RD SHARMA

Solutions

Class 9 Maths

Chapter 9

Ex *9.1*

Q1) In a ΔABC, if $\angle A = 55^0$, $\angle B = 40^0$, Find $\angle C$.

Solution:

Given Data:

$\angle B = 55^0$, $\angle B = 40^0$, then $\angle C$ =?

We know that

In a ΔABC sum of all angles of a triangle is 180^0

i.e., $\angle A + \angle B + \angle C = 180^0$

$\Rightarrow 55^0 + 40^0 + \angle C = 180^0$

$\Rightarrow 95^0 + \angle C = 180^0$

$\Rightarrow \angle C = 180^0 - 95^0$

$\Rightarrow \angle C = 85^0$

Q2) If the angles of a triangle are in the ratio 1:2:3, determine three angles.

Solution:

Given that,

Angles of a triangle are in the ratio 1:2:3

Let the angles be x, 2x, 3x

∴ We know that,

Sum of all angles of triangles is 180^0

x+2x+3x = 180^0

=>6x = 180^0

=>$x = \frac{180^0}{6}$

=>x=30^0

Since x=30^0

2x = $2(30)^0$ = 60^0

3x = $3(30)^0$ = 90^0

Therefore, angles are $30^0, 60^0, 90^0$

Q3) The angles of a triangle are $(x - 40^0), (x - 20^0)$ and $(\frac{1}{2}x - 10^0)$. *Find the value of x.*

Solution:

Given that,

The angles of a triangle are

$(x - 40^0), (x - 20^0)$ and $(\frac{1}{2}x - 10^0)$

We know that,

Sum of all angles of triangle is 180^0

$\therefore (x - 40^0) + (x - 20^0) + (\frac{1}{2}x - 10^0) = 180^0$

$2x + \frac{1}{2}x - 70^0 = 180^0$

$\frac{5}{2}x = 180^0 + 70^0$

$5x = 2(250)^0$

$x = \frac{500^0}{5}$

$\therefore x = 100^0$

Q4) The angles of a triangle are arranged in ascending order of magnitude. If the difference between two consecutive angles is 10^0, find the three angles.

Solution:

Given that,

The difference between two consecutive angles is 10^0

Let x, x+10^0, x+20^0 be the consecutive angles that differ by 10^0

We know that,

Sum of all angles in a triangle is 180^0

x+x+10^0+x+20^0 = 180^0

3x+30^0 = 180^0

=>3x = $180^0 - 30^0$

=>3x = 150^0

=>x = 50^0

Therefore, the required angles are

x = 50^0

x+10^0 = 50^0 + 10^0 = 60^0

x+20^0 = 50^0 + 20^0 = 70^0

As the difference between two consecutive angles is 10^0, the three angles are 50^0, 60^0, 70^0.

Q5) Two angles of a triangle are equal and the third angle is greater than each of those angles by 30^0. Determine all the angles of the triangle.

Solution:

Given that,

Two angles of a triangle are equal and the third angle is greater than each of those angles by 30^0.

Let x, x, x+30^0 be the angles of a triangle

We know that,

Sum of all angles in a triangle is 180^0

x + x + x + 30^0 = 180^0

3x + 30^0 = 180^0

3x = $180^0 - 30^0$

3x = 150^0

x = 50^0

Therefore, the three angles are $50^0,\ 50^0,\ 80^0$.

Q6) If one angle of a triangle is equal to the sum of the other two, show that the triangle is a right angle triangle.

Solution:

If one angle of a triangle is equal to the sum of the other two angles

=>$\angle B = \angle A + \angle C$

In ΔABC,

Sum of all angles of a triangle is 180^0

=>$\angle A + \angle B + \angle C = 180^0$

=>$\angle B + \angle B = 180^0$[∠ B=∠ A+∠C]

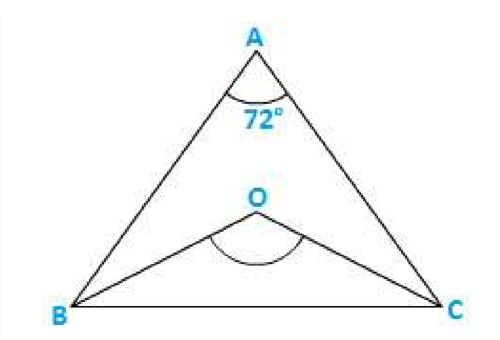
=>$2\angle B = 180^0$

=>$\angle B = \frac{180^0}{2}$

=>$\angle B = 90^0$

Therefore, ABC is a right angled triangle.

