RD SHARMA

Solutions

Class 9 Maths

Chapter 5

Ex 5.1

Q1 . $x^3 + x - 3x^2 - 3$

SOLUTION :

Taking x common in $x^3 + x$

$= x(x^2 + 1) - 3x^2 - 3$

Taking -3 common in $-3x^2 - 3$

$= x(x^2 + 1) - 3(x^2 + 1)$

Now , we take $(x^2 + 1)$ common

$= (x^2 + 1)(x - 3)$

$\therefore x^3 + x - 3y^2 - 3 = (x^2 + 1)(x - 3)$

Q2 . $a(a + b)^3 - 3a^2b(a + b)$

SOLUTION :

Taking (a + b) common in the two terms

$= (a + b)\{a(a + b)^2 - 3a^2b\}$

Now, using $(a + b)^2 = a^2 + b^2 + 2ab$

$= (a + b)\{a(a^2 + b^2 + 2ab) - 3a^2b\}$

$= (a + b)\{a^3 + ab^2 + 2a^2b - 3a^2b\}$

$= (a + b)\{a^3 + ab^2 - a^2b\}$

$= (a + b)\, p\,\{a^2 + b^2 - ab\}$

$= p(a + b)(a^2 + b^2 - ab)$

$\therefore\ a(a + b)^3 - 3a^2b(a + b) = a(a + b)(a^2 + b^2 - ab)$

Q3 . $x(x^3 - y^3) + 3xy(x - y)$

SOLUTION :

Elaborating $x^3 - y^3$ using the identity $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$= x(x - y)(x^2 + xy + y^2) + 3xy(x - y)$

Taking common x(x-y) in both the terms

$= x(x - y)(x^2 + xy + y^2 + 3y)$

$\therefore\ x(x^3 - y^3) + 3xy(x - y) = x(x - y)(x^2 + xy + y^2 + 3y)$

Q4 . $a^2x^2 + (ax^2 + 1)x + a$

SOLUTION :

We multiply $x(ax^2 + 1) = ax^3 + x$

=$a^2x^2 + ax^3 + x + a$

Taking common ax^2 in $(a^2x^2 + ax^3)$ and 1in (x + a)

=$ax^2(a + x) + 1(x + a)$

=$ax^2(a + x) + 1(a + x)$

Taking (a + x) common in both the terms

=$(a + x)(ax^2 + 1)$

$\therefore a^2x^2 + (ax^2 + 1)x + a$ =$(a + x)(ax^2 + 1)$

Q5 . $x^2 + y - xy - x$

SOLUTION :

On rearranging

$x^2 - xy - x + y$

Taking x common in the $(x^2 - xy)$ and -1 in(-x+y)

=x(x – y) – 1 (x – y)

Taking (x – y) common in the terms

=(x – y)(x – 1)

latex]$\therefore$[/latex]$x^2 + y - xy - x$ =(x – y)(x – 1)

Q6 . $x^3 - 2x^2b + 3xy^2 - 6y^3$

SOLUTION :

Taking x^2 common in $(x^3 - 2x^2y)$ and +$3y^2$ common in $(3xy^2 - 6y^3)$

= $x^2(x - 2y) + 3y^2(x - 2y)$

Taking (x – 2y) common in the terms

= $(x - 2y)(x^2 + 3y^2)$

latex]$\therefore$[/latex] $x^3 - 2x^2y + 3xy^2 - 6y^3$ = $(x - 2y)(x^2 + 3y^2)$

Q7 . $6ab - b^2 + 12ac - 2bc$

SOLUTION :

Taking b common in $(6ab - b^2)$ and 2c in (12ac – 2bc)

=b(6a – b) + 2c (6a – b)

Taking (6a – b) common in the terms

=(6a – b)(b + 2c)

latex]∴[/latex]$6ab - b^2 + 12ac - 2bc$ =(6a – b)(b + 2c)

Q8 . $[x^2 + \frac{1}{x^2}] - 4[x + \frac{1}{x}] + 6$

SOLUTION :

=$x^2 + \frac{1}{x^2} - 4x - \frac{4}{x} + 4 + 2$

=$x^2 + \frac{1}{x^2} + 4 + 2 - \frac{4}{x} - 4x$

=$(x^2) + (\frac{1}{x})^2 + (-2)^2 + 2 \times x \times \frac{1}{x} + 2 \times \frac{1}{x} \times (-2) + 2(-2)x$

Using identity

$x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

We get,

=$[x + \frac{1}{x} + (-2)]^2$

=$[x + \frac{1}{x} - 2]^2$

=$[x + \frac{1}{x} - 2][x + \frac{1}{x} - 2]$

$\therefore [x^2 + \frac{1}{x^2}] - 4[x + \frac{1}{x}] + 6$ =$[x + \frac{1}{x} - 2][x + \frac{1}{x} - 2]$

Q9 . x(x- 2)(x – 4) + 4x – 8

SOLUTION :

=x(x – 2)(x – 4) + 4(x – 2)

Taking (x – 2) common in both the terms

=(x – 2){x(x – 4) + 4}

=(x – 2) $\{x^2 - 4x + 4\}$

Now splitting the middle term of $x^2 - 4x + 4$

=(x – 2){ $x^2 - 2x - 2x + 4$}

=(x – 2){ x(x – 2) -2(x -2)}

$=(x-2)\{(x-2)(x-2)\}$

$=(x-2)(x-2)(x-2)$

$=(x-2)^3$

$\therefore x(x-2)(x-4)+4x-8=(x-2)^3$

Q10 . $(x+2)(x^2+25)-10x^2-20x$

SOLUTION :

$(x+2)(x^2+25)-10x(x+2)$

Taking $(x+2)$ common in both the terms

$=(x+2)(x^2+25-10x)$

$=(x+2)(x^2-10x+25)$

Splitting the middle term of $(x^2-10x+25)$

$=(x+2)(x^2-5x-5x+25)$

$=(x+2)\{x(x-5)-5(x-5)\}$

$=(x+2)(x-5)(x-5)$

$\therefore (x+2)(x^2+25)-10x^2-20x=(x+2)(x-5)(x-5)$

Q11 . $2a^2+2\sqrt{6}ab+3b^2$

SOLUTION :

$=(\sqrt{2}a)^2+2\times\sqrt{2}a\times\sqrt{3}b+(\sqrt{3}b)^2$

Using the identity $(p+q)^2=p^2+q^2+2pq$

$=(\sqrt{2}a+\sqrt{3}b)^2$

$=(\sqrt{2}a+\sqrt{3}b)(\sqrt{2}a+\sqrt{3}b)$

$\therefore 2a^2+2\sqrt{6}ab+3b^2=(\sqrt{2}a+\sqrt{3}b)(\sqrt{2}a+\sqrt{3}b)$

Q 12 . $(a-b+c)^2+(b-c+a)^2+2(a-b+c)\times(b-c+a)$

SOLUTION :

