Exercise - 1A

1. What do you mean by Euclid's division algorithm?

Sol:

Euclid's division algorithm states that for any two positive integers a and b, there exit unique integers q and r, such that $a = bq + r$. where $0 \leq r \leq b$.

2. A number when divided by 61 gives 27 as quotient and 32 as remainder. Find the number.

Sol:

We know, Dividend = Divisor × Quotient + Remainder

Given: Divisor = 61, Quotient = 27, Remainder = 32

Let the Dividend be x.

$\therefore x = 61 \times 27 + 32$

$= 1679$

Hence, the required number is 1679.

3. By what number should be 1365 be divided to get 31 as quotient and 32 as remainder?

Sol:

Given: Dividend = 1365, Quotient = 31, Remainder = 32

Let the divisor be x.

Dividend = Divisor × Quotient + Remainder

$1365 = x \times 31 + 32$

$\Rightarrow 1365 - 32 = 31x$

$\Rightarrow 1333 = 31x$

$\Rightarrow x = \frac{1333}{31} = 43$

Hence, 1365 should be divided by 43 to get 31 as quotient and 32 as remainder.

4. Using Euclid's algorithm, find the HCF of

(i) 405 and 2520

(ii) 504 and 1188

(iii) 960 and 1575

Sol:

(i)

405)2520(6
- 2430
90)405(4
- 360
45)90(2
- 90
0

On applying Euclid's algorithm, i.e. dividing 2520 by 405, we get:

Quotient = 6, Remainder = 90

$\therefore 2520 = 405 \times 6 + 90$

Again on applying Euclid's algorithm, i.e. dividing 405 by 90, we get:

Quotient = 4, Remainder = 45

$\therefore 405 = 90 \times 4 + 45$

Again on applying Euclid's algorithm, i.e. dividing 90 by 45, we get:

$\therefore 90 = 45 \times 2 + 0$

Hence, the HCF of 2520 and 405 is 45.

(ii)

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504 ) 1188 ( 2
    -  1008
      -----
       180 ) 504 ( 2
           - 360
             -----
             144 ) 180 ( 1
                 - 144
                   -----
                    36 ) 144 ( 4
                       -  144
                         -----
                            0
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On applying Euclid's algorithm, i.e. dividing 1188 by 504, we get:

Quotient = 2, Remainder = 180

$\therefore 1188 = 504 \times 2 + 180$

Again on applying Euclid's algorithm, i.e. dividing 504 by 180, we get:

Quotient = 2, Remainder = 144

$\therefore 504 = 180 \times 2 + 144$

Again on applying Euclid's algorithm, i.e. dividing 180 by 144, we get:

Quotient = 1, Remainder = 36

$\therefore 180 = 144 \times 1 + 36$

Again on applying Euclid's algorithm, i.e. dividing 144 by 36, we get:

$\therefore 144 = 36 \times 4 + 0$

Hence, the HCF of 1188 and 504 is 36.

(iii)

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960 ) 1575 (1
     - 960
      615) 960 (1
          - 615
           345) 615 (1
               - 345
                 270 ) 345 (1
                      - 270
                        75 ) 270 (3
                            - 225
                              45 ) 75 (1
                                  - 45
                                   30) 45 (1
                                      - 30
                                       15 ) 30 (2
                                           - 30
                                             0
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On applying Euclid's algorithm, i.e. dividing 1575 by 960, we get:

Quotient = 1, Remainder = 615

$\therefore 1575 = 960 \times 1 + 615$

Again on applying Euclid's algorithm, i.e. dividing 960 by 615, we get:

Quotient = 1, Remainder = 345

$\therefore 960 = 615 \times 1 + 345$

Again on applying Euclid's algorithm, i.e. dividing 615 by 345, we get:

Quotient = 1, Remainder = 270

$\therefore 615 = 345 \times 1 + 270$

Again on applying Euclid's algorithm, i.e. dividing 345 by 270, we get:

Quotient = 1, Remainder = 75

$\therefore 345 = 270 \times 1 + 75$

Again on applying Euclid's algorithm, i.e. dividing 270 by 75, we get:

Quotient = 3, Remainder = 45

$\therefore 270 = 75 \times 3 + 45$

Again on applying Euclid's algorithm, i.e. dividing 75 by 45, we get:

Quotient = 1, Remainder = 30

$\therefore 75 = 45 \times 1 + 30$

Again on applying Euclid's algorithm, i.e. dividing 45 by 30, we get:

Quotient = 1, Remainder = 15

$\therefore 45 = 30 \times 1 + 15$

Again on applying Euclid's algorithm, i.e. dividing 30 by 15, we get:

Quotient = 2, Remainder = 0

$\therefore 30 = 15 \times 2 + 0$

Hence, the HCF of 960 and 1575 is 15.

5. Show that every positive integer is either even or odd?

Sol:

Let us assume that there exist a smallest positive integer that is neither odd nor even, say n. Since n is least positive integer which is neither even nor odd, $n - 1$ must be either odd or even.

Case 1: If $n - 1$ is even, $n - 1 = 2k$ for some k.

But this implies $n = 2k + 1$

this implies n is odd.

Case 2: If $n - 1$ is odd, $n - 1 = 2k + 1$ for some k.

But this implies $n = 2k + 2\ (k+1)$

this implies n is even.

In both ways we have a contradiction.

Thus, every positive integer is either even or odd.

6. Show that every positive even integer is of the form (6m+1) or (6m+3) or (6m+5)where m is some integer.

Sol:

Let n be any arbitrary positive odd integer.

On dividing n by 6, let m be the quotient and r be the remainder. So, by Euclid's division lemma, we have

$n = 6m + r$, where $0 \le r < 6$.

As $0 \le r < 6$ and r is an integer, r can take values 0, 1, 2, 3, 4, 5.

$\Rightarrow n = 6m$ or $n = 6m + 1$ or $n = 6m + 2$ or $n = 6m + 3$ or $n = 6m + 4$ or $n = 6m + 5$

But $n \neq 6m$ or $n \neq 6m + 2$ or $n \neq 6m + 4$ ($\because 6m, 6m + 2, 6m + 4$ are multiples of 2, so an even integer whereas n is an odd integer)

$\Rightarrow n = 6m + 1$ or $n = 6m + 3$ or $n = 6m + 5$

Thus, any positive odd integer is of the form $(6m + 1)$ or $(6m + 3)$ or $(6m + 5)$, where m is some integer.

7. Show that every positive even integer is of the form 4m and that every positive odd integer is of the form 4m + 1 for some integer m.

Sol:

Let n be any arbitrary positive odd integer.

On dividing n by 4, let m be the quotient and r be the remainder. So, by Euclid's division lemma, we have

$n = 4m + r$, where $0 \le r < 4$.

As $0 \leq r < 4$ and r is an integer, r can take values 0, 1, 2, 3.
$\Rightarrow n = 4m$ or $n = 4m + 1$ or $n = 4m + 2$ or $n = 4m + 3$
But $n \neq 4m$ or $n \neq 4m + 2$ ($\because 4m$, $4m + 2$ are multiples of 2, so an even integer whereas n is an odd integer)
$\Rightarrow n = 4m + 1$ or $n = 4m + 3$
Thus, any positive odd integer is of the form $(4m + 1)$ or $(4m + 3)$, where m is some integer.