Exercise – 3C

1.

Sol:

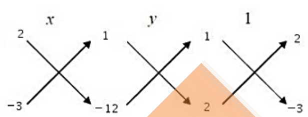
The given equations are:

$x + 2y + 1 = 0$ …….(i)

$2x – 3y – 12 = 0$ …….(ii)

Here $a_1 = 1$, $b_1 = 2$, $c_1 = 1$, $a_2 = 2$, $b_2 = -3$ and $c_2 = -12$

By cross multiplication, we have:



$\therefore \frac{x}{[2 \times (-12) - 1 \times (-3)]} = \frac{y}{[1 \times 2 - 1 \times (-12)]} = \frac{1}{[1 \times (-3) - 2 \times 2]}$

$\Rightarrow \frac{x}{(-24+3)} = \frac{y}{(2+12)} = \frac{1}{(-3-4)}$

$\Rightarrow \frac{x}{(-21)} = \frac{y}{(14)} = \frac{1}{(-7)}$

$\Rightarrow x = \frac{-21}{-7} = 3$, $y = \frac{14}{-7} = -2$

Hence, $x = 3$ and $y = -2$ is the required solution.

2.

Sol:

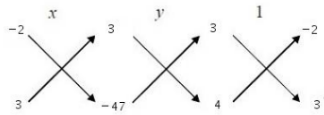
The given equations are:

$3x - 2y + 3 = 0$ ....(i)

$4x + 3y - 47 = 0$ ....(ii)

Here $a_1 = 3$, $b_1 = -2$, $c_1 = 3$, $a_2 = 4$, $b_2 = 3$ and $c_2 = -47$

By cross multiplication, we have:



$$\therefore \frac{x}{[(-2)\times(-47)-3\times 3]} = \frac{y}{[3\times 4-(-47)\times 3]} = \frac{1}{[3\times 3-(-2)\times 4]}$$

$$\Rightarrow \frac{x}{(94-9)} = \frac{y}{(12+141)} = \frac{1}{(9+8)}$$

$$\Rightarrow \frac{x}{(85)} = \frac{y}{(153)} = \frac{1}{(17)}$$

$$\Rightarrow x = \frac{85}{17} = 5,\ y = \frac{153}{17} = 9$$

Hence, x = 5 and y = 9 is the required solution.

3.

Sol:

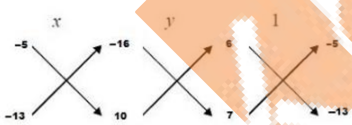
The given equations are:

$6x - 5y - 16 = 0$ ....(i)

$7x - 13y + 10 = 0$ ....(ii)

Here $a_1 = 6$, $b_1 = -5$, $c_1 = -16$, $a_2 = 7$, $b_2 = -13$ and $c_2 = 10$

By cross multiplication, we have:



$$\therefore \frac{x}{[(-5)\times 10-(-16)\times(-13)]} = \frac{y}{[(-16)\times 7-10\times 6]} = \frac{1}{[6\times(-13)-(-5)\times 7]}$$

$$\Rightarrow \frac{x}{(-50-208)} = \frac{y}{(-112-60)} = \frac{1}{(-78+35)}$$

$$\Rightarrow \frac{x}{(-258)} = \frac{y}{(-172)} = \frac{1}{(43)}$$

$$\Rightarrow x = \frac{-258}{-43} = 6,\ y = \frac{-172}{-43} = 4$$

Hence, x = 6 and y = 4 is the required solution.

4.

