Exponents
Exercise 5C

Q1

Answer :

(d) 24

$$\left(6^{-1}-8^{-1}\right)^{-1}=\left(\tfrac{1}{6}-\tfrac{1}{8}\right)^{-1}$$
$$=\left(\tfrac{4-3}{24}\right)^{-1} \quad \text{[since L.C.M. of 6 and 8 is 24]}$$
$$=\left(\tfrac{1}{24}\right)^{-1}$$
$$=\left(\tfrac{24}{1}\right)^{1}=24 \quad \left[s\,\text{ince}\left(\tfrac{a}{b}\right)^{-1}=\left(\tfrac{b}{a}\right)^{1}\right]$$

Q2

Answer :

(c) 15

We have:

$$\left(5^{-1}\times 3^{-1}\right)^{-1}=\left(\tfrac{1}{5}\times\tfrac{1}{3}\right)^{-1}$$
$$=\left(\tfrac{1}{15}\right)^{-1}$$
$$=\left(\tfrac{15}{1}\right)^{1}=15 \quad \left[s\,\text{ince}\left(\tfrac{a}{b}\right)^{-1}=\left(\tfrac{b}{a}\right)^{1}\right]$$

Q3

Answer :

(c) $\tfrac{1}{16}$

We have:

$$\left(2^{-1}-4^{-1}\right)^{2}=\left(\tfrac{1}{2}-\tfrac{1}{4}\right)^{2}$$
$$=\left(\tfrac{2-1}{4}\right)^{2} \quad \text{[since L.C.M. of 2 and 4 is 4]}$$
$$=\left(\tfrac{1}{4}\right)^{2}$$
$$=\left(\tfrac{1}{4}\times\tfrac{1}{4}\right)=\tfrac{1}{16}$$

Q4

Answer :

(b) 29

We have:

$$\left(\tfrac{1}{2}\right)^{-2} + \left(\tfrac{1}{3}\right)^{-2} + \left(\tfrac{1}{4}\right)^{-2} = \left(\tfrac{2}{1}\right)^{2} + \left(\tfrac{3}{1}\right)^{2} + \left(\tfrac{4}{1}\right)^{2} \qquad \left[since\ \left(\tfrac{a}{b}\right)^{-1} = \left(\tfrac{b}{a}\right)^{1}\right]$$

$$= (2^2 + 3^2 + 4^2)$$
$$= (4 + 9 + 16)$$
$$= 29$$

Q5

Answer :

(c) $\tfrac{6}{5}$

We have:

$$\left\{6^{-1} + \left(\tfrac{3}{2}\right)^{-1}\right\}^{-1} = \left(\tfrac{1}{6} + \tfrac{2}{3}\right)^{-1}$$

$$= \left(\tfrac{1+4}{6}\right)^{-1} \quad \text{[since L.C.M. of 3 and 6 is 6]}$$

$$= \left(\tfrac{5}{6}\right)^{-1}$$

$$= \left(\tfrac{6}{5}\right)^{1} = \left(\tfrac{6}{5}\right) \qquad \left[since\ \left(\tfrac{a}{b}\right)^{-1} = \left(\tfrac{b}{a}\right)^{1}\right]$$

Q6

Answer :

(b) 64

We have:

$$\left(\tfrac{-1}{2}\right)^{-6} = \left(\tfrac{2}{-1}\right)^{6} \qquad \left[since\ \left(\tfrac{a}{b}\right)^{-n} = \left(\tfrac{b}{a}\right)^{n}\right]$$

$$= (-2)^6$$
$$= (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2)$$
$$= 64$$

Q7

Answer :

(b) $\tfrac{-3}{8}$

$$\left\{\left(\tfrac{3}{4}\right)^{-1} - \left(\tfrac{1}{4}\right)^{-1}\right\}^{-1} = \left(\tfrac{4}{3} - \tfrac{4}{1}\right)^{-1}$$

$$= \left(\tfrac{4-12}{3}\right)^{-1} \quad \text{[since L.C.M. of 1 and 3 is 3]}$$

$$= \left(\tfrac{-8}{3}\right)^{-1}$$

$$= \left(\tfrac{3}{-8}\right)^{1} \qquad \left[since\ \left(\tfrac{a}{b}\right)^{-1} = \left(\tfrac{b}{a}\right)^{1}\right]$$

$$= \left(\tfrac{3 \times -1}{-8 \times -1}\right) = \tfrac{-3}{8}$$

Q8

Answer :

(a) $\frac{1}{16}$

$$\left[\left\{\left(-\frac{1}{2}\right)^2\right\}^{-2}\right]^{-1} = \left[\left(-\frac{1}{2}\right)^{2\times -2}\right]^{-1} \qquad \left[s\,ince\left\{\left(\frac{a}{b}\right)^m\right\}^n = \left(\frac{a}{b}\right)^{mn}\right]$$

$$= \left[\left(-\frac{1}{2}\right)^{-4}\right]^{-1}$$

$$= \left(-\frac{1}{2}\right)^{(-4)\times(-1)}$$

$$= \left(-\frac{1}{2}\right)^4 = \frac{(-1)^4}{(2)^4}$$

$$= \frac{1}{16}$$

Q9

Answer :

(c) 1

$(a)^0 = 1$

$\therefore \left(\frac{5}{6}\right)^0 = 1$

Q10

Answer :

(b) $\frac{243}{32}$

$$\left(\frac{2}{3}\right)^{-5} = \left(\frac{3}{2}\right)^5 \qquad \left[s\,ince\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$$

$$= \frac{3^5}{2^5} = \frac{3\times3\times3\times3\times3}{2\times2\times2\times2\times2} = \frac{243}{32}$$

Q11

Answer :

(b) $\left(\frac{1}{3}\right)^8$

$$\left\{\left(\frac{1}{3}\right)^2\right\}^4 = \left(\frac{1}{3}\right)^{2\times4} = \left(\frac{1}{3}\right)^8 \qquad \left[s\,ince\left\{\left(\frac{a}{b}\right)^m\right\}^n = \left(\frac{a}{b}\right)^{mn}\right]$$

Q12

Answer :

(b) $\frac{-2}{3}$

We have:

$$\left(\frac{-3}{2}\right)^{-1} = \left(\frac{2}{-3}\right)^1 \qquad \left[s\,ince\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$$

$$= \frac{-2}{3}$$

Q13

Answer :

(d) $\frac{135}{8}$

$$\left(3^2 - 2^2\right) \times \left(\frac{2}{3}\right)^{-3} = (9-4)\times\left(\frac{3}{2}\right)^3 \qquad \left[s\,ince\left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^1\right]$$

$$= 5 \times \frac{3^3}{2^3} = 5 \times \frac{27}{8} = \frac{135}{8}$$

Q14

Answer :

(a) $\frac{19}{64}$

We have:

$$\left\{\left(\tfrac{1}{3}\right)^{-3}-\left(\tfrac{1}{2}\right)^{-3}\right\}\div\left(\tfrac{1}{4}\right)^{-3}=\left\{\left(\tfrac{3}{1}\right)^{3}-\left(\tfrac{2}{1}\right)^{3}\right\}\div\left(\tfrac{4}{1}\right)^{3}$$

$$\left[since\ \left(\tfrac{a}{b}\right)^{-1}=\left(\tfrac{b}{a}\right)^{1}\right]$$

$$=\left\{\left(3^{3}\right)-(2)^{3}\right\}\div(4)^{3}$$
$$=(27-8)\div 64$$
$$=19\div 64$$
$$=19\times\tfrac{1}{64}=\tfrac{19}{64}$$

Q15

Answer :

(c) $(-5)^{5}$

We have:

$$\left(\tfrac{-1}{5}\right)^{3}\div\left(\tfrac{-1}{5}\right)^{8}=\left(\tfrac{-1}{5}\right)^{3-8}\qquad\left[since\ a^{m}\div a^{n}=a^{m-n}\right]$$
$$=\left(\tfrac{-1}{5}\right)^{-5}$$
$$=\left(\tfrac{5}{-1}\right)^{5}\qquad\left[Since\ \left(\tfrac{a}{b}\right)^{-1}=\left(\tfrac{b}{a}\right)^{1}\right]$$
$$=\left(\tfrac{5\times-1}{-1\times-1}\right)^{5}=\left(\tfrac{-5}{1}\right)^{5}=(-5)^{5}$$

Q16

Answer :

(a) $\frac{4}{25}$

$$\left(\tfrac{-2}{5}\right)^{7}\div\left(\tfrac{-2}{5}\right)^{5}=\left(\tfrac{-2}{5}\right)^{7-5}\qquad\left[since\ a^{m}\div a^{n}=a^{m-n}\right]$$
$$=\left(\tfrac{-2}{5}\right)^{2}$$
$$=\tfrac{(-2)^{2}}{(5)^{2}}=\tfrac{4}{25}$$

Q17

Answer :

(c) $\frac{4}{9}$

$$\left(\tfrac{-2}{3}\right)^{2}=\tfrac{-2}{3}\times\tfrac{-2}{3}=\tfrac{4}{9}$$

Q18

Answer :

(b) $\frac{-1}{8}$

We have:

$$\left(\tfrac{-1}{2}\right)^{3}=\tfrac{-1}{2}\times\tfrac{-1}{2}\times\tfrac{-1}{2}=\tfrac{-1}{8}$$

Q19

Answer :

(c) $\frac{3}{4}$

$$\left(\frac{5}{3}\right)^{-5} \times \left(\frac{5}{3}\right)^{11} = \left(\frac{5}{3}\right)^{8x}$$
$$\Rightarrow \left(\frac{5}{3}\right)^{-5+11} = \left(\frac{5}{3}\right)^{8x} \quad [\text{ since } a^m \times a^n = a^{m+n}]$$
$$\Rightarrow \left(\frac{5}{3}\right)^{6} = \left(\frac{5}{3}\right)^{8x}$$

On equating the coefficients:

$6 = 8x$

$\therefore x = \frac{6}{8} = \frac{3}{4}$

Q20

Answer :

(c) $\frac{-4}{5}$

Let the required number be x.

$(-8)^{-1} \times x = (10)^{-1}$

$\Rightarrow \frac{1}{-8} \times x = \frac{1}{10}$

$\therefore x = \frac{1}{10} \times \left(-8\right) = \frac{-4}{5}$

Hence, the required number is $\frac{-4}{5}$.

Q21

Answer :

(c) 2.156×10^6

A given number is said to be in standard form if it can be expressed as $k \times 10^n$, where k is a real number such that $1 \le k < 10$ and n is a positive integer.

For example: 2.156×10^6