = 2 \times 2$

$6 = 2 \times 3$

$8 = 2 \times 2 \times 2$

$10 = 2 \times 5$

$12 = 2 \times 2 \times 3$

∴ LCM (2, 4, 6, 8, 10, 12) = $2^3 \times 3 \times 5 = 120$

Hence, after every 120minutes (i.e. 2 hours), they will toll together.

∴ Required number of times = $\left(\frac{30}{2} + 1\right) = 16$

23. Find the missing numbers in the following factorization:

Answer:

660 = 2 × 2 × 3 × 5 × 11