Q7) ABC is a triangle in which $\angle A = 72^0$, the internal bisectors of angles B and C meet in O. Find the magnitude of $\angle BOC$.



Solution:

Given,

ABC is a triangle where $\angle A = 72^0$ and the internal bisector of angles B and C meeting O.

In ΔABC,

$\angle A + \angle B + \angle C = 180^0$

=>$72^0 + \angle B + \angle C = 180^0$

=>$\angle B + \angle C = 180^0 - 72^0$

Dividing both sides by '2'

=>$\frac{\angle B}{2} + \frac{\angle C}{2} = \frac{108^0}{2}$

=>$\angle OBC + \angle OCB = 54^0$

Now, $\text{In } \Delta BOC \Rightarrow \angle OBC + \angle OCB + \angle BOC = 180^0$

=>$54^0 + \angle BOC = 180^0$

=>$\angle BOC = 180^0 - 54^0 = 126^0$

$\therefore \angle BOC = 126^0$

Q8) The bisectors of base angles of a triangle cannot enclose a right angle in any case.

Solution:

In ΔXYZ,

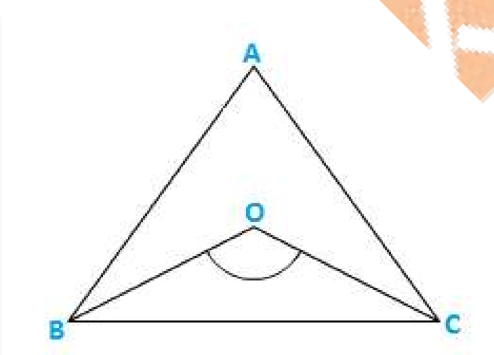
Sum of all angles of a triangle is 180^0

i.e., $\angle X + \angle Y + \angle Z = 180^0$

Dividing both sides by '2'

=>$\frac{1}{2}\angle X + \frac{1}{2}\angle Y + \frac{1}{2}\angle Z = 180^0$

=>$\frac{1}{2}\angle X + \angle OYZ + \angle OYZ = 90^0$ [∵ OY, OZ,∠ Y and ∠ Z]



$\Rightarrow \angle OYZ + \angle OZY = 90^0 - \frac{1}{2}\angle X$

Now in ΔYOZ

$\therefore \angle YOZ + \angle OYZ + \angle OZY = 180^0$

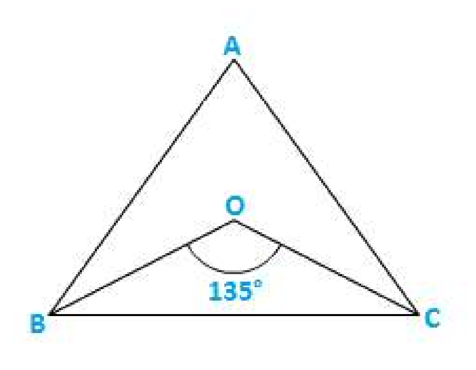
$\Rightarrow \angle YOZ + 90^0 - \frac{1}{2}\angle X = 180^0$

$\Rightarrow \angle YOZ = 90^0 - \frac{1}{2}\angle X$

Therefore, the bisectors of a base angle cannot enclosure right angle.

Q9) If the bisectors of the base angles of a triangle enclose an angle of 135^0, prove that the triangle is a right angle.

Solution:



Given the bisectors of the base angles of a triangle enclose an angle of 135^0

i.e., $\angle BOC = 135^0$

But, We know that

$\angle BOC = 90^0 + \frac{1}{2}\angle A$

=>$135^0 = 90^0 + \frac{1}{2}\angle A$

=>$\frac{1}{2}\angle A = 135^0 - 90^0$

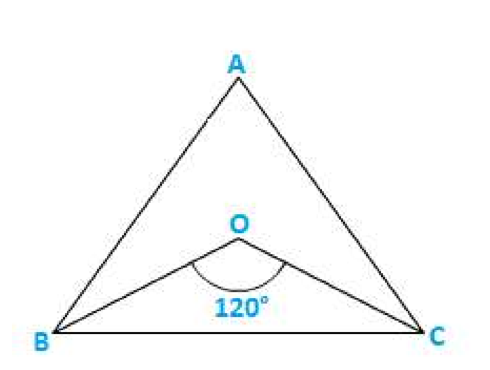
=>$\angle A = 45^0(2)$

=>$\angle A = 90^0$

Therefore, ΔABC is a right angle triangle that is right angled at A.