Sol:

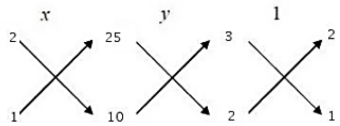
The given equations are:

$3x + 2y + 25 = 0$ ....(i)

2x + y + 10 = 0 …….(ii)

Here $a_1 = 3$, $b_1 = 2$, $c_1 = 25$, $a_2 = 2$, $b_2 = 1$ and $c_2 = 10$

By cross multiplication, we have:



$\therefore \frac{x}{[2\times10-25\times1]} = \frac{y}{[25\times2-10\times3]} = \frac{1}{[3\times1-2\times2]}$

$\Rightarrow \frac{x}{(20-25)} = \frac{y}{(50-30)} = \frac{1}{(3-4)}$

$\Rightarrow \frac{x}{(-5)} = \frac{y}{20} = \frac{1}{(-1)}$

$\Rightarrow x = \frac{-5}{-1} = 5$, $y = \frac{20}{(-1)} = -20$

Hence, x = 5 and y = -20 is the required solution.

5.

Sol:

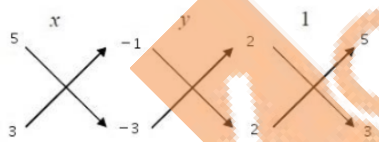
The given equations may be written as:

2x + 5y – 1 = 0 …….(i)

2x + 3y – 3 = 0 …….(ii)

Here $a_1 = 2$, $b_1 = 5$, $c_1 = -1$, $a_2 = 2$, $b_2 = 3$ and $c_2 = -3$

By cross multiplication, we have:



$\therefore \frac{x}{[5\times(-3)-3\times(-1)]} = \frac{y}{[(-1)\times2-(-3)\times2]} = \frac{1}{[2\times3-2\times5]}$

$\Rightarrow \frac{x}{(-15+3)} = \frac{y}{(-2+6)} = \frac{1}{(6-10)}$

$\Rightarrow \frac{x}{-12} = \frac{y}{4} = \frac{1}{-4}$

$\Rightarrow x = \frac{-12}{-4} = 3$, $y = \frac{4}{-4} = -1$

Hence, x = 3 and y = -1 is the required solution.

6.

Sol:

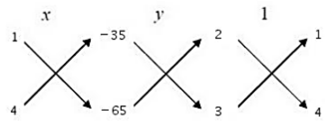
The given equations may be written as:

2x + y – 35 = 0 …….(i)

$3x + 4y - 65 = 0$ ....(ii)

Here $a_1 = 2$, $b_1 = 1$, $c_1 = -35$, $a_2 = 3$, $b_2 = 4$ and $c_2 = -65$

By cross multiplication, we have:



$$\therefore \frac{x}{[1\times(-65) - 4\times(-35)]} = \frac{y}{[(-35)\times 3 - (-65)\times 2]} = \frac{1}{[2\times 4 - 3\times 1]}$$

$$\Rightarrow \frac{x}{(-65+140)} = \frac{y}{(-105+130)} = \frac{1}{(8-3)}$$

$$\Rightarrow \frac{x}{75} = \frac{y}{25} = \frac{1}{5}$$

$$\Rightarrow x = \frac{75}{5} = 15,\ y = \frac{25}{5} = 5$$

Hence, x = 15 and y = 5 is the required solution.

7.

Sol:

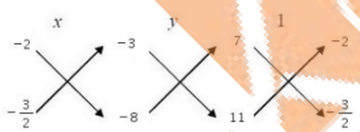
The given equations may be written as:

$7x - 2y - 3 = 0$ ....(i)

$11x - \frac{3}{2}y - 8 = 0$ ....(ii)

Here $a_1 = 7$, $b_1 = -2$, $c_1 = -3$, $a_2 = 11$, $b_2 = -\frac{3}{2}$ and $c_2 = -8$

By cross multiplication, we have:



$$\therefore \frac{x}{\left[(-2)\times(-8) - \left(-\frac{3}{2}\right)\times(-3)\right]} = \frac{y}{[(-3)\times 11 - (-8)\times 7]} = \frac{1}{\left[7\times\left(-\frac{3}{2}\right) - 11\times(-2)\right]}$$

$$\Rightarrow \frac{x}{(16-\frac{9}{2})} = \frac{y}{(-33+56)} = \frac{1}{(-\frac{21}{2}+22)}$$

$$\Rightarrow \frac{x}{\left(\frac{23}{2}\right)} = \frac{y}{23} = \frac{1}{\left(\frac{23}{2}\right)}$$

$$\Rightarrow x = \frac{\frac{23}{2}}{\frac{23}{2}} = 1,\ y = \frac{23}{\frac{23}{2}} = 2$$

Hence, x = 1 and y = 2 is the required solution.

