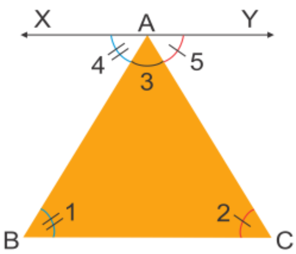
Properties of Triangles
Exercise 15D

Properties of Triangles

Angle Sum Property of a Triangle



The sum of the interior angles of a triangle is 180°.

Proof:

Draw $XY \parallel BC$

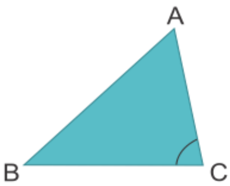
$\angle 1 = \angle 4$
$\angle 2 = \angle 5$ {*Alternate Interior angles are equal*}

$\Rightarrow \angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 5 + \angle 3$

But, $\angle 4 + \angle 5 + \angle 3 = 180^\circ$ *(By linearity property)*

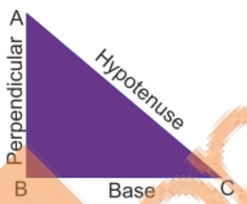
$\therefore \angle 1 + \angle 2 + \angle 3 = 180^\circ$

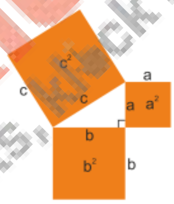
Triangle Inequality Property



- The sum of any two sides of a triangle is always greater than its third side. $AB + BC > AC$
- The angle opposite to the longest side is the largest angle.
- The angle opposite to the smallest side is the smallest angle

Pythagoras Theorem





Pythagoras' Theorem
In a right angled triangle, the square of the hypotenuse is equal to the sum of squares of its remaining two sides. $(AC)^2 = (AB)^2 + (BC)^2$ or $c^2 = a^2 + b^2$

Pythagorean triplets
Three positive numbers a, b, c in this order, are said to form a Pythagorean triplet, if $c^2 = a^2 + b^2$

Converse of Pythagoras' Theorem
If a triangle has sides of length a, b and c units such that $a^2 + b^2 = c^2$, then the triangle is right angled

Q1

Answer :

Suppose the length of the hypotenuse is *a* cm.

Then, by Pythagoras theorem:

$a^2 = 9^2 + 12^2$
=> $a^2 = 81 + 144$
=> $a^2 = 225$
=> $a = \sqrt{225}$
=> $a = 15$

Hence, the length of the hypotenuse is 15 cm.

Q2

Answer :

Suppose the length of the other side is *a* cm.

Then, by Pythagoras theorem:

$26^2 = 10^2 + a^2$
$\Rightarrow a^2 = 676 - 100$
$\Rightarrow a^2 = 576$
$\Rightarrow a = \sqrt{576}$
$\Rightarrow a = 24$

Hence, the length of the other side is 24 cm.

Q3

Answer :

Suppose the length of the other side is *a* cm.

Then, by Pythagoras theorem:

$4.5^2 + a^2 = 7.5^2$
$\Rightarrow a^2 = 56.25 - 20.25$
$\Rightarrow a^2 = 36$
$\Rightarrow a = \sqrt{36}$
$\Rightarrow a = 6$

Hence, the length of the other side of the triangle is 6 cm.

Q4

Answer :

Suppose the length of the two legs of the right triangle are *a* cm and *a* cm.
Then, by Pythagoras theorem:
$a^2 + a^2 = 50$
$\Rightarrow 2a^2 = 50$
$\Rightarrow a^2 = 25$
$\Rightarrow a = \sqrt{25}$
$\Rightarrow a = 5$

Hence, the length of each leg is 5 cm.

Q5

Answer :

The largest side of the triangle is 39 cm.

$15^2 + 36^2$
$= 225 + 1296 = 1521$

Also, $39^2 = 1521$
$\therefore 15^2 + 36^2 = 39^2$

Sum of the square of the two sides is equal to the square of the third side.

Hence, the triangle is right angled.