Let $(a-b+c)=x$ and $(b-c+a)=y$

$= x^2 + y^2 + 2xy$

Using the identity $(a + b)^2 = a^2 + b^2 + 2ab$

$= (x + y)^2$

Now , substituting x and y

$(a - b + c + b - c + a)^2$

Cancelling −b , +b & +c , -c

$= (2a)^2$

$= 4a^2$

$\therefore (a - b + c)^2 + (b - c + a)^2 + 2(a - b + c) \times (b - c + a) = 4a^2$

Q13 . $a^2 + b^2 + 2(ab + bc + ca)$

SOLUTION :

$= a^2 + b^2 + 2ab + 2bc + 2ca$

Using the identity $(p + q)^2 = p^2 + q^2 + 2pq$

We get,

= $(a + b)^2$ + 2bc + 2ca

= $(a + b)^2$ + 2c(b + a)

Or $(a + b)^2$ + 2c(a + b)

Taking (a + b) common

= (a + b)(a + b + 2c)

$\therefore a^2 + b^2 + 2(ab + bc + ca)$ = (a + b)(a + b + 2c)

Q14 . $4(x - y)^2 - 12(x - y)(x + y) + 9(x + y)^2$

SOLUTION :

Let (x − y) = x,(x + y) = y

= $4x^2 - 12xy + 9y^2$

Splitting the middle term − 12 = -6 -6 also $4 \times 9 = -6 \times -6$

=$4x^2 - 6xy - 6xy + 9y^2$

=2x(2x − 3y) -3y(2x − 3y)

=(2x − 3y) (2x − 3y)

=$(2x - 3y)^2$

Substituting $x = x - y$ & $y = x + y$

$=[2(x-y)-3(x+y)]^2 = [2x - 2y - 3x - 3y]^2$

$=(2x-3x-2y-3y)^2$

$=[-x-5y]^2$

$=[(-1)(x+5y)]^2$

$=(x+5y)^2$ [$\because (-1)^2 = 1$]

$\therefore 4(x-y)^2 - 12(x-y)(x+y) + 9(x+y)^2 = (x+5y)^2$

Q 15. $a^2 - b^2 + 2bc - c^2$

SOLUTION :

$a^2 - (b^2 - 2bc + c^2)$

Using the identity $(a-b)^2 = a^2 + b^2 - 2ab$

$= a^2 - (b-c)^2$

Using the identity $a^2 - b^2 = (a+b)(a-b)$

$=(a+b-c)(a-(b-c))$

$=(a+b-c)(a-b+c)$

$\therefore a^2 - b^2 + 2bc - c^2 = (a+b-c)(a-b+c)$

Q16. $a^2 + 2ab + b^2 - c^2$

SOLUTION :

Using the identity $(p+q)^2 = p^2 + q^2 + 2pq$

$=(a+b)^2 - c^2$

Using the identity $p^2 - q^2 = (p+q)(p-q)$

$=(a+b+c)(a+b-c)$

$\therefore a^2 + 2ab + b^2 - c^2 = (a+b+c)(a+b-c)$

Q 17. $a^2 + 4b^2 - 4ab - 4c^2$

SOLUTION :

On rearranging

$= a^2 - 4ab + 4b^2 - 4c^2$

$= (a)^2 - 2 \times a \times 2b + (2b)^2 - 4c^2$

Using the identity $(a-b)^2 = a^2 + b^2 - 2ab$

$=(a-2b)^2 - 4c^2$

$=(a-2b)^2 - (2c)^2$

Using the identity $a^2 - b^2 = (a+b)(a-b)$

$=(a - 2b - 2c)(a - 2b + 2c)$

$\therefore a^2 + 4b^2 - 4ab - 4c^2 = (a - 2b - 2c)(a - 2b + 2c)$

Q18 . $xy^9 - yx^9$

SOLUTION :

$=xy(y^8 - x^8)$

$=xy((y^4)^2 - (x^4)^2)$

Using the identity $p^2 - q^2 = (p+q)(p-q)$

$=xy(y^4 + x^4)(y^4 - x^4)$

$=xy(y^4 + x^4)((y^2)^2 - (x^2)^2)$

Using the identity $p^2 - q^2 = (p+q)(p-q)$

$=xy(y^4 + x^4)(y^2 + x^2)(y^2 - x^2)$

$= xy(y^4 + x^4)(y^2 + x^2)(y + x)(y - x)$

$= xy(x^4 + y^4)(x^2 + y^2)(x + y)(-1)(x - y)$

$\because (y - x) = -1(x - y)$

$=-xy(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$

$\therefore xy^9 - yx^9 = -xy(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$

Q 19 . $x^4 + x^2y^2 + y^4$

SOLUTION :

Adding x^2y^2 and subtracting x^2y^2 to the given equation

$= x^4 + x^2y^2 + y^4 + x^2y^2 - x^2y^2$

$= x^4 + 2x^2y^2 + y^4 - x^2y^2$

$= (x^2)^2 + 2 \times x^2 \times y^2 + (y^2)^2 - (xy)^2$

Using the identity $(p+q)^2 = p^2 + q^2 + 2pq$

$= (x^2 + y^2)^2 - (xy)^2$

Using the identity $p^2 - q^2 = (p+q)(p-q)$

$= (x^2 + y^2 + xy)(x^2 + y^2 - xy)$

$\therefore x^4 + x^2y^2 + y^4 = (x^2 + y^2 + xy)(x^2 + y^2 - xy)$

Q20 . $x^2 - y^2 - 4xz + 4z^2$

SOLUTION :

On rearranging the terms

$= x^2 - 4xz + 4z^2 - y^2$

$= (x)^2 - 2 \times x \times 2z + (2z)^2 - y^2$

Using the identity $x^2 - 2xy + y^2 = (x-y)^2$

$= (x - 2z)^2 - y^2$

Using the identity $p^2 - q^2 = (p+q)(p-q)$

$= (x - 2z + y)(x - 2z - y)$

$\therefore x^2 - y^2 - 4xz + 4z^2 = (x - 2z + y)(x - 2z - y)$

Q21 . $x^2 + 6\sqrt{2}x + 10$

SOLUTION :

Splitting the middle term ,

$= x^2 + 5\sqrt{2}x + \sqrt{2}x + 10$ $\quad [\because 6\sqrt{2} = 5\sqrt{2} + \sqrt{2}$ and $5\sqrt{2} \times \sqrt{2} = 10]$

$= x(x + 5\sqrt{2}) + \sqrt{2}(x + 5\sqrt{2})$

$= (x + 5\sqrt{2})(x + \sqrt{2})$

$\therefore x^2 + 6\sqrt{2}x + 10 = (x + 5\sqrt{2})(x + \sqrt{2})$

Q22 . $x^2 - 2\sqrt{2}x - 30$

SOLUTION :

Splitting the middle term,

$= x^2 - 5\sqrt{2}x + 3\sqrt{2}x - 30$

$[\because -2\sqrt{2} = -5\sqrt{2} + 3\sqrt{2} \text{ also } -5\sqrt{2} \times 3\sqrt{2} = -30]$

$= x(x - 5\sqrt{2}) + 3\sqrt{2}(x - 5\sqrt{2})$

$= (x - 5\sqrt{2})(x + 3\sqrt{2})$

$\therefore x^2 - 2\sqrt{2}x - 30 = (x - 5\sqrt{2})(x + 3\sqrt{2})$

Q23 . $x^2 - \sqrt{3}x - 6$

SOLUTION :

Splitting the middle term,

$= x^2 - 2\sqrt{3}x + \sqrt{3}x - 6$ $\quad [\because -\sqrt{3} = -2\sqrt{3} + \sqrt{3} \text{ also } -2\sqrt{3} \times \sqrt{3} = -6]$

$= x(x - 2\sqrt{3}) + \sqrt{3}(x - 2\sqrt{3})$

$= (x - 2\sqrt{3})(x + \sqrt{3})$

$\therefore x^2 - \sqrt{3}x - 6 = (x - 2\sqrt{3})(x + \sqrt{3})$

Q24 . $x^2 + 5\sqrt{5}x + 30$

SOLUTION :

Splitting the middle term,

$= x^2 + 2\sqrt{5}x + 3\sqrt{5}x + 30$ $\quad [\because 5\sqrt{5} = 2\sqrt{5} + 3\sqrt{5} \text{ also } 2\sqrt{5} \times 3\sqrt{5} = 30]$

$= x(x + 2\sqrt{5}) + 3\sqrt{5}(x + 2\sqrt{5})$

$= (x + 2\sqrt{5})(x + 3\sqrt{5})$

$\therefore x^2 + 5\sqrt{5}x + 30 = (x + 2\sqrt{5})(x + 3\sqrt{5})$

Q25 . $x^2 + 2\sqrt{3}x - 24$

SOLUTION :

Splitting the middle term,

$= x^2 + 4\sqrt{3}x - 2\sqrt{3}x - 24$ $\quad [\because 2\sqrt{3} = 4\sqrt{3} - 2\sqrt{3}$ also $4\sqrt{3}(-2\sqrt{3}) = -24]$

$= x(x + 4\sqrt{3}) - 2\sqrt{3}(x + 4\sqrt{3})$

$= (x + 4\sqrt{3})(x - 2\sqrt{3})$

$\therefore x^2 + 2\sqrt{3}x - 24 = (x + 4\sqrt{3})(x - 2\sqrt{3})$

Q26 . $2x^2 - \frac{5}{6}x + \frac{1}{12}$

SOLUTION :

Splitting the middle term,

$= 2x^2 - \frac{x}{2} - \frac{x}{3} + \frac{1}{12}$ $\quad [\because -\frac{5}{6} = -\frac{1}{2} - \frac{1}{3}$ also $-\frac{1}{2} \times -\frac{1}{3} = 2 \times \frac{1}{12}]$

$= x(2x - \frac{1}{2}) - \frac{1}{6}(2x - \frac{1}{2})$

$= (2x - \frac{1}{2})(x - \frac{1}{6})$

$\therefore 2x^2 - \frac{5}{6}x + \frac{1}{12} = (2x - \frac{1}{2})(x - \frac{1}{6})$

Q27 . $x^2 + \frac{12}{35}x + \frac{1}{35}$

SOLUTION :

Splitting the middle term,

$= x^2 + \frac{5}{35}x + \frac{7}{35}x + \frac{1}{35}$ $\quad [\because \frac{12}{35} = \frac{5}{35} + \frac{7}{35}$ and $\frac{5}{35} \times \frac{7}{35} = \frac{1}{35}]$

$= x^2 + \frac{x}{7} + \frac{x}{5} + \frac{1}{35}$

$= x(x + \frac{1}{7}) + \frac{1}{5}(x + \frac{1}{7})$

$= (x + \frac{1}{7})(x + \frac{1}{5})$

$\therefore x^2 + \frac{12}{35}x + \frac{1}{35} = (x + \frac{1}{7})(x + \frac{1}{5})$

Q28 . $21x^2 - 2x + \frac{1}{21}$

SOLUTION :

$= (\sqrt{21}x)^2 - 2\sqrt{21}x \times \frac{1}{\sqrt{21}} + (\frac{1}{\sqrt{21}})^2$

Using the identity $(x-y)^2 = x^2 + y^2 - 2xy$

$= (\sqrt{21}x - \frac{1}{\sqrt{21}})^2$

$\therefore 21x^2 - 2x + \frac{1}{21} = (\sqrt{21}x - \frac{1}{\sqrt{21}})^2$

Q29 . $5\sqrt{5}x^2 + 20x + 3\sqrt{5}$

SOLUTION :

Splitting the middle term,

$= 5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5}$ $[\because 20 = 15 + 5 \text{ and } 15 \times 5 = 5\sqrt{5} \times 3\sqrt{5}]$

$= 5x(\sqrt{5}x + 3) + \sqrt{5}(\sqrt{5}x + 3)$

$= (\sqrt{5}x + 3)(5x + \sqrt{5})$

$\therefore 5\sqrt{5}x^2 + 20x + 3\sqrt{5} = (\sqrt{5}x + 3)(5x + \sqrt{5})$

Q30 . $2x^2 + 3\sqrt{5}x + 5$

SOLUTION :

Splitting the middle term,

$= 2x^2 + 2\sqrt{5}x + \sqrt{5}x + 5$

$= 2x(x + \sqrt{5}) + \sqrt{5}(x + \sqrt{5})$

$= (x + \sqrt{5})(2x + \sqrt{5})$

$\therefore 2x^2 + 3\sqrt{5}x + 5 = (x + \sqrt{5})(2x + \sqrt{5})$

Q31 . $9(2a - b)^2 - 4(2a - b) - 13$

SOLUTION :

Let $2a - b = x$

$= 9x^2 - 4x - 13$

Splitting the middle term,

$= 9x^2 - 13x + 9x - 13$

$= x(9x - 13) + 1(9x - 13)$

$= (9x - 13)(x + 1)$

Substituting $x = 2a - b$

$= [9(2a - b) - 13](2a - b + 1)$

$= (18a - 9b - 13)(2a - b + 1)$

$\therefore 9(2a - b)^2 - 4(2a - b) - 13 = (18a - 9b - 13)(2a - b + 1)$

Q 32 . $7(x - 2y)^2 - 25(x - 2y) + 12$

SOLUTION :

Let $x-2y = P$

$= 7P^2 - 25P + 12$

Splitting the middle term,

$= 7P^2 - 21P - 4P + 12$

$= 7P(P - 3) - 4(P - 3)$

$= (P - 3)(7P - 4)$

Substituting $P = x - 2y$

$= (x - 2y - 3)(7(x - 2y) - 4)$

$= (x - 2y - 3)(7x - 14y - 4)$

$\therefore 7(x - 2y)^2 - 25(x - 2y) + 12 = (x - 2y - 3)(7x - 14y - 4)$

Q33 . $2(x + y)^2 - 9(x + y) - 5$

SOLUTION :

Let $x+y = z$

$= 2z^2 - 9z - 5$

Splitting the middle term,

$= 2z^2 - 10z + z - 5$

$= 2z(z - 5) + 1(z - 5)$

$= (z - 5)(2z + 1)$

Substituting $z = x + y$

$= (x + y - 5)(2(x + y) + 1)$

= $(x + y - 5)(2x + 2y + 1)$

$\therefore 2(x + y)^2 - 9(x + y) - 5 = (x + y - 5)(2x + 2y + 1)$

Q34 . Give the possible expression for the length & breadth of the rectangle having $35y^2 - 13y - 12$ *as its area.*

SOLUTION :

Area is given as $35y^2 - 13y - 12$

Splitting the middle term,

Area = $35y^2 + 218y - 15y - 12$

= $7y(5y + 4) - 3(5y + 4)$

= $(5y + 4)(7y - 3)$

We also know that area of rectangle = length ×breadth

∴ Possible length = $(5y + 4)$ and breadth=$(7y - 3)$

Or possible length = $(7y - 3)$ and breadth= $(5y + 4)$

Q35 . What are the possible expression for the cuboid having volume $3x^2 - 12x$.

SOLUTION :

Volume = $3x^2 - 12x$

= $3x(x - 4)$

= $3 \times x(x - 4)$

Also volume = Length ×Breadth×Height

∴ Possible expression for dimensions of cuboid are = 3 , x , $(x - 4)$

RD SHARMA
Solutions
Class 9 Maths
Chapter 5
Ex 5.2

Q 1 . $p^3 + 27$

SOLUTION :

= $p^3 + 3^3$ $\because [a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

= $(p + 3)(p^2 - 3p - 9)$

$\therefore p^3 + 27 = (p + 3)(p^2 - 3p - 9)$

Q 2 . $y^3 + 125$

SOLUTION :

= $y^3 + 5^3$ $\because [a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

= $(y+5)(y^2 - 5y + 5^2)$

= $(y+5)(y^2 - 5y + 25)$

$\therefore y^3 + 125 = (y+5)(y^2 - 5y + 25)$

Q 3 . $1 - 27a^3$

SOLUTION :

= $(1)^3 - (3a)^3$

= $(1-3a)(1^2 + 1 \times 3a + (3a)^2)$ $\because [a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$

= $(1-3a)(1^2 + 3a + 9a^2)$

$\therefore 1 - 27a^3 = (1-3a)(1^2 + 3a + 9a^2)$

Q 4 . $8x^3y^3 + 27a^3$

SOLUTION :

= $(2xy)^3 + (3a)^3$

= $(2xy + 3a)((2xy)^2 - 2xy \times 3a + (3a)^2)$ $\because [a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

= $(2xy + 3a)(4x^2y^2 - 6xya + 9a^2)$

$\therefore 8x^3y^3 + 27a^3 = (2xy + 3a)(4x^2y^2 - 6xya + 9a^2)$

Q 5 . $64a^3 - b^3$

SOLUTION :

= $(4a)^3 - b^3$

= $(4a - b)((4a)^2 + 4a \times b + b^2)$ $\because [a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$

=$(4a - b)(16a^2 + 4ab + b^2)$

$\therefore\ 64a^3 - b^3 = (4a - b)(16a^2 + 4ab + b^2)$

Q 6 . $\frac{x^3}{216} - 8y^3$

SOLUTION :

$= \left(\frac{x}{6}\right)^3 - (2y)^3$

$= \left(\frac{x}{6} - 2y\right)\left(\left(\frac{x}{6}\right)^2 + \frac{x}{6} \times 2y + (2y)^2\right)$ $\quad \because [x^3 - y^3 = (x - y)(x^2 + xy + y^2)]$

$= \left(\frac{x}{6} - 2y\right)\left(\frac{x^2}{36} + \frac{xy}{3} + 4y^2\right)$

$\therefore\ \frac{x^3}{216} - 8y^3 = \left(\frac{x}{6} - 2y\right)\left(\frac{x^2}{36} + \frac{xy}{3} + 4y^2\right)$

Q 7 . $10x^4y - 10xy^4$

SOLUTION :

$= 10xy(x^3 - y^3)$

$= 10xy(x - y)(x^2 + xy + y^2)$ $\quad \because [x^3 - y^3 = (x - y)(x^2 + xy + y^2)]$

$\therefore 10x^4y - 10xy^4 = 10xy(x - y)(x^2 + xy + y^2)$

Q 8 . $54x^6y + 2x^3y^4$

SOLUTION :

$= 2x^3y(27x^3 + y^3)$

$= 2x^3y((3x)^3 + y^3)$

$= 2x^3y(3x + y)((3x)^2 - 3x \times y + y^2)$ $\quad \because [a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$

$= 2x^3y(3x + y)(9x^2 - 3xy + y^2)$

$\therefore 54x^6y + 2x^3y^4 = 2x^3y(3x + y)(9x^2 - 3xy + y^2)$

Q 9 . $32a^3 + 108b^3$

SOLUTION :

$= 4(8a^3 + 27b^3)$

$= 4((2a)^3 + (3b)^3)$

$= 4[(2a + 3b)((2a)^2 - 2a \times 3b + (3b)^2)]$

$\because\ [a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$

$= 4(2a + 3b)(4a^2 - 6ab + 9b^2)$

$\therefore 32a^3 + 108b^3 = 4(2a + 3b)(4a^2 - 6ab + 9b^2)$

Q 10 . $(a - 2b)^3 - 512b^3$

SOLUTION :

$= (a - 2b)^3 - (8b)^3$

$= (a - 2b - 8b)((a - 2b)^2 + (a - 2b)8b + (8b)^2)$

$\because [a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$=(a - 10b)(a^2 + 4b^2 - 4ab + 8ab - 16b^2 + 64b^2)$

$=(a - 10b)(a^2 + 52b^2 + 4ab)$

$\therefore (a - 2b)^3 - 512b^3 = (a - 10b)(a^2 + 52b^2 + 4ab)$

Q 11 . $(a + b)^3 - 8(a - b)^3$

SOLUTION :

$= (a + b)^3 - [2(a - b)]^3$

$= (a + b)^3 - [2a - 2b]^3$

$= (a + b - (2a - 2b))((a + b)^2 + (a + b)(2a - 2b) + (2a - 2b)^2)$

$\because [a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$=(a + b - 2a + 2b)(a^2 + b^2 + 2ab + (a + b)(2a - 2b) + (2a - 2b)^2)$

$=(a + b - 2a + 2b)(a^2 + b^2 + 2ab + 2a^2 - 2ab + 2ab - 2b^2 + (2a - 2b)^2)$

$=(3b - a)(3a^2 + 2ab - b^2 + (2a - 2b)^2)$

$=(3b - a)(3a^2 + 2ab - b^2 + 4a^2 + 4b^2 - 8ab)$

$=(3b - a)(3a^2 + 4a^2 - b^2 + 4b^2 - 8ab + 2ab)$

$=(3b - a)(7a^2 + 3b^2 - 6ab)$

$\therefore (a + b)^3 - 8(a - b)^3 = (3b - a)(7a^2 + 3b^2 - 6ab)$

Q 12 . $(x + 2)^3 + (x - 2)^3$

SOLUTION :

$= (x + 2 + x - 2)((x + 2)^2 - (x + 2)(x - 2) + (x - 2)^2)$

$\because [a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$

$=2x(x^2+4x+4-(x+2)(x-2)+x^2-4x+4)$

$=2x(2x^2+8-(x^2-2^2))$ $[\because (a+b)(a-b)=a^2-b^2]$

$=2x(2x^2+8-x^2+4)$

$=2x(x^2+12)$

$\therefore (x+2)^3+(x-2)^3=2x(x^2+12)$

Q 13 . $8x^2y^3-x^5$

SOLUTION :

$=x^2((2y)^3-x^3)$

$=x^2(2y-x)((2y)^2+2y\times x+x^2)$ $[\because x^3-y^3=(x-y)(x^2+xy+y^2)]$

$=x^2(2y-x)(4y^2+2xy+x^2)$

$\therefore 8x^2y^3-x^5=x^2(2y-x)(4y^2+2xy+x^2)$

Q 14 . $1029-3x^3$

SOLUTION :

$=3(343-x^3)$

$=3((7)^3-x^3)$

$=3(7-x)(7^2+7x+x^2)$ $[\because a^3-b^3=(a-b)(a^2+ab+b^2)]$

$=3(7-x)(49+7x+x^2)$

$\therefore 1029-3x^3=3(7-x)(49+7x+x^2)$

Q 15 . x^6+y^6

SOLUTION :

$=(x^2)^3+(y^2)^3$

$=(x^2+y^2)((x^2)^2-x^2y^2+(y^2)^2)$

$=(x^2+y^2)(x^4-x^2y^2+y^4)$ $[\because a^3+b^3=(a+b)(a^2-ab+b^2)]$

$\therefore x^6+y^6=(x^2+y^2)(x^4-x^2y^2+y^4)$

Q 16 . x^3y^3+1

SOLUTION :

$=(xy)^3+1^3$

$= (xy + 1)((xy)^2 + xy + 1^2)$ $[\because x^3 + y^3 = (x + y)(x^2 - xy + y^2)]$

$= (xy + 1)(x^2y^2 - xy + 1)$

$\therefore x^3y^3 + 1 = (xy + 1)(x^2y^2 - xy + 1)$

Q 17 . $x^4y^4 - xy$

SOLUTION :

$= xy(x^3y^3 - 1)$

$= xy((xy)^3 - 1^3)$

$= xy(xy - 1)((xy)^2 + xy \times 1 + 1^2)$ $\because [x^3 - y^3 = (x - y)(x^2 + xy + y^2)]$

$= xy(xy - 1)(x^2y^2 + xy + 1)$

$\therefore x^4y^4 - xy = xy(xy - 1)(x^2y^2 + xy + 1)$

Q 18 . $a^{12} + b^{12}$

SOLUTION :

$= (a^4)^3 + (b^4)^3$

$= (a^4 + b^4)((a^4)^2 - a^4 \times b^4 + (b^4)^2)$ $\because [a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$

$= (a^4 + b^4)(a^8 - a^4b^4 + b^8)$

$\therefore a^{12} + b^{12} = (a^4 + b^4)(a^8 - a^4b^4 + b^8)$

Q 19 . $x^3 + 6x^2 + 12x + 16$

SOLUTION :

$= x^3 + 6x^2 + 12x + 8 + 8$

$= x^3 + 3 \times x^2 \times 2 + 3 \times x \times 2^2 + 2^3 + 8$

$= (x + 2)^3 + 8$ $[\because a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3]$

$= (x + 2)^3 + 2^3$

$= (x + 2 + 2)((x + 2)^2 - 2(x + 2) + 2^2)$ $\because [a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$

$= (x + 2 + 2)(x^2 + 4 + 4x - 2x - 4 + 4)$ $[\because (a + b)^2 = a^2 + b^2 + 2ab]$

$= (x + 4)(x^2 + 4 + 2x)$

$\therefore x^3 + 6x^2 + 12x + 16 = (x + 4)(x^2 + 4 + 2x)$

Q 20 . $a^3 + b^3 + a + b$

SOLUTION :

= $(a^3 + b^3) + 1\,(a + b)$

= $(a + b)\,(a^2 - ab + b^2) + 1\,(a + b)$ $[\because a^3 + b^3 = (a + b)\,(a^2 - ab + b^2)]$

=$(a + b)\,(a^2 - ab + b^2 + 1)$

$\therefore a^3 + b^3 + a + b$ = $(a + b)\,(a^2 - ab + b^2 + 1)$

Q 21 . $a^3 - \frac{1}{a^3} - 2a + \frac{2}{a}$

SOLUTION :

= $(a^3 - \frac{1}{a^3}) - 2\,(a - \frac{1}{a})$

= $(a^3 - (\frac{1}{a})^3) - 2\,(a - \frac{1}{a})$

= $(a - \frac{1}{a})\,(a^2 + a \times \frac{1}{a} + (\frac{1}{a})^2) - 2\,(a - \frac{1}{a})$ $[\because a^3 - b^3 = (a - b)\,(a^2 + ab + b^2)]$

=$(a - \frac{1}{a})\,(a^2 + 1 + \frac{1}{a^2}) - 2\,(a - \frac{1}{a})$

=$(a - \frac{1}{a})\,(a^2 + 1 + \frac{1}{a^2} - 2)$

=$(a - \frac{1}{a})\,(a^2 + \frac{1}{a^2} - 1)$

$\therefore a^3 - \frac{1}{a^3} - 2a + \frac{2}{a}$ = $(a - \frac{1}{a})\,(a^2 + \frac{1}{a^2} - 1)$

Q 22 . $a^3 + 3a^2b + 3ab^2 + b^3 - 8$

SOLUTION :

= $(a + b)^3 - 8$ $[\because a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3]$

=$(a + b)^3 - 2^3$

=$(a + b - 2)\,((a + b)^2 + (a + b) \times 2 + 2^2)$

= $(a + b - 2)(a^2 + 2ab + b^2 + 2a + 2b + 4)$

$\therefore a^3 + 3a^2b + 3ab^2 + b^3 - 8$ = $(a + b - 2)(a^2 + 2ab + b^2 + 2a + 2b + 4)$

Q 23 . $8a^3 - b^3 - 4ax + 2bx$

SOLUTION :

= $(2a)^3 - b^3 - 2x\,(2a - b)$

$= (2a - b)((2a)^2 + 2a \times b + b^2) - 2x(2a - b)$ $[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$=(2a - b)(4a^2 + 2ab + b^2 - 2x)$

$\therefore 8a^3 - b^3 - 4ax + 2bx = (2a - b)(4a^2 + 2ab + b^2 - 2x)$

Q 24 . i . $\frac{173\times173\times173+127\times127\times127}{173\times173-173\times127+127\times127}$

SOLUTION :

$= \frac{173^3+127^3}{173^2-173\times127+127^2}$

$= \frac{(173+127)(173^2-173\times127+127^2)}{173^2-173\times127+127^2}$ $\because [a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$

$=(173 + 127)$

=300

Q 24 . ii . $\frac{1.2\times1.2\times1.2-0.2\times0.2\times0.2}{1.2\times1.2+1.2\times0.2+0.2\times0.2}$

SOLUTION :

$= \frac{1.2^3-0.2^3}{1.2^2+1.2\times0.2+0.2^2}$

$= \frac{(1.2-0.2)((1.2)^2+1.2\times0.2+(0.2)^2)}{1.2^2+1.2\times0.2+0.2^2}$ $[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$=(1.2 - 0.2)$

=1.0

Q 24 . iii . $\frac{155\times155\times155-55\times55\times55}{155\times155+155\times55+55\times55}$

SOLUTION :

$= \frac{155^3-55^3}{155^2+155\times55+55^2}$

$= \frac{(155-55)(155^2+155\times55+55^2)}{155^2+155\times55+55^2}$ $[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

= (155 – 55)

= 100

RD SHARMA

Solutions

Class 9 Maths

Chapter 5

Ex 5.3

Q 1. $64a^3 + 125b^3 + 240a^2b + 300ab^2$

SOLUTION :

$= (4a)^3 + (5b)^3 + 3(4a)^2(5b) + 3(4a)(5b)^2$

$[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a+b)^3]$

$= (4a + 5b)^3$

$= (4a + 5b)(4a + 5b)(4a + 5b)$

$\therefore 64a^3 + 125b^3 + 240a^2b + 300ab^2 = (4a + 5b)(4a + 5b)(4a + 5b)$

Q 2. $125x^3 - 27y^3 - 225x^2y + 135xy^2$

SOLUTION :

$= (5x)^3 - (3y)^3 - 3(5x)^2(3y) + 3(5x)(3y)^2$ $[\because a^3 - b^3 - 3a^2b + 3ab^2 = (a-b)^3]$

$=(5x - 3y)^3$

$=(5x - 3y)(5x - 3y)(5x - 3y)$

$\therefore 125x^3 - 27y^3 - 225x^2y + 135xy^2 = (5x - 3y)(5x - 3y)(5x - 3y)$

Q 3. $\frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x$

SOLUTION :

$= (\frac{2}{3}x^3)^3 + 1^3 + 3 \times (\frac{2}{3}x)^2 \times 1 + 3(1)^2 \times (\frac{2}{3}x)$

$=(\frac{2}{3}x + 1)^3$ $[\because x^3 + b^3 + 3x^2b + 3xb^2 = (x+b)^3]$

$=(\frac{2}{3}x + 1)(\frac{2}{3}x + 1)(\frac{2}{3}x + 1)$

$\therefore \frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x = (\frac{2}{3}x + 1)(\frac{2}{3}x + 1)(\frac{2}{3}x + 1)$

Q 4. $8x^3 + 27y^3 + 36x^2y + 54xy^2$

SOLUTION :

$= (2x)^3 + (3y)^3 + 3 \times (2x)^2 \times 3y + 3 \times (2x)(3y)^2$

$= (2x + 3y)^3$ $[\because$

$a^3 + b^3 + 3a^2b + 3ab^2 = (a + b)^3]$

$=(2x+3y)(2x+3y)(2x+3y)$

$\therefore 8x^3+27y^3+36x^2y+54xy^2=(2x+3y)(2x+3y)(2x+3y)$

Q 5. $a^3-3a^2b+3ab^2-b^3+8$

SOLUTION :

$=(a-b)^3+2^3$ $\quad [\because a^3-b^3-3a^2b+3ab^2=(a-b)^3]$

$=(a-b+2)((a-b)^2-(a-b)2+2^2)$ $\quad \because [a^3+b^3=(a+b)(a^2-ab+b^2)]$

$=(a-b+2)(a^2+b^2-2ab-2(a-b)+4)$

$=(a-b+2)(a^2+b^2-2ab-2a+2b+4)$

$\therefore a^3-3a^2b+3ab^2-b^3+8=(a-b+2)(a^2+b^2-2ab-2a+2b+4)$

Q 6. $x^3+8y^3+6x^2y+12xy^2$

SOLUTION :

$=(x)^3+(2y)^3+3\times x^2\times 2y+3\times x\times(2y)^2$

$=(x+2y)^3$ $\quad [\because x^3+y^3+3x^2y+3xy^2=(x+y)^3]$

$=(x+2y)(x+2y)(x+2y)$

$\therefore x^3+8y^3+6x^2y+12xy^2=(x+2y)(x+2y)(x+2y)$

Q 7. $8x^3+y^3+12x^2y+6xy^2$

SOLUTION :

$=(2x)^3+(y)^3+3\times(2x)^2\times y+3(2x)\times y^2$

$=(2x+y)^3$ $\quad [\because a^3+b^3+3a^2b+3ab^2=(a+b)^3]$

$=(2x+y)(2x+y)(2x+y)$

$\therefore 8x^3+y^3+12x^2y+6xy^2=(2x+y)(2x+y)(2x+y)$

Q 8. $8a^3+27b^3+36a^2b+54ab^2$

SOLUTION :

$=(2a)^3+(3b)^3+3\times(2a)^2\times 3b+3\times 2a\times(3b)^2$

$= (2a + 3b)^3$ $[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a + b)^3]$

$=(2a + 3b)(2a + 3b)(2a + 3b)$

$\therefore 8a^3 + 27b^3 + 36a^2b + 54ab^2 = (2a + 3b)(2a + 3b)(2a + 3b)$

Q 9 . $8a^3 - 27b^3 - 36a^2b + 54ab^2$

SOLUTION :

$= (2a)^3 - (3b)^3 - 3 \times (2a)^2 \times 3b + 3 \times 2a \times (3b)^2$

$= (2a - 3b)^3$ $[\because a^3 - b^3 - 3a^2b + 3ab^2 = (a - b)^3]$

$=(2a - 3b)(2a - 3b)(2a - 3b)$

$\therefore 8a^3 - 27b^3 - 36a^2b + 54ab^2 = (2a - 3b)(2a - 3b)(2a - 3b)$

Q 10 . $x^3 - 12x(x - 4) - 64$

SOLUTION :

$= x^3 - 12x^2 + 48x - 64$

$= x^3 - 3 \times x^2 \times 4 + 3 \times 4^2 \times x - 4^3$

$= (x - 4)^3$ $[\because a^3 - b^3 - 3a^2b + 3ab^2 = (a - b)^3]$

$=(x - 4)(x - 4)(x - 4)$

$\therefore x^3 - 12x(x - 4) - 64 = (x - 4)(x - 4)(x - 4)$

Q 11 . $a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3$

SOLUTION :

$= (ax)^3 - 3(ax)^2 \times b + 3(ax) \times b^2 - b^3$

$= (ax - b)^3$ $[\because a^3 - b^3 - 3a^2b + 3ab^2 = (a - b)^3]$

$=(ax - b)(ax - b)(ax - b)$

$\therefore a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3 = (ax - b)(ax - b)(ax - b)$

RD SHARMA

Solutions

Class 9 Maths

Chapter 5

Ex 5.4

Q 1 . $a^3 + 8b^3 + 64c^3 - 24abc$

SOLUTION :

$= (a)^3 + (2b)^3 + (4c)^3 - 3 \times a \times 2b \times 4c$

$= (a + 2b + 4c)(a^2 + (2b)^2 + (4c)^2 - a \times 2b - 2b \times 4c - 4c \times a)$

$[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$

$= (a + 2b + 4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ac)$

$\therefore\ a^3 + 8b^3 + 64c^3 - 24abc = (a + 2b + 4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ac)$

Q 2 . $x^3 - 8y^3 + 27z^3 + 18xyz$

SOLUTION :

$= x^3 - (2y)^3 + (3z)^3 - 3 \times x \times (-2y)(3z)$

$= (x + (-2y) + 3z)(x^2 + (-2y)^2 + (3z)^2 - x(-2y) - (-2y)(3z) - 3z(x))$

$[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$

$=(x + (-2y) + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx)$

$\therefore\ x^3 - 8y^3 + 27z^3 + 18xyz =(x + (-2y) + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx)$

Q 3 . $\frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz$

SOLUTION :

$= (\frac{x}{3})^3 + (-y)^3 + (5z)^3 - 3 \times \frac{x}{3}(-y)(5z)$

$= (\frac{x}{3} + (-y) + 5z)((\frac{x}{3})^2 + (-y)^2 + (5z)^2 - \frac{x}{3}(-y) - (-y)5z - 5z(\frac{x}{3}))$

$= (\frac{x}{3} - y + 5z)(\frac{x^2}{9} + y^2 + 25z^2 + \frac{xy}{3} + 5yz - \frac{5zx}{3})$

$\therefore\ \frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz = (\frac{x}{3} - y + 5z)(\frac{x^2}{9} + y^2 + 25z^2 + \frac{xy}{3} + 5yz - \frac{5zx}{3})$

Q 4 . $8x^3 + 27y^3 - 216z^3 + 108xyz$

SOLUTION :

$= (2x)^3 + (3y)^3 + (-6y)^3 - 3(2x)(3y)(-6z)$

$= (2x + 3y + (-6z))((2x)^2 + (3y)^2 + (-6z)^2 - 2x \times 3y - 3y(-6z) - (-6z)2x)$

$= (2x + 3y + (-6z))(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx)$

$\therefore\ 8x^3 + 27y^3 - 216z^3 + 108xyz = (2x + 3y + (-6z))(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx)$

Q 5. $125 + 8x^3 - 27y^3 + 90xy$

SOLUTION :

$= (5)^3 + (2x)^3 + (-3y)^3 - 3 \times 5 \times 2x \times (-3y)$

$= (5 + 2x + (-3y))(5^2 + (2x)^2 + (-3y)^2 - 5(2x) - 2x(-3y) - (-3y)5)$

$= (5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y)$

$\therefore\ 125 + 8x^3 - 27y^3 + 90xy = (5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y)$

Q 6. $(3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3$

SOLUTION :

Let $(3x - 2y) = a$, $(2y - 4z) = b$, $(4z - 3x) = c$

$\therefore a + b + c = 3x - 2y + 2y - 4z + 4z - 3x = 0$

$\because a + b + c = 0 \therefore a^3 + b^3 + c^3 = 3abc$

$= 3(3x - 2y)(2y - 4z)(4z - 3x)$

$\therefore\ (3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3 = 3(3x - 2y)(2y - 4z)(4z - 3x)$

Q 7. $(2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3$

SOLUTION :

Let $2x - 3y = a$, $4z - 2x = b$, $3y - 4z = c$

$\therefore a + b + c = 2x - 3y + 4z - 2x + 3y - 4z = 0$

$\because a + b + c = 0 \therefore a^3 + b^3 + c^3 = 3abc$

$= 3(2x - 3y)(4z - 2x)(3y - 4z)$

$\therefore\ (2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3 = 3(2x - 3y)(4z - 2x)(3y - 4z)$

Q 8. $[\frac{x}{2} + y + \frac{z}{3}]^3 + [\frac{x}{3} - \frac{2y}{3} + z]^3 + [-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}]^3$

SOLUTION :

Let $[\frac{x}{2}+y+\frac{z}{3}]=a, [\frac{x}{3}-\frac{2y}{3}+z]=b, [-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}]=c$

$a+b+c=\frac{x}{2}+y+\frac{z}{3}+\frac{x}{3}-\frac{2y}{3}+z-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}$

$a+b+c=(\frac{x}{2}+\frac{x}{3}-\frac{5x}{6})+(y-\frac{2y}{3}-\frac{y}{3})+(\frac{z}{3}+z-\frac{4z}{3})$

$a+b+c=\frac{3x}{6}+\frac{2x}{6}-\frac{5x}{6}+\frac{3y}{3}-\frac{2y}{3}-\frac{y}{3}+\frac{z}{3}+\frac{3z}{3}-\frac{4z}{3}$

$a+b+c=\frac{5x-5x}{6}+\frac{3y-3y}{3}+\frac{4z-4z}{3}$

$a+b+c=0$

$\because a+b+c=0 \qquad \therefore a^3+b^3+c^3=3abc$

$=3(\frac{x}{2}+y+\frac{z}{3})(\frac{x}{3}-\frac{2y}{3}+z)(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3})$

$\therefore [\frac{x}{2}+y+\frac{z}{3}]^3+[\frac{x}{3}-\frac{2y}{3}+z]^3+[-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}]^3=3(\frac{x}{2}+y+\frac{z}{3})(\frac{x}{3}-\frac{2y}{3}+z)(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3})$

Q 9. $(a-3b)^3+(3b-c)^3+(c-a)^3$

SOLUTION :

Let $a-3b=x, 3b-c=y, c-a=z$

$x+y+z=a-3b+3b-c+c-a=0$

$(\because x+y+z=0) \qquad \therefore x^3+y^3+z^3=3xyz$

$=3(a-3b)(3b-c)(c-a)$

$\therefore (a-3b)^3+(3b-c)^3+(c-a)^3=3(a-3b)(3b-c)(c-a)$

Q 10. $2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6abc}$

SOLUTION :

$=(\sqrt{2}a)^3+(\sqrt{3}b)^3+c^3-3\times\sqrt{2a}\times\sqrt{3b}\times c$

$=(\sqrt{2a}+\sqrt{3b}+c)((\sqrt{2a})^2+(\sqrt{3b})^2+c^2-(\sqrt{2a})(\sqrt{3b})-(\sqrt{3b})c-(\sqrt{2ac}))$

$=(\sqrt{2a}+\sqrt{3b}+c)(2a^2+3b^2+c^2-\sqrt{6}ab-\sqrt{3}bc-\sqrt{2}ac)$

$\therefore 2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6abc}=(\sqrt{2a}+\sqrt{3b}+c)(2a^2+3b^2+c^2-\sqrt{6}ab-\sqrt{3}bc-\sqrt{2}ac)$

Q 11. $3\sqrt{3}a^3-b^3-5\sqrt{5}c^3-3\sqrt{15}abc$

SOLUTION :

$= (\sqrt{3}a)^3 + (-b)^3 - (\sqrt{5}c)^3 - 3(\sqrt{3}a)(-b)(-\sqrt{5}c)$

=

$(\sqrt{3}a + (-b) + (-\sqrt{5}c))((\sqrt{3}a)^2 + (-b)^2 + (-\sqrt{5}c)^2 - \sqrt{3}a(-b) - (-b)(-\sqrt{5}c)$

$-(-\sqrt{5}c)\sqrt{3}a)$

$=(\sqrt{3}a - b - \sqrt{5}c)(3a^2 + b^2 + 5c^2 + \sqrt{3}ab - \sqrt{5}bc + \sqrt{15}ac)$

$\therefore\ 3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc = (\sqrt{3}a - b - \sqrt{5}c)(3a^2 + b^2 + 5c^2 + \sqrt{3}ab - \sqrt{5}bc + \sqrt{15}ac)$

Q 12. $8x^3 - 125y^3 + 216 + 180xy$

SOLUTION :

$= (2x)^3 + (-5y)^3 + 6^3 - 3 \times (2x)(-5y)(6)$

$= (2x + (-5y) + 6)((2x)^2 + (-5y)^2 + 6^2 - 2x \times (-5y) - (-5y)6 - 6(2x))$

$= (2x - 5y + 6)(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x)$

$\therefore\ 8x^3 - 125y^3 + 216 + 180xy = (2x - 5y + 6)(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x)$

Q 13. $2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$

SOLUTION :

$= (\sqrt{2}a)^3 + (2\sqrt{2}b)^3 + c^3 - 3 \times \sqrt{2}a \times 2\sqrt{2}b \times c$

$= (\sqrt{2}a + 2\sqrt{2}b + c)((\sqrt{2}a)^2 + (2\sqrt{2}b)^2 + c^2 - (\sqrt{2}a)(2\sqrt{2}b) - (2\sqrt{2}b)c - (\sqrt{2}a)c)$

$= (\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ac)$

$\therefore\ 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc = (\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ac)$

Q 14. Find the value of $x^3 + y^3 - 12xy + 64$ *when x + y = -4.*

SOLUTION :

$= x^3 + y^3 + 64 - 12xy$

$= x^3 + y^3 + 4^3 - 3(x)(y)(4)$

$= (x + y + 4)(x^2 + y^2 + 4^2 - xy - y \times 4 - 4 \times x)$

$= (-4 + 4)(x^2 + y^2 + 16 - xy - 4y - 4x)$ $[\because x + y = -4]$

$= 0$

$\therefore\ x^3 + y^3 - 12xy + 64 = 0$

Q 15 . MULTIPLY :

(i) . $x^2 + y^2 + z^2 - xy + xz + yz$ by $x + y - z$

SOLUTION :

$= (x^2 + y^2 + z^2 - xy + xz + yz)(x + y - z)$

$= x^3 + y^3 + z^3 - 3xyz$

(ii) . $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ by $x - 2y - z$

SOLUTION :

$x^2 + (-2y)^2 + (-z)^2 - (-2y)(-z) - (-z)(x) = x^3 + (-2y)^3 + (-z)^3 - 3x(-2y)(-z)$

$\Rightarrow x^2 + 4y^2 + z^2 + 2xy - 2yz + zx = x^3 - 8y^3 - z^3 - 6xyz$

(iii) . $x^2 + 4y^2 + 2xy - 3x + 6y + 9$ *by* $(x - 2y + 3)$

SOLUTION :

$(x)^2 + (-2y)^2 + (3)^2 - (x)(-2y) - (-2y)(3) - 3(x) = (x)^3 + (-2y)^3 + 3^3 - 3(x)(-2y)(3)$

$\Rightarrow x^2 + 4y^2 + 9 + 2xy + 6y - 3x = x^3 - 8y^3 + 27 + 18xy$

(iv) . $9x^2 + 25y^2 + 15xy + 12x - 20y + 16$ by $3x - 5y + 4$

SOLUTION :

$(3x)^2 + (5y)^2 + 4^2 - (-3x)(5y) - (5y)(4) - (4)(-3x) = (-3x)^3 + (5y)^3 + 4^3$
$- 3(-3x)(5y)(4)$

$\Rightarrow 9x^2 + 25y^2 + 16 + 15xy - 20y + 12x = -27x^3 + 125y^3 + 64 + 180xy$