RD SHARMA

Solutions

Class 9 Maths

Chapter 6

Ex 6.1

Q1. Which of the following expressions are polynomials in one variable and which are not?

State the reasons for your answers

1. $3x^2 - 4x + 15$
2. $y^2 + 2\sqrt{3}$
3. $3\sqrt{x} + \sqrt{2}x$
4. $x - \frac{4}{x}$
5. $x^{12} + y^2 + t^{50}$

Sol :

1. $3x^2 - 4x + 15$ – it is a polynomial of x
2. $y^2 + 2\sqrt{3}$ – it is a polynomial of y
3. $3\sqrt{x} + \sqrt{2}x$ – it is not a polynomial since the exponent of $3\sqrt{x}$ is not a positive term
4. $x - \frac{4}{x}$ – it is not a polynomial since the exponent of $-\frac{4}{x}$ is not a positive term
5. $x^{12} + y^2 + t^{50}$ – it is a three variable polynomial which variables of x, y, t

Q2. Write the coefficients of x^2 in each of the following

1. $17 - 2x + 7x^2$
2. $9 - 12x + x^2$
3. $\frac{\Pi}{6}x^2 - 3x + 4$
4. $\sqrt{3}x - 7$

Sol :

Given , to find the coefficients of x^2

1. $17 - 2x + 7x^2$ – the coefficient is 7
2. $9 - 12x + x^2$ – the coefficient is 0
3. $\frac{\Pi}{6}x^2 - 3x + 4$ – the coefficient is $\frac{\Pi}{6}$
4. $\sqrt{3}x - 7$ – the coefficient is 0

Q3. Write the degrees of each of the following polynomials :

1. $7x^3 + 4x^2 - 3x + 12$
2. $12 - x + 2x^2$
3. $5y - \sqrt{2}$
4. $7 - 7x^0$
5. 0

Sol :

Given , to find degrees of the polynomials

Degree is highest power in the polynomial

1. $7x^3 + 4x^2 - 3x + 12$ – the degree is 3
2. $12 - x + 2x^3$ – the degree is 3
3. $5y - \sqrt{2}$ – the degree is 1
4. $7 - 7x^0$ – the degree is 0

5. 0 – the degree of 0 is not defined

Q4. Classify the following polynomials as linear, quadratic, cuboc and biquadratic polynomials :

1. $x + x^2 + 4$
2. 3x – 2
3. $2x + x^2$
4. 3y
5. $t^2 + 1$

f . $7t^4 + 4t^2 + 3t - 2$

Sol :

Given

1. $x + x^2 + 4$ – it is a quadratic polynomial as its degree is 2
2. 3x – 2 – it is a linear polynomial as its degree is 1
3. $2x + x^2$ – it is a quadratic polynomial as its degree is 2
4. 3y – it is a linear polynomial as its degree is 1
5. $t^2 + 1$ – it is a quadratic polynomial as its degree is 2

f . $7t^4 + 4t^2 + 3t - 2$ – it is a bi- quadratic polynomial as its degree is 4

Q5. Classify the following polynomials as polynomials in one variables, two – variables etc :

1. $x^2 - xy + 7y^2$
2. $x^2 - 2tx + 7t^2 - x + t$
3. $t^3 - 3t^2 + 4t - 5$
4. xy + yz + zx

Sol :

Given

1. $x^2 - xy + 7y^2$ – it is a polynomial in two variables x and y
2. $x^2 - 2tx + 7t^2 - x + t$ – it is a polynomial in two variables x and t
3. $t^3 - 3t^2 + 4t - 5$– it is a polynomial in one variable t
4. $xy + yz + zx$ – it is a polynomial in 3 variables in x , y and z

Q6. Identify the polynomials in the following :

1. $f(x) = 4x^3 - x^2 - 3x + 7$
2. b . $g(x) = 2x^3 - 3x^2 + \sqrt{x} - 1$
3. $p(x) = \frac{2}{3}x^2 + \frac{7}{4}x + 9$
4. $q(x) = 2x^2 - 3x + \frac{4}{x} + 2$
5. $h(x) = x^4 - x^{\frac{3}{2}} + x - 1$
6. $f(x) = 2 + \frac{3}{x} + 4x$

Sol :

Given

1. \(f(x) = [latex]4x^{3} − x^{2} -3x + 7\)\(4x^{3} − x^{2} -3x + 7\]”> – it is a polynomial

2. b . [latex]g(x) = 2x^{3} − 3x^{2} + \sqrt{x} − 1\) – it is not a polynomial since the exponent of $\sqrt{x}$ is a negative integer
3. \(p(x) = [latex]\frac{2}{3}x^{2} + \frac{7}{4}x + 9\)\(\frac{2}{3}x^{2} + \frac{7}{4}x + 9\]”> – it is a polynomial as it has positive integers as exponents
4. [latex]q(x) = 2x^{2} − 3x + \frac{4}{x} + 2\) – it is not a polynomial since the exponent of $\frac{4}{x}$ is a negative integer
5. h(x) = $x^4 - x^{\frac{3}{2}} + x - 1$ – it is not a polynomial since the exponent of $- x^{\frac{3}{2}}$ is a negative integer
6. f(x) = 2 + $\frac{3}{x} + 4x$ – it is not a polynomial since the exponent of $\frac{3}{x}$ is a negative integer

Q7. Identify constant , linear , quadratic abd cubic polynomial from the following polynomials :

1. $f(x) = 0$
2. $g(x) = 2x^3 - 7x + 4$
3. $h(x) = -3x + \frac{1}{2}$
4. $p(x) = 2x^2 - x + 4$
5. $q(x) = 4x + 3$
6. $r(x) = 3x^3 + 4x^2 + 5x - 7$

Sol :

Given ,

1. $f(x) = 0$ – as 0 is constant , it is a constant variable
2. $g(x) = 2x^3 - 7x + 4$ – since the degree is 3 , it is a cubic polynomial
3. $h(x) = -3x + \frac{1}{2}$ – since the degree is 1 , it is a linear polynomial
4. $p(x) = 2x^2 - x + 4$ – since the degree is 2 , it is a quadratic polynomial
5. $q(x) = 4x + 3$ – since the degree is 1 , it is a linear polynomial
6. $r(x) = 3x^3 + 4x^2 + 5x - 7$ – since the degree is 3 , it is a cubic polynomial

Q8. Give one example each of a binomial of degree 25, and of a monomial of degree 100

Sol :

Given , to write the examples for binomial and monomial with the given degrees

Example of a binomial with degree 25 – $7x^{35} - 5$

Example of a monomial with degree 100 – $2t^{100}$

RD SHARMA

Solutions

Class 9 Maths

Chapter 6

Ex 6.2

Q1. If $f(x) = 2x^3 – 13x^2 + 17x + 12$, Find

1. **f(2)**
2. **f(-3)**
3. **f(0)**

Sol :

The given polynomial is $f(x) = 2x^3 – 13x^2 + 17x + 12$

1. f(2)

we need to substitute the ' 2 ' in f(x)

$f(2) = 2(2)^3 – 13(2)^2 + 17(2) + 12$

= (2 * 8) – (13 * 4) + (17 * 2) + 12

= 16 – 52 + 34 + 12

= 10

therefore f(2) = 10

2. f(-3)

we need to substitute the ' (-3) ' in f(x)

f(-3) =$2(-3)^3 – 13(-3)^2 + 17(-3) + 12$

= (2 * -27) – (13 * 9) – (17 * 3) + 12

= -54 – 117 – 51 + 12

= -210

therefore f(-3) = -210

3. f(0)

we need to substitute the ' (0) ' in f(x)

$f(0) = 2(0)^3 – 13(0)^2 + 17(0) + 12$

= (2 * 0) – (13 * 0) + (17 * 0) + 12

= 0 – 0 + 0 + 12

= 12

therefore f(0) = 12

Q2. Verify whether the indicated numbers are zeros of the polynomial corresponding to them in the following cases :

1. $f(x) = 3x + 1, x = \frac{-1}{3}$
2. $f(x) = x^2 – 1, x = (1, -1)$
3. $g(x) = 3x^2 – 2$ **, x = ($\frac{2}{\sqrt{3}}$, $\frac{-2}{\sqrt{3}}$)**
4. $p(x) = x^3 – 6x^2 + 11x – 6$ **, x = 1, 2, 3**
5. $f(x) = 5x – \pi, x = \frac{4}{5}$

6. $f(x) = x^2$ **, x = 0**
7. $f(x) = lx + m, x = \frac{-m}{l}$
8. $f(x) = 2x + 1, x = \frac{1}{2}$

Sol :

(1) $f(x) = 3x + 1, x = \frac{-1}{3}$

we know that ,

f(x) = 3x + 1

substitute x = $\frac{-1}{3}$ in f(x)

f($\frac{-1}{3}$) = 3($\frac{-1}{3}$) + 1

= -1 + 1

= 0

Since, the result is 0 x = $\frac{-1}{3}$ is the root of 3x + 1

(2) $f(x) = x^2 - 1, x = (1, -1)$

we know that,

f(x) =x^2 − 1

Given that x = (1 , -1)

substitute x = 1 in f(x)

f(1) = 1^2 − 1

= 1 − 1

= 0

Now , substitute x = (-1) in f(x)

f(-1) = $(-1)^2$ − 1

= 1 − 1

= 0

Since , the results when x = (1 , -1) are 0 they are the roots of the polynomial f(x) = x^2 − 1

(3) $g(x) = 3x^2 - 2$, x = ($\frac{2}{\sqrt{3}}$, $\frac{-2}{\sqrt{3}}$)

Sol :

We know that

g(x) = $3x^2 - 2$

Given that , x = ($\frac{2}{\sqrt{3}}$, $\frac{-2}{\sqrt{3}}$)

Substitute x = $\frac{2}{\sqrt{3}}$ in g(x)

g($\frac{2}{\sqrt{3}}$) = 3($\frac{2}{\sqrt{3}}$)^2 − 2

= $3(\frac{4}{3})$ − 2

= 4 − 2

= 2 ≠ 0

Now, Substitute x = $\frac{-2}{\sqrt{3}}$ in g(x)

g($\frac{-2}{\sqrt{3}}$) = 3($\frac{-2}{\sqrt{3}}$)$^2 - 2$

= $3(\frac{4}{3})$ − 2

= 4 − 2

= 2 ≠ 0

Since, the results when x = ($\frac{2}{\sqrt{3}}$, $\frac{-2}{\sqrt{3}}$) are not 0, they are roots of $3x^2 - 2$

(4) $p(x) = x^3 - 6x^2 + 11x - 6$, x = 1, 2, 3

Sol :

We know that ,

p(x) = $x^3 - 6x^2 + 11x - 6$

given that the values of x are 1, 2 , 3

substitute x = 1 in p(x)

p(1) = $1^3 - 6(1)^2 + 11(1) - 6$

= 1 − (6 * 1) + 11 − 6

= 1 − 6 + 11 − 6

= 0

Now, substitute x = 2 in p(x)