Q10) In a ΔABC, $\angle ABC = \angle ACB$ and the bisectors of $\angle ABC$ and $\angle ACB$ intersect at O such that $\angle BOC = 120^0$. Show that $\angle A = \angle B = \angle C = 60^0$.

Solution:



Given,

In ΔABC,

$\angle ABC = \angle ACB$

Dividing both sides by '2'

$\frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

=>$\angle OBC = \angle OCB$ $\quad$ [$\therefore$ OB, OC bisects $\angle B$ and $\angle C$]

Now,

$\angle BOC = 90^0 + \frac{1}{2}\angle A$

=>$120^0 - 90^0 = \frac{1}{2}\angle A$

=>$30^0 * (2) = \angle A$

=>$\angle A = 60^0$

Now in ΔABC

$\angle A + \angle ABC + \angle ACB = 180^0$ $\quad$ (Sum of all angles of a triangle)

=>$60^0 + 2\angle ABC = 180^0$ $\quad$ [$\therefore \angle ABC = \angle ACB$]

=>$2\angle ABC = 180^0 - 60^0$

=>$\angle ABC = \frac{120^0}{2} = 60^0$

=>$\angle ABC = \angle ACB$

$\therefore \angle ACB = 60^0$

Hence Proved.

Q11) Can a triangle have:

(i) Two right angles?

(ii) Two obtuse angles?

(iii) Two acute angles?

(iv) All angles more than 60°?

(v) All angles less than 60°?

(vi) All angles equal to 60"?

Justify your answer in each case.

Sol:

(i) No,

Two right angles would up to 180°. So the third angle becomes zero. This is not possible, so a triangle cannot have two right angles. [Since sum of angles in a triangle is 180^0]

(ii) No,

A triangle can't have 2 obtuse angles. Obtuse angle means more than 90° So that the sum of the two sides will exceed 180° which is not possible. As the sum of all three angles of a triangle is 180°.

(iii) Yes

A triangle can have 2 acute angles. Acute angle means less the 90" angle.

(iv) No

Having angles more than 60^0 make that sum more than 180^0. This is not possible. [Since the sum of all the internal angles of a triangle is 180^0]

(v) No

Having all angles less than 60^0 will make that sum less than 180^0 which is not possible.[Therefore, the sum of all the internal angles of a triangle is 180^0]

(vi) Yes

A triangle can have three angles equal to 60^0 . Then the sum of three angles equal to the 180^0. Such triangles are called as equilateral triangle. [Since, the sum of all the internal angles of a triangle is180^0]

Q12) If each angle of a triangle is less than the sum of the other two, show that the triangle is acute angled.

Solution

Given each angle of a triangle less than the sum of the other two

$\therefore \angle X + \angle Y + \angle Z$

=>$\angle X + \angle X < \angle X + \angle Y + \angle Z$

=>$2\angle X < 180^0$ [Sum of all the angles of a triangle]

=>$\angle X < 90^0$

Similarly $\angle Y < 90^0$ and $\angle Z < 90^0$

Hence, the triangles are acute angled.

RD SHARMA

Solutions

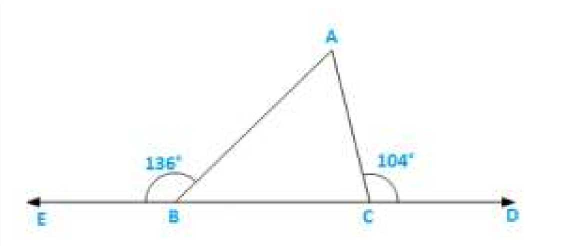
Class 9 Maths

Chapter 9

Ex 9.2

Q1) The exterior angles, obtained on producing the base of a triangle both ways are 104^0 and 136^0. Find all the angles of the triangle.

