Mark the correct alternative in each of the following :

Question 1.

If 7th and 13th terms of an A.P. be 34 and 64 respectively, then its 18th term is

(a) 87
(b) 88
(c) 89
(d) 90

Solution:

(c) 7th term (a_7) = a + 6d = 34
13th term (a_{13}) = a + 12d = 64
Subtracting, 6d = 30 => d = 5
and a + 12 x 5 = 64 => a + 60 = 64 => a = 64 – 60 = 4
18th term (a_{18}) = a + 17d = 4 + 17 x 5 = 4 + 85 = 89

Question 2.

If the sum of p terms of an A.P. is q and the sum of q terms is p, then the sum of (p + q) terms will be

(a) 0
(b) p – q
(c) p + q
(d) – (p + q)

Solution:

(d)

Sum of p terms $= q$

i.e., $S_p = \frac{p}{2}[2a + (p-1)d] = q$

$\Rightarrow p[2a + (p-1)d] = 2q$

$2ap + p(p-1)d = 2q$ ....(i)

and sum of q terms $= p$

i.e., $S_q = \frac{q}{2}[2a + (q-1)d] = p$

$\Rightarrow q[2a + (q-1)d] = 2p$

$= 2aq + q(q-1)d = 2p$ ....(ii)

Subtracting (ii) from (i)

$2a(p-q) + \{p^2 - p - q^2 + q\}d = 2q - 2p$

$\Rightarrow 2a(p-q) + \{p^2 - q^2 - (p-q)\}d = -2(p-q)$

$\Rightarrow 2a(p-q) + \{(p+q)(p-q) - (p-q)\}d = -2(p-q)$

$\Rightarrow 2a(p-q) + (p-q)[p+q-1]d = -2(p-q)$

Dividing by $(p-q)$

$2a + (p+q-1)d = -2$ ....(i)

$S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$

$= \frac{p+q}{2}[-2]$ [From (i)]

$= -(p+q)$

Question 3.

If the sum of n terms of an A.P. be $3n^2 + n$ and its common difference is 6, then its first term is

(a) 2

(b) 3

(c) 1

(d) 4

Solution:

(d)

Sum of n terms of an A.P. $= 3n^2 + n$

and common difference $(d) = 6$

Let first term be a, then

$$\therefore\ S_n = \frac{n}{2}[2a + (n-1)d] = 3n^2 + n$$

$$\Rightarrow \frac{n}{2}[2a + (n-1)6] = 3n^2 + n$$

$$2a + 6n - 6 = (3n^2 + n) \times \frac{2}{n} = n\frac{(3n+1)\times 2}{n}$$

$$\Rightarrow 2a + 6n - 6 = (3n + 1)2 = 6n + 2$$

$$\Rightarrow 2a = 6n + 2 - 6n + 6 = 8$$

$$a = \frac{8}{2} = 4$$

Question 4.

The first and last terms of an A.P. are 1 and 11. If the sum of its terms is 36, then the number of terms will be

(a) 5

(b) 6

(c) 7

(d) 8

Solution:

(b) First term of an A.P. (a) = 1

Last term (l) = 11

and sum of its terms = 36

Let n be the number of terms and d be the common difference, then

$$a_n = 1 = a + (n-1)d = 11$$

$$\Rightarrow 1 + (n-1)d = 11 \Rightarrow (n-1)d = 11 - 1 = 10 \quad ....(i)$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = 36$$

$$\Rightarrow \frac{n}{2}[2 \times 1 + 10] = 36 \quad \text{[From } (i)\text{]}$$

$$\Rightarrow n(2 + 10) = 72 \Rightarrow 12n = 72$$

$$\Rightarrow n = \frac{72}{12} = 6$$

Question 5.
If the sum of n terms of an A.P. is $3n^2 + 5n$ then which of its terms is 164 ?
(a) 26th
(b) 27th
(c) 28th
(d) none of these
Solution:
(b)