P(2) =$2^3 - 6(2)^2 + 11(2) - 6$

= (2 * 3) − (6 * 4) + (11 * 2) − 6

= 8 − 24 − 22 − 6

= 0

Now, substitute x = 3 in p(x)

P(3) = $3^3 - 6(3)^2 + 11(3) - 6$

= (3 * 3) − (6 * 9) + (11 * 3) − 6

= 27 − 54 + 33 − 6

= 0

Since , the result is 0 for x = 1, 2, 3 these are the roots of$x^3 - 6x^2 + 11x - 6$

(5) $f(x) = 5x - \pi, x = \frac{4}{5}$

we know that ,

f(x) =$5x - \pi$

Given that , x = $\frac{4}{5}$

Substitute the value of x in f(x)

f($\frac{4}{5}$) = 5($\frac{4}{5}$) − π

= 4 − π

≠ 0

Since , the result is not equal to zero , x = $\frac{4}{5}$ is not the root of the polynomial 5x − π

(6) $f(x) = x^2$, x = 0

Sol :

we know that , f(x) =x^2

Given that value of x is ' 0 '

Substitute the value of x in f(x)

f(0) =0^2

= 0

Since, the result is zero , x = 0 is the root of x^2

(7) $f(x) = lx + m, x = \frac{-m}{l}$

Sol :

We know that,

f(x) = lx + m

Given , that x = $\frac{-m}{l}$

Substitute the value of x in f(x)

f($\frac{-m}{l}$) = l($\frac{-m}{l}$) + m

= -m + m

= 0

Since, the result is 0 , x = $\frac{-m}{l}$ is the root of lx + m

(8) $f(x) = 2x + 1, x = \frac{1}{2}$

Sol :

We know that ,

f(x) = 2x + 1

Given that x = $\frac{1}{2}$

Substitute the value of x and f(x)

f($\frac{1}{2}$) = 2($\frac{1}{2}$) + 1

= 1 + 1

= 2 ≠ 0

Since , the result is not equal to zero

x = $\frac{1}{2}$ is the root of 2x + 1

Q3. If x = 2 is a root of the polynomial f(x) = $2x^2 - 3x + 7a$, Find the value of a

Sol :

We know that , f(x) = $2x^2 - 3x + 7a$

Given that x = 2 is the root of f(x)

Substitute the value of x in f(x)

f(2) = $2(2)^2 - 3(2) + 7a$

= (2 * 4) − 6 + 7a

= 8 − 6 + 7a

= 7a + 2

Now, equate 7a + 2 to zero

=> 7a + 2 = 0

=> 7a = -2

=> a = $\frac{-2}{7}$

The value of a = $\frac{-2}{7}$

Q4. If x = $\frac{-1}{2}$ is zero of the polynomial p(x) = $8x^3 - ax^2 - x + 2$, Find the value of a

Sol :

We know that , p(x) = $8x^3 - ax^2 - x + 2$

Given that the value of x = $\frac{-1}{2}$

Substitute the value of x in f(x)

p($\frac{-1}{2}$) = 8$(\frac{-1}{2})^3 - a(\frac{-1}{2})^2 - (\frac{-1}{2}) + 2$

$= -8(\frac{1}{8}) - a(\frac{1}{4}) + \frac{1}{2} + 2$

= -1 − ($\frac{a}{4} + \frac{1}{2}$ + 2

= 1 − ($\frac{a}{4} + \frac{1}{2}$

= $\frac{3}{2} - \frac{a}{4}$

To , find the value of a , equate p($\frac{-1}{2}$) to zero

p($\frac{-1}{2}$) = 0

$\frac{3}{2} - \frac{a}{4}$ = 0

On taking L.C.M

$\frac{6-a}{4}$ = 0

=> 6 − a = 0

=> a = 6

Q5. If x = 0 and x = -1 are the roots of the polynomial f(x) = $2x^3 - 3x^2 + ax + b$, Find the of a and b.

Sol :

We know that , f(x) = $2x^3 - 3x^2 + ax + b$

Given , the values of x are 0 and -1

Substitute x = 0 in f(x)

f(0) = $2(0)^3 - 3(0)^2 + a(0) + b$

= 0 − 0 + 0 + b

= b ——— 1

Substitute x = (-1) in f(x)

f(-1) = $2(-1)^3 - 3(-1)^2 + a(-1) + b$

= -2 − 3 − a + b

= -5 − a + b ————- 2

We need to equate equations 1 and 2 to zero

b = 0 and -5 − a + b = 0

since, the value of b is zero

substitute b = 0 in equation 2

=> -5 − a = -b

=> -5 − a = 0

a = -5

the values of a and b are -5 and 0 respectively

Q6. Find the integral roots of the polynomial f(x) = $x^3 + 6x^2 + 11x + 6$

Sol :

Given , that f(x) = $x^3 + 6x^2 + 11x + 6$

Clearly we can say that, the polynomial f(x) with an integer coefficient and the highest degree term coefficient which is known as leading factor is 1.

So, the roots of f(x) are limited to integer factor of 6, they are

±1, ±2, ±3, ±6

Let x = -1

f(-1) = $(-1)^3 + 6(-1)^2 + 11(-1) + 6$

= -1 + 6 -11 + 6

= 0

Let x = -2

f(-2) = $(-2)^3 + 6(-2)^2 + 11(-2) + 6$

= -8 − (6 * 4) − 22 + 6

= -8 + 24 − 22 + 6

= 0

Let x = -3

f(-3) = $(-3)^3 + 6(-3)^2 + 11(-3) + 6$

= -27 − (6 * 9) − 33 + 6

= -27 + 54 − 33 + 6

= 0

But from all the given factors only -1 , -2 , -3 gives the result as zero .

So, the integral multiples of $x^3 + 6x^2 + 11x + 6$ are -1 , -2 , -3

Q7. Find the rational roots of the polynomial f(x) = $2x^3 + x^2 - 7x - 6$

Sol :

Given that f(x) = $2x^3 + x^2 - 7x - 6$

f(x) is a cubic polynomial with an integer coefficient . If the rational root in the form of $\frac{p}{q}$, the values of p are limited to factors of 6 which are ±1, ±2, ±3, ±6

and the values of q are limited to the highest degree coefficient i.e 2 which are ±1, ±2

here, the possible rational roots are

±1, ±2, ±3, ±6, $\pm\frac{1}{2}, \pm\frac{3}{2}$

Let , x = -1

f(-1) = $2(-1)^3 + (-1)^2 - 7(-1) - 6$

= -2 + 1 +7 − 6

= -8 + 8

= 0

Let , x = 2

f(-2) = $2(2)^3 + (2)^2 – 7(2) – 6$

= (2 * 8) + 4 -14 − 6

= 16 + 4 -14 − 6

= 20 − 20

= 0

Let , x = $\frac{-3}{2}$

f($\frac{-3}{2}$) = $2(\frac{-3}{2})^3$ + ($\frac{-3}{2})^2$ − $7(\frac{-3}{2})$ − 6

= $2(\frac{-27}{8}) + \frac{9}{4} - 7(\frac{-3}{2}) – 6$

= $(\frac{-27}{4}) + \frac{9}{4} - (\frac{-21}{2}) – 6$

= -6.75 + 2.25 + 10.5 − 6

= 12.75 − 12.75

= 0

But from all the factors only -1 , 2 and $\frac{-3}{2}$ gives the result as zero

So, the rational roots of $2x^3 + x^2 – 7x – 6$ are -1 , 2 and $\frac{-3}{2}$

RD SHARMA
Solutions
Class 9 Maths
Chapter 6
Ex 6.3

In each of the following , using the remainder theorem, find the remainder when f(x) is divided by g(x) and verify the by actual division : (1 – 8)

Q1. f(x) = $x^3 + 4x^2 - 3x + 10$, g(x) = x + 4

Sol :

Here, f(x) = $x^3 + 4x^2 - 3x + 10$

g(x) = x + 4

from, the remainder theorem when f(x) is divided by g(x) = x – (-4) the remainder will be equal to f(-4)

Let , g(x) = 0

=> x + 4 = 0

=> x = -4

Substitute the value of x in f(x)

f(-4) = $(-4)^3 + 4(-4)^2 - 3(-4) + 10$

= -64 + (4 * 16) + 12 + 10

= -64 + 64 + 12 + 10

= 12 + 10

= 22

Therefore, the remainder is 22

Q2. f(x) = $4x^4 - 3x^3 - 2x^2 + x - 7$, g(x) = x – 1

Sol :

Here, f(x) = $4x^4 - 3x^3 - 2x^2 + x - 7$

g(x) = x – 1

from, the remainder theorem when f(x) is divided by g(x) = x – (-1) the remainder will be equal to f(1)

Let , g(x) = 0

=> x – 1 = 0

=> x = 1

Substitute the value of x in f(x)

f(1) = $4(1)^4 - 3(1)^3 - 2(1)^2 + 1 - 7$

= 4 – 3 – 2 + 1 – 7

= 5 – 12

= -7

Therefore, the remainder is 7

Q3. f(x) = $2x^4 - 6x^3 + 2x^2 - x + 2$, g(x) = x + 2

Sol :

Here, f(x) = $2x^4 - 6x^3 + 2x^2 - x + 2$

g(x) = x + 2

from, the remainder theorem when f(x) is divided by g(x) = x − (-2) the remainder will be equal to f(-2)

Let , g(x) = 0

=> x + 2 = 0

=> x = -2

Substitute the value of x in f(x)

f(-2) = $2(-2)^4 - 6(-2)^3 + 2(-2)^2 - (-2) + 2$

= (2 * 16) − (6 * (-8)) + (2 * 4) + 2 + 2

= 32 + 48 + 8 + 2 + 2

= 92

Therefore, the remainder is 92

Q4. f(x) = $4x^3 - 12x^2 + 14x - 3$, g(x) = 2x − 1

Sol:

Here, f(x) = $4x^3 - 12x^2 + 14x - 3$

g(x) = 2x − 1

from, the remainder theorem when f(x) is divided by g(x) = $2(x - \frac{1}{2})$, the remainder is equal to $f(\frac{1}{2})$

Let , g(x) = 0

=> 2x − 1 = 0

=> 2x = 1

=> x = $\frac{1}{2}$

Substitute the value of x in f(x)

$f(\frac{1}{2}) = 4(\frac{1}{2})^3 - 12(\frac{1}{2})^2 + 14(\frac{1}{2} - 3$

= $4(\frac{1}{8}) - 12(\frac{1}{4}) + 4(\frac{1}{2}) - 3$

= $(\frac{1}{2}) - 3 + 7 - 3$

= $(\frac{1}{2}) + 1$

Taking L.C.M

= $(\frac{2+1}{2})$

= $(\frac{3}{2})$

Therefore, the remainder is $(\frac{3}{2})$

Q5. f(x) = $x^3 - 6x^2 + 2x - 4$, g(x) = 1 − 2x

Sol :