8.

Sol:

The given equations may be written as:

$\frac{x}{6} + \frac{y}{15} - 4 = 0$ …….(i)

$\frac{x}{3}$ - $\frac{y}{12} - \frac{19}{4} = 0$ …….(ii)

Here $a_1 = \frac{1}{6}$, $b_1 = \frac{1}{15}$, $c_1 = -4$, $a_2 = \frac{1}{3}$, $b_2 = -\frac{1}{12}$ and $c_2 = -\frac{19}{4}$

By cross multiplication, we have:

$\therefore \frac{x}{\left[\frac{1}{15}\times\left(-\frac{19}{4}\right)-\left(-\frac{1}{12}\right)\times(-4)\right]} = \frac{y}{\left[(-4)\times\frac{1}{3}-\left(\frac{1}{6}\right)\times\left(-\frac{19}{4}\right)\right]} = \frac{1}{\left[\frac{1}{6}\times\left(-\frac{1}{12}\right)\times\frac{1}{3}\times\frac{1}{15}\right]}$

$\Rightarrow \frac{x}{\left(-\frac{19}{60}-\frac{1}{3}\right)} = \frac{y}{\left(-\frac{4}{3}+\frac{19}{34}\right)} = \frac{1}{\left(-\frac{1}{72}-\frac{1}{45}\right)}$

$\Rightarrow \frac{x}{\left(-\frac{39}{60}\right)} = \frac{y}{\left(-\frac{13}{24}\right)} = \frac{1}{\left(-\frac{13}{360}\right)}$

$\Rightarrow x = \left[\left(-\frac{39}{60}\right)\times\left(-\frac{360}{13}\right)\right] = 18$, $y = \left[\left(-\frac{13}{24}\right)\times\left(-\frac{360}{13}\right)\right] = 15$

Hence, x = 18 and y = 15 is the required solution.

9.

Sol:

Taking $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the given equations become:

$u + v = 7$

$2u + 3v = 17$

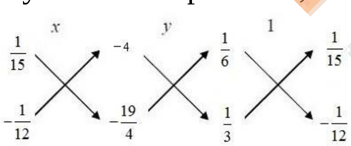
The given equations may be written as:

$u + v - 7 = 0$ …..(i)

$2u + 3v - 17 = 0$ ……(ii)

Here, $a_1 = 1$, $b_1 = 1$, $c_1 = -7$, $a_2 = 2$, $b_2 = 3$ and $c_2 = -17$

By cross multiplication, we have:



$\therefore \frac{u}{[1\times(-17)-3\times(-7)]} = \frac{v}{[(-7)\times 2 - 1\times(-17)]} = \frac{1}{[3-2]}$

$\Rightarrow \frac{u}{(-17+21)} = \frac{v}{(-14+17)} = \frac{1}{(1)}$

$\Rightarrow \frac{u}{4} = \frac{v}{3} = \frac{1}{1}$

$\Rightarrow u = \frac{4}{1} = 4$, $v = \frac{3}{1} = 3$

$\Rightarrow \frac{1}{x} = 4$, $\frac{1}{y} = 3$

$\Rightarrow x = \frac{1}{4}$, $y = \frac{1}{3}$

Hence, $x = \frac{1}{4}$ and $y = \frac{1}{3}$ is the required solution.

10.