Q6

Answer :

Suppose the length of the hypotenuse is *c* cm.
Then, by Pythagoras theorem:

$a^2 + b^2 = c^2$
$\Rightarrow c^2 = 6^2 + 4.5^2$
$\Rightarrow c^2 = 36 + 20.25$
$\Rightarrow c^2 = 56.25$
$\Rightarrow c = \sqrt{56.25}$
$\Rightarrow c = 7.5$

Hence, the length of its hypotenuse is 7.5 cm.

Q7

Answer :

(i) Largest side, c = 25 cm

We have:
$a^2 + b^2$ = 225 + 400 = 625

Also, c^2 = 625

$\therefore a^2 + b^2 = c^2$
Hence, the given triangle is right angled using the Pythagoras theorem.

(ii) Largest side, c = 16 cm
We have:
$a^2 + b^2$ = 81 + 144 = 225

Also, c^2 = 256

Here, $a^2 + b^2 \neq c^2$

Therefore, the given triangle is not right angled.

Q8

Answer :

We have:
$\angle B$ = 35° and $\angle C$ = 55°

$\therefore \angle B$ = 180 - 35 -55 = 90° (since sum of the angles of any triangle is 180°)

We know that the side opposite to the right angle is the hypotenuse.

By Pythagoras theorem:
$BC^2 = AB^2 + AC^2$

Hence, (iii) is true.

Q9

Answer :

By Pythagoras theorem in △ABC:
AB2 = AC2 + BC2
152 = x2 +122⇒x2 = 225 - 144 ⇒x2= 81⇒x2=92⇒x=9
∴ x = 9 cm

Hence, the distance of the foot of the ladder from the wall is 9 cm.

Q10

Answer :

Suppose the foot of the ladder is *x* m far from the wall.

Let the ladder is represented by AB, the height at which it reaches the wall be AC and the distance between the foot of ladder and wall be BC.

Then, by Pythagoras theorem:

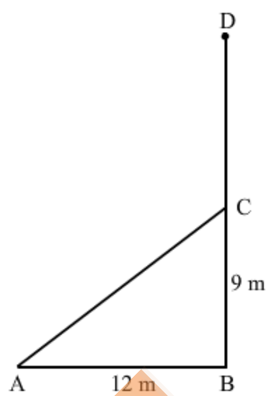
$$AB^2 = AC^2 + BC^2$$
$$\Rightarrow 5^2 = 4.8^2 + x^2$$
$$\Rightarrow x^2 = 25 - 23.04$$
$$\Rightarrow x^2 = 1.96$$
$$\Rightarrow x^2 = (1.4)^2$$
$$\Rightarrow x = 1.4$$

Hence, the foot of the ladder is 1.4 m far from the wall.

Q11

Answer :

Let BD be the height of the tree broken at point C and suppose CD take the position CA



Now as per given conditions we have AB = 9 m , BC = 12 m

By Pythagoras theorem:

$$AC^2 = AB^2 + BC^2$$

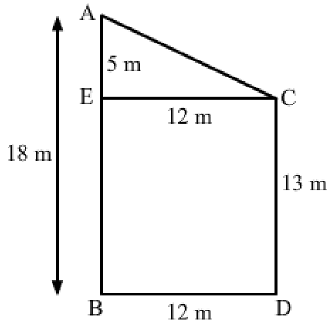
$$\Rightarrow AC^2 = 12^2 + 9^2$$
$$\Rightarrow AC^2 = 144 + 81$$
$$\Rightarrow AC^2 = 225$$
$$\Rightarrow AC^2 = 15^2$$
$$\Rightarrow AC = 15$$

Length of the tree before it broke = AC + AB
= 15 + 9
= 24 m

Q12

Answer :

Suppose, the two poles are AB and CD, having the length of 18 m and 13 m, respectively.
Distance between them, BD, is equal to 12 m.
We need to find AC.



From C, draw CE⊥AB.

AE=AB-EB
= AB-CD   (CD = EB)
= 18-13
= 5 cm
EC = BD = 12 m

Now, by Pythagoras theorem in $\triangle AEC$:

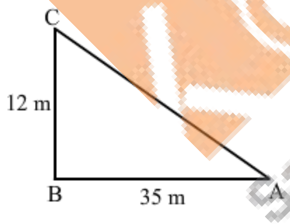
$$\mathbf{AC^2 = AE^2 + EC^2}$$
$$\Rightarrow \mathbf{AC^2} = 5^2 + 12^2$$
$$\Rightarrow \mathbf{AC^2} = 25 + 144$$
$$\Rightarrow \mathbf{AC^2} = 169$$
$$\Rightarrow \mathbf{AC^2} = 13^2$$
$$\Rightarrow \mathbf{AC} = 13$$

Hence, the distance between their tops is 13 m.