Here, f(x) = $x^3 - 6x^2 + 2x - 4$

g(x) = 1 − 2x

from, the remainder theorem when f(x) is divided by g(x) = -2(x − $\frac{1}{2}$) , the remainder is equal to f($\frac{1}{2}$)

Let , g(x) = 0

=> 1 − 2x = 0

=> -2x = -1

=> 2x = 1

=> x = $\frac{1}{2}$

Substitute the value of x in f(x)

f($\frac{1}{2}$) = ($\frac{1}{2}$)³ − 6($\frac{1}{2}$)² + 2($\frac{1}{2}$) − 4

= $\frac{1}{8} - 8(\frac{1}{4}) + 2(\frac{1}{2}) - 4$

= $\frac{1}{8} - (\frac{1}{2}) + 1 - 4$

= $\frac{1}{8} - (\frac{1}{2}) - 3$

Taking L.C.M

= $\frac{1-4+8-32}{8}$

= $\frac{1-36}{8}$

= $\frac{1-36}{8}$

= $\frac{-35}{8}$

Therefore, the remainder is $\frac{-35}{8}$

Q6. f(x) = $x^4 - 3x^2 + 4$, g(x) = x − 2

Sol :

Here, f(x) = $x^4 - 3x^2 + 4$

g(x) = x − 2

from, the remainder theorem when f(x) is divided by g(x) = x − 2 the remainder will be equal to f(2)

let , g(x) = 0

=> x − 2 = 0

=> x = 2

Substitute the value of x in f(x)

f(2) = $2^4 - 3(2)^2 + 4$

= 16 − (3* 4) + 4

= 16 − 12 + 4

= 20 − 12

= 8

Therefore, the remainder is 8

Q7. f(x) = $9x^3 - 3x^2 + x - 5$, g(x) = $x - \frac{2}{3}$

Sol :

Here, f(x) = $9x^3 - 3x^2 + x - 5$

g(x) = $x - \frac{2}{3}$

from, the remainder theorem when f(x) is divided by g(x) = x − $\frac{2}{3}$ the remainder will be equal to f($\frac{2}{3}$)

substitute the value of x in f(x)

f($\frac{2}{3}$) = 9($\frac{2}{3}$) − 3($\frac{2}{3}$)² + ($\frac{2}{3}$) − 5

= $9(\frac{8}{27}) - 3(\frac{4}{9}) + \frac{2}{3} - 5$

= $(\frac{8}{3}) - (\frac{4}{3}) + \frac{2}{3} - 5$

= $\frac{8-4+2-15}{3}$

= $\frac{10-19}{3}$

= $\frac{-9}{3}$

= -3

Therefore, the remainder is -3

Q8. f(x) = $3x^4 + 2x^3 - \frac{x^3}{3} - \frac{x}{9} + \frac{2}{27}$, g(x) = $x + \frac{2}{3}$

Sol :

Here, f(x) = $3x^4 + 2x^3 - \frac{x^3}{3} - \frac{x}{9} + \frac{2}{27}$

g(x) = $x + \frac{2}{3}$

from remainder theorem when f(x) is divided by g(x) = $x - (-\frac{2}{3})$, the remainder is equal to f($-\frac{2}{3}$)

substitute the value of x in f(x)

f($-\frac{2}{3}$) = 3($-\frac{2}{3}$)^4 + 2($-\frac{2}{3}$)³ − $\frac{(-\frac{2}{3})^3}{3}$ − \(\frac{(-\frac{2}{3})}{9} + [latex]\frac{2}{27}\)\(\frac{2}{27}\]”>

= [latex]3(\frac{16}{81}) + 2(\frac{-8}{27}) − \frac{4}{(9 * 3)} − (\frac{-2}{(9 * 3)}) + \frac{2}{27}\)

= $(\frac{16}{27}) - (\frac{16}{27}) - \frac{4}{27} + (\frac{2}{27}) + \frac{2}{27}$

= $(\frac{4}{27}) - (\frac{4}{27})$

= 0

Therefore, the remainder is 0

Q9. If the polynomial $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 4x + a$ leave the same remainder when divided by x − 2 , Find the value of a

Sol :

Given , the polymials are

f(x) = $2x^3 + ax^2 + 3x - 5$

p(x) = $x^3 + x^2 - 4x + a$

The remainders are f(2) and p(2) when f(x) and p(x) are divided by x − 2

We know that,

f(2) = p(2)   (given in problem)

we need to calculate f(2) and p(2)

for, f(2)

substitute (x = 2) in f(x)

f(2) = $2(2)^3 + a(2)^2 + 3(2) - 5$

= (2 * 8) + a4 + 6 − 5

= 16 + 4a + 1

= 4a + 17 —–- 1

for, p(2)

substitute (x = 2) in p(x)

p(2) = $2^3 + 2^2 - 4(2) + a$

= 8 + 4 − 8 + a

= 4 + a ——— 2

Since, f(2) = p(2)

Equate eqn 1 and 2

=> 4a + 17 = 4 + a

=> 4a − a = 4 − 17

=> 3a = -13

=> a = $\frac{-13}{3}$

The value of a = $\frac{-13}{3}$

Q10. If polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by (x − 4) leave the remainders as R_1 and R_2 respectively. Find the values of a in each of the following cases, if

1. $R_1 = R_2$
2. $R_1 + R_2 = 0$
3. $2R_1 - R_2 = 0$

Sol :

Here, the polynomials are

$f(x) = ax^3 + 3x^2 - 3$

$p(x) = 2x^3 - 5x + a$

let,

R_1 is the remainder when f(x) is divided by x − 4

=> $R_1 = f(4)$

=> $R_1 = a(4)^3 + 3(4)^2 - 3$

= 64a + 48 − 3

= 64a + 45 ——— 1

Now , let

R_2 is the remainder when p(x) is divided by x − 4

=> $R_2 = p(4)$

=> $R_2 = 2(4)^3 - 5(4) + a$

= 128 − 20 + a

= 108 + a ——— 2

1. Given , $R_1 = R_2$

=> 64a + 45 = 108 + a

=> 63a = 63

=> a= 1

2. Given, $R_1 + R_2 = 0$

=> 64a + 45 + 108 + a = 0

=> 65a + 153 = 0

=> $a = \frac{-153}{65}$

3.Given, $2R_1 - R_2 = 0$

=> 2(64a + 45) − 108 − a = 0

=> 128a + 90 − 108 − a = 0

=> 127a − 18 = 0

=> $a = \frac{18}{127}$

Q11. If the polynomials $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + a$ when divided by (x − 2) leave the same remainder, Find the value of a

Sol :

Here , the polynomials are

f(x) = $ax^3 + 3x^2 - 13$

p(x) = $2x^3 - 5x + a$

equate , x − 2 = 0

x = 2

substitute the value of x in f(x) and p(x)

f(2) = $(2)^3 + 3(2)^2 - 13$

= 8a + 12 − 13

= 8a − 1 ———- 1

p(2) = $2(2)^3 - 5(2) + a$

= 16 − 10 + a

= 6 + a ———– 2

f(2) = p(2)

=> 8a − 1 = 6 + a

=> 8a − a = 6 + 1

=> 7a = 7

=> a = 1

The value of a = 1

Q12. Find the remainder when $x^3 + 3x^3 + 3x + 1$ is divided by,

1. x + 1
2. x − $\frac{1}{2}$
3. x
4. x + π
5. 5 + 2x

Sol :

Here, f(x) = $x^3 + 3x^2 + 3x + 1$

by remainder theorem

1. => x + 1 = 0

=> x = -1

substitute the value of x in f(x)

f(-1) = $(-1)^3 + 3(-1)^2 + 3(-1) + 1$

$= -1 + 3 - 3 + 1$

$= 0$

2. $x - \frac{1}{2}$

Sol :

Here, f(x) = $x^3 + 3x^2 + 3x + 1$

By remainder theorem

=> $x - \frac{1}{2} = 0$

=> $x = \frac{1}{2}$

substitute the value of x in f(x)

$f(\frac{1}{2}) = (\frac{1}{2})^3 + 3(\frac{1}{2})^2 + 3(\frac{1}{2}) + 1$

$= (\frac{1}{2})^3 + 3(\frac{1}{2})^2 + 3(\frac{1}{2}) + 1$

$= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1$

$= \frac{1+6+12+8}{8}$

$= \frac{27}{8}$

3. x

Sol :

Here, f(x) = $x^3 + 3x^2 + 3x + 1$

by remainder theorem

=> $x = 0$

substitute the value of x in f(x)

$f(0) = 0^3 + 3(0)^2 + 3(0) + 1$

$= 0 + 0 + 0 + 1$

$= 1$

4. $x + \pi$

Sol :

Here, f(x) = $x^3 + 3x^2 + 3x + 1$

by remainder theorem

=> $x + \pi = 0$

=> $x = -\pi$

Substitute the value of x in f(x)

$f(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$

= $-(\pi)^3 + 3(\pi)^2 - 3(\pi) + 1$

5. 5 + 2x

Sol :

Here, f(x) = $x^3 + 3x^2 + 3x + 1$

by remainder theorem

5 + 2x = 0

2x = -5

x = $\frac{-5}{2}$

substitute the value of x in f(x)

f($\frac{-5}{2}$) = ($\frac{-5}{2}$)³ + 3($\frac{-5}{2}$)² + 3($\frac{-5}{2}$) + 1

= $\frac{-125}{8} + 3(\frac{25}{4}) + 3(\frac{-5}{2}) + 1$

= $\frac{-125}{8} + \frac{75}{4} - \frac{15}{2} + 1$

Taking L.C.M

= $\frac{-125+150-50+8}{8}$

= $\frac{-27}{8}$

RD SHARMA

Solutions

Class 9 Maths

Chapter 6

Ex *6.4*

In each of the following, use factor theorem to find whether polynomial g(x) is a factor of polynomial f(x) , or not : (1 – 7)

Q1. $f(x) = x^3 - 6x^2 + 11x - 6, g(x) = x - 3$

Sol :

Here , $f(x) = x^3 - 6x^2 + 11x - 6$

g(x) = x – 3

To prove that g(x) is the factor of f(x) ,

we should show => f(3) = 0

here , x – 3 = 0

=> x = 3

Substitute the value of x in f(x)

$f(3) = 3^3 - 6 * (3)^2 + 11(3) - 6$

= 27 – (6*9) + 33 – 6

= 27 – 54 + 33 – 6

= 60 – 60

= 0

Since, the result is 0 g(x) is the factor of f(x)

Q2. $f(x) = 3x^4 + 17x^3 + 9x^2 - 7x - 10, g(x) = x + 5$

Sol :