Sol:

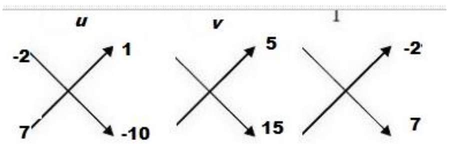
Taking $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$, the given equations become:

$5u - 2v + 1 = 0$ …..(i)

$15u + 7v - 10 = 0$ ……(ii)

Here, $a_1 = 5$, $b_1 = -2$, $c_1 = 1$, $a_2 = 15$, $b_2 = -7$ and $c_2 = -10$

By cross multiplication, we have:



$\therefore \frac{u}{[-2\times(-10) - 1\times 7]} = \frac{v}{[1 \times 15 - (-10)\times 5]} = \frac{1}{[35+30]}$

$\Rightarrow \frac{u}{(20-7)} = \frac{v}{(15+50)} = \frac{1}{65}$

$\Rightarrow \frac{u}{13} = \frac{v}{65} = \frac{1}{65}$

$\Rightarrow u = \frac{13}{65} = \frac{1}{5}$, $v = \frac{65}{65} = 1$

$\Rightarrow \frac{1}{x+y} = \frac{1}{5}, \frac{1}{x-y} = 1$

So, $(x + y) = 5$ …….(iii)

and $(x - y) = 1$ ……(iv)

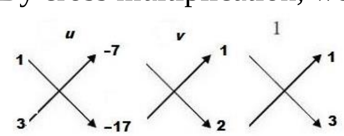
Again, the above equations (ii) and (iv) may be written as:

$x + y - 5 = 0$ …..(i)

$x - y - 1 = 0$ ……(ii)

Here, $a_1 = 1$, $b_1 = 1$, $c_1 = -5$, $a_2 = 1$, $b_2 = -1$ and $c_2 = -1$

By cross multiplication, we have:



$\therefore \frac{x}{[1\times(-1) - (-5)\times(-1)]} = \frac{y}{[(-5)\times 1 - (-1)\times 1]} = \frac{1}{[1\times(-1) - 1\times 1]}$

$\Rightarrow \frac{x}{(-1-5)} = \frac{y}{(-5+1)} = \frac{1}{(-1-1)}$

$\Rightarrow \frac{x}{-6} = \frac{v}{-4} = \frac{1}{-2}$

$\Rightarrow x = \frac{-6}{-2} = 3$, $y = \frac{-4}{-2} = 2$

Hence, $x = 3$ and $y = 2$ is the required solution.

11.

Sol:

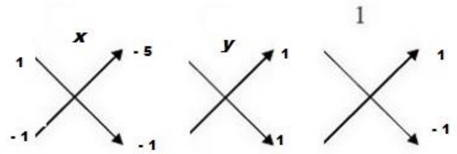
The given equations may be written as:

$\frac{ax}{b} - \frac{by}{a} - (a + b) = 0$ ……(i)

$ax - by - 2ab = 0$ ……(ii)

Here, $a_1 = \frac{a}{b}$, $b_1 = \frac{-b}{a}$, $c_1 = -(a + b)$, $a_2 = a$, $b_2 = -b$ and $c_2 = -2ab$

By cross multiplication, we have:



$$\therefore \frac{x}{\left[\left(-\frac{b}{a}\right)\times(-2ab) - (-b)\times(-(a+b))\right]} = \frac{y}{\left[-(a+b)\times a - (-2ab)\times\frac{a}{b}\right]} = \frac{1}{\left[\frac{a}{b}\times(-b) - a\times\left(-\frac{b}{a}\right)\right]}$$

$$\Rightarrow \frac{x}{(2b^2 - b(a+b))} = \frac{y}{-a(a+b)+2a^2} = \frac{1}{-a+b}$$

$$\Rightarrow \frac{x}{2b^2 - ab - b^2} = \frac{y}{-a^2 - ab + 2a^2} = \frac{1}{-a+b}$$

$$\Rightarrow \frac{x}{b^2 - ab} = \frac{y}{a^2 - ab} = \frac{1}{-(a-b)}$$

$$\Rightarrow \frac{x}{-b(a-b)} = \frac{y}{a(a-b)} = \frac{1}{-(a-b)}$$

$$\Rightarrow x = \frac{-b(a-b)}{-(a-b)} = b,\ y = \frac{a(a-b)}{-(a-b)} = -a$$

Hence, $x = b$ and $y = -a$ is the required solution.