Solution:



$\angle ACD = \angle ABC + \angle BAC$ [Exterior angle property]

Now $\angle ABC = 180^0 - 136^0 = 44^0$ [Linera pair]

$\angle ACB = 180^0 - 104^0 = 76^0$ [Linera pair]

Now,

In ΔABC

$\angle A + \angle ABC + \angle ACB = 180^0$ [Sum of all angles of a triangle]

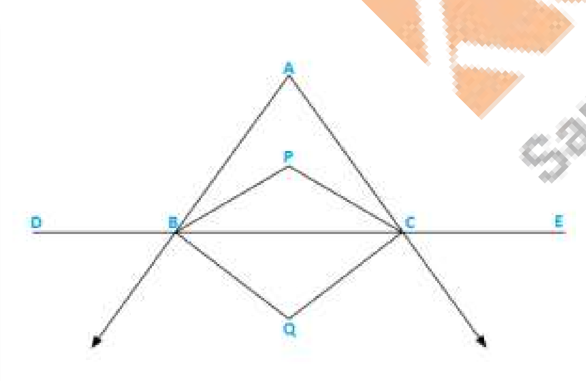
$\Rightarrow \angle A + 44^0 + 76^0 = 180^0$

$\Rightarrow \angle A = 180^0 - 44^0 - 76^0$

$\Rightarrow \angle A = 60^0$

Q2) In a triangle ABC, the internal bisectors of $\angle B$ and $\angle C$ meet at P and the external bisectors of $\angle B$ and $\angle C$ meet at Q. Prove that $\angle BPC + \angle BQC = 180^0$.

Solution:



Let $\angle ABD = 2x$ and $\angle ACE = 2y$

$\angle ABC = 180^0 - 2x$ [Linera pair]

$\angle ACB = 180^0 - 2y$ [Linera pair]

$\angle A + \angle ABC + \angle ACB = 180^0$ [Sum of all angles of a triangle]

$\Rightarrow \angle A + 180^0 - 2x + 180^0 - 2y = 180^0$

$\Rightarrow -\angle A + 2x + 2y = 180^0$

$\Rightarrow x + y = 90^0 + \frac{1}{2}\angle A$

Now in ΔBQC

$x + y + \angle BQC = 180^0$ [Sum of all angles of a triangle]

$\Rightarrow 90^0 + \frac{1}{2}\angle A + \angle BQC = 180^0$

$\Rightarrow \angle BQC = 90^0 - \frac{1}{2}\angle A \ldots.(i)$

and we know that $\angle BPC = 90^0 + \frac{1}{2}\angle A \ldots.(ii)$

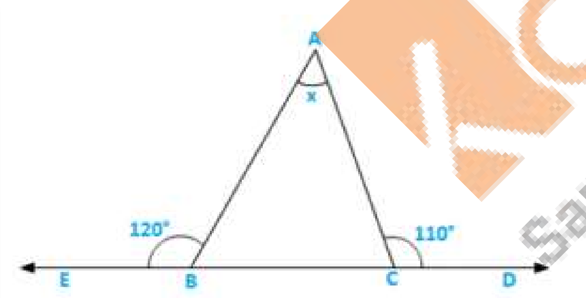
Adding (i) and (ii) we get

$\angle BPC + \angle BQC = 180^0$

Hence proved.

Q3) In figure 9.30, the sides BC, CA and AB of a triangle ABC have been produced to D, E and F respectively. If $\angle ACD = 105^0$ and $\angle EAF = 45^0$, ***find all the angles of the triangle ABC.***

Solution:



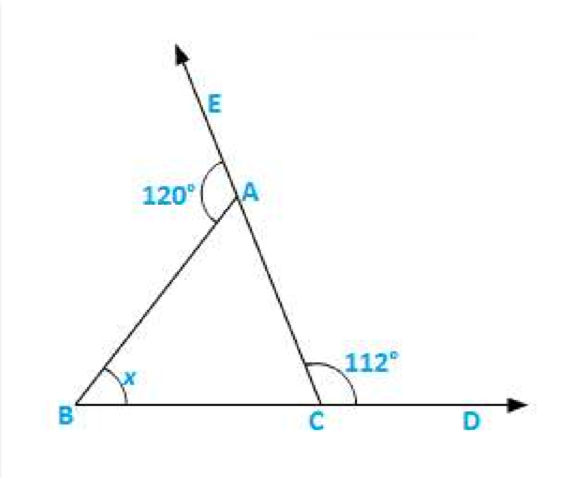
$\angle BAC = \angle EAF = 45^0$ [Vertically opposite angles]