Q13

Answer :

Suppose the man starts at point A and goes 35 m towards west, say AB. He then goes 12 m north, say BC.



We need to find AC.

By Pythagoras theorem:

$$\mathbf{AC^2 = BC^2 + AB^2}$$
$$\Rightarrow \mathbf{AC^2} = 35^2 + 12^2$$
$$\Rightarrow \mathbf{AC^2} = 1225 + 144$$
$$\Rightarrow \mathbf{AC^2} = 1369$$
$$\Rightarrow \mathbf{AC^2} = 37^2$$
$$\Rightarrow \mathbf{AC = 37\ m}$$

Hence, the man is 37 m far from the starting point.

Q14

Answer :

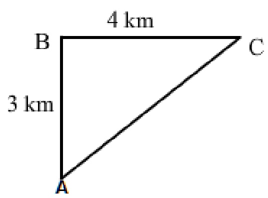
Suppose the man starts from A and goes 3 km north and reaches B.
He then goes 4 km towards east and reaches C.

∴ AB = 3 km
BC = 4 km
We have to find AC.

By Pythagoras theorem:

$$\Rightarrow AC^2 = AB^2 + BC^2$$
$$\Rightarrow AC^2 = 3^2 + 4^2$$
$$\Rightarrow AC^2 = 25$$
$$\Rightarrow AC^2 = 5^2$$
$$\Rightarrow AC = 5 \text{ km}$$

Hence, he is 5 km far from the initial position.



Q15

Answer :

Suppose the sides are *x* and *y* of lengths 16 cm and 12 cm, respectively.

Let the diagonal be *z* cm.

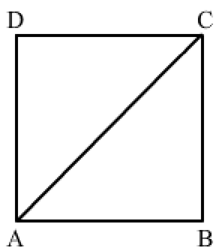
Clearly, the diagonal is the hypotenuse of the right triangle with legs *x* and *y*.

By Pythagoras theorem:
$$z^2 = x^2 + y^2$$
$$\Rightarrow z^2 = 16^2 + 12^2$$
$$\Rightarrow z^2 = 256 + 144$$
$$\Rightarrow z^2 = 400$$
$$\Rightarrow z^2 = 20^2$$
$$\Rightarrow z = 20$$

Hence, the length of the diagonal is 20 cm.

Q16

Answer :



AB = 40 cm
Diagonal, AC = 41 cm

Then, by Pythagoras theorem in right $\triangle ABC$:
$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow BC^2 = 41^2 - 40^2$$
$$\Rightarrow BC^2 = 1681 - 1600$$
$$\Rightarrow BC^2 = 81$$
$$\Rightarrow BC^2 = 9^2$$
$$\Rightarrow BC = 9 \text{ cm}$$

∴ Length = 40 cm
Breadth = 9 cm

∴ Perimeter of the rectangle = 2(length + breadth)
= 2(40+9)
= 98 cm

Q17

Answer :

We know that the diagonals of a rhombus bisect each other at right angles.
Therefore, in right triangle AOB, we have:
AO = 8 cm
BO = 15 cm

By Pythagoras theorem in ΔAOB:

$$AB^2 = AO^2 + BO^2$$
$$\Rightarrow AB^2 = 8^2 + 15^2$$
$$\Rightarrow AB^2 = 64 + 225$$
$$\Rightarrow AB^2 = 289$$
$$\Rightarrow AB^2 = 17^2$$
$$\Rightarrow AB = 17 \text{ cm}$$

Now, as we know that all sides of a rhombus are equal.
∴ Perimeter of the rhombus = 4(side)
= 4(17)
= 68 cm

Q18

Answer :

(i) In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
(ii) If the square of one side of a triangle s equal to the sum of the squares of the other two sides then the triangle is right angled.
(iii) Of all the line segments that can be drawn to a given line from a given point outside it, the perpendicular is the shortest.