12.

Sol:

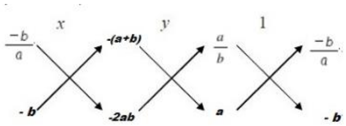
The given equations may be written as:

$2ax + 3by - (a + 2b) = 0$ ……(i)

$3ax + 2by - (2a + b) = 0$ ……(ii)

Here, $a_1 = 2a$, $b_1 = 3b$, $c_1 = -(a + 2b)$, $a_2 = 3a$, $b_2 = 2b$ and $c_2 = -(2a + b)$

By cross multiplication, we have:



$$\therefore \frac{x}{[3b\times(-(2a+b)) - 2b\times(-(a+2b))]} = \frac{y}{[-(a+2b)\times 3a - 2a\times(-(2a+b))]} = \frac{1}{[2a\times 2b - 3a\times 3b]}$$

$$\Rightarrow \frac{x}{(-6ab - 3b^2 + 2ab + 4b^2)} = \frac{y}{(-3a^2 - 6ab + 4a^2 + 2ab)} = \frac{1}{4ab - 9ab}$$

$\Rightarrow \frac{x}{b^2-4ab} = \frac{y}{a^2-4ab} = \frac{1}{-5ab}$

$\Rightarrow \frac{x}{-b(4a-b)} = \frac{y}{-a(4b-a)} = \frac{1}{-5ab}$

$\Rightarrow x = \frac{-b(4a-b)}{-5ab} = \frac{(4a-b)}{5a}$, $y = \frac{-a(4b-a)}{-5ab} = \frac{(4b-a)}{5b}$

Hence, $x = \frac{(4a-b)}{5a}$ and $y = \frac{(4b-a)}{5b}$ is the required solution.

13.

Sol:

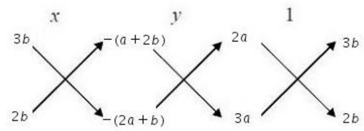
Substituting $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in the given equations, we get

$au - bv + 0 = 0$ …….(i)

$ab^2u + a^2bv - (a^2 + b^2) = 0$ ……(ii)

Here, $a_1 = a$, $b_1 = -b$, $c_1 = 0$, $a_2 = ab^2$, $b_2 = a^2b$ and $c_2 = -(a^2 + b^2)$.

So, by cross-multiplication, we have



$\frac{u}{b_1c_2 - b_2c_1} = \frac{v}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\Rightarrow \frac{u}{(-b)[-(a^2+b^2)]-(a^2b)(0)} = \frac{v}{(0)(ab^2)-(-a^2-b^2)(a)} = \frac{1}{(a)(a^2b)-(ab^2)(-b)}$

$\Rightarrow \frac{u}{b(a^2+b^2)} = \frac{v}{a(a^2+b^2)} = \frac{1}{ab(a^2+b^2)}$

$\Rightarrow u = \frac{b(a^2+b^2)}{ab(a^2+b^2)}$, $v = \frac{a(a^2+b^2)}{ab(a^2+b^2)}$

$\Rightarrow u = \frac{1}{a}$, $v = \frac{1}{b}$

$\Rightarrow \frac{1}{x} = \frac{1}{a}, \frac{1}{y} = \frac{1}{b}$

$\Rightarrow x = a, y = b$

Hence, $x = a$ and $y = b$.

Exercise – 3D

1.

Sol:

The given system of equations is: