Exercise – 3D

1.

Sol:

The given system of equations is:

3x + 5y = 12
5x + 3y = 4
These equations are of the forms:
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 3$, $b_1= 5$, $c_1 = -12$ and $a_2 = 5$, $b_2 = 3$, $c_2 = -4$
For a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, i.e., $\frac{3}{5} \neq \frac{5}{3}$
Hence, the given system of equations has a unique solution.
Again, the given equations are:
3x + 5y = 12 …..(i)
5x + 3y = 4 …..(ii)
On multiplying (i) by 3 and (ii) by 5, we get:
9x + 15y = 36 …….(iii)
25x + 15y = 20 ……(iv)
On subtracting (iii) from (iv), we get:
16x = -16
⇒x = -1
On substituting x = -1 in (i), we get:
3(-1) + 5y = 12
⇒5y = (12 + 3) = 15
⇒y = 3
Hence, x = -1 and y = 3 is the required solution.

2.

Sol:
The given system of equations is:
2x - 3y - 17 = 0 …..(i)
4x + y - 13 = 0 …..(ii)
The given equations are of the form
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= -3$, $c_1 = -17$ and $a_2 = 4$, $b_2 = 1$, $c_2 = -13$
Now,

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$ and $\frac{b_1}{b_2} = \frac{-3}{1} = -3$
Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, therefore the system of equations has unique solution.
Using cross multiplication method, we have

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \frac{x}{-3(-13)-1\times(-17)} = \frac{y}{-17\times4-(-13)\times2} = \frac{1}{2\times1-4\times(-3)}$$

$$\Rightarrow \frac{x}{39+17} = \frac{y}{-68+26} = \frac{1}{2+12}$$

$$\Rightarrow \frac{x}{56} = \frac{y}{-42} = \frac{1}{14}$$

$\Rightarrow$ x = $\frac{56}{14}$, y = $\frac{-42}{14}$

$\Rightarrow$ x = 4, y = -3

Hence, x = 4 and y = -3.

3.

Also, find the solution of the given system of equations.

Sol:

The given system of equations is:

$\frac{x}{3} + \frac{y}{2} = 3$

$\Rightarrow \frac{2x+3y}{6} = 3$

2x + 3y = 18

$\Rightarrow$2x + 3y – 18 = 0 ….(i)

and

x – 2y = 2

x – 2y – 2 = 0 …..(ii)

These equations are of the forms:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1= 3$, $c_1 = -18$ and $a_2 = 1$, $b_2 = -2$, $c_2 = -2$

For a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, i.e., $\frac{2}{1} \neq \frac{3}{-2}$

Hence, the given system of equations has a unique solution.

Again, the given equations are:

2x + 3y – 18 = 0 …..(iii)

x – 2y – 2 = 0 …..(iv)

On multiplying (i) by 2 and (ii) by 3, we get:

4x + 6y – 36 = 0 …….(v)

3x - 6y – 6 = 0 ……(vi)

On adding (v) from (vi), we get:

7x = 42

$\Rightarrow$x = 6

On substituting x = 6 in (iii), we get:

2(6) + 3y = 18

⇒3y = (18 - 12) = 6

⇒y = 2

Hence, x = 6 and y = 2 is the required solution.

4.

Sol:

The given system of equations are

2x + 3y – 5 = 0

kx - 6y - 8 = 0

This system is of the form:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1= 3$, $c_1 = -5$ and $a_2 = k$, $b_2 = -6$, $c_2 = -8$

Now, for the given system of equations to have a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{2}{k} \neq \frac{3}{-6}$

⇒ k ≠ -4

Hence, k ≠ -4

5.

Sol:

The given system of equations are

x - ky – 2 = 0

3x + 2y + 5 = 0

This system of equations is of the form:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = 1$, $b_1= -k$, $c_1 = -2$ and $a_2 = 3$, $b_2 = 2$, $c_2 = 5$

Now, for the given system of equations to have a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{1}{3} \neq \frac{-k}{2}$

$\Rightarrow k \neq -\frac{2}{3}$

Hence, $k \neq -\frac{2}{3}$.

6.

Sol:

The given system of equations are

$5x - 7y - 5 = 0$ ….(i)

$2x + ky - 1 = 0$ …(ii)

This system is of the form:

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

where, $a_1 = 5$, $b_1= -7$, $c_1 = -5$ and $a_2 = 2$, $b_2 = k$, $c_2 = -1$

Now, for the given system of equations to have a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{5}{2} \neq \frac{-7}{k}$

$\Rightarrow k \neq -\frac{14}{5}$

Hence, $k \neq -\frac{14}{5}$.

7.

Sol:

The given system of equations are

$4x + ky + 8 = 0$

$x + y + 1 = 0$

This system is of the form:

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

where, $a_1 = 4$, $b_1= k$, $c_1 = 8$ and $a_2 = 1$, $b_2 = 1$, $c_2 = 1$

For the given system of equations to have a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{4}{1} \neq \frac{k}{1}$

$\Rightarrow k \neq 4$

Hence, $k \neq 4$.

8.

Sol:

The given system of equations are

$4x - 5y = k$

$\Rightarrow 4x - 5y - k = 0$ ….(i)

And, $2x - 3y = 12$

$\Rightarrow 2x - 3y - 12 = 0$ …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

Here, $a_1 = 4$, $b_1= -5$, $c_1 = -k$ and $a_2 = 2$, $b_2 = -3$, $c_2 = -12$

For a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e., $\frac{4}{2} \neq \frac{-5}{-3}$

$\Rightarrow 2 \neq \frac{5}{3} \Rightarrow 6 \neq 5$

Thus, for all real values of k, the given system of equations will have a unique solution.

9.

Sol:

The given system of equations:

$kx + 3y = (k – 3)$

$\Rightarrow kx + 3y – (k - 3) = 0$ ….(i)

And, $12x + ky = k$

$\Rightarrow 12x + ky - k = 0$ …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

Here, $a_1 = k$, $b_1= 3$, $c_1 = -(k – 3)$ and $a_2 = 12$, $b_2 = k$, $c_2 = -k$

For a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e., $\frac{k}{12} \neq \frac{3}{k}$

$\Rightarrow k^2 \neq 36 \Rightarrow k \neq \pm 6$

Thus, for all real values of k, other than ± 6, the given system of equations will have a unique solution.

10.

Sol:

The given system of equations: 2x - 3y = 5

$\Rightarrow$ 2x - 3y – 5 = 0 ….(i)

6x - 9y = 15

$\Rightarrow$6x - 9y - 15 = 0 …(ii)

These equations are of the following forms:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

Here, $a_1 = 2$, $b_1= -3$, $c_1 = -5$ and $a_2 = 6$, $b_2 = -9$, $c_2 = -15$

$\therefore \frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{-5}{-15} = \frac{1}{3}$

Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given system of equations has an infinite number of solutions.

11.

Sol:

The given system of equations can be written as

6x + 5y – 11 = 0 ….(i)

$\Rightarrow 9x + \frac{15}{2}y - 21 = 0$ …(ii)

This system is of the form

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

Here, $a_1 = 6$, $b_1= 5$, $c_1 = -11$ and $a_2 = 9$, $b_2 = \frac{15}{2}$, $c_2 = -21$

Now,

$\frac{a_1}{a_2} = \frac{6}{9} = \frac{2}{3}$

$\frac{b_1}{b_2} = \frac{5}{\frac{15}{2}} = \frac{2}{3}$

$\frac{c_1}{c_2} = \frac{-11}{-21} = \frac{11}{21}$

Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, therefore the given system has no solution.

12.

Sol:

The given system of equations:

kx + 2y = 5

$\Rightarrow$ kx + 2y - 5 = 0 ….(i)

3x - 4y = 10

$\Rightarrow$3x - 4y - 10 = 0 …(ii)

These equations are of the forms:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1= 2$, $c_1 = -5$ and $a_2 = 3$, $b_2 = -4$, $c_2 = -10$

(i) For a unique solution, we must have:

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., $\frac{k}{3} \neq \frac{2}{-4} \Rightarrow k \neq \frac{-3}{2}$

Thus for all real values of k other than $\frac{-3}{2}$, the given system of equations will have a unique solution.

(ii) For the given system of equations to have no solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{3} = \frac{2}{-4} \neq \frac{-5}{-10}$

$\Rightarrow \frac{k}{3} = \frac{2}{-4}$ and $\frac{k}{3} \neq \frac{1}{2}$

$\Rightarrow k = \frac{-3}{2}, k \neq \frac{3}{2}$

Hence, the required value of k is $\frac{-3}{2}$.

13.

Sol:

The given system of equations:

x + 2y = 5

$\Rightarrow$ x + 2y - 5 = 0 ….(i)

3x + ky + 15 = 0 …(ii)

These equations are of the forms:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = 1$, $b_1= 2$, $c_1 = -5$ and $a_2 = 3$, $b_2 = k$, $c_2 = 15$

(i) For a unique solution, we must have:

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., $\frac{1}{3} \neq \frac{2}{k} \Rightarrow k \neq 6$

Thus for all real values of k other than 6, the given system of equations will have a unique solution.

(ii) For the given system of equations to have no solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{3} = \frac{2}{k} \neq \frac{-5}{15}$

$\Rightarrow \frac{1}{3} = \frac{2}{k}$ and $\frac{2}{k} \neq \frac{-5}{15}$

$\Rightarrow$k = 6, k $\neq$ -6

Hence, the required value of k is 6.

14.

Sol:

The given system of equations:

x + 2y = 3

$\Rightarrow$ x + 2y - 3 = 0 ….(i)

And, 5x + ky + 7 = 0 …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 1$, $b_1= 2$, $c_1 = -3$ and $a_2 = 5$, $b_2 = k$, $c_2 = 7$

(i) For a unique solution, we must have:

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., $\frac{1}{5} \neq \frac{2}{k} \Rightarrow$ k $\neq$ 10

Thus for all real values of k other than 10, the given system of equations will have a unique solution.

(ii) In order that the given system of equations has no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{5} \neq \frac{2}{k} \neq \frac{-3}{7}$

$\Rightarrow \frac{1}{5} \neq \frac{2}{k}$ and $\frac{2}{k} \neq \frac{-3}{7}$

$\Rightarrow$k = 10, k $\neq \frac{14}{-3}$

Hence, the required value of k is 10.

There is no value of k for which the given system of equations has an infinite number of solutions.

15.

Sol:

The given system of equations:

2x + 3y = 7,

$\Rightarrow$ 2x + 3y - 7 = 0 ….(i)

And, (k – 1)x + (k + 2)y = 3k

$\Rightarrow$(k – 1)x + (k + 2)y - 3k = 0 …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = (k – 1)$, $b_2 = (k + 2)$, $c_2 = -3k$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{2}{(k-1)} = \frac{3}{(k+2)} = \frac{-7}{-3k}$

$\Rightarrow \frac{2}{(k-1)} = \frac{3}{(k+2)} = \frac{7}{3k}$

Now, we have the following three cases:

Case I:

$\frac{2}{(k-1)} = \frac{3}{k+2}$

$\Rightarrow$ 2(k + 2) = 3(k – 1) $\Rightarrow$ 2k + 4 = 3k – 3 $\Rightarrow$ k = 7

Case II:

$\frac{3}{(k+2)} = \frac{7}{3k}$

$\Rightarrow$ 7(k + 2) = 9k $\Rightarrow$ 7k + 14 = 9k $\Rightarrow$ 2k = 14 $\Rightarrow$ k = 7

Case III:

$\frac{2}{(k-1)} = \frac{7}{3k}$

$\Rightarrow$ 7k – 7 = 6k $\Rightarrow$ k = 7

Hence, the given system of equations has an infinite number of solutions when k is equal to 7.

16.

Sol:

The given system of equations:

$2x + (k – 2)y = k$

$\Rightarrow 2x + (k – 2)y - k = 0$ ….(i)

And, $6x + (2k - 1) y = (2k + 5)$

$\Rightarrow 6x + (2k - 1) y - (2k + 5) = 0$ …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0, a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1 = (k – 2)$, $c_1 = -k$ and $a_2 = 6$, $b_2 = (2k - 1)$, $c_2 = -(2k + 5)$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{2}{6} = \frac{(k-2)}{(2k-1)} = \frac{-k}{-(2k+5)}$

$\Rightarrow \frac{1}{3} = \frac{(k-2)}{(2k-1)} = \frac{k}{(2k+5)}$

Now, we have the following three cases:

Case I:

$\frac{1}{3} = \frac{(k-2)}{(2k-1)}$

$\Rightarrow (2k – 1) = 3(k – 2)$

$\Rightarrow 2k - 1 = 3k – 6 \Rightarrow k = 5$

Case II:

$\frac{(k-2)}{(2k-1)} = \frac{k}{(2k+5)}$

$\Rightarrow (k - 2) (2k + 5) = k(2k - 1)$

$\Rightarrow 2k^2 + 5k – 4k – 10 = 2k^2 – k$

$\Rightarrow k + k = 10 \Rightarrow 2k = 10 \Rightarrow k = 5$

Case III:

$\frac{1}{3} = \frac{k}{(2k+5)}$

$\Rightarrow 2k + 5 = 3k \Rightarrow k = 5$

Hence, the given system of equations has an infinite number of solutions when k is equal to 5.

17.

Sol:

The given system of equations:

$kx + 3y = (2k + 1)$

$\Rightarrow kx + 3y - (2k + 1) = 0$ ….(i)

And, $2(k + 1)x + 9y = (7k + 1)$

$\Rightarrow$ $2(k + 1)x + 9y - (7k + 1) = 0$ …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1= 3$, $c_1 = -(2k + 1)$ and $a_2 = 2(k + 1)$, $b_2 = 9$, $c_2 = -(7k + 1)$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i.e., $\frac{k}{2(k+1)} = \frac{3}{9} = \frac{-(2k+1)}{-(7k+1)}$

$\Rightarrow \frac{k}{2(k+1)} = \frac{1}{3} = \frac{(2k+1)}{(7k+1)}$

Now, we have the following three cases:

Case I:

$\frac{k}{2(k+1)} = \frac{1}{3}$

$\Rightarrow 2(k + 1) = 3k$

$\Rightarrow 2k + 2 = 3k$

$\Rightarrow k = 2$

Case II:

$\frac{1}{3} = \frac{(2k+1)}{(7k+1)}$

$\Rightarrow (7k + 1) = 6k + 3$

$\Rightarrow k = 2$

Case III:

$\frac{k}{2(k+1)} = \frac{(2k+1)}{(7k+1)}$

$\Rightarrow k(7k + 1) = (2k + 1) \times 2(k + 1)$

$\Rightarrow 7k^2 + k = (2k + 1)(2k + 2)$

$\Rightarrow 7k^2 + k = 4k^2 + 4k + 2k + 2$

$\Rightarrow 3k^2 – 5k – 2 = 0$

$\Rightarrow 3k^2 – 6k + k – 2 = 0$

$\Rightarrow 3k(k – 2) + 1(k – 2) = 0$

$\Rightarrow (3k + 1)(k – 2) = 0$

$\Rightarrow k = 2$ or $k = \frac{-1}{3}$

Hence, the given system of equations has an infinite number of solutions when k is equal to 2.

18.

Sol:

The given system of equations:

5x + 2y = 2k

⇒ 5x + 2y - 2k = 0 ….(i)

And, 2(k + 1)x + ky = (3k + 4)

⇒ 2(k + 1)x + ky - (3k + 4) = 0 …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 5$, $b_1= 2$, $c_1 = -2k$ and $a_2 = 2(k + 1)$, $b_2 = k$, $c_2 = -(3k + 4)$

For an infinite number of solutions, we must have:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{5}{2(k+1)} = \frac{2}{k} = \frac{-2k}{-(3k+4)}$$

$$\Rightarrow \frac{5}{2(k+1)} = \frac{2}{k} = \frac{2k}{(3k+4)}$$

Now, we have the following three cases:

Case I:

$$\frac{5}{2(k+1)} = \frac{2}{k}$$

⇒ 2 × 2(k + 1) = 5k

⇒ 4(k + 1) = 5k

⇒ 4k + 4 = 5k

⇒ k = 4

Case II:

$$\frac{2}{k} = \frac{2k}{(3k+4)}$$

⇒ $2k^2 = 2 \times (3k + 4)$

⇒ $2k^2 = 6k + 8 \Rightarrow 2k^2 – 6k – 8 = 0$

⇒ $2(k^2 – 3k – 4) = 0$

⇒ $k^2 – 4k + k – 4 = 0$

⇒ k(k – 4) + 1(k – 4) = 0

⇒ (k + 1) (k – 4) = 0

⇒ (k + 1) = 0 or (k – 4) = 0

$\Rightarrow$ k = -1 or k = 4

Case III:

$\frac{5}{2(k+1)} = \frac{2k}{(3k+4)}$

$\Rightarrow$ 15k + 20 = $4k^2$ + 4k

$\Rightarrow$ $4k^2 - 11k - 20 = 0$

$\Rightarrow$ $4k^2 - 16k + 5k - 20 = 0$

$\Rightarrow$ 4k(k – 4) + 5(k – 4) = 0

$\Rightarrow$ (k – 4) (4k + 5) = 0

$\Rightarrow$ k = 4 or k = $\frac{-5}{4}$

Hence, the given system of equations has an infinite number of solutions when k is equal to 4.

19.

Sol:

The given system of equations:

(k – 1)x – y = 5

$\Rightarrow$ (k – 1)x – y – 5 = 0 ….(i)

And, (k + 1)x + (1 – k)y = (3k + 1)

$\Rightarrow$ (k + 1)x + (1 – k)y – (3k + 1) = 0 …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = (k – 1)$, $b_1= -1$, $c_1 = -5$ and $a_2 = (k + 1)$, $b_2 = (1 – k)$, $c_2 = -(3k + 1)$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i.e., $\frac{(k-1)}{(k+1)} = \frac{-1}{-(k-1)} = \frac{-5}{-(3k+1)}$

$\Rightarrow \frac{(k-1)}{(k+1)} = \frac{1}{(k-1)} = \frac{5}{(3k+1)}$

Now, we have the following three cases:

Case I:

$\frac{(k-1)}{(k+1)} = \frac{1}{(k-1)}$

$\Rightarrow$ $(k - 1)^2 = (k + 1)$

$\Rightarrow$ $k^2 + 1 - 2k = k + 1$

$\Rightarrow k^2 - 3k = 0 \Rightarrow k(k-3) = 0$

$\Rightarrow k = 0$ or $k = 3$

Case II:

$\frac{1}{(k-1)} = \frac{5}{(3k+1)}$

$\Rightarrow 3k + 1 = 5k - 5$

$\Rightarrow 2k = 6 \Rightarrow k = 3$

Case III:

$\frac{(k-1)}{(k+1)} = \frac{5}{(3k+1)}$

$\Rightarrow (3k + 1)(k - 1) = 5(k + 1)$

$\Rightarrow 3k^2 + k - 3k - 1 = 5k + 5$

$\Rightarrow 3k^2 - 2k - 5k - 1 - 5 = 0$

$\Rightarrow 3k^2 - 7k - 6 = 0$

$\Rightarrow 3k^2 - 9k + 2k - 6 = 0$

$\Rightarrow 3k(k - 3) + 2(k - 3) = 0$

$\Rightarrow (k - 3)(3k + 2) = 0$

$\Rightarrow (k - 3) = 0$ or $(3k + 2) = 0$

$\Rightarrow k = 3$ or $k = \frac{-2}{3}$

Hence, the given system of equations has an infinite number of solutions when k is equal to 3.

20.

Sol:

The given system of equations can be written as

$(k - 3)x + 3y - k = 0$

$kx + ky - 12 = 0$

This system is of the form:

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1= 3$, $c_1 = -k$ and $a_2 = k$, $b_2 = k$, $c_2 = -12$

For the given system of equations to have a unique solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{k-3}{k} = \frac{3}{k} = \frac{-k}{-12}$

$\Rightarrow k - 3 = 3$ and $k^2 = 36$

$\Rightarrow$ k = 6 and k = $\pm$6
$\Rightarrow$ k = 6
Hence, k = 6.

21.

Sol:
The given system of equations can be written as
(a – 1) x + 3y = 2
$\Rightarrow$(a – 1) x + 3y – 2 = 0 ….(i)
and 6x + (1 – 2b)y = 6
$\Rightarrow$6x + (1 – 2b)y – 6 = 0 ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = (a – 1)$, $b_1= 3$, $c_1 = -2$ and $a_2 = 6$, $b_2 = (1 – 2b)$, $c_2 = -6$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\Rightarrow \frac{a-1}{6} = \frac{3}{(1-2b)} = \frac{-2}{-6}$
$\Rightarrow \frac{a-1}{6} = \frac{3}{(1-2b)} = \frac{1}{3}$
$\Rightarrow \frac{a-1}{6} = \frac{1}{3}$ and $\frac{3}{(1-2b)} = \frac{1}{3}$
$\Rightarrow$ 3a – 3 = 6 and 9 = 1 – 2b
$\Rightarrow$ 3a = 9 and 2b = -8
$\Rightarrow$ a = 3 and b = -4
$\therefore$ a = 3 and b = -4

22.

Sol:
The given system of equations can be written as
(2a – 1) x + 3y = 5
$\Rightarrow$(2a – 1) x + 3y – 5 = 0 ….(i)
and 3x + (b – 1)y = 2
$\Rightarrow$3x + (b – 1)y – 2 = 0 ….(ii)
These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = (2a – 1)$, $b_1= 3$, $c_1 = -5$ and $a_2 = 3$, $b_2 = (b – 1)$, $c_2 = -2$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\Rightarrow \frac{(2a-1)}{3} = \frac{3}{(b-1)} = \frac{-5}{-2}$
$\Rightarrow \frac{(2a-1)}{6} = \frac{3}{(b-1)} = \frac{5}{2}$
$\Rightarrow \frac{(2a-1)}{6} = \frac{5}{2}$ and $\frac{3}{(b-1)} = \frac{5}{2}$
$\Rightarrow 2(2a – 1) = 15$ and $6 = 5(b – 1)$
$\Rightarrow 4a – 2 = 15$ and $6 = 5b – 5$
$\Rightarrow 4a = 17$ and $5b = 11$
$\therefore a = \frac{17}{4}$ and $b = \frac{11}{5}$

23.

Sol:
The given system of equations can be written as
$2x - 3y = 7$
$\Rightarrow 2x - 3y - 7 = 0$ ….(i)
and $(a + b)x - (a + b – 3)y = 4a + b$
$\Rightarrow (a + b)x - (a + b – 3)y - 4a + b = 0$ ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
Here, $a_1 = 2$, $b_1= -3$, $c_1 = -7$ and $a_2 = (a + b)$, $b_2 = -(a + b - 3)$, $c_2 = -(4a + b)$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\frac{2}{a+b} = \frac{-3}{-(a+b-3)} = \frac{-7}{-(4a+b)}$
$\Rightarrow \frac{2}{a+b} = \frac{3}{(a+b-3)} = \frac{7}{(4a+b)}$
$\Rightarrow \frac{2}{a+b} = \frac{7}{(4a+b)}$ and $\frac{3}{(a+b-3)} = \frac{7}{(4a+b)}$
$\Rightarrow 2(4a + b) = 7(a + b)$ and $3(4a + b) = 7(a + b - 3)$
$\Rightarrow 8a + 2b = 7a + 7b$ and $12a + 3b = 7a + 7b - 21$
$\Rightarrow 4a = 17$ and $5b = 11$
$\therefore a = 5b$ …….(iii)
and $5a = 4b – 21$ ……(iv)
On substituting $a = 5b$ in (iv), we get:

25b = 4b – 21

⇒21b = -21

⇒b = -1

On substituting b = -1 in (iii), we get:

a = 5(-1) = -5

∴a = -5 and b = -1.

24.

Sol:

The given system of equations can be written as

2x + 3y = 7

⇒2x + 3y - 7 = 0 ….(i)

and (a + b + 1)x - (a + 2b + 2)y = 4(a + b) + 1

(a + b + 1)x - (a + 2b + 2)y – [4(a + b) + 1] = 0 ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = (a + b + 1)$, $b_2 = (a + 2b + 2)$, $c_2 = –[4(a + b) + 1]$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{2}{(a+b+1)} = \frac{3}{(a+2b+2)} = \frac{-7}{-[4(a+b)+1]}$

$\Rightarrow \frac{2}{(a+b+1)} = \frac{3}{(a+2b+2)} = \frac{7}{[4(a+b)+1]}$

$\Rightarrow \frac{2}{(a+b+1)} = \frac{3}{(a+2b+2)}$ and $\frac{3}{(a+2b+2)} = \frac{7}{[4(a+b)+1]}$

⇒ 2(a + 2b + 2) = 3(a + b + 1) and 3[4(a + b) + 1] = 7(a + 2b + 2)

⇒ 2a + 4b + 4 = 3a + 3b + 3 and 3(4a + 4b + 1) = 7a + 14b + 14

⇒ a – b - 1=0 and 12a + 12b + 3 = 7a + 14b + 14

⇒ a - b = 1 and 5a – 2b = 11

a = (b + 1) …….(iii)

5a - 2b = 11 ……(iv)

On substituting a = (b + 1) in (iv), we get:

5(b + 1) – 2b = 11

⇒5b + 5 – 2b = 11

⇒ 3b = 6

⇒ b = 2

On substituting b = 2 in (iii), we get:

a = 3

∴a = 3 and b = 2.

25.

Sol:

The given system of equations can be written as

2x + 3y - 7 = 0 ….(i)

(a + b)x + (2a - b)y – 21 = 0 ….(ii)

This system is of the form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = a + b$, $b_2 = 2a - b$, $c_2 = -21$

For the given system of linear equations to have an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{a+b} = \frac{3}{2a-b} = \frac{-7}{-21}$

$\Rightarrow \frac{2}{a+b} = \frac{-7}{-21} = \frac{1}{3}$ and $\frac{3}{2a-b} = \frac{-7}{-21} = \frac{1}{3}$

⇒ a + b = 6 and 2a –b = 9

Adding a + b = 6 and 2a – b = 9 ,we get

$3a = 15 \Rightarrow a = \frac{15}{3} = 3$

Now substituting a = 5 in a + b = 6, we have

5 + b = 6 ⇒ b = 6 – 5 = 1

Hence, a = 5 and b = 1.

26.

Sol:

The given system of equations can be written as

2x + 3y - 7 = 0 ….(i)

2ax + (a + b)y – 28 = 0 ….(ii)

This system is of the form:

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = 2a$, $b_2 = a + b$, $c_2 = -28$

For the given system of linear equations to have an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{-28}$

$\Rightarrow \frac{2}{2a} = \frac{-7}{-28} = \frac{1}{4}$ and $\frac{3}{a+b} = \frac{-7}{-28} = \frac{1}{4}$

$\Rightarrow$ a = 4 and a + b = 12

Substituting a = 4 in a + b = 12, we get

4 + b = 12 $\Rightarrow$ b = 12 – 4 = 8

Hence, a = 4 and b = 8.

27.

Sol:

The given system of equations:

8x + 5y = 9

8x + 5y - 9 = 0 ….(i)

kx + 10y = 15

kx + 10y - 15 = 0 ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 8$, $b_1= 5$, $c_1 = -9$ and $a_2 = k$, $b_2 = 10$, $c_2 = -15$

In order that the given system has no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e., $\frac{8}{k} = \frac{5}{10} \neq \frac{-9}{-15}$

i.e., $\frac{8}{k} = \frac{1}{2} \neq \frac{3}{5}$

$\frac{8}{k} = \frac{1}{2}$ and $\frac{8}{k} \neq \frac{3}{5}$

$\Rightarrow$ k = 16 and k $\neq \frac{40}{3}$

Hence, the given system of equations has no solutions when k is equal to 16.

28.

Sol:

The given system of equations:

kx + 3y = 3

kx + 3y - 3 = 0 ….(i)

12x + ky = 6

12x + ky - 6 = 0 ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = k$, $b_1= 3$, $c_1 = -3$ and $a_2 = 12$, $b_2 = k$, $c_2 = -6$
In order that the given system has no solution, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
i.e., $\frac{k}{12} = \frac{3}{k} \neq \frac{-3}{-6}$
$\frac{k}{12} = \frac{3}{k}$ and $\frac{3}{k} \neq \frac{1}{2}$
$\Rightarrow k^2 = 36$ and $k \neq 6$
$\Rightarrow k = \pm 6$ and $k \neq 6$
Hence, the given system of equations has no solution when k is equal to -6.

29.

Sol:
The given system of equations:
$3x - y - 5 = 0$ ….(i)
And, $6x - 2y + k = 0$ ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 3$, $b_1= -1$, $c_1 = -5$ and $a_2 = 6$, $b_2 = -2$, $c_2 = k$
In order that the given system has no solution, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
i.e., $\frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{k}$
$\Rightarrow \frac{-1}{-2} \neq \frac{-5}{k} \Rightarrow k \neq -10$
Hence, equations (i) and (ii) will have no solution if $k \neq -10$.

30.

Sol:
The given system of equations can be written as
$kx + 3y + 3 - k = 0$ ….(i)
$12x + ky - k = 0$ ….(ii)
This system of the form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = k$, $b_1= 3$, $c_1 = 3 - k$ and $a_2 = 12$, $b_2 = k$, $c_2 = -k$

For the given system of linear equations to have no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{12} = \frac{3}{k} \neq \frac{3-k}{-k}$

$\Rightarrow \frac{k}{12} = \frac{3}{k}$ and $\frac{3}{k} \neq \frac{3-k}{-k}$

$\Rightarrow k^2 = 36$ and $-3 \neq 3 - k$

$\Rightarrow k = \pm 6$ and $k \neq 6$

$\Rightarrow k = -6$

Hence, k = -6.

31.

Sol:

The given system of equations:

5x - 3y = 0 ….(i)

2x + ky = 0 ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 5$, $b_1 = -3$, $c_1 = 0$ and $a_2 = 2$, $b_2 = k$, $c_2 = 0$

For a non-zero solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\Rightarrow \frac{5}{2} = \frac{-3}{k}$

$\Rightarrow 5k = -6 \Rightarrow k = \frac{-6}{5}$

Hence, the required value of k is $\frac{-6}{5}$.

Linear equations in two variables – 3E

32.

Sol:

Let the cost of a chair be ₹ x and that of a table be ₹ y, then

5x + 4y = 5600 ……..(i)

4x + 3y = 4340 ……..(ii)

Multiplying (i) by 3 and (ii) by 4, we get

15x – 16x = 16800 – 17360

$\Rightarrow$ -x = -560

$\Rightarrow$ x = 560

Substituting x = 560 in (i), we have

$5 \times 560 + 4y = 5600$

$\Rightarrow 4y = 5600 – 2800$

$\Rightarrow y = \frac{2800}{4} = 700$

Hence, the cost of a chair and that a table are respectively ₹ 560 and ₹ 700.

33.

Sol:

Let the cost of a spoon be Rs.x and that of a fork be Rs.y. Then

$23x + 17y = 1770$ ……….(i)

$17x + 23y = 1830$ ………(ii)

Adding (i) and (ii), we get

$40x + 40y = 3600$

$\Rightarrow x + y = 90$ ………..(iii)

Now, subtracting (ii) from (i), we get

$6x – 6y = -60$

$\Rightarrow x – y = -10$ ………..(iv)

Adding (iii) and (iv), we get

$2x = 80 \Rightarrow x = 40$

Substituting x = 40 in (iii), we get

$40 + y = 90 \Rightarrow y = 50$

Hence, the cost of a spoon that of a fork is Rs.40 and Rs.50 respectively.

34.

Sol:

Let x and y be the number of 50-paisa and 25-paisa coins respectively. Then

$x + y = 50$ ……….(i)

$0.5x + 0.25y = 19.50$ ………(ii)

Multiplying (ii) by 2 and subtracting it from (i), we get

$0.5y = 50 - 39$

$\Rightarrow y = \frac{11}{0.5} = 22$

Subtracting y = 22 in (i), we get

$x + 22 = 50$

$\Rightarrow x = 50 – 22 = 28$

Hence, the number of 25-paisa and 50-paisa coins is 22 and 28 respectively.

35.

Sol:

Let the larger number be x and the smaller number be y.

Then, we have:

$x + y = 137$ ……….(i)

$x - y = 43$ ………(ii)

On adding (i) and (ii), we get

$2x = 180 \Rightarrow x = 90$

On substituting x = 90 in (i), we get

$90 + y = 137$

$\Rightarrow y = (137 - 90) = 47$

Hence, the required numbers are 90 and 47.

36.

Sol:

Let the first number be x and the second number be y.

Then, we have:

$2x + 3y = 92$ ……….(i)

$4x - 7y = 2$ ………(ii)

On multiplying (i) by 7 and (ii) by 3, we get

$14x + 21y = 644$ ………..(iii)

$12x - 21y = 6$ ………..(iv)

On adding (iii) and (iv), we get

$26x = 650$

$\Rightarrow x = 25$

On substituting x = 25 in (i), we get

$2 \times 25 + 3y = 92$

$\Rightarrow 50 + 3y = 92$

$\Rightarrow 3y = (92 - 50) = 42$

$\Rightarrow y = 14$

Hence, the first number is 25 and the second number is 14.

37.

Sol:

Let the first number be x and the second number be y.

Then, we have:

$3x + y = 142$ ……….(i)

$4x - y = 138$ ………(ii)

On adding (i) and (ii), we get

$7x = 280$

$\Rightarrow x = 40$

On substituting x = 40 in (i), we get:

$3 \times 40 + y = 142$

$\Rightarrow y = (142 - 120) = 22$

$\Rightarrow y = 22$

Hence, the first number is 40 and the second number is 22.

38.

Sol:

Let the greater number be x and the smaller number be y.

Then, we have:

$25x - 45 = y$ or $2x - y = 45$ ……….(i)

$2y - 21 = x$ or $-x + 2y = 21$ ………(ii)

On multiplying (i) by 2, we get:

$4x - 2y = 90$ ………..(iii)

On adding (ii) and (iii), we get

$3x = (90 + 21) = 111$

$\Rightarrow x = 37$

On substituting x = 37 in (i), we get

$2 \times 37 - y = 45$

$\Rightarrow 74 - y = 45$

$\Rightarrow y = (74 - 45) = 29$

Hence, the greater number is 37 and the smaller number is 29.

39.

Sol:

We know:

Dividend = Divisor $\times$ Quotient + Remainder

Let the larger number be x and the smaller be y.

Then, we have:

$3x = y \times 4 + 8$ or $3x - 4y = 8$ ……….(i)

$5y = x \times 3 + 5$ or $-3x + 5y = 5$ ………(ii)

On adding (i) and (ii), we get:

$y = (8 + 5) = 13$

On substituting y = 13 in (i) we get

$3x - 4 \times 13 = 8$

$\Rightarrow 3x = (8 + 52) = 60$

$\Rightarrow x = 20$

Hence, the larger number is 20 and the smaller number is 13.

40.

Sol:

Let the required numbers be x and y.

Now, we have:

$\frac{x+2}{y+2} = \frac{1}{2}$

By cross multiplication, we get:

$2x + 4 = y + 2$

$\Rightarrow 2x - y = -2$ ……(i)

Again, we have:

$\frac{x-4}{y-4} = \frac{5}{11}$

By cross multiplication, we get:

$11x - 44 = 5y - 20$

$\Rightarrow 11x - 5y = 24$ ……(ii)

On multiplying (i) by 5, we get:

$10x - 5y = -10$

On subtracting (iii) from (ii), we get:

$x = (24 + 10) = 34$

On substituting x = 34 in (i), we get:

$2 \times 34 - y = -2$

$\Rightarrow 68 - y = -2$

$\Rightarrow y = (68 + 2) = 70$

Hence, the required numbers are 34 and 70.

41.

Sol:

Let the larger number be x and the smaller number be y.

Then, we have:

$x - y = 14$ or $x = 14 + y$ ……….(i)

$x^2 - y^2 = 448$ ………(ii)

On substituting $x = 14 + y$ in (ii) we get

$(14 + y)^2 - y^2 = 448$

$\Rightarrow 196 + y^2 + 28y - y^2 = 448$

$\Rightarrow 196 + 28y = 448$

$\Rightarrow 28y = (448 - 196) = 252$

$\Rightarrow y = \frac{252}{28} = 9$

On substituting $y = 9$ in (i), we get:

$x = 14 + 9 = 23$

Hence, the required numbers are 23 and 9.

42.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Required number = $(10x + y)$

$x + y = 12$ ……….(i)

Number obtained on reversing its digits = $(10y + x)$

$\therefore (10y + x) - (10x + y) = 18$

$\Rightarrow 10y + x - 10x - y = 18$

$\Rightarrow 9y - 9x = 18$

$\Rightarrow y - x = 2$ ……..(ii)

On adding (i) and (ii), we get:

$2y = 14$

$\Rightarrow y = 7$

On substituting $y = 7$ in (i) we get

$x + 7 = 12$

$\Rightarrow x = (12 - 7) = 5$

Number = $(10x + y) = 10 \times 5 + 7 = 50 + 7 = 57$

Hence, the required number is 57.

43.

Sol:
Let the tens and the units digits of the required number be x and y, respectively.

Required number = $(10x + y)$

$10x + y = 7(x + y)$

$10x + 7y = 7x + 7y$ or $3x - 6y = 0$ ……….(i)

Number obtained on reversing its digits = $(10y + x)$

$(10x + y) - 27 = (10y + x)$

$\Rightarrow 10x - x + y - 10y = 27$

$\Rightarrow 9x - 9y = 27$

$\Rightarrow 9(x - y) = 27$

$\Rightarrow x - y = 3$ ……..(ii)

On multiplying (ii) by 6, we get:

$6x - 6y = 18$ ………(iii)

On subtracting (i) from (ii), we get:

$3x = 18$

$\Rightarrow x = 6$

On substituting $x = 6$ in (i) we get

$3 \times 6 - 6y = 0$

$\Rightarrow 18 - 6y = 0$

$\Rightarrow 6y = 18$

$\Rightarrow y = 3$

Number = $(10x + y) = 10 \times 6 + 3 = 60 + 3 = 63$

Hence, the required number is 63.

44.

Sol:
Let the tens and the units digits of the required number be x and y, respectively.

Required number = $(10x + y)$

$x + y = 15$ ……….(i)

Number obtained on reversing its digits = $(10y + x)$

$\therefore (10y + x) - (10x + y) = 9$

$\Rightarrow 10y + x - 10x - y = 9$

$\Rightarrow 9y - 9x = 9$

$\Rightarrow y - x = 1$ ……..(ii)

On adding (i) and (ii), we get:

$2y = 16$

$\Rightarrow y = 8$

On substituting y = 8 in (i) we get

$x + 8 = 15$

$\Rightarrow x = (15 - 8) = 7$

Number = $(10x + y) = 10 \times 7 + 8 = 70 + 8 = 78$

Hence, the required number is 78.

45.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Required number = $(10x + y)$

$10x + y = 4(x + y) + 3$

$\Rightarrow 10x + y = 4x + 4y + 3$

$\Rightarrow 6x - 3y = 3$

$\Rightarrow 2x - y = 1$ ……….(i)

Again, we have:

$10x + y + 18 = 10y + x$

$\Rightarrow 9x - 9y = -18$

$\Rightarrow x - y = -2$ ……..(ii)

On subtracting (ii) from (i), we get:

$x = 3$

On substituting x = 3 in (i) we get

$2 \times 3 - y = 1$

$\Rightarrow y = 6 - 1 = 5$

Required number = $(10x + y) = 10 \times 3 + 5 = 30 + 5 = 35$

Hence, the required number is 35.

46.

Sol:

We know:

Dividend = Divisor $\times$ Quotient + Remainder

Let the tens and the units digits of the required number be x and y, respectively.
Required number = (10x + y)

$10x + y = (x + y) \times 6 + 0$

$\Rightarrow 10x - 6x + y - 6y = 0$

$\Rightarrow 4x - 5y = 0$ …….(i)

Number obtained on reversing its digits = (10y + x)

$\therefore 10x + y - 9 = 10y + x$

$\Rightarrow 9x - 9y = 9$

$\Rightarrow x - y = 1$ ……..(ii)

On multiplying (ii) by 5, we get:

$5x - 5y = 5$ ……..(iii)

On subtracting (i) from (iii), we get:

x = 5

On substituting x = 5 in (i) we get

$4 \times 5 - 5y = 0$

$\Rightarrow 20 - 5y = 0$

$\Rightarrow y = 4$

$\therefore$ The number = $(10x + y) = 10 \times 5 + 4 = 50 + 4 = 54$

Hence, the required number is 54.

47.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Then, we have:

$xy = 35$ …….(i)

Required number = (10x + y)

Number obtained on reversing its digits = (10y + x)

$\therefore (10x + y) + 18 = 10y + x$

$\Rightarrow 9x - 9y = -18$

$\Rightarrow 9(y - x) = 18$

$\Rightarrow y - x = 2$ ……..(ii)

We know:
$(y + x)^2 - (y - x)^2 = 4xy$

$\Rightarrow (y + x) = \pm \sqrt{(y-x)^2 + 4xy}$

$\Rightarrow (y + x) = \pm \sqrt{4 + 4 \times 35} = \pm \sqrt{144} = \pm 12$

$\Rightarrow y + x = 12$ ……..(iii) ($\because$ x and y cannot be negative)
On adding (ii) and (iii), we get:
$2y = 2 + 12 = 14$

$\Rightarrow y = 7$
On substituting y = 7in (ii) we get
$7 - x = 2$

$\Rightarrow x = (7 - 2) = 5$

$\therefore$ The number = $(10x + y) = 10 \times 5 + 7 = 50 + 7 = 57$
Hence, the required number is 57.

48.

Sol:
Let the tens and the units digits of the required number be x and y, respectively.
Then, we have:
$xy = 18$ …….(i)
Required number = $(10x + y)$
Number obtained on reversing its digits = $(10y + x)$
$\therefore (10x + y) - 63 = 10y + x$

$\Rightarrow 9x - 9y = 63$

$\Rightarrow 9(x - y) = 63$

$\Rightarrow x - y = 7$ ……..(ii)
We know:
$(x + y)^2 - (x - y)^2 = 4xy$

$\Rightarrow (x + y) = \pm \sqrt{(x-y)^2 + 4xy}$

$\Rightarrow (x + y) = \pm \sqrt{49 + 4 \times 18}$

$= \pm \sqrt{49 + 72}$

$= \pm \sqrt{121} = \pm 11$

$\Rightarrow x + y = 11$ ……..(iii) ($\because$ x and y cannot be negative)
On adding (ii) and (iii), we get:

2x = 7 +11 = 18

⇒x = 9

On substituting x = 9in (ii) we get

9 – y = 7

⇒ y = (9 – 7) = 2

∴ Number = (10x + y) = 10 × 9 + 2 = 90 + 2 = 92

Hence, the required number is 92.

49.

Sol:

Let x be the ones digit and y be the tens digit. Then

Two digit number before reversing = 10y + x

Two digit number after reversing = 10x + y

As per the question

(10y + x) + (10x + y) = 121

⇒11x + 11y = 121

⇒x + y = 11 …….(i)

Since the digits differ by 3, so

x – y = 3 ……….(ii)

Adding (i) and (ii), we get

2x = 14 ⇒ x = 7

Putting x = 7 in (i), we get

7 + y = 11 ⇒ y = 4

Changing the role of x and y, x = 4 and y = 7

Hence, the two-digit number is 74 or 47.

50.

Sol:

Let the required fraction be $\frac{x}{y}$.

Then, we have:

x + y = 8 ……(i)

And, $\frac{x+3}{y+3} = \frac{3}{4}$

⇒4(x + 3) = 3(y + 3)

⇒4x + 12 = 3y + 9

$\Rightarrow$ 4x – 3y = -3 ……(ii)
On multiplying (i) by 3, we get:
3x + 3y = 24
On adding (ii) and (iii), we get:
7x = 21
$\Rightarrow$ x = 3
On substituting x = 3 in (i), we get:
3 + y = 8
$\Rightarrow$ y = (8 – 3) = 5
$\therefore$ x = 3 and y = 5
Hence, the required fraction is $\frac{3}{5}$.

51.

Sol:
Let the required fraction be $\frac{x}{y}$.
Then, we have:
$\frac{x+2}{y} = \frac{1}{2}$
$\Rightarrow$ 2(x + 2) = y
$\Rightarrow$ 2x + 4 = y
$\Rightarrow$2x - y = -4 ……(i)
Again, $\frac{x}{y-1} = \frac{1}{3}$
$\Rightarrow$3x = 1(y – 1)
$\Rightarrow$3x – y = -1 ……(ii)
On subtracting (i) from (ii), we get:
x = (-1 + 4) = 3
On substituting x = 3 in (i), we get:
2 $\times$ 3 – y = -4
$\Rightarrow$ 6 – y = -4
$\Rightarrow$ y = (6 + 4) = 10
$\therefore$ x = 3 and y = 10
Hence, the required fraction is $\frac{3}{10}$.

52.

Sol:

Let the required fraction be $\frac{x}{y}$.

Then, we have:

y = x + 11

$\Rightarrow$ y – x = 11 ……(i)

Again, $\frac{x+8}{y+8} = \frac{3}{4}$

$\Rightarrow$4(x + 8) = 3(y + 8)

$\Rightarrow$4x + 32 = 3y + 24

$\Rightarrow$ 4x – 3y = -8 ……(ii)

On multiplying (i) by 4, we get:

4y – 4x = 44

On adding (ii) and (iii), we get:

y = (-8 + 44) = 36

On substituting y = 36 in (i), we get:

36 – x = 11

$\Rightarrow$ x = (36 – 11) = 25

$\therefore$ x = 25 and y = 36

Hence, the required fraction is $\frac{25}{36}$.

53.

Sol:

Let the required fraction be $\frac{x}{y}$.

Then, we have:

$\frac{x-1}{y+2} = \frac{1}{2}$

$\Rightarrow$ 2(x – 1) = 1(y + 2)

$\Rightarrow$2x – 2 = y + 2

$\Rightarrow$2x – y = 4 ……(i)

Again, $\frac{x-7}{y-2} = \frac{1}{3}$

$\Rightarrow$3(x – 7) = 1(y – 2)

$\Rightarrow 3x - 21 = y - 2$

$\Rightarrow 3x - y = 19$ ……(ii)

On subtracting (i) from (ii), we get:

$x = (19 - 4) = 15$

On substituting x = 15 in (i), we get:

$2 \times 15 - y = 4$

$\Rightarrow 30 - y = 4$

$\Rightarrow y = 26$

$\therefore$ x = 15 and y = 26

Hence, the required fraction is $\frac{15}{26}$.

54.

Sol:

Let the required fraction be x/y

As per the question

$x + y = 4 + 2x$

$\Rightarrow y - x = 4$ ……(i)

After changing the numerator and denominator

New numerator = x + 3

New denominator = y + 3

Therefore

$\frac{x+3}{y+3} = \frac{2}{3}$

$\Rightarrow 3(x + 3) = 2(y + 3)$

$\Rightarrow 3x + 9 = 2y + 6$

$\Rightarrow 2y - 3x = 3$ ……(ii)

Multiplying (i) by 3 and subtracting (ii), we get:

$3y - 2y = 12 - 3$

$\Rightarrow y = 9$

Now, putting y = 9 in (i), we get:

$9 - x = 4 \Rightarrow x = 9 - 4 = 5$

Hence, the required fraction is $\frac{5}{9}$.

55.

Sol:

Let the larger number be x and the smaller number be y.

Then, we have:

$x + y = 16$ ……(i)

And, $\frac{1}{x} + \frac{1}{y} = \frac{1}{3}$ ……(ii)

$\Rightarrow 3(x + y) = xy$

$\Rightarrow 3 \times 16 = xy$ [Since from (i), we have: $x + y = 16$]

$\therefore xy = 48$ …….(iii)

We know:

$(x - y)^2 = (x + y)^2 - 4xy$

$(x - y)^2 = (16)^2 - 4 \times 48 = 256 - 192 = 64$

$\therefore (x - y) = \pm\sqrt{64} = \pm 8$

Since x is larger and y is smaller, we have:

$x - y = 8$ ………(iv)

On adding (i) and (iv), we get:

$2x = 24$

$\Rightarrow x = 12$

On substituting $x = 12$ in (i), we get:

$12 + y = 16 \Rightarrow y = (16 - 12) = 4$

Hence, the required numbers are 12 and 4.

56.

Sol:

Let the number of students in classroom A be x

Let the number of students in classroom B be y.

If 10 students are transferred from A to B, then we have:

$x - 10 = y + 10$

$\Rightarrow x - y = 20$ …..(i)

If 20 students are transferred from B to A, then we have:

$2(y - 20) = x + 20$

$\Rightarrow 2y - 40 = x + 20$

$\Rightarrow -x + 2y = 60$ …..(ii)

On adding (i) and (ii), we get:
y = (20 + 60) = 80
On substituting y = 80 in (i), we get:
x – 80 = 20

$\Rightarrow$x = (20 + 80) = 100
Hence, the number of students in classroom A is 100 and the number of students in classroom B is 80.

57.

Sol:
Let fixed charges be Rs.x and rate per km be Rs.y.
Then as per the question
x + 80y = 1330 ………(i)
x + 90y = 1490 ……..(ii)
Subtracting (i) from (ii), we get
$10y = 160 \Rightarrow y = \frac{160}{10} = 16$
Now, putting y = 16, we have
$x + 80 \times 16 = 1330$

$\Rightarrow$x = 1330 – 1280 = 50

Hence, the fixed charges be Rs.50 and the rate per km is Rs.16.

58.

Sol:
Let the fixed charges be Rs.x and the cost of food per day be Rs.y.
Then as per the question
x + 25y = 4500 ………(i)
x + 30y = 5200 ……..(ii)
Subtracting (i) from (ii), we get
$5y = 700 \Rightarrow y = \frac{700}{5} = 140$
Now, putting y = 140, we have
$x + 25 \times 140 = 4500$

$\Rightarrow$x = 4500 – 3500 = 1000

Hence, the fixed charges be Rs.1000 and the cost of the food per day is Rs.140.

59.

Sol:

Let the amounts invested at 10% and 8% be Rs.x and Rs.y respectively.

Then as per the question

$\frac{x \times 10 \times 1}{100} = \frac{y \times 8 \times 1}{100}$ = 1350

10x + 8y = 135000 …………..(i)

After the amounts interchanged but the rate being the same, we have

$\frac{x \times 8 \times 1}{100} = \frac{y \times 10 \times 1}{100}$ = 1350 – 45

8x + 10y = 130500 …………….(ii)

Adding (i) and (ii) and dividing by 9, we get

2x + 2y = 29500 ……………(iii)

Subtracting (ii) from (i), we get

2x – 2y = 4500

Now, adding (iii) and (iv), we have

4x = 34000

x = $\frac{34000}{4}$ = 8500

Putting x = 8500 in (iii), we get

$2 \times 8500 + 2y = 29500$

2y = 29500 – 17000 = 12500

y = $\frac{12500}{2}$ = 6250

Hence, the amounts invested are Rs. 8,500 at 10% and Rs. 6,250 at 8%.

60.

Sol:

Let the monthly income of A and B are Rs.x and Rs.y respectively.

Then as per the question

$\frac{x}{y} = \frac{5}{4}$

$\Rightarrow$ y = $\frac{4x}{5}$

Since each save Rs.9,000, so

Expenditure of A = Rs.(x – 9000)

Expenditure of B = Rs.(y – 9000)

The ratio of expenditures of A and B are in the ratio 7:5.

$\therefore \frac{x-9000}{y-9000} = \frac{7}{5}$

$\Rightarrow 7y - 63000 = 5x - 45000$

$\Rightarrow 7y - 5x = 18000$

From (i), substitute $y = \frac{4x}{5}$ in (ii) to get

$7 \times \frac{4x}{5} - 5x = 18000$

$\Rightarrow 28x - 25x = 90000$

$\Rightarrow 3x = 90000$

$\Rightarrow x = 30000$

Now, putting x = 30000, we get

$y = \frac{4 \times 30000}{5} = 4 \times 6000 = 24000$

Hence, the monthly incomes of A and B are Rs. 30,000 and Rs.24,000.

61.

Sol:

Let the cost price of the chair and table be Rs.x and Rs.y respectively.

Then as per the question

Selling price of chair + Selling price of table = 1520

$\frac{100+25}{100} \times x + \frac{100+10}{100} \times y = 1520$

$\Rightarrow \frac{125}{100} x + \frac{110}{100} y = 1520$

$\Rightarrow 25x + 22y - 30400 = 0$ ……….(i)

When the profit on chair and table are 10% and 25% respectively, then

$\frac{100+10}{100} \times x + \frac{100+25}{100} \times y = 1535$

$\Rightarrow \frac{110}{100} x + \frac{125}{100} y = 1535$

$\Rightarrow 22x + 25y - 30700 = 0$ ……….(ii)

Solving (i) and (ii) by cross multiplication, we get

$\frac{x}{(22)(-30700)-(25)(-30400)} = \frac{y}{(-30400)(22)-(-30700)(25)} = \frac{1}{(25)(25)-(22)(22)}$

$\Rightarrow \frac{x}{7600-6754} = \frac{y}{7675-6688} = \frac{100}{3 \times 47}$

$\Rightarrow \frac{x}{846} = \frac{y}{987} = \frac{100}{3 \times 47}$

$\Rightarrow x = \frac{100 \times 846}{3 \times 47}, y = \frac{100 \times 987}{3 \times 47}$

$\Rightarrow$ x = 600, y = 700

Hence, the cost of chair and table are Rs.600 and Rs.700 respectively.

62.

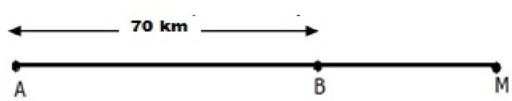
Sol:

Let X and Y be the cars starting from points A and B, respectively and let their speeds be x km/h and y km/h, respectively.

Then, we have the following cases:

Case I: When the two cars move in the same direction

In this case, let the two cars meet at point M.



Distance covered by car X in 7 hours = 7x km

Distance covered by car Y in 7 hours = 7y km

$\therefore$ AM = (7x) km and BM = (7y) km

$\Rightarrow$(AM – BM) = AB

$\Rightarrow$(7x – 7y) = 70

$\Rightarrow$7(x – y) = 70

$\Rightarrow$(x – y) = 10 ……..(i)

Case II: When the two cars move in opposite directions

In this case, let the two cars meet at point N.

Distance covered by car X in 1 hour = x km

Distance covered by car Y in 1 hour = y km

$\therefore$ AN = x km and BN = y km

$\Rightarrow$ AN + BN = AB

$\Rightarrow$ x + y = 70 ………(ii)

On adding (i) and (ii), we get:

2x = 80

$\Rightarrow$x = 40

On substituting x = 40 in (i), we get:

40 – y = 10

$\Rightarrow$y = (40 – 10) = 30

Hence, the speed of car X is 40km/h and the speed of car Y is 30km/h.

63.

Sol:
Let the original speed be x kmph and let the time taken to complete the journey be y hours.
∴ Length of the whole journey = (xy) km
Case I:
When the speed is (x + 5) kmph and the time taken is (y – 3) hrs:
Total journey = (x + 5) (y – 3) km

$\Rightarrow (x + 5)(y - 3) = xy$

$\Rightarrow xy + 5y - 3x - 15 = xy$

$\Rightarrow 5y - 3x = 15$ ………(i)

Case II:
When the speed is (x – 4) kmph and the time taken is (y + 3) hrs:
Total journey = (x – 4) (y + 3) km

$\Rightarrow (x - 4)(y + 3) = xy$

$\Rightarrow xy - 4y + 3x - 12 = xy$

$\Rightarrow 3x - 4y = 12$ ………(ii)

On adding (i) and (ii), we get:
y = 27
On substituting y = 27 in (i), we get:

$5 \times 27 - 3x = 15$

$\Rightarrow 135 - 3x = 15$

$\Rightarrow 3x = 120$

$\Rightarrow x = 40$

∴ Length of the journey = (xy) km = (40 × 27) km = 1080 km

64.

Sol:
Let the speed of the train and taxi be x km/h and y km/h respectively. Then as per the question

$\frac{3}{x} + \frac{2}{y} = \frac{11}{200}$ …………(i)

When the speeds of the train and taxi are 260 km and 240 km respectively, then

$\frac{260}{x} + \frac{240}{y} = \frac{11}{2} + \frac{6}{60}$

$\Rightarrow \frac{13}{x} + \frac{12}{y} = \frac{28}{100}$ …………..(ii)

Multiplying (i) by 6 and subtracting (ii) from it, we get

$\frac{18}{x} - \frac{13}{x} = \frac{66}{200} - \frac{28}{100}$

$\Rightarrow \frac{5}{x} = \frac{10}{200} \Rightarrow x = 100$

Putting x = 100 in (i), we have

$\frac{3}{100} + \frac{2}{y} = \frac{11}{200}$

$\Rightarrow \frac{2}{y} = \frac{11}{200} - \frac{3}{100} = \frac{1}{40}$

$\Rightarrow y = 80$

Hence, the speed of the train and that of the taxi are 100 km/h and 80 km/h respectively.

65.

Sol:

Let the speed of the car A and B be x km/h and y km/h respectively. Let x > y.

Case-1: When they travel in the same direction

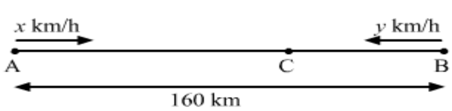
From the figure

AC – BC = 160

$\Rightarrow x \times 8 - y \times 8 = 160$

$\Rightarrow x - y = 20$

Case-2: When they travel in opposite direction



From the figure

AC + BC = 160

$\Rightarrow x \times 2 + y \times 2 = 160$

$\Rightarrow x + y = 80$

Adding (i) and (ii), we get

$2x = 100 \Rightarrow x = 50$ km/h

Putting x = 50 in (ii), we have

$50 + y = 80 \Rightarrow y = 80 - 50 = 30$ km/h

Hence, the speeds of the cars are 50 km/h and 30 km/h.

66.

Sol:

Let the speed of the sailor in still water be x km/h and that of the current y km/h.

Speed downstream = (x + y) km/h

Speed upstream = (x – y) km/h

As per the question

$(x + y) \times \frac{40}{60} = 8$

$\Rightarrow x + y = 12$ ………(i)

When the sailor goes upstream, then

$(x - y) \times 1 = 8$

$x - y = 8$ ………(ii)

Adding (i) and (ii), we get

$2x = 20 \Rightarrow x = 10$

Putting x = 10 in (i), we have

$10 + y = 12 \Rightarrow y = 2$

Hence, the speeds of the sailor in still water and the current are 10 km/h and 2 km/h respectively.

67.

Sol:

Let the speed of the boat in still water be x km/h and the speed of the stream be y km/h.

Then we have

Speed upstream = (x – y) km/hr

Speed downstream = (x + y) km/hr

Time taken to cover 12 km upstream = $\frac{12}{(x-y)}$ hrs

Time taken to cover 40 km downstream = $\frac{40}{(x+y)}$ hrs

Total time taken = 8 hrs

$\therefore \frac{12}{(x-y)} + \frac{40}{(x+y)} = 8$ ……….(i)

Again, we have:

Time taken to cover 16 km upstream = $\frac{16}{(x-y)}$ hrs

Time taken to cover 32 km downstream = $\frac{32}{(x+y)}$ hrs

Total time taken = 8 hrs

$\therefore \frac{16}{(x-y)} + \frac{32}{(x+y)} = 8$ ……….(ii)

Putting $\frac{1}{(x-y)}$ = u and $\frac{1}{(x+y)}$ = v in (i) and (ii), we get:

12u + 40v = 8

3u + 10v = 2 ………(a)

And, 16u + 32v = 8

⇒2u + 4v = 1 ……….(b)

On multiplying (a) by 4 and (b) by 10, we get:

12u + 40v = 8 …….(iii)

And, 20u + 40v = 10 …….(iv)

On subtracting (iii) from (iv), we get:

8u = 2

⇒ u = $\frac{2}{8} = \frac{1}{4}$

On substituting u = $\frac{1}{4}$ in (iii), we get:

40v = 5

⇒ v = $\frac{5}{40} = \frac{1}{8}$

Now, we have:

u = $\frac{1}{4}$

⇒ $\frac{1}{(x-y)} = \frac{1}{4}$ ⇒ x – y = 4 ……..(v)

v = $\frac{1}{8}$

⇒ $\frac{1}{(x+y)} = \frac{1}{8}$ ⇒ x + y = 8 ……..(vi)

On adding (v) and (vi), we get:

2x = 12

⇒ x = 6

On substituting x = 6 in (v), we get:

6 – y = 4

y = (6 – 4) = 2

∴ Speed of the boat in still water = 6km/h

And, speed of the stream = 2 km/h

68.

Sol:
Let us suppose that one man alone can finish the work in x days and one boy alone can finish it in y days.
$\therefore$ One man's one day's work $= \frac{1}{x}$
And, one boy's one day's work $= \frac{1}{y}$
2 men and 5 boys can finish the work in 4 days.
$\therefore$ (2 men's one day's work) + (5 boys' one day's work) $= \frac{1}{4}$

$\Rightarrow \frac{2}{x} + \frac{5}{y} = \frac{1}{4}$
$\Rightarrow 2u + 5v = \frac{1}{4}$ ....(i) Here, $\frac{1}{x} = u$ and $\frac{1}{y} = v$
Again, 3 men and 6 boys can finish the work in 3days.
$\therefore$ (3 men's one day's work) + (6 boys' one day's work) $= \frac{1}{3}$
$\Rightarrow \frac{3}{x} + \frac{6}{y} = \frac{1}{3}$
$\Rightarrow 3u + 6v = \frac{1}{3}$ ....(ii) Here, $\frac{1}{x} = u$ and $\frac{1}{y} = v$
On multiplying (iii) from (iv), we get:
$3u = \left(\frac{5}{3} - \frac{6}{4}\right) = \frac{2}{12} = \frac{1}{6}$
$\Rightarrow u = \frac{1}{6 \times 3} = \frac{1}{18} \Rightarrow \frac{1}{x} = \frac{1}{18} \Rightarrow x = 18$
On substituting $u = \frac{1}{18}$ in (i), we get:
$2 \times \frac{1}{18} + 5v = \frac{1}{4} \Rightarrow 5v = \left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36}$
$\Rightarrow v = \left(\frac{5}{36} \times \frac{1}{5}\right) = \frac{1}{36} \Rightarrow \frac{1}{y} = \frac{1}{36} \Rightarrow y = 36$
Hence, one man alone can finish the work is 18days and one boy alone can finish the work in 36 days.

69.

Sol:
Let the length of the room be x meters and he breadth of the room be y meters.
Then, we have:
Area of the room = xy
According to the question, we have:
x = y + 3

⇒ x – y = 3 …….(i)

And, (x + 3) (y – 2) = xy

⇒ xy – 2x + 3y – 6 = xy

⇒ 3y – 2x = 6 ……..(ii)

On multiplying (i) by 2, we get:

2x – 2y = 6 ……….(iii)

On adding (ii) and (iii), we get:

y = (6 + 6) = 12

On substituting y = 12 in (i), we get:

x – 12 = 3

⇒ x = (3 + 12) = 15

Hence, the length of the room is 15 meters and its breadth is 12 meters.

70.

Sol:

Let the length and the breadth of the rectangle be x m and y m, respectively.

∴ Area of the rectangle = (xy) sq.m

Case 1:

When the length is reduced by 5m and the breadth is increased by 3 m:

New length = (x – 5) m

New breadth = (y + 3) m

∴ New area = (x – 5) (y + 3) sq.m

∴ xy – (x – 5) (y + 3) = 8

⇒ xy – [xy – 5y + 3x – 15] = 8

⇒ xy – xy + 5y – 3x + 15 = 8

⇒ 3x – 5y = 7 ………(i)

Case 2:

When the length is increased by 3 m and the breadth is increased by 2 m:

New length = (x + 3) m

New breadth = (y + 2) m

∴ New area = (x + 3) (y + 2) sq.m

⇒ (x + 3) (y + 2) – xy = 74

⇒ [xy + 3y + 2x + 6] – xy = 74

⇒ 2x + 3y = 68 ………(ii)

On multiplying (i) by 3 and (ii) by 5, we get:
$9x - 15y = 21$ ……….(iii)
$10x + 15y = 340$ ………(iv)
On adding (iii) and (iv), we get:
$19x = 361$

$\Rightarrow x = 19$
On substituting x = 19 in (iii), we get:

$9 \times 19 - 15y = 21$

$\Rightarrow 171 - 15y = 21$

$\Rightarrow 15y = (171 - 21) = 150$

$\Rightarrow y = 10$
Hence, the length is 19m and the breadth is 10m.

71.

Sol:
Let the length and the breadth of the rectangle be x m and y m, respectively.
Case 1: When length is increased by 3m and the breadth is decreased by 4m:
$xy - (x + 3)(y - 4) = 67$

$\Rightarrow xy - xy + 4x - 3y + 12 = 67$

$\Rightarrow 4x - 3y = 55$ ………(i)
Case 2: When length is reduced by 1m and breadth is increased by 4m:
$(x - 1)(y + 4) - xy = 89$

$\Rightarrow xy + 4x - y - 4 - xy = 89$

$\Rightarrow 4x - y = 93$ ………(ii)
Subtracting (i) and (ii), we get:

$2y = 38 \Rightarrow y = 19$
On substituting y = 19 in (ii), we have
$4x - 19 = 93$

$\Rightarrow 4x = 93 + 19 = 112$

$\Rightarrow x = 28$
Hence, the length = 28m and breadth = 19m.

72.

Sol:

Let the basic first class full fare be Rs.x and the reservation charge be Rs.y.

Case 1: One reservation first class full ticket cost Rs.4, 150

x + y = 4150 ………(i)

Case 2: One full and one and half reserved first class tickets cost Rs.6,255

$(x + y) + \left(\frac{1}{2}x + y\right) = 6255$

$\Rightarrow$ 3x + 4y = 12510 ………(ii)

Substituting y = 4150 – x from (i) in (ii), we get

3x + 4(4150 – x) = 12510

$\Rightarrow$3x – 4x + 16600 = 12510

$\Rightarrow$x = 16600 – 12510 = 4090

Now, putting x = 4090 in (i), we have

4090 + y = 4150

$\Rightarrow$ y = 4150 – 4090 = 60

Hence, cost of basic first class full fare = Rs.4,090 and reservation charge = Rs.60.

73.

Sol:

Let the present age of the man be x years and that of his son be y years.

After 5 years man's age = x + 5

After 5 years ago son's age = y + 5

As per the question

x + 5 = 3(y + 5)

$\Rightarrow$ x – 3y = 10 ……………(i)

5 years ago man's age = x – 5

5 years ago son's age = y – 5

As per the question

x – 5 = 7(y – 5)

$\Rightarrow$ x – 7y = -30 …….(ii)

Subtracting (ii) from (i), we have

4y = 40 $\Rightarrow$ y = 10

Putting y = 10 in (i), we get

x – 3 × 10 = 10

$\Rightarrow$ x = 10 + 30 = 40

Hence, man's present age = 40 years and son's present age = 10 years.

74.

Sol:

Let the man's present age be x years.
Let his son's present age be y years.
According to the question, we have:
Two years ago:
Age of the man = Five times the age of the son

$\Rightarrow (x-2) = 5(y-2)$

$\Rightarrow x - 2 = 5y - 10$

$\Rightarrow x - 5y = -8$ ....(i)

Two years later:
Age of the man = Three times the age of the son + 8

$\Rightarrow (x+2) = 3(y+2) + 8$

$\Rightarrow x + 2 = 3y + 6 + 8$

$\Rightarrow x - 3y = 12$ ....(ii)

Subtracting (i) from (ii), we get:

$2y = 20$

$\Rightarrow y = 10$

On substituting y = 10 in (i), we get:

$x - 5 \times 10 = -8$

$\Rightarrow x - 50 = -8$

$\Rightarrow x = (-8 + 50) = 42$

Hence, the present age of the man is 42 years and the present age of the son is 10 years.

75.

Sol:

Let the mother's present age be x years.
Let her son's present age be y years.
Then, we have:

$x + 2y = 70$ ....(i)

And, $2x + y = 95$ ......(ii)

On multiplying (ii) by 2, we get:

$4x + 2y = 190$ ....(iii)

On subtracting (i) from (iii), we get:

$3x = 120$

$\Rightarrow x = 40$

On substituting x = 40 in (i), we get:

$40 + 2y = 70$

$\Rightarrow 2y = (70 - 40) = 30$

$\Rightarrow y = 15$

Hence, the mother's present age is 40 years and her son's present age is 15 years.

76.

Sol:

Let the woman's present age be x years.

Let her daughter's present age be y years.

Then, we have:

$x = 3y + 3$

$\Rightarrow x - 3y = 3$ ....(i)

After three years, we have:

$(x + 3) = 2(y + 3) + 10$

$\Rightarrow x + 3 = 2y + 6 + 10$

$\Rightarrow x - 2y = 13$ ......(ii)

Subtracting (ii) from (i), we get:

$-y = (3 - 13) = -10$

$\Rightarrow y = 10$

On substituting y = 10 in (i), we get:

$x - 3 \times 10 = 3$

$\Rightarrow x - 30 = 3$

$\Rightarrow x = (3 + 30) = 33$

Hence, the woman's present age is 33 years and her daughter's present age is 10 years.

77.

Sol:

Let the actual price of the tea and lemon set be Rs.x and Rs.y respectively.
When gain is Rs.7, then
$\frac{y}{100} \times 15 - \frac{x}{100} \times 5 = 7$
$\Rightarrow$ 3y – x = 140 ……..(i)
When gain is Rs.14, then
$\frac{y}{100} \times 5 + \frac{x}{100} \times 10 = 14$
$\Rightarrow$ y + 2x = 280 ……..(ii)
Multiplying (i) by 2 and adding with (ii), we have
7y = 280 + 280
$\Rightarrow y = \frac{560}{7} = 80$
Putting y = 80 in (ii), we get
80 + 2x = 280
$\Rightarrow x = \frac{200}{2} = 100$
Hence, actual price of the tea set and lemon set are Rs.100 and Rs.80 respectively.

78.

Sol:
Let the fixed charge be Rs.x and the charge for each extra day be Rs.y.
In case of Mona, as per the question
x + 4y = 27 ………(i)
In case of Tanvy, as per the question
x + 2y = 21 ………(ii)
Subtracting (ii) from (i), we get
2y = 6 $\Rightarrow$ y = 3
Now, putting y = 3 in (ii), we have
$x + 2 \times 3 = 21$
$\Rightarrow$ x = 21 – 6 = 15

Hence, the fixed charge be Rs.15 and the charge for each extra day is Rs.3.

79.

Sol:
Let x litres and y litres be the amount of acids from 50% and 25% acid solutions respectively.
As per the question
50% of x + 25% of y = 40% of 10

$\Rightarrow 0.50x + 0.25y = 4$

$\Rightarrow 2x + y = 16$ ………(i)

Since, the total volume is 10 liters, so

$x + y = 10$

Subtracting (ii) from (i), we get

$x = 6$

Now, putting x = 6 in (ii), we have

$6 + y = 10 \Rightarrow y = 4$

Hence, volume of 50% acid solution = 6litres and volume of 25% acid solution = 4litres.

80.

Sol:

Let x g and y g be the weight of 18-carat and 12- carat gold respectively.

As per the given condition

$\frac{18x}{24} + \frac{12y}{24} = \frac{120 \times 16}{24}$

$\Rightarrow 3x + 2y = 320$ …………(i)

And

$x + y = 120$ …………(ii)

Multiplying (ii) by 2 and subtracting from (i), we get

$x = 320 – 240 = 80$

Now, putting x = 80 in (ii), we have

$80 + y = 120 \Rightarrow y = 40$

Hence, the required weight of 18-carat and 12-carat gold bars are 80 g and 40 g respectively.

81.

Sol:

Let x litres and y litres be respectively the amount of 90% and 97% pure acid solutions.

As per the given condition

$0.90x + 0.97y = 21 \times 0.95$

$\Rightarrow 0.90x + 0.97y = 21 \times 0.95$ ……….(i)

And

$x + y = 21$

From (ii), substitute y = 21 – x in (i) to get

$0.90x + 0.97(21 – x) = 21 \times 0.95$

$\Rightarrow 0.90x + 0.97 \times 21 – 0.97x = 21 \times 0.95$

$\Rightarrow 0.07x = 0.97 \times 21 - 21 \times 0.95$

$\Rightarrow x = \frac{21 \times 0.02}{0.07} = 6$

Now, putting x = 6 in (ii), we have

$6 + y = 21 \Rightarrow y = 15$

Hence, the request quantities are 6 litres and 15 litres.

82.

Sol:

Let x and y be the supplementary angles, where x > y.

As per the given condition

$x + y = 180^0$ …….(i)

And

$x - y = 18^0$ ……..(ii)

Adding (i) and (ii), we get

$2x = 198^0 \Rightarrow x = 99^0$

Now, substituting $x = 99^0$ in (ii), we have

$99^0 - y = 18^0 \Rightarrow x = 99^0 - 18^0 = 81^0$

Hence, the required angles are $99^0$and 81^0.

83.

Sol:

$\because \angle C - \angle B = 9^0$

$\therefore y^0 - (3x - 2)^0 = 9^0$

$\Rightarrow y^0 - 3x^0 + 2^0 = 9^0$

$\Rightarrow y^0 - 3x^0 = 7^0$

The sum of all the angles of a triangle is 180^0, therefore

$\angle A + \angle B + \angle C = 180^0$

$\Rightarrow x^0 + (3x - 2)^0 + y^0 = 180^0$

$\Rightarrow 4x^0 + y^0 = 182^0$

Subtracting (i) from (ii), we have

$7x^0 = 182^0 - 7^0 = 175^0$

$\Rightarrow x^0 = 25^0$

Now, substituting $x^0 = 25^0$ in (i), we have

$y^0 = 3x^0 + 7^0 = 3 \times 25^0 + 7^0 = 82^0$

Thus

$\angle A = x^0 = 25^0$