Here , $f(x) = 3x^4 + 17x^3 + 9x^2 - 7x - 10$

g(x) = x + 5

To prove that g(x) is the factor of f(x) ,

we should show => f(-5) = 0

here , x + 5 = 0

=> x = -5

Substitute the value of x in f(x)

$f(-5) = 3(-5)^4 + 17(-5)^3 + 9(-5)^2 - 7(-5) - 10$

= (3 * 625) + (12 * (-125)) +(9*25) + 35 – 10

= 1875 – 2125 + 225 + 35 – 10

= 2135 – 2135

= 0

Since, the result is 0 g(x) is the factor of f(x)

Q3. $f(x) = x^5 + 3x^4 - x^3 - 3x^2 + 5x + 15, g(x) = x + 3$

Sol :

Here , $f(x) = x^5 + 3x^4 - x^3 - 3x^2 + 5x + 15$

g(x) = x + 3

To prove that g(x) is the factor of f(x) ,

we should show => f(-3) = 0

here , x + 3 = 0

=> x = -3

Substitute the value of x in f(x)

$f(-3) = (-3)^5 + 3(-3)^4 - (-3)^3 - 3(-3)^2 + 5(-3) + 15$

= -243 + 243 + 27 − 27 − 15 + 15

= 0

Since, the result is 0 g(x) is the factor of f(x)

Q4. $f(x) = x^3 - 6x^2 - 19x + 84, g(x) = x - 7$

Sol :

Here , $f(x) = x^3 - 6x^2 - 19x + 84$

g(x) = x − 7

To prove that g(x) is the factor of f(x) ,

we should show => f(7) = 0

here , x − 7 = 0

=> x = 7

Substitute the value of x in f(x)

$f(7) = 7^3 - 6(7)^2 - 19(7) + 84$

= 343 − (6*49) − (19*7) + 84

= 342 − 294 − 133 + 84

= 427 − 427

= 0

Since, the result is 0 g(x) is the factor of f(x)

Q5. $f(x) = 3x^3 + x^2 - 20x + 12, g(x) = 3x - 2$

Sol :

Here , $f(x) = 3x^3 + x^2 - 20x + 12$

g(x) = 3x − 2

To prove that g(x) is the factor of f(x) , we should show => $f(\frac{2}{3}) = 0$

here , $3x - 2 = 0$

=> $3x = 2$

=> $x = \frac{2}{3}$

Substitute the value of x in f(x)

$f(\frac{2}{3}) = 3(\frac{2}{3})$^{3} + $(\frac{2}{3})^2 - 20(\frac{2}{3}) + 12$

$= 3(\frac{8}{27}) + \frac{4}{9} - \frac{40}{3} + 12$

$= \frac{8}{9} + \frac{4}{9} - \frac{40}{3} + 12$

$= \frac{12}{9} - \frac{40}{3} + 12$

Taking L.C.M

$= \frac{12 - 120 + 108}{9}$

$= \frac{120 - 120}{9}$

$= 0$

Since, the result is 0 g(x) is the factor of f(x)

Q6. $f(x) = 2x^3 - 9x^2 + x + 13, g(x) = 3 - 2x$

Sol :

Here , $f(x) = 2x^3 - 9x^2 + x + 13$

$g(x) = 3 - 2x$

To prove that g(x) is the factor of f(x) ,

To prove that g(x) is the factor of f(x) ,

we should show => $f(\frac{3}{2}) = 0$

here , $3 - 2x = 0$

=> $-2x = -3$

=> $2x = 3$

=> $x = \frac{3}{2}$

Substitute the value of x in f(x)

$f(\frac{3}{2}) = 2(\frac{3}{2})^3 - 9(\frac{3}{2})^2 + (\frac{3}{2}) + 13$

$= 2(\frac{27}{8}) - 9(\frac{9}{4}) + \frac{3}{2} + 12$

$= (\frac{27}{4}) - (\frac{81}{4}) + \frac{3}{2} + 12$

Taking L.C.M

= $\frac{21-81+6+48}{4}$

= $\frac{81-81}{4}$

= 0

Since, the result is 0 g(x) is the factor of f(x)

Q7. $f(x) = x^3 - 6x^2 + 11x - 6, g(x) = x^2 - 3x + 2$

Sol :

Here , $f(x) = x^3 - 6x^2 + 11x - 6$

$g(x) = x^2 - 3x + 2$

First we need to find the factors of $x^2 - 3x + 2$

=>$x^2 - 2x - x + 2$

=> x(x − 2) -1(x − 2)

=> (x − 1) and (x − 2) are the factors

To prove that g(x) is the factor of f(x) ,

The results of f(1) and f(2) should be zero

Let , x − 1 = 0

x = 1

substitute the value of x in f(x)

$f(1) = 1^3 - 6(1)^2 + 11(1) - 6$

= 1 − 6 + 11 − 6

= 12 − 12

= 0

Let , x − 2 = 0

x = 2

substitute the value of x in f(x)

$f(2) = 2^3 - 6(2)^2 + 11(2) - 6$

= 8 − (6 * 4) + 22 − 6

= 8 − 24 + 22 − 6

= 30 − 30

= 0

Since, the results are 0 g(x) is the factor of f(x)

Q8. Show that (x − 2) , (x + 3) and (x − 4) are the factors of $x^3 - 3x^2 - 10x + 24$

Sol :

Here , f(x) = $x^3 - 3x^2 - 10x + 24$

The factors given are (x − 2) , (x + 3) and (x − 4)

To prove that g(x) is the factor of f(x) ,

The results of f(2) , f(-3) and f(4) should be zero

Let , x − 2 = 0

=> x = 2

Substitute the value of x in f(x)

f(2) = $2^3 - 3(2)^2 - 10(2) + 24$

= 8 − (3 * 4) − 20 + 24

= 8 − 12 − 20 + 24

= 32 − 32

= 0

Let , x + 3 = 0

=> x = -3

Substitute the value of x in f(x)

f(-3) = $(-3)^3 - 3(-3)^2 - 10(-3) + 24$

= -27 − 3(9) + 30 + 24

= -27 − 27 + 30 + 24

= 54 − 54

= 0

Let , x − 4 = 0

=> x = 4

Substitute the value of x in f(x)

f(4) = $(4)^3 - 3(4)^2 - 10(4) + 24$

= 64 − (3*16) − 40 + 24

= 64 − 48 − 40 + 24

= 84 − 84

= 0

Since, the results are 0 g(x) is the factor of f(x)

Q9. Show that (x + 4) , (x − 3) and (x − 7) are the factors of $x^3 - 6x^2 - 19x + 84$

Sol :

Here , f(x) = $x^3 - 6x^2 - 19x + 84$

The factors given are (x + 4) , (x − 3) and (x − 7)

To prove that g(x) is the factor of f(x) ,

The results of f(-4) , f(3) and f(7) should be zero

Let, x + 4 = 0

=> x = -4

Substitute the value of x in f(x)

f(-4) = $(-4)^3 - 6(-4)^2 - 19(-4) + 84$

= -64 − (6 * 16) − (19 * (-4)) + 84

= -64 − 96 + 76 + 84

= 160 − 160

= 0

Let, x − 3 = 0

=> x = 3

Substitute the value of x in f(x)

f(3) = $(3)^3 - 6(3)^2 - 19(3) + 84$

= 27 − (6 * 9) − (19 * 3) + 84

= 27 − 54 − 57 + 84

= 111 − 111

= 0

Let, x − 7 = 0

=> x = 7

Substitute the value of x in f(x)

f(7) = $(7)^3 - 6(7)^2 - 19(7) + 84$

= 343 − (6 * 49) − (19 * 7) + 84

= 343 − 294 − 133 + 84

= 427 − 427

= 0

Since, the results are 0 g(x) is the factor of f(x)

Q10. For what value of a is (x − 5) a factor of $x^3 - 3x^2 + ax - 10$

Sol :

Here, f(x) = $x^3 - 3x^2 + ax - 10$

By factor theorem

If (x − 5) is the factor of f(x) then , f(5) = 0

=> x − 5 = 0

=> x = 5

Substitute the value of x in f(x)

f(5) = $5^3 - 3(5)^2 + a(5) - 10$

= 125 − (3 * 25) + 5a − 10

= 125 − 75 + 5a − 10

= 5a + 40

Equate f(5) to zero

f(5) = 0

=> 5a + 40 = 0

=> 5a = -40

=> a = $\frac{-40}{5}$

= -8

When a= -8 , (x − 5) will be factor of f(x)

Q11. Find the value of a such that (x − 4) is a factor of $5x^3 - 7x^2 - ax - 28$

Sol :

Here, f(x) = $5x^3 - 7x^2 - ax - 28$

By factor theorem

If (x − 4) is the factor of f(x) then , f(4) = 0

=> x − 4 = 0

=> x = 4

Substitute the value of x in f(x)

f(4) = $5(4)^3 - 7(4)^2 - a(4) - 28$

= 5(64) − 7(16) − 4a − 28

= 320 − 112 − 4a − 28

= 180 − 4

Equate f(4) to zero, to find a

f(4) = 0

=> 180 − 4a = 0

=> -4a = -180

=> 4a = 180

=> a = $\frac{180}{4}$

=> a = 45

When a = 45 , (x − 4) will be factor of f(x)

Q12. Find the value of a, if (x + 2) is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$

Sol :

Here, f(x) = $4x^4 + 2x^3 - 3x^2 + 8x + 5a$

By factor theorem

If (x + 2) is the factor of f(x) then , f(-2) = 0

=> x + 2 = 0

=> x = -2

Substitute the value of x in f(x)

f(-2) = $4(-2)^4 + 2(-2)^3 - 3(-2)^2 + 8(-2) + 5a$

= 4(16) + 2(-8) − 3(4) − 16 + 5a

= 64 − 16 − 12 − 16 + 5a

= 5a + 20

equate f(-2) to zero

f(-2) = 0

=> 5a + 20 = 0

=> 5a = -20

=> a = $\frac{-20}{5}$

=> a = -4

When a = -4 , (x + 2) will be factor of f(x)

Q13. Find the value of k if x − 3 is a factor of $k^2x^3 - kx^2 + 3kx - k$

Sol :

Let f(x) = $k^2x^3 - kx^2 + 3kx - k$

From factor theorem if x − 3 is the factor of f(x) then f(3) = 0

=> x − 3 = 0

=> x = 3

Substitute the value of x in f(x)

f(3) = $k^2(3)^3 - k(3)^2 + 3k(3) - k$

= $27k^2 - 9k + 9k$ − k

= $27k^2 - k$

= k(27k − 1)

Equate f(3) to zero, to find k

=> f(3) = 0

=> k(27k − 1) = 0

=> k = 0 and 27k − 1 = 0

=> k = 0 and 27k = 1

=> k = 0 and k = $\frac{1}{27}$

When k = 0 and $\frac{1}{27}$, (x − 3) will be the factor of f(x)

Q14. Find the values of a and b, if x^2 − 4 is a factor of $ax^4 + 2x^3 - 3x^2 + bx - 4$

Sol :

Given , f(x) = $ax^4 + 2x^3 - 3x^2 + bx - 4$

g(x) = x^2 − 4

first we need to find the factors of g(x)

=> x^2 − 4

=> x^2 = 4

=> x = $\sqrt{4}$

=> x = ±2

(x − 2) and (x + 2) are the factors

By factor therorem if (x − 2) and (x + 2) are the factors of f(x) the result of f(2) and f(-2) should be zero

Let , x − 2 = 0

=> x = 2

Substitute the value of x in f(x)

f(2) = $a(2)^4 + 2(2)^3 - 3(2)^2 + b(2) - 4$

= 16a + 2(8) − 3(4) + 2b − 4

= 16a + 2b + 16 − 12 − 4

= 16a + 2b

Equate f(2) to zero

=> 16a + 2b = 0

=> 2(8a + b) = 0

=> 8a + b = 0 ———- 1

Let , x + 2 = 0

=> x = -2

Substitute the value of x in f(x)

f(-2) = $a(-2)^4 + 2(-2)^3 - 3(-2)^2 + b(-2) - 4$

= 16a + 2(-8) − 3(4) − 2b − 4

= 16a − 2b − 16 − 12 − 4

= 16a − 2b − 32

= 16a − 2b − 32

Equate f(2) to zero

=> 16a − 2b − 32 = 0

=> 2(8a − b) = 32

=> 8a − b = 16 ----- 2

Solve equation 1 and 2

8a + b = 0

8a − b = 16

16a = 16

a = 1

substitute a value in eq 1

8(1) + b = 0

=> b = -8

The values are a = 1 and b = -8

Q15. Find α , β if (x + 1) and (x + 2) are the factors of $x^3 + 3x^2 - 2\alpha x + \beta$

Sol:

Given, f(x) = $x^3 + 3x^2 - 2\alpha x + \beta$ and the factors are (x + 1) and (x + 2)

From factor theorem, if they are tha factors of f(x) then results of f(-2) and f(-1) should be zero

Let , x + 1 = 0

=> x = -1

Substitute value of x in f(x)

f(-1) = $(-1)^3 + 3(-1)^2 - 2\alpha(-1) + \beta$

$= -1 + 3 + 2\alpha + \beta$

$= 2\alpha + \beta$ + 2 ----- 1

Let , x + 2 = 0

=> x = -2

Substitute value of x in f(x)

f(-2) = $(-2)^3 + 3(-2)^2 - 2\alpha(-2) + \beta$

$= -8 + 12 + 4\alpha + \beta$

$= 4\alpha + \beta$ + 4 ------ 2

Solving 1 and 2 i.e (1 − 2)

=> $2\alpha + \beta + 2 - (4\alpha + \beta + 4) = 0$

=> $-2\alpha - 2 = 0$

=> $2\alpha = -2$

=> $\alpha = -1$

Substitute α= -1 in equation 1

=> $2(-1) + \beta$ = -2

=> β= -2 + 2

=> β= 0

The values are $\alpha = -1 \text{and} \beta = 0$

Q16. Find the values of p and q so that $x^4 + px^3 + 2x^2 - 3x + q$ is divisible by (x^2 – 1)

Sol :

Here , f(x) = $x^4 + px^3 + 2x^2 - 3x + q$

g(x) = $x^2 - 1$

first, we need to find the factors of $x^2 - 1$

=> $x^2 - 1$ = 0

=> $x^2 = 1$

=> x = ±1

=> (x + 1) and (x – 1)

From factor theorem , if x = 1, -1 are the factors of f(x) then f(1) = 0 and f(-1) = 0

Let us take , x + 1

=> x + 1 = 0

=> x = -1

Substitute the value of x in f(x)

f(-1) = $(-1)^4 + p(-1)^3 + 2(-1)^2 - 3(-1) + q$

= 1 – p + 2 + 3 + q

= -p + q + 6 ——- 1

Let us take , x – 1

=> x – 1 = 0

=> x = 1

Substitute the value of x in f(x)

f(1) = $(1)^4 + p(1)^3 + 2(1)^2 - 3(1) + q$

= 1 + p + 2 – 3 + q

= p + q ——- 2

Solve equations 1 and 2

-p + q = -6

p + q = 0

2q = -6

q = -3

substitute q value in equation 2

p + q = 0

p − 3 = 0

p = 3

the values of are p = 3 and q = -3

Q17. Find the values of a and b so that (x + 1) and (x − 1) are the factors of $x^4 + ax^3 - 3x^2 + 2x + b$

Sol :

Here, f(x) = $x^4 + ax^3 - 3x^2 + 2x + b$

The factors are (x + 1) and (x − 1)

From factor theorem , if x = 1, -1 are the factors of f(x) then f(1) = 0 and f(-1) = 0

Let , us take x + 1

=> x + 1 = 0

=> x = -1

Substitute value of x in f(x)

f(-1) = $(-1)^4 + a(-1)^3 - 3(-1)^2 + 2(-1) + b$

= 1 − a − 3 − 2 + b

= -a + b − 4 ——- 1

Let , us take x − 1

=> x − 1 = 0

=> x = 1

Substitute value of x in f(x)

f(1) = $(1)^4 + a(1)^3 - 3(1)^2 + 2(1) + b$

= 1 + a − 3 + 2 + b

= a + b ——- 2

Solve equations 1 and 2

-a + b = 4

a + b = 0

2b = 4

b = 2

substitute value of b in eq 2

a + 2 = 0

a = -2

the values are a = -2 and b = 2

Q18. If $x^3 + ax^2 - bx + 10$ is divisible by $x^3 - 3x + 2$, find the values of a and b

Sol :

Here , f(x) = $x^3 + ax^2 - bx + 10$

g(x) = $x^3 - 3x + 2$

first, we need to find the factors of g(x)

g(x) = $x^3 - 3x + 2$

= $x^3 - 2x - x + 2$

= x(x − 2) -1(x − 2)

= (x − 1) and (x − 2) are the factors

From factor theorem , if x = 1, 2 are the factors of f(x) then f(1) = 0 and f(2) = 0

Let, us take x − 1

=> x − 1 = 0

=> x = 1

Substitute the value of x in f(x)

f(1) = $1^3 + a(1)^2 - b(1) + 10$

= 1 + a − b + 10

= a − b + 11 —–- 1

Let, us take x − 2

=> x − 2 = 0

=> x = 2

Substitute the value of x in f(x)

f(2) = $2^3 + a(2)^2 - b(2) + 10$

= 8 + 4a − 2b + 10

= 4a − 2b + 18

Equate f(2) to zero

=> 4a − 2b + 18 = 0

=> 2(2a − b + 9) = 0

=> 2a − b + 9 ——- 2

Solve 1 and 2

a − b = -11

2a − b = -9

(-) (+) (+)

-a = -2

a = 2

substitute a value in eq 1

=> 2 − b = -11

=> − b = -11 − 2

=> -b = -13

=> b = 13

The values are a = 2 and b = 13

Q19. If both (x + 1) and (x − 1) are the factors of $ax^3 + x^2 - 2x + b$, Find the values of a and b

Sol:

Here, f(x) = $ax^3 + x^2 - 2x + b$

(x + 1) and (x − 1) are the factors

From factor theorem , if x = 1, -1 are the factors of f(x) then f(1) = 0 and f(-1) = 0

Let, x − 1= 0

=> x = -1

Substitute x value in f(x)

f(1) = $a(1)^3 + (1)^2 - 2(1) + b$

= a + 1 − 2 + b

= a + b − 1 ———- 1

Let, x + 1= 0

=> x = -1

Substitute x value in f(x)

f(-1) = $a(-1)^3 + (-1)^2 - 2(-1) + b$

= -a + 1 + 2 + b

= -a + b + 3 ———- 2

Solve equations 1 and 2

a + b = 1

-a + b = -3

2b = -2

=> b = -1

substitute b value in eq 1

=> a − 1 = 1

=> a = 1 + 1

=> a = 2

The values are a= 2 and b = -1

Q20. What must be added to $x^3 - 3x^2 - 12x + 19$ so that the result is exactly divisible by $x^2 + x - 6$

Sol :

Here , p(x) = $x^3 - 3x^2 - 12x + 19$

g(x) = $x^2 + x - 6$

by division algorithm, when p(x) is divided by g(x) , the remainder wiil be a linear expression in x

let, r(x) = ax + b is added to p(x)

=> f(x) = p(x) + r(x)

= $x^3 - 3x^2 - 12x + 19$ + ax + b

f(x) = $x^3 - 3x^2 + x(a - 12) + 19$ + b

We know that , g(x) = $x^2 + x - 6$

First, find the factors for g(x)

g(x) = $x^2 + 3x - 2x - 6$

= x(x + 3) -2(x + 3)

= (x + 3) (x − 2) are the factors

From, factor theorem when (x + 3) and (x − 2) are the factors of f(x) the f(-3) = 0 and f(2) = 0

Let, x + 3 = 0

=> x = -3

Substitute the value of x in f(x)

f(-3) = $(-3)^3 - 3(-3)^2 + (-3)(a - 12) + 19$ + b

= -27 − 27 − 3a + 24 + 19 + b

= -3a + b + 1 −−− 1

Let, x − 2 = 0

=> x = 2

Substitute the value of x in f(x)

f(2) = $(2)^3 - 3(2)^2 + (2)(a - 12) + 19$ + b

= 8 − 12 + 2a − 24 + b

= 2a + b − 9 −−− 2

Solve equations 1 and 2

-3a + b = -1

2a + b = 9

(-) (-) (-)

-5a = − 10

a = 2

substitute the value of a in eq 1

=> -3(2) + b = -1

=> -6 + b = -1

=> b = -1 + 6

=> b = 5

∴ r(x) = ax + b

= 2x + 5

∴ $x^3 - 3x^2 - 12x + 19$ is divided by $x^2 + x - 6$ when it is added by 2x + 5

Q21. What must be added to $x^3 - 6x^2 - 15x + 80$ so that the result is exactly divisible by $x^2 + x - 12$

Sol :

Let, p(x) = $x^3 - 6x^2 - 15x + 80$

q(x) = $x^2 + x - 12$

by division algorithm, when p(x) is divided by q(x) the remainder is a linear expression in x.

so, let r(x) = ax + b is subtracted from p(x), so that p(x) − q(x) is divisible by q(x)

let f(x) = p(x) − q(x)

q(x) = $x^2 + x - 12$

= $x^2 + 4x - 3x - 12$

= x(x + 4) (-3)(x + 4)

= (x+4) , (x − 3)

clearly, (x − 3) and (x + 4) are factors of q(x)

so, f(x) wiil be divisible by q(x) if (x − 3) and (x + 4) are factors of q(x)

from , factor theorem

f(-4) = 0 and f(3) = 0

=> f(3) = $3^3 - 6(3)^2 - 3(a + 15) + 80$ − b = 0

= 27 − 54 -3a -45 + 80 −b

= -3a −b + 8 ——— 1

Similarly,

f(-4) = 0

=> f(-4) => $(-4)^3 - 6(-4)^2 - (-4)(a + 15) + 80$ – b = 0

=> -64 – 96 -4a + 60 + 80 –b = 0

=> 4a – b – 20 = 0 ——- 2

Substract eq 1 and 2

=> 4a – b – 20 – 8 + 3a + b = 0

=> 7a – 28 = 0

=> a = $\frac{28}{7}$

=> a= 4

Put a = 4 in eq 1

=> -3(4) – b = -8

=> -b – 12 = -8

=> -b = -8 + 12

=> b = -4

Substitute a and b values in r(x)

=> r(x) = ax + b

= 4x – 4

Hence, p(x) is divisible by q(x) , if r(x) = 4x – 4 is subtracted from it

Q22. What must be added to $3x^3 + x^2 - 22x + 9$ so that the result is exactly divisible by $3x^2 + 7x - 6$

Sol :

Let, p(x) = $3x^3 + x^2 - 22x + 9$ and q(x) = $3x^2 + 7x - 6$

By division theorem, when p(x) is divided by q(x) , the remainder is a linear equation in x.

Let, r(x) = ax + b is added to p(x) , so that p(x) + r(x) is divisible by q(x)

f(x) = p(x) + r(x)

=> f(x) = $3x^3 + x^2 - 22x + 9(ax + b)$

=> = $3x^3 + x^2 + x(a - 22) + b + 9$

We know that,

q(x) = $3x^2 + 7x - 6$

= $3x^2 + 9x - 2x - 6$

= 3x(x+3) – 2(x+3)

= (3x-2) (x+3)

So, f(x) is divided by q(x) if (3x-2) and (x+3) are the factors of f(x)

From, factor theorem

$f(\frac{2}{3}) = 0$ and $f(-3) = 0$

let , $3x - 2 = 0$

$3x = 2$

$x = \frac{2}{3}$

=> $f(\frac{2}{3}) = 3(\frac{2}{3})^3 + (\frac{2}{3})$^{2} + $(\frac{2}{3})(a - 22) + b + 9$

= $3(\frac{8}{27}) + \frac{4}{9} + \frac{2}{3}a - \frac{44}{3} + b + 9$

= $\frac{12}{9} + \frac{2}{3}a - \frac{44}{3} + b + 9$

= $\frac{12+6a-132+9b+81}{9}$

Equate to zero

=> $\frac{12+6a-132+9b+81}{9} = 0$

=> $6a + 9b - 39 = 0$

=> $3(2a + 3b - 13) = 0$

=> $2a + 3b - 13 = 0$ ——–- 1

Similarly,

Let, $x + 3 = 0$

=> $x = -3$

=> $f(-3) = 3(-3)^3 + (-3)^2 + (-3)(a-22) + b + 9$

= -81 + 9 -3a + 66 + b + 9

= -3a + b + 3

Equate to zero

-3a + b + 3 = 0

Multiply by 3

-9a + 3b + 9 = 0 ——– 2

Substact eq 1 from 2

=> -9a + 3b + 9 -2a – 3b + 13 = 0

=> -11a + 22 = 0

=> -11a = -22

=> $a = \frac{22}{11}$

=> a = 2

Substitute a value in eq 1

=> -3(2) + b = -3

=> -6 + b = -3

=> b = -3 + 6

=> b = 3

Put the values in r(x)

r(x) = ax + b

= 2x + 3

Hence, p(x) is divisible by q(x) , if r(x) = 2x + 3 is added to it

Q23. If x − 2 is a factor of each of the following two polynomials , find the value of a in each case :

1. $x^3 - 2ax^2 + ax - 1$
2. $x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$

Sol :

(1) let f(x) = $x^3 - 2ax^2 + ax - 1$

from factor theorem

if (x − 2) is the factor of f(x) the f(2) = 0

let , x − 2 = 0

=> x = 2

Substitute x value in f(x)

f(2) = $2^3 - 2a(2)^2 + a(2) - 1$

= 8 − 8a + 2a − 1

= -6a + 7

Equate f(2) to zero

=> -6a + 7 = 0

=> -6a = -7

=> a= $\frac{7}{6}$

When , (x − 2) is the factor of f(x) then a= $\frac{7}{6}$

(2) Let, f(x) = $x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$

from factor theorem

if (x − 2) is the factor of f(x) the f(2) = 0

let , x − 2 = 0

=> x = 2

Substitute x value in f(x)

f(2) = $2^5 - 3(2)^4 - a(2)^3 + 3a(2)^2 + 2a(2) + 4$

= 32 − 48 − 8a + 12 + 4a + 4

= 8a − 12

Equate f(2) to zero

=> 8a − 12 = 0

=> 8a = 12

=> a = $\frac{12}{8}$

= $\frac{3}{2}$

So, when (x − 2) is a factor of f(x) then a= $\frac{3}{2}$

Q24. In each of the following two polynomials , find the value of a, if (x − a) is a factor :

1. $x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$
2. $x^5 - a^2x^3 + 2x + a + 1$

Sol :

(1) $x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$

let , f(x) = $x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$

here , x − a = 0

=> x = a

Substitute the value of x in f(x)

f(a) = $a^6 - a(a)^5 + (a)^4 - a(a)^3 + 3(a) - a + 2$

= $a^6 - a^6 + (a)^4 - a^4 + 3(a) - a + 2$

= 2a + 2

Equate to zero

=> 2a + 2 = 0

=> 2(a + 1) = 0

=> a = -1

So, when (x − a) is a factor of f(x) then a = -1

(2) $x^5 - a^2x^3 + 2x + a + 1$

let, $f(x) = x^5 - a^2x^3 + 2x + a + 1$

here , x − a = 0

=> x = a

Substitute the value of x in f(x)

f(a) = $a^5 - a^2a^3 + 2(a) + a + 1$

= $a^5 - a^5 + 2a + a + 1$

= 3a + 1

Equate to zero

=> 3a + 1 = 0

=> 3a = -1

=> a= $\frac{-1}{3}$

So, when (x − a) is a factor of f(x) then a = $\frac{-1}{3}$

Q25. In each of the following two polynomials , find the value of a, if (x + a) is a factor :

1. $x^3 + ax^2 - 2x + a + 4$
2. $x^4 - a^2x^2 + 3x - a$

Sol :

(1) $x^3 + ax^2 - 2x + a + 4$

let, f(x) = $x^3 + ax^2 - 2x + a + 4$

here , x + a = 0

=> x = -a

Substitute the value of x in f(x)

f(-a) = $(-a)^3 + a(-a)^2 - 2(-a) + a + 4$

= $(-a)^3 + a^3 - 2(-a) + a + 4$

= 3a + 4

Equate to zero

=> 3a + 4 = 0

=> 3a = -4

=> a = $\frac{-4}{3}$

So, when (x + a) is a factor of f(x) then a = $\frac{-4}{3}$

(2) $x^4 - a^2x^2 + 3x - a$

let, $f(x) = x^4 - a^2x^2 + 3x - a$

here , x + a = 0

=> x = -a

Substitute the value of x in f(x)

f(-a) = $(-a)^4 - a^2(-a)^2 + 3(-a) - a$

= $a^4 - a^4 - 3(a) - a$

= -4a

Equate to zero

=> -4a = 0

=> a = 0

So, when (x + a) is a factor of f(x) then a = 0

RD SHARMA

Solutions

Class 9 Maths

Chapter 6

Ex 6.5

Using factor theorem, factorize each of the following polynomials :

Q1. $x^3 + 6x^2 + 11x + 6$

Sol:

Given polynomial, f(x) = $x^3 + 6x^2 + 11x + 6$

The constant term in f(x) is 6

The factors of 6 are ±1, ±2, ±3, ±6

Let, x + 1 = 0

=> x = -1

Substitute the value of x in f(x)

f(-1) = $)(-1)^3 + 6(-1)^2 + 11(-1) + 6$

= -1 + 6 -11 + 6

= 12 − 12

= 0

So, (x + 1) is the factor of f(x)

Similarly, (x + 2) and (x + 3) are also the factors of f(x)

Since, f(x) is a polynomial having a degree 3, it cannot have more than three linear factors.

∴ f(x) = k(x + 1)(x + 2)(x + 3)

=> $x^3 + 6x^2 + 11x + 6$ = k(x + 1)(x + 2)(x + 3)

Substitute x = 0 on both the sides

=> 0 + 0 + 0 + 6 = k(0 +1)(0 + 2)(0 + 3)

=>   6 = k(1*2*3)

=> 6 = 6k

=> k = 1

Substitute k value in f(x) = k(x + 1)(x + 2)(x + 3)

=> f(x) = (1)(x + 1)(x + 2)(x + 3)

=> f(x) = (x + 1)(x + 2)(x + 3)

∴ $x^3 + 6x^2 + 11x + 6$ = (x + 1)(x + 2)(x + 3)

Q2. $x^3 + 2x^2 - x - 2$

Sol:

Given, f(x) = $x^3 + 2x^2 - x - 2$

The constant term in f(x) is -2

The factors of (-2) are ±1, ±2

Let , x − 1 = 0

=> x = 1

Substitute the value of x in f(x)

f(1) = $(1)^3 + 2(1)^2 - 1 - 2$

= 1 + 2 − 1 − 2

= 0

Similarly , the other factors (x + 1) and (x + 2) of f(x)

Since, f(x) is a polynomial having a degree 3, it cannot have more than three linear factors.

∴ f(x) = k(x − 1)(x + 2)(x + 1)

$x^3 + 2x^2 - x - 2$ = k(x − 1)(x + 2)(x + 1)

Substitute x = 0 on both the sides

0 + 0 − 0 − 2 = k(-1)(1)(2)

=>   − 2 = -2k

=>   k = 1

Substitute k value in f(x) = k(x − 1)(x + 2)(x + 1)

f(x) = (1)(x − 1)(x + 2)(x + 1)

=> f(x) = (x − 1)(x + 2)(x + 1)

So, $x^3 + 2x^2 - x - 2$ = (x − 1)(x + 2)(x + 1)

Q3. $x^3 - 6x^2 + 3x + 10$

Sol:

Let, f(x) = $x^3 - 6x^2 + 3x + 10$

The constant term in f(x) is 10

The factors of 10 are ±1, ±2, ±5, ±10

Let , x + 1 = 0

=> x = -1

Substitute the value of x in f(x)

f(-1) = $(-1)^3 - 6(-1)^2 + 3(-1) + 10$

= -1 − 6 − 3 + 10

= 0

Similarly , the other factors (x − 2) and (x − 5) of f(x)

Since, f(x) is a polynomial having a degree 3, it cannot have more than three linear factors.

∴ f(x) = k(x + 1)(x − 2)(x − 5)

Substitute x = 0 on both sides

=> $x^3 - 6x^2 + 3x + 10$ = k(x + 1)(x − 2)(x − 5)

=> 0 − 0 + 0 + 10 = k(1)(-2)(-5)

=> 10 = k(10)

=> k = 1

Substitute k = 1 in f(x) = k(x + 1)(x − 2)(x − 5)

f(x) = (1)(x + 1)(x − 2)(x − 5)

so, $x^3 - 6x^2 + 3x + 10$ = (x + 1)(x − 2)(x − 5)

Q4. $x^4 - 7x^3 + 9x^2 + 7x - 10$

Sol:

Given, f(x) = $x^4 - 7x^3 + 9x^2 + 7x - 10$

The constant term in f(x) is 10

The factors of 10 are ±1, ±2, ±5, ±10

Let , x − 1 = 0

=> x = 1

Substitute the value of x in f(x)

f(x) = $1^4 - 7(1)^3 + 9(1)^2 + 7(1) - 10$

= 1 − 7 + 9 + 7 − 10

= 10 − 10

= 0

(x − 1) is the factor of f(x)

Simarly, the other factors are (x + 1) ,(x − 2) , (x − 5)

Since, f(x) is a polynomial of degree 4. So, it cannot have more than four linear factor.

So, f(x) = k(x − 1)(x + 1)(x − 2)(x − 5)

=> $x^4 - 7x^3 + 9x^2 + 7x - 10$ = k(x − 1)(x + 1)(x − 2)(x − 5)

Put x = 0 on both sides

0 − 0 + 0 − 10 = k(-1)(1)(-2)(-5)

− 10 = k(-10)

=> k = 1

Substitute k = 1 in f(x) = k(x − 1)(x + 1)(x − 2)(x − 5)

f(x) = (1)(x − 1)(x + 1)(x − 2)(x − 5)

= (x − 1)(x + 1)(x − 2)(x − 5)

So, $x^4 - 7x^3 + 9x^2 + 7x - 10$ = (x − 1)(x + 1)(x − 2)(x − 5)

Q5. $x^4 - 2x^3 - 7x^2 + 8x + 12$

Sol:

Given , f(x) = $x^4 – 2x^3 – 7x^2 + 8x + 12$

The constant term f(x) is equal is 12

Tha factors of 12 are ±1, ±2, ±3, ±4, ±6, ±12

Let, x + 1 = 0

=> x = -1

Substitute the value of x in f(x)

f(-1) = $(-1)^4 – 2(-1)^3 – 7(-1)^2 + 8(-1) + 12$

= 1 + 2 − 7 − 8 + 12

= 0

So, x + 1 is factor of f(x)

Similarly, (x + 2), (x − 2), (x − 3) are also the factors of f(x)

Since, f(x) is a polynomial of degree 4 , it cannot have more than four linear factors.

=> f(x) = k(x + 1)(x + 2)(x − 3)(x − 2)

=> $x^4 – 2x^3 – 7x^2 + 8x + 12$ = k(x + 1)(x + 2)(x − 3)(x − 2)

Substitute x = 0 on both sides,

=> 0 − 0 − 0 + 12 = k(1)(2)(-2)(-3)

=> 12 = k12

=> k = 1

Substitute k = 1 in f(x) = k(x − 2)(x + 1)(x + 2)(x − 3)

f(x) = (x − 2)(x + 1)(x + 2)(x − 3)

so, $x^4 – 2x^3 – 7x^2 + 8x + 12$ = (x − 2)(x + 1)(x + 2)(x − 3)

Q6. $x^4 + 10x^3 + 35x^2 + 50x + 24$

Sol:

Given, f(x) = $x^4 + 10x^3 + 35x^2 + 50x + 24$

The constant term in f(x) is equal to 24

The factors of 24 are ±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24

Let, x + 1 = 0

=> x = -1

Substitute the value of x in f(x)

f(-1) = $(-1)^4 + 10(-1)^3 + 35(-1)^2 + 50(-1) + 24$

= 1 − 10 + 35 − 50 + 24

= 0

=> (x + 1) is the factor of f(x)

Similarly, (x + 2),(x + 3),(x + 4) are also the factors of f(x)

Since, f(x) is a polynomial of degree 4, it cannot have more than four linear factors.

=> f(x) = k(x + 1)(x + 2)(x + 3)(x + 4)

=> $x^4 + 10x^3 + 35x^2 + 50x + 24$ = k(x + 1)(x + 2)(x + 3)(x + 4)

Substitute x = 0 on both sides

=> 0 + 0 + 0 + 0 + 24 = k(1)(2)(3)(4)

=> 24 = k(24)

=> k = 1

Substitute k = 1 in f(x) = k(x + 1)(x + 2)(x + 3)(x + 4)

f(x) = (1)(x + 1)(x + 2)(x + 3)(x + 4)

f(x) = (x + 1)(x + 2)(x + 3)(x + 4)

hence, $x^4 + 10x^3 + 35x^2 + 50x + 24$ = (x + 1)(x + 2)(x + 3)(x + 4)

Q7. $2x^4 - 7x^3 - 13x^2 + 63x - 45$

Sol :

Given, f(x) = $2x^4 - 7x^3 - 13x^2 + 63x - 45$

The factors of constant term -45 are ±1, ±3, ±5, ±9, ±15, ±45

The factors of the coefficient of x^{4} is 2. Hence possible rational roots of f(x) are

±1, ±3, ±5, ±9, ±15, ±45, $\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{9}{2}, \pm\frac{15}{2}, \pm\frac{45}{2}$

Let, x − 1 = 0

=> x= 1

f(1) = $2(1)^4 - 7(1)^3 - 13(1)^2 + 63(1) - 45$

= 2 − 7 − 13 + 63 − 45

= 0

Let, x − 3 = 0

=> x = 3

f(3) = $2(3)^4 - 7(3)^3 - 13(3)^2 + 63(3) - 45$

= 162 − 189 − 117 + 189 − 45

= 0

So, (x − 1) and (x − 3) are the roots of f(x)

=> x^2 − 4x + 3 is the factor of f(x)

Divide f(x) with x^2 − 4x + 3 to get other three factors

By long division,

$2x^2$ + x − 15

$x^2 - 4x + 3 \quad 2x^4 - 7x^3 - 13x^2 + 63x - 45$

$2x^4 - 8x^3 + 6x^2$

(-) (+) (-)

$x^3 - 19x^2 + 63x$

$x^3 - 4x^2 + 3x$

(-) (+) (-)

$-15x^2 + 60x - 45$

$-15x^2 + 60x - 45$

(+) (-) (+)

0

=> $2x^4 - 7x^3 - 13x^2 + 63x - 45 = (x^{2} - 4x + 3)(2x^{2} + x - 15)$

=> $2x^4 - 7x^3 - 13x^2 + 63x - 45 = (x - 1)(x - 3)(2x^{2} + x - 15)$

Now,

$2x^2 + x - 15 = 2x^{2} + 6x - 5x - 15$

$= 2x(x + 3) - 5(x + 3)$

$= (2x - 5)(x + 3)$

So, $2x^4 - 7x^3 - 13x^2 + 63x - 45 = (x - 1)(x - 3)(x + 3)(2x - 5)$

Q8. $3x^3 - x^2 - 3x + 1$

Sol :

Given , $f(x) = 3x^3 - x^2 - 3x + 1$

The factors of constant term 1 is ±1

The factors of the coefficient of $x^2 = 3$

The possible rational roots are $\pm 1, \frac{1}{3}$

Let, $x - 1 = 0$

=> $x = 1$

$f(1) = 3(1)^3 - (1)^2 - 3(1) + 1$

$= 3 - 1 - 3 + 1$

$= 0$

So, $x - 1$ is tha factor of $f(x)$

Now, divide $f(x)$ with $(x - 1)$ to get other factors

By long division method,

$3x^2 + 2x - 1$

$x - 1 \quad 3x^3 - x^2 - 3x + 1$

$3x^3 - x^2$

(-) (+)

$2x^2 - 3x$

$2x^2 - 2x$

(-) (+)

$-x + 1$

$-x + 1$

(+) (-)

0

=> $3x^3 - x^2 - 3x + 1 = (x - 1)(3x^2 + 2x - 1)$

Now,

$3x^2 + 2x - 1 = 3x^2 + 3x - x - 1$

$= 3x(x + 1) - 1(x + 1)$

$= (3x - 1)(x + 1)$

Hence , $3x^3 - x^2 - 3x + 1 = (x - 1)(3x - 1)(x + 1)$

Q9. $x^3 - 23x^2 + 142x - 120$

Sol :

Let, $f(x) = x^3 - 23x^2 + 142x - 120$

The constant term in f(x) is -120

The factors of -120 are ±1, ±2, ±3, ±4, ±5, ±6, ±8, ±10, ±12, ±15, ±20, ±24, ±30, ±40, ±60, ±120

Let, $x - 1 = 0$

=> $x = 1$

$f(1) = (1)^3 - 23(1)^2 + 142(1) - 120$

$= 1 - 23 + 142 - 120$

$= 0$

So, $(x - 1)$ is the factor of f(x)

Now, divide f(x) with $(x - 1)$ to get other factors

By long division,

$x^2 - 22x + 120$

$x - 1 \quad x^3 - 23x^2 + 142x - 120$

$x^3 - x^2$

(-) (+)

$-22x^2 + 142x$

$-22x^2 + 22x$

(+) (-)

$120x - 120$

$120x - 120$

(-) (+)

0

=> $x^3 - 23x^2 + 142x - 120 = (x - 1)(x^2 - 22x + 120)$

Now,

$x^2 - 22x + 120 = x^2 - 10x - 12x + 120$

$= x(x - 10) - 12(x - 10)$

$= (x - 10)(x - 12)$

Hence, $x^3 - 23x^2 + 142x - 120 = (x - 1)(x - 10)(x - 12)$

Q10. $y^3 - 7y + 6$

Sol :

Given, $f(y) = y^3 - 7y + 6$

The constant term in f(y) is 6

The factors are ±1, ±2, ±3, ±6

Let , $y - 1 = 0$

=> $y = 1$

$f(1) = (1)^3 - 7(1) + 6$

$= 1 - 7 + 6$

$= 0$

So, $(y - 1)$ is the factor of f(y)

Similarly, $(y - 2)$ and $(y + 3)$ are also the factors

Since, f(y) is a polynomial which has degree 3, it cannot have more than 3 linear factors

=> $f(y) = k(y - 1)(y - 2)(y + 3)$

=> $y^3 - 7y + 6 = k(y - 1)(y - 2)(y + 3)$ ---- 1

Substitute k = 0 in eq 1

=> $0 - 0 + 6 = k(-1)(-2)(3)$

=> $6 = 6k$

=> $k = 1$

$y^3 - 7y + 6 = (1)(y - 1)(y - 2)(y + 3)$

$y^3 - 7y + 6 = (y - 1)(y - 2)(y + 3)$

Hence, $y^3 - 7y + 6$ = (y − 1)(y − 2)(y + 3)

Q11. $x^3 - 10x^2 - 53x - 42$

Sol :

Given , f(x) = $x^3 - 10x^2 - 53x - 42$

The constant in f(x) is -42

The factors of -42 are ±1, ±2, ±3, ±6, ±7, ±14, ±21,±42

Let, x + 1 = 0

=> x = -1

f(-1) = $(-1)^3 - 10(-1)^2 - 53(-1) - 42$

= -1 − 10 + 53 − 42

= 0

So., (x + 1) is the factor of f(x)

Now, divide f(x) with (x + 1) to get other factors

By long division,

$x^2 - 11x - 42$

$x + 1 \quad x^3 - 10x^2 - 53x - 42$

$x^3 + x^2$

(-) (-)

$-11x^2 - 53x$

$-11x^2 - 11x$

(+) (+)

-42x − 42

-42x − 42

(+) (+)

0

=> $x^3 - 10x^2 - 53x - 42 = (x + 1)(x^2 - 11x - 42)$

Now,

$x^2 - 11x - 42 = x^2 - 14x + 3x - 42$

= x(x − 14) + 3(x − 14)

= (x + 3)(x − 14)

Hence, $x^3 - 10x^2 - 53x - 42 = (x + 1)(x + 3)(x - 14)$

Q12. $y^3 - 2y^2 - 29y - 42$

Sol :

Given, f(x) = $y^3 - 2y^2 - 29y - 42$

The constant in f(x) is -42

The factors of -42 are ±1, ±2, ±3, ±6, ±7, ±14, ±21,±42

Let, y + 2 = 0

=> y = -2

f(-2) = $(-2)^3 - 2(-2)^2 - 29(-2) - 42$

= -8 -8 + 58 – 42

= 0

So, (y + 2) is the factor of f(y)

Now, divide f(y) with (y + 2) to get other factors

By, long division

$y^2 - 4y - 21$

y + 2 $y^3 - 2y^2 - 29y - 42$

$y^3 + 2y^2$

(-)   (-)

$-4y^2 - 29y$

$-4y^2 - 8y$

(+)   (+)

-21y – 42

-21y – 42

(+)   (+)

0

=> $y^3 - 2y^2 - 29y - 42 = (y + 2)(y^2 - 4y - 21)$

Now,

$y^2 - 4y - 21 = y^2 - 7y + 3y - 21$

= y(y – 7) +3(y – 7)

= (y – 7)(y + 3)

Hence, $y^3 - 2y^2 - 29y - 42 = (y + 2)(y - 7)(y + 3)$

Q13. $2y^3 - 5y^2 - 19y + 42$

Sol :

Given, f(x) = $2y^3 - 5y^2 - 19y + 42$

The constant in f(x) is +42

The factors of 42 are ±1, ±2, ±3, ±6, ±7, ±14, ±21,±42

Let, y – 2 = 0

=> y = 2

f(2) = $2(2)^3 - 5(2)^2 - 19(2) + 42$

= 16 – 20 – 38 + 42

= 0

So, (y – 2) is the factor of f(y)

Now, divide f(y) with (y – 2) to get other factors

By, long division method

$2y^2 - y - 21$

$y - 2 \quad 2y^3 - 5y^2 - 19y + 42$

$2y^3 - 4y^2$

(-) (+)

$-y^2 - 19y$

$-y^2 + 2y$

(+) (-)

-21y + 42

-21y + 42

(+) (-)

0

=> $2y^3 - 5y^2 - 19y + 42 = (y - 2)(2y^2 - y - 21)$

Now,

$2y^2 - y - 21$

The factors are (y + 3) (2y – 7)

Hence, $2y^3 - 5y^2 - 19y + 42 = (y - 2)(y + 3)(2y - 7)$

Q14. $x^3 + 13x^2 + 32x + 20$

Sol:

Given, f(x) = $x^3 + 13x^2 + 32x + 20$

The constant in f(x) is 20

The factors of 20 are ±1, ±2, ±4, ±5, ±10, ±20

Let, x + 1 = 0

=> x = -1

f(-1) = $(-1)^3 + 13(-1)^2 + 32(-1) + 20$

= -1 + 13 – 32 + 20

= 0

So, (x + 1) is the factor of f(x)

Divide f(x) with (x + 1) to get other factors

By, long division

$x^2 + 12x + 20$

$x + 1 \quad x^3 + 13x^2 + 32x + 20$

$x^3 + x^2$

(-) (-)

$12x^2 + 32x$

$12x^2 + 12x$

(-) (-)

20x – 20

20x – 20

(-) (-)

0

=> $x^3 + 13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20)$

Now,

$x^2 + 12x + 20 = x^2 + 10x + 2x + 20$

= x(x + 10) + 2(x + 10)

The factors are (x + 10) and (x + 2)

Hence, $x^3 + 13x^2 + 32x + 20 = (x + 1)(x + 10)(x + 2)$

Q15. $x^3 - 3x^2 - 9x - 5$

Sol :

Given, f(x) = $x^3 - 3x^2 - 9x - 5$

The constant in f(x) is -5

The factors of -5 are ±1, ±5

Let, x + 1 = 0

=> x = -1

f(-1) = $(-1)^3 - 3(-1)^2 - 9(-1) - 5$

= -1 – 3 + 9 – 5

= 0

So, (x + 1) is the factor of f(x)

Divide f(x) with (x + 1) to get other factors

By, long division

$x^2 - 4x - 5$

$x + 1 \quad x^3 - 3x^2 - 9x - 5$

$x^3 + x^2$

(-)   (-)

$-4x^2 - 9x$

$-4x^2 - 4x$

(+)   (+)

$-5x - 5$

$-5x - 5$

(+)   (+)

0

=> $x^3 - 3x^2 - 9x - 5 = (x + 1)(x^2 - 4x - 5)$

Now,

$x^2 - 4x - 5 = x^2 - 5x + x - 5$

$= x(x - 5) + 1(x - 5)$

The factors are $(x - 5)$ and $(x + 1)$

Hence, $x^3 - 3x^2 - 9x - 5 = (x + 1)(x - 5)(x + 1)$

Q16. $2y^3 + y^2 - 2y - 1$

Sol :

Given , $f(y) = 2y^3 + y^2 - 2y - 1$

The constant term is 2

The factors of 2 are $\pm1, \pm\frac{1}{2}$

Let, $y - 1 = 0$

=> $y = 1$

$f(1) = 2(1)^3 + (1)^2 - 2(1) - 1$

$= 2 + 1 - 2 - 1$

$= 0$

So, $(y - 1)$ is the factor of $f(y)$

Divide $f(y)$ with $(y - 1)$ to get other factors

By, long division

$2y^2 + 3y + 1$

$y - 1 \quad 2y^3 + y^2 - 2y - 1$

$2y^3 - 2y^2$

(-) (+)

$3y^2 - 2y$

$3y^2 - 3y$

(-) (+)

y – 1

y – 1

(-) (+)

0

=> $2y^3 + y^2 - 2y - 1 = (y - 1)(2y^2 + 3y + 1)$

Now,

$2y^2 + 3y + 1 = 2y^2 + 2y + y + 1$

$= 2y(y + 1) + 1(y + 1)$

$= (2y + 1)(y + 1)$ are the factors

Hence, $2y^3 + y^2 - 2y - 1 = (y - 1)(2y + 1)(y + 1)$

Q17. $x^3 - 2x^2 - x + 2$

Sol :

Let, f(x) = $x^3 - 2x^2 - x + 2$

The constant term is 2

The factors of 2 are ±1, $\pm\frac{1}{2}$

Let, x – 1= 0

=> x = 1

f(1) = $(1)^3 - 2(1)^2 - (1) + 2$

= 1 – 2 – 1 + 2

= 0

So, (x – 1) is the factor of f(x)

Divide f(x) with (x – 1) to get other factors

By, long division

$x^2 - x - 2$

$x - 1$ $x^3 - 2x^2 - y + 2$

$x^3 - x^2$

(-) (+)

$-x^2 - x$

$-x^2 + x$

(+) (-)

$-2x + 2$

$-2x + 2$

(+) (-)

0

=> $x^3 - 2x^2 - y + 2 = (x - 1)(x^2 - x - 2)$

Now,

$x^2 - x - 2 = x^2 - 2x + x - 2$

$= x(x - 2) + 1(x - 2)$

$=(x - 2)(x + 1)$ are the factors

Hence, $x^3 - 2x^2 - y + 2 = (x - 1)(x + 1)(x - 2)$

Q18. Factorize each of the following polynomials :

1. $x^3 + 13x^2 + 31x - 45$ **given that x + 9 is a factor**
2. $4x^3 + 20x^2 + 33x + 18$ **given that 2x + 3 is a factor**

Sol :

1. $x^3 + 13x^2 + 31x - 45$ given that x + 9 is a factor

let, f(x) = $x^3 + 13x^2 + 31x - 45$

given that (x + 9) is the factor

divide f(x) with (x + 9) to get other factors

by , long division

$x^2 + 4x - 5$

$x + 9 \quad x^3 + 13x^2 + 31x - 45$

$x^3 + 9x^2$

(-) (-)

$4x^2 + 31x$

$4x^2 + 36x$

(-) (-)

$-5x - 45$

$-5x - 45$

(+) (+)

0

=> $x^3 + 13x^2 + 31x - 45 = (x + 9)(x^2 + 4x - 5)$

Now,

$x^2 + 4x - 5 = x^2 + 5x - x - 5$

$= x(x + 5) - 1(x + 5)$

$= (x + 5)(x - 1)$ are the factors

Hence, $x^3 + 13x^2 + 31x - 45 = (x + 9)(x + 5)(x - 1)$

2. $4x^3 + 20x^2 + 33x + 18$ given that 2x + 3 is a factor

let, $f(x) = 4x^3 + 20x^2 + 33x + 18$

given that 2x + 3 is a factor

divide f(x) with (2x + 3) to get other factors

by, long division

$2x^2 + 7x + 6$

$2x + 3 \quad 4x^3 + 20x^2 + 33x + 18$

$4x^3 + 6x^2$

(-) (-)

$14x^2 - 33x$

$14x^2 - 21x$

(-) (+)

$12x + 18$

$12x + 18$

(-) (-)

0

$\Rightarrow 4x^3 + 20x^2 + 33x + 18 = (2x + 3)(2x^2 + 7x + 6)$

Now,

$2x^2 + 7x + 6 = 2x^2 + 4x + 3x + 6$

$= 2x(x + 2) + 3(x + 2)$

$= (2x + 3)(x + 2)$ are the factors

Hence, $4x^3 + 20x^2 + 33x + 18 = (2x + 3)(2x + 3)(x + 2)$