$\angle ABC = 105^0 - 45^0 = 60^0$ [Exterior angle property]

$\angle ACD = 180^0 - 105^0 = 75^0$ [Linear pair]

Q4) Compute the value of x in each of the following figures:

(i)



Solution:

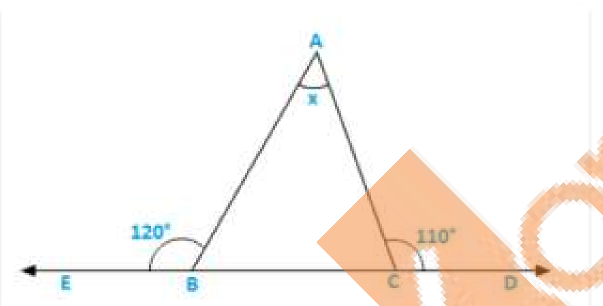
$\angle BAC = 180^0 - 120^0 = 60^0$ [Linear pair]

$\angle ACB = 180^0 - 112^0 = 68^0$ [Linear pair]

$\therefore x = 180^0 - \angle BAC - \angle ACB = 180^0 - 60^0 - 68^0 = 52^0$

[Sum of all angles of a triangle]

(ii)



Solution:

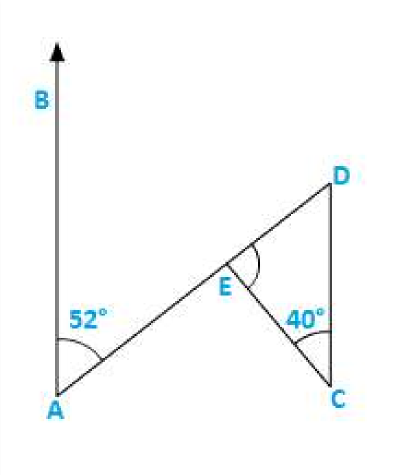
$\angle ABC = 180^0 - 120^0 = 60^0$ [Linear pair]

$\angle ACB = 180^0 - 110^0 = 70^0$ [Linear pair]

$\therefore e\angle BAC = x = 180^0 - \angle ABC - \angle ACB$

$= 180^0 - 60^0 - 70^0 = 50^0$ [Sum of all angles of a triangle]

(iii)



Solution:

$\angle BAE = \angle EDC = 52^0$ [Alternate angles]

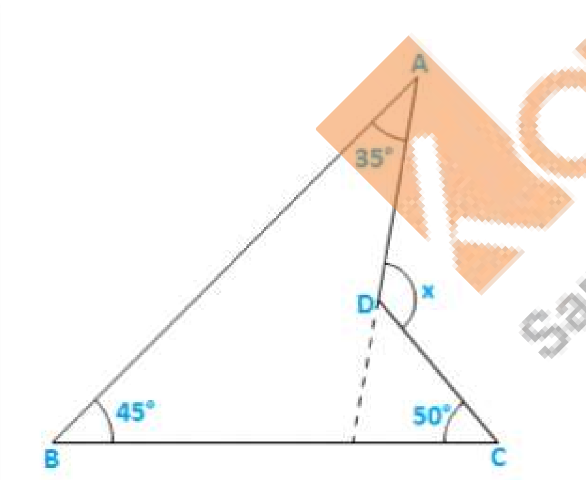
$\therefore \angle DEC = x = 180^0 - 40^0 - \angle EDC$

$= 180^0 - 40^0 - 52^0$

$= 180^0 - 92^0$

$= 88^0$ [Sum of all angles of a triangle]

(iv)



Solution:

CD is produced to meet AB at E.

$\angle BEC = 180^0 - 45^0 - 50^0$

$= 85^0$ [Sum of all angles of a triangle]

$\angle AEC = 180^0 - 85^0 = 95^0$ [Linear pair]

$\therefore x = 95^0 + 35^0 = 130^0$ [Exterior angle property]

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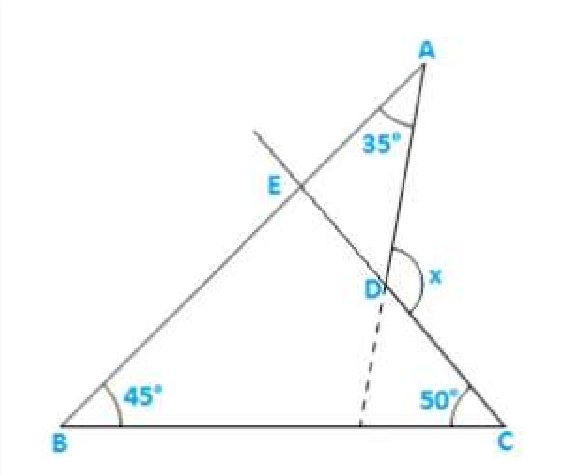
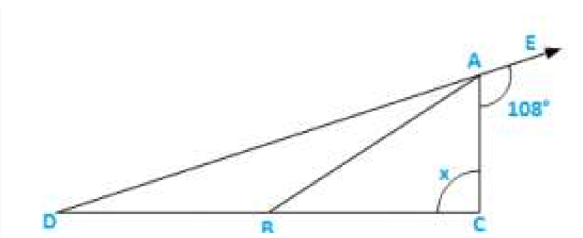


Q5) In figure 9.35, AB divides $\angle DAC$ in the ratio 1 : 3 and AB = DB. Determine the value of x.



Solution:

Let $\angle BAD = Z, \angle BAC = 3Z$

$\Rightarrow \angle BDA = \angle BAD = Z \quad (\because AB = DB)$

Now $\angle BAD + \angle BAC + 108^0 = 180^0$ [Linear pair]

$\Rightarrow Z + 3Z + 108^0 = 180^0$

$\Rightarrow 4Z = 72^0$

$\Rightarrow Z = 18^0$

Now, In ΔADC

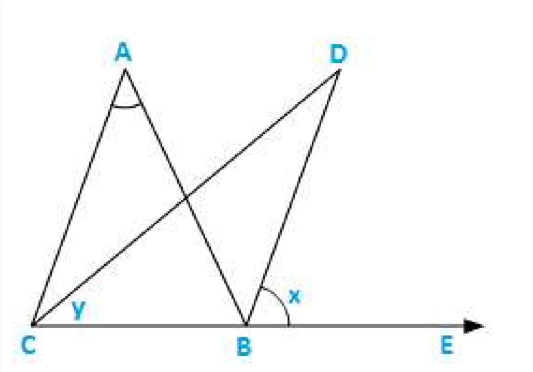
$\angle ADC + \angle ACD = 108^0$ [Exterior angle property]

$\Rightarrow x + 18^0 = 180^0$

$\Rightarrow x = 90^0$

Q6) ABC is a triangle. The bisector of the exterior angle at B and the bisector of $\angle C$ intersect each other at D. Prove that $\angle D = \frac{1}{2}\angle A$.

Solution:



$$\text{Let } \angle ABE = 2x \text{ and } \angle ACB = 2y$$
$$\angle ABC = 180^0 - 2x \quad [\text{Linear pair}]$$
$$\therefore \angle A = 180^0 - \angle ABC - \angle ACB \quad [\text{Angle sum property}]$$
$$= 180^0 - 180^0 + 2x + 2y$$
$$= 2(x - y) \quad \ldots\ldots(i)$$
$$\text{Now, } \angle D = 180^0 - \angle DBC - \angle DCB$$
$$\Rightarrow \angle D = 180^0 - (x + 180^0 - 2x) - y$$
$$\Rightarrow \angle D = 180^0 - x - 180^0 + 2x - y$$
$$= (x - y)$$
$$= \tfrac{1}{2}\angle A \quad \ldots\ldots \text{from(i)}$$
$$\text{Hence, } \angle D = \tfrac{1}{2}\angle A$$

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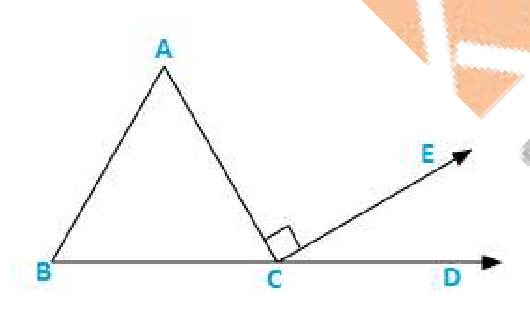
Q7) In figure 9.36, AC ⊥ CE and $\angle A : \angle B : \angle C = 3 : 2 : 1$, find



Solution:

$\angle A : \angle B : \angle C = 3 : 2 : 1$

Let the angles be 3x, 2x and x

$\Rightarrow 3x + 2x + x = 180^0$ [Angle sum property]

$\Rightarrow 6x = 180^0$

$\Rightarrow x = 30^0 = \angle ACB$

$\therefore \angle ECD = 180^0 - \angle ACB - 90^0$ [Linear pair]

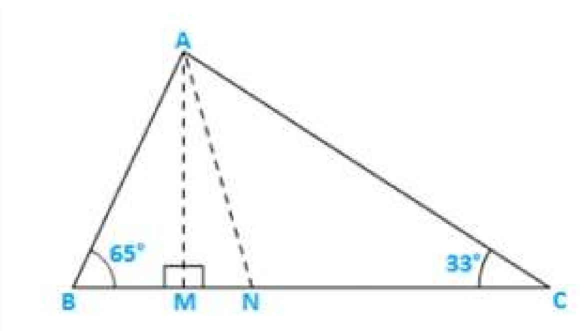
$= 180^0 - 30^0 - 90^0$

$= 60^0$

$\therefore \angle ECD = 60^0$

Q8) In figure 9.37,

AM $\perp$ BC and AN is the bisector of $\angle A$. If $\angle B = 65^0$ and $\angle C = 33^0$, find $\angle MAN$..



Solution:

Let $\angle BAN = \angle NAC = x$ [$\because$ AN bisects $\angle A$]

$\therefore \angle ANM = x + 33^0$ [Exterior angle property]

In ΔAMB

$\angle BAM = 90^0 - 65^0 = 25^0$ [Exterior angle property]

$\therefore \angle MAN = \angle BAN - \angle BAM = (x - 25)^0$

Now in ΔMAN,

$(x - 25)^0 + (x + 33)^0 + 90^0 = 180^0$ [Angle sum property]

$\Rightarrow 2x + 8^0 = 90^0$

$\Rightarrow 2x = 82^0$

$\Rightarrow x = 41^0$

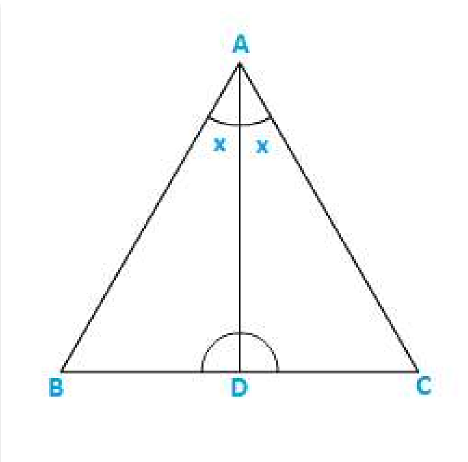
$\therefore MAN = x - 25^0$

$= 41^0 - 25^0$

$= 16^0$

Q9) In a triangle ABC, AD bisects $\angle A$ and $\angle C > \angle B$. Prove that $\angle ADB > \angle ADC$..

Solution:



$\because \angle C > \angle B$ [Given]

$\Rightarrow \angle C + x > \angle B + x$ [Adding x on both sides]

$\Rightarrow$ 180° − ∠ ADC>180^{0} − ∠ ADB

$\Rightarrow$ -∠ ADC> − ∠ ADB

$\Rightarrow$ ∠ ADB > ∠ ADC

Hence proved.

Q10) In triangle ABC,
$BD \perp AC$ and $CE \perp AB$. If BD and CE intersect at O, prove that $\angle BOC = 180^0 - \angle A$

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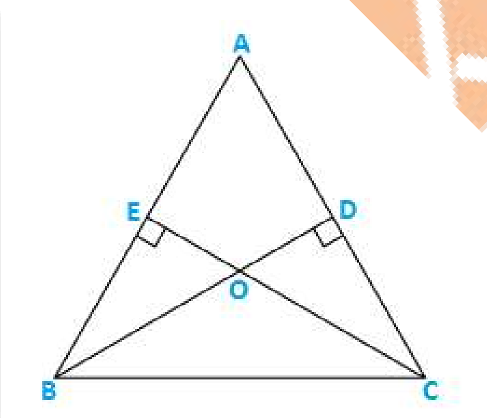
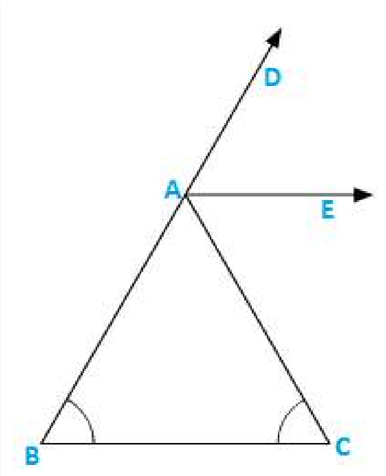
Solution:



In quadrilateral AEOD

$$\angle A + \angle AEO + \angle EOD + \angle ADO = 360^0$$
$$\Rightarrow \angle A + 90^0 + 90^0 + \angle EOD = 360^0$$
$$\Rightarrow \angle A + \angle BOC = 180^0 \quad [\because \angle EOD = \angle BOC \text{ vertically opposite angles}]$$
$$\Rightarrow \angle BOC = 180^0 - \angle A$$

Q11) In figure 9.38, AE bisects $\angle CAD$ and $\angle B = \angle C$. Prove that $AE \parallel BC$.



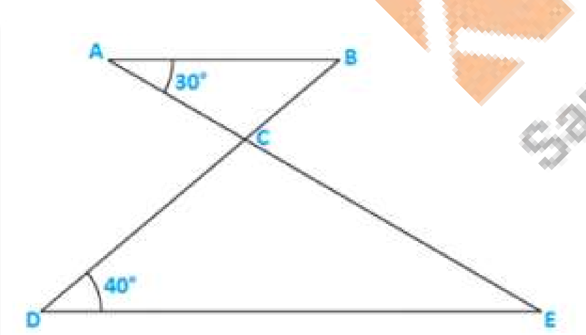
Solution:

$$\text{Let } \angle B = \angle C = x$$
$$\text{Then,}$$
$$\angle CAD = \angle B + \angle C = 2x \quad \text{(exterior angle)}$$
$$\Rightarrow \frac{1}{2}\angle CAD = x$$
$$\Rightarrow \angle EAC = x$$
$$\Rightarrow \angle EAC = \angle C$$

These are alternate interior angles for the lines AE and BC

$\therefore AE \parallel BC$

Q12) In figure 9.39, $AB \parallel DE$. Find $\angle ACD$.



Solution:

Since $AB \parallel DE$

$\therefore \angle ABC = \angle CDE = 40^0$ [Alternate angles]

$\therefore \angle ACB = 180^0 - \angle ABC - \angle BAC$

$= 180^0 - 40^0 - 30^0$

$= 110^0$

$\therefore \angle ACD = 180^0 - 110^0$ [Linear pair]

$= 70^0$

Q13) . Which of the following statements are true (T) and which are false (F) :

(i) Sum of the three angles of a triangle is 180°.

(ii) A triangle can have two right angles.

(iii) All the angles of a triangle can be less than 60°.

(iv) All the angles of a triangle can be greater than 60°.

(v) All the angles of a triangle can be equal to 60°.

(vi) A triangle can have two obtuse angles.

(vii) A triangle can have at most one obtuse angles.

(viii) If one angle of a triangle is obtuse, then it cannot be a right angled triangle.

(ix) An exterior angle of a triangle is less than either of its interior opposite angles.

(x) An exterior angle of a triangle is equal to the sum of the two interior opposite angles.

(xi) An exterior angle of a triangle is greater than the opposite interior angles.

Solution:

(i) T

(ii) F

(iii) F

(iv) F

(v) T

(vi) F

(vii) T

(viii) T

(ix) F

(x) T

(xi) T

Q14) Fill in the blanks to make the following statements true:

(i) Sum of the angles of a triangle is _______ .

(ii) An exterior angle of a triangle is equal to the two ________ opposite angles.

(iii) An exterior angle of a triangle is always ________ than either of the interior opposite angles.

(iv) A triangle cannot have more than _______ right angles.

(v) A triangles cannot have more than _______ obtuse angles.

Solution:

(i) 180^0

(ii) Interior

(iii) Greater

(iv) One

(v) One