Sum of n terms of an A.P. = $3n^2 + 5n$
Let a be the first term and d be the common difference
$S_n = 3n^2 + 5n$
$\therefore S_1 = 3(1)^2 + 5 \times 1 = 3 + 5 = 8$
$S_2 = 3(2)^2 + 5 \times 2 = 12 + 10 = 22$
$\therefore$ First term $(a) = 8$
$a_2 = S_2 - S_1 = 22 - 8 = 14$
$\therefore d = a_2 - a_1 = 14 - 8 = 6$
Now $a_n = a + (n-1)d$
$\Rightarrow 164 = 8 + (n-1) \times 6$
$6n - 6 = 164 - 8 = 156$
$6n = 156 + 6 = 162$
$n = \frac{162}{6} = 27$
$\therefore$ 168 is 27th term

Question 6.
If the sum of it terms of an A.P. is $2n^2 + 5n$, then its nth term is
(a) 4n – 3
(b) 3n – 4
(c) 4n + 3
(d) 3n + 4
Solution:

(c)

Let a be the first term and d be the common difference of an A.P. and

$S_n = 2n^2 + 5n$

$\therefore$ $S_1 = 2(1)^2 + 5 \times 1 = 2 + 5 = 7$

$S_2 = 2(2)^2 + 5 \times 2 = 8 + 10 = 18$

$\therefore$ First term $(a_1) = 7$

and second term $a_2 = S_2 - S_1 = 18 - 7 = 11$

$\therefore$ $d = a_2 - d_1 = 11 - 7 = 4$

Now $a_n = a + (n - 1) d$

$= 7 + (n - 1) 4 = 7 + 4n - 4$

$= 4n + 3$

Question 7.

If the sum of three consecutive terms of an increasing A.P. is 51 and the product of the first and third of these terms is 273, then the third term is :

(a) 13
(b) 9
(c) 21
(d) 17

Solution:

(c)

Let three consecutive terms of an increasing A.P. be $a - d$, d, $a + d$ where a is the first term and d be the common difference

$\therefore$ $a - d + a + a + d = 51$

$\Rightarrow$ $3a + 51 \Rightarrow a = \dfrac{51}{3} = 17$

and product of the first and third terms

$= (a - d)(a + d) = 273$

$\Rightarrow$ $a^2 - d^2 = 273 \Rightarrow (17)^2 - d^2 = 273$

$\Rightarrow$ $289 - d^2 = 273$

$\Rightarrow$ $d^2 = 289 - 273 = 16 = (\pm 4)^2$

$\therefore$ $d = \pm 4$

$\because$ The A.P. is increasing

$\therefore$ $d = 4$

Now third term $= a + d$

$= 17 + 4 = 21$

Question 8.
If four numbers in A.P. are such that their sum is 50 and the greatest number is 4 times the least, then the numbers are
(a) 5, 10, 15, 20
(b) 4, 10, 16, 22
(c) 3, 7, 11, 15
(d) None of these
Solution:
(a)
4 numbers are in A.P.
Let the numbers be
a – 3d, a – d, a + d, a + 3d
Where a is the first term and 2d is the common difference
Now their sum = 50
a – 3d + a – d + a + d + a + 3d = 50
and greatest number is 4 times the least number
a + 3d = 4 (a – 3d)
a + 3d = 4a – 12d
4a – a = 3d + 12d
=> 3a = 15d

$$\Rightarrow a = 5d$$

$$\Rightarrow \frac{25}{2} = 5d \Rightarrow d = \frac{25}{2 \times 5} = \frac{5}{2}$$

$\therefore$ Numbers are

$$\frac{25}{2} - 3 \times \frac{5}{2}, \frac{25}{2} - \frac{5}{2}, \frac{25}{2} + \frac{5}{2}, \frac{25}{2} + 3 \times \frac{5}{2}$$

$$\Rightarrow \frac{10}{2}, \frac{20}{2}, \frac{30}{2}, \frac{40}{2}$$

$$\Rightarrow 5, 10, 15, 20$$

Question 9.
Let S denotes the sum of n terms of an A.P. whose first term is a. If the common difference d is given by d = S_n – k S_{n-1} + S_{n-2} then k =
(a) 1
(b) 2
(c) 3
(d) None of these
Solution:

(b)

S_n is the sum of n terms of an A.P.
a is its first term and d is common difference

$d = S_n - kS_{n-1} + S_{n-2}$

$\Rightarrow kS_{n-1} = S_n + S_{n-2} - d$

$= (a_n + S_{n-1}) + (S_{n-1} - a_{n-1} - 1) - d$

$$\left\{\begin{array}{l} \because\ S_n = S_{n-1} + a_n \\ \text{and } S_{n-1} = a_{n-1} + S_{n-2} \\ \Rightarrow S_{n-2} = S_{n-1} - a_{n-1} \end{array}\right.$$

$= a_n + 2S_{n-1} - a_{n-1} - d$

$= 2S_{n-1} + a_n - a_{n-1} - d$

$= 2S_{n-1} + d - d \qquad (\because a_n - a_{n-1} = d)$

$= 2S_{n-1}$

$\therefore\ k = 2$

Question 10.
The first and last term of an A.P. are a and l respectively. If S is the sum of all the terms of the A.P. and the common difference is given by

$\frac{l^2 - a^2}{k - (l + a)}$ then $k =$

(a) S
(b) 2S
(c) 3S
(d) None of these

Solution:

(b)

$$S = \frac{n}{2}(l + a),\ l = a + (n - 1)\ d$$

$$d = \frac{l^2 - a^2}{k - (l + a)} \text{ and also } d = \frac{l - a}{n - 1}$$

$$\therefore \frac{l - a}{n - 1} = \frac{(l + a)(l - a)}{k - (l + a)}$$

$$\Rightarrow \frac{1}{n - 1} = \frac{l + a}{k - (l + a)} \Rightarrow k - (l + a) = (n - 1)$$

$(l + a)$

$\Rightarrow k = (n - 1)(l + a) + (l + a)$

$\Rightarrow k = (l + a)(n - 1 + 1) = n(l + a)$

$$= 2 \times \frac{n}{2}(l + a) = 2 \times S \quad \left\{\because \frac{n}{2}(l + a) = S\right\}$$

$= 2S$

Question 11.
If the sum of first n even natural number is equal to k times the sum of first n odd natural numbers, then k =

(a) $\frac{1}{n}$ (b) $\frac{n-1}{n}$

(c) $\frac{n+1}{2n}$ (d) $\frac{n+1}{n}$

Solution:
(d)

Sum of n even natural number $= n(n + 1)$
and sum of n odd natural numbers $= n^2$

$\therefore n(n + 1) = kn^2$

$$\Rightarrow k = \frac{n(n+1)}{n^2} = \frac{n+1}{n}$$

Question 12.
If the first, second and last term of an A.P. are a, b and 2a respectively, its sum is

(a) $\frac{ab}{2(b-a)}$ (b) $\frac{ab}{b-a}$

(c) $\frac{3ab}{2(b-a)}$ (d) None of these

Solution:
(c)

First term $(a_1) = a$
Second term $(a_2) = b$
and last term $(l) = 2a$
$\therefore d =$ Second term – first term $= b - a$
$\therefore l = a_n = a + (n-1)d$
$\Rightarrow 2a = a + (n-1)(b-a) \Rightarrow (n-1)(b-a) = a$

$$\Rightarrow n - 1 = \frac{a}{b-a} \Rightarrow n = \frac{a}{b-a} + 1 = \frac{a+b-a}{b-a}$$

$$= \frac{b}{b-a}$$

$$\therefore S_n = \frac{n}{2}[a + l] = \frac{b}{2(b-a)}[a + 2a]$$

$$= \frac{3ab}{2(b-a)}$$

Question 13.
If S_1 is the sum of an arithmetic progression of 'n' odd number of terms and S_2 is the sum of the terms of the

series in odd places, then $\frac{S_1}{S_2} =$

(a) $\frac{2n}{n+1}$ (b) $\frac{n}{n+1}$

(c) $\frac{n+1}{2n}$ (d) $\frac{n-1}{n}$

Solution:
(a)

Odd numbers are 1, 3, 5, 7, 9, 11, 13, ... n

$\therefore$ S_1 = Sum of odd numbers = n^2

S_2 = Sum of number at odd places

3, 7, 11, 15, ...

$a = 3, d = 7 - 3 = 4$ and number of term $= \frac{n}{2}$

$$S_2 = \frac{n}{2 \times 2}\left[2 \times 3 + \left(\frac{n}{2} - 1\right) \times 4\right]$$

$$= \frac{n}{4}[6 + 2n - 4] = \frac{n}{4}[2n + 2] = \frac{n(n+1)}{2}$$

$$\therefore \frac{S_1}{S_2} = \frac{n^2 \times 2}{n(n+1)} = \frac{2n}{n+1}$$

Question 14.

If in an A.P., $S_n = n^2p$ and $S_m = m^2p$, where S denotes the sum of r terms of the A.P., then S_p is equal to

(a) 12 p^3

(b) mnp

(c) p^3

(d) (m + n) p^2

Solution:

(c)

$S_n = n^2p$, $S_m = m^2p$

$\therefore$ $S_r = r^2p$ and $S_p = p^2q = p^3$

Hence $S_p = p^3$

Question 15.

If Sn denote the sum of the first n terms of an A.P. If $S_{2n} = 3S_n$, then $S_{3n} : S_n$ is equal to

(a) 4

(b) 6

(c) 8

(d) 10

Solution:

(b)

S_n = Sum of n terms of an A.P.
and $S_{2n} = 3\ S_n$

$S_n = \frac{n}{2}\ [2a + (n-1)\ d]$, $S_{2n} = \frac{2n}{2}\ [2a + (2n - 1)\ d]$ and $S_{3n} = \frac{3n}{2}\ [2a + (3n-1)\ d]$

We know that $S_{3n} = 3\ (S_{2n} - S_n)$ and $S_{2n} = 3S_n$

$$\frac{S_{3n}}{S_n} = \frac{3(S_{2n} - S_n)}{S_n} = \frac{3(3Sn - Sn)}{Sn}$$

$$= \frac{3 \times 2Sn}{Sn} = \frac{6}{1}$$

$\therefore\ S_{3n} : S_n = 6$

Question 16.
In an AP, $S_p = q$, $S_q = p$ and S denotes the sum of first r terms. Then, S_{p+q} is equal to
(a) 0
(b) – (p + q)
(c) p + q
(d) pq
Solution:
(c) In an A.P. $S_p = q$, $S_q = p$
S_{p+q} = Sum of (p + q) terms = Sum of p term + Sum of q terms = q + p

Question 17.
If S_n denotes the sum of the first r terms of an A.P. Then, $S_{3n} : (S_{2n} - S_n)$ is
(a) n
(b) 3n
(b) 3
(d) None of these
Solution:

(c)

$S_n = \frac{n}{2}[2a + (n-1)d]$, $S_{2n} = \frac{2n}{2}[2a + (2n - 1)d]$

and $S_{3n} = \frac{3n}{2}[2a + (3n-1)d]$

Now $S_{2n} - S_n = \frac{2n}{2}[2a + (2n-1)d] - \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[4a + (4n-2)d] - [2a + (n-1)d]$

$= \frac{n}{2}[4a - 2a + (4n - 2 - n + 1)d] = \frac{n}{2}[2a + (3n-1)d]$

$= \frac{1}{3}(S_{3n})$

$\therefore S_{3n} : (S_{3n} - S_n) = 3 : 1$ or $\frac{3}{1} = 3$

Question 18.
If the first term of an A.P. is 2 and common difference is 4, then the sum of its 40 term is
(a) 3200
(b) 1600
(c) 200
(d) 2800
Solution:
(a)

In an A.P.
$a = 2$ and $d = 4$, $n = 40$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d] = \frac{40}{2}[2 \times 2 + (40 - 1) \times 4]$

$= 20[4 + 39 \times 4] = 20 \times (4 + 156) = 20 \times 160 = 3200$

Question 19.
The number of terms of the A.P. 3, 7,11, 15, … to be taken so that the sum is 406 is
(a) 5
(b) 10
(c) 12
(d) 14
Solution:
(d)

The A.P. is 3, 7, 11, 15, ...
Where $a = 3$, $d = 7 - 3 = 4$ and sum $S_n = 406$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow 406 = \frac{n}{2}[2 \times 3$
$\times (n-1) \times 4]$
$\Rightarrow 812 = n(6 + 4n - 4) = 812 = n(4n + 2)$
$\Rightarrow 4n^2 + 2n - 812 = 0 \Rightarrow 2n^2 + n - 406 = 0$
$\Rightarrow 2n^2 + 29n - 28n - 406 = 0 \Rightarrow n(2n + 29) -$
$14(2n + 29) = 0$
$\Rightarrow (2n + 29)(n - 14) = 0$

$\therefore n = 14$ or $\frac{-29}{2}$

But $n = \frac{-29}{2}$ is not possible

Question 20.
Sum of n terms of the series
$\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots$ is

(a) $\frac{n(n+1)}{2}$ **(b)** $2n(n+1)$

(c) $\frac{n(n+1)}{\sqrt{2}}$ **(d)** 1

Solution:
(c)

The series is given

$$\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots..$$

$$\Rightarrow \sqrt{2} + 2\sqrt{2} + 3\sqrt{2} + 4\sqrt{2} + \ldots..$$

Here $a = \sqrt{2}$ and $d = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2\sqrt{2} + (n-1)\sqrt{2}]$$

$$= \frac{n}{2}[2\sqrt{2} + \sqrt{2}n - \sqrt{2}]$$

$$= \frac{n}{2}(\sqrt{2}n + \sqrt{2})$$

$$= \frac{n\sqrt{2}}{2}(n+1) = \frac{n(n+1)}{\sqrt{2}}$$

Question 21.

The 9th term of an A.P. is 449 and 449th term is 9. The term which is equal to zero is

(a) 50th

(b) 502th

(c) 508th

(d) None of these

Solution:

(d)

$a_n = a + (n-1)d$

$a_9 = 449 = a + (9-1)d = a + 8d \quad \ldots.(i)$

$a_{449} = 9 = a + (449-1)d = a + 448d$

$\ldots.(ii)$

Subtracting $440d = -440 \Rightarrow d = \frac{-440}{440} = -1$

and $a + 8d = 449 \Rightarrow a \times 8 \times (-1) = 449$

$\Rightarrow a = 449 + 8 = 457$

$\therefore 0 = a + (n-1)d$

$0 = 457 + (n-1)(-1) \Rightarrow 0 = 457 - n + 1$

$\Rightarrow n = 458$

$\therefore$ 458th term $= 0$

Question 22.

22. If $\frac{1}{x+2}, \frac{1}{x+3}, \frac{1}{x+5}$ are in A.P. then $x =$

(a) 5 (b) 3
(c) 1 (d) 2

Solution:
(c)

$\frac{1}{x+2}, \frac{1}{x+3}, \frac{1}{x+5}$ and in A.P.

$$\therefore \frac{1}{x+3} - \frac{1}{x+2} = \frac{1}{x+5} - \frac{1}{x+3}$$

$$\Rightarrow \frac{x+2-x-3}{(x+3)(x+2)} = \frac{x+3-x-5}{(x+5)\ (x+3)}$$

$$\Rightarrow \frac{-1}{(x+3)\ (x+2)} = \frac{-2}{(x+5)\ (x+3)}$$

$$\Rightarrow \frac{-1}{x+2} = \frac{-2}{x+5}$$

$\Rightarrow -2x - 4 = -x - 5$
$\Rightarrow -2x + x = -5 + 4 \Rightarrow -x = -1$
$\therefore x = 1$

Question 23.

The n^{th} term of an A.P., the sum of whose n terms is S_n, is

(a) $S_n + S_{n-1}$ (b) $S_n - S_{n-1}$
(c) $S_n + S_{n+1}$ (d) $S_n - S_{n+1}$

Solution:
(b) S_n is the sum of first n terms
Last term nth term = $S_n - S_{n-1}$

Question 24.
The common difference of an A.P., the sum of whose n terms is S_n, is

(a) $S_n - 2S_{n-1} + S_{n-2}$ (b) $S_n - 2S_{n-1} - S_{n-2}$
(c) $S_n - S_{n-2}$ (d) $S_n - S_{n-1}$

Solution:

(a)

Sum of n terms $= S_n$

$\therefore a_n = S_n - S_{n-1}$

and $a_{n-1} = S_{n-1} - S_{n-2}$

$\therefore$ Common difference $(d) = a_n - a_{n-1}$

$= (S_n - S_{n-1}) - (S_{n-1} - S_{n-2})$

$= S_n - S_{n-1} - S_{n-1} + S_{n-2}$

$= S_n - 2S_{n-1} + S_{n-2}$

Question 25.

If the sums of n terms of two arithmetic progressions are in the ratio $\frac{3n+5}{5n+7}$, then their n^{th} terms are in the ratio

(a) $\frac{3n-1}{5n-1}$ **(b)** $\frac{3n+1}{5n+1}$

(c) $\frac{5n+1}{3n+1}$ **(d)** $\frac{5n-1}{3n-1}$

Solution:

(b)

In first A.P. let its first term be a_1 and common difference d_1 and in second A.P., first term be a_2 and common difference d_2, then

$$\frac{S_n}{S_n} = \frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2}$$

$$\therefore \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{3n+5}{5n+7}$$

Substituting $n = 2n - 1$, then

$$\frac{2a_1 + (2n-2)d_1}{2a_2 + (2n-2)d_2} = \frac{3(2n-1)+5}{5(2n-1)+7}$$

$$\Rightarrow \frac{a_1 + (n-1)d_1}{a_2 + (n-1)d_2} = \frac{6n-3+5}{10n-5+7}$$

(Dividing by 2)

$$\Rightarrow \frac{a_{1n}}{a_{2n}} = \frac{6n+2}{10n+2} = \frac{3n+1}{5n+1}$$

Question 26.

If S_n denote the sum of n terms of an A.P. with first term a and common difference d such that $\frac{S_x}{S_{kx}}$ is independent of x, then

(a) $d = a$ (b) $d = 2a$

(c) $a = 2d$ (d) $d = -a$

Solution:

(b)

S_n is the sum of first n terms a is the first term and d is the common difference

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\frac{S_x}{S_{kx}} = \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{kx}{2}[2a+(kx-1)d]}$$

$\because \frac{S_x}{S_{kx}}$ is independent of x

$\therefore \frac{\frac{n}{2}[2a+(x-1)d]}{\frac{kx}{2}[2a+(kx-1)d]}$ is independent of x

$\therefore \frac{\frac{n}{2}[2a+xd-d]}{\frac{kx}{2}[2a+kdx-d]}$ is indepenet of x

$\Rightarrow \frac{2a-d}{k(2a-d)}$ is in dependent of x if $2a-d \neq 0$

If $2a - d = 0$, then $d = 2a$

Question 27.

If the first term of an A.P. is a and nth term is b, then its common difference is

(a) $\frac{b-a}{n+1}$ (b) $\frac{b-a}{n-1}$

(c) $\frac{b-a}{n}$ (d) $\frac{b+a}{n-1}$

Solution:

(b)

In the given A.P.

First term = a and nth term = b

$\therefore a + (n - 1) d = b$

$\Rightarrow (n - 1) d = b - a$

$$\Rightarrow d = \frac{b-a}{n-1}$$

Question 28.

The sum of first n odd natural numbers is

(a) $2n - 1$

(b) $2n + 1$

(c) n^2

(d) $n^2 - 1$

Solution:

(c)

1, 3, 5, 7, ... are n odd numbers

Where $a = 1$, and $d = 2$

$$\therefore S_n = \frac{n}{2} [2a + (n - 1)]$$

$$= \frac{n}{2} [2 \times 1 + (n - 1) \times 2]$$

$$= \frac{n}{2} [2 + 2n - 2] = \frac{n}{2} \times 2n$$

$$= n^2$$

Question 29.

Two A.P.'s have the same common difference. The first term of one of these is 8 and that of the other is 3. The difference between their 30th terms is

(a) 11

(b) 3

(c) 8

(d) 5

Solution:

(d) In two A.P.'s common-difference is same
Let A and a are two A.P. 's
First term of A is 8 and first term of a is 3
$A_{30} - a_{30} = 8 + (30 - 1) d - 3 - (30 - 1) d$
$= 5 + 29d - 29d = 5$

Question 30.
If 18, a, b – 3 are in A.P., the a + b =
(a) 19
(b) 7
(c) 11
(d) 15
Solution:
(d) 18, a, b – 3 are in A.P., then a – 18 = -3 – b
=> a + b = -3 + 18 = 15

Question 31.
The sum of n terms of two A.P.'s are in the ratio 5n + 4 : 9n + 6. Then, the ratio of their 18th term is
(a) $\frac{179}{321}$ (b) $\frac{178}{321}$
(c) $\frac{175}{321}$ (d) $\frac{176}{321}$
Solution:
(a)

Let a_1, d_2 be the first terms of two ratios S and S′ and d_1, d_2 be their common difference respectively

Then, $S_n = \frac{n}{2}[2a_1 + (n-1)d_1]$ and

$S'_n = \frac{n}{2}[2a_2 + (n-1)d_2]$

Now $\frac{S_n}{S'_n} = \frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]}$

$= \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2}$

But $\frac{S_n}{S'_n} = \frac{5n+9}{9n+6}$

$\therefore \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{5n+9}{9n+6}$

Now we have to find the ratios in 18th term

Here $n = 18$

$\therefore \frac{2a_1+(18-1)d_1}{2a_2+(18-1)d_2} = \frac{5(2n-1)+4}{9(2n-1)+6}$

$= \frac{5(2\times18-1)+4}{9(2\times18-1)+6} = \frac{5\times35+4}{9\times35+6}$

$= \frac{175+4}{315+6} = \frac{179}{321}$

Question 32.

If $\frac{5+9+13+\ldots \text{ to } n \text{ terms}}{7+9+11+\ldots \text{ to } (n+1) \text{ terms}} = \frac{17}{16}$, then $n =$

(a) 8 **(b) 7**

(c) 10 **(d) 11**

Solution:

(b)

Sum of 5 + 9 + 13 + ... to n terms

$$= \frac{n}{2}\left[2a + (n-1)d\right]$$

Here $a = 5$, $d = 9 - 5 = 4$

$$\therefore \text{ Sum} = \frac{n}{2}\left[2 \times 5 + (n-1) \times 4\right]$$

$$= \frac{n}{2}\left[10 + 4n - 4\right]$$

$$= \frac{n}{2}\left[6 + 4n\right] = n(3 + 2n)$$

and sum of 7 + 9 + 11 + ... to $(n + 1)$ terms

$$= \frac{n+1}{2}\left[2 \times 7 + (n + 1 - 1) 2\right]$$

$$= \frac{n+1}{2}\left[14 + 2n\right] = (n + 1)(7 + n)$$

$$\therefore \frac{5+9+13+\ldots \text{ to } n \text{ terms}}{7+9+11+\ldots \text{ to } (n+1) \text{ terms}} = \frac{17}{16}$$

$$\Rightarrow \frac{n(3+2n)}{(n+1)(7+n)} = \frac{17}{16}$$

$\Rightarrow 16n\ (3 + 2n) = 17\ (n + 1)\ (7 + n)$

$\Rightarrow 48n + 32n^2 = 17\ (n^2 + 8n + 7)$

$\Rightarrow 48n + 32n^2 = 17n^2 + 136n + 119$

$\Rightarrow 48n + 32n^2 - 17n^2 - 136n - 119 = 0$

$\Rightarrow 15n^2 - 88n - 119 = 0$

$\Rightarrow 15n^2 - 105n + 17n - 119 = 0$

$$\left\{\begin{array}{l} \because 15 \times (-119) = 1785 \\ \quad -1785 = 17 \times (105) \\ \quad -88 = 17 - 105 \end{array}\right\}$$

$\Rightarrow 15n\ (n - 7) + 17\ (n - 7) = 0$

$\Rightarrow (n - 7)\ (15n + 17) = 0$

Either $n - 7 = 0$, then $n = 7$

or $15n + 13 = 0$, then $n = \frac{-13}{15}$ which is not possible being fraction

$\therefore n = 7$

Question 33.

The sum of n terms of an A.P. is $3n^2 + 5n$, then 164 is its

(a) 24th term

(b) 27th term

(c) 26th term

(d) 25th term

Solution:

(b)

$$\text{Sum of } n \text{ terms } (S_n) = 3n^2 + 5n$$

$\therefore$ Sum of $(n-1)$ terms $(S_{n-1}) = 3(n-1)^2 + 5(n-1)$

$= 3(n^2 - 2n + 1) + 5n - 5$

$= 3n^2 - 6n + 3 + 5n - 5$

$= 3n^2 - n - 2$

$\therefore$ nth term $= S_n - S_{n-1}$

$\Rightarrow a_n = 3n^2 + 5n - 3n^2 + n + 2$

$a_n = 6n + 2$, But $a_n = 164$

$\Rightarrow 6n + 2 = 164 \Rightarrow 6n = 164 - 2 = 162$

$\therefore n = \frac{162}{6} = 27$

$\therefore$ 27th term

Question 34.
If the nth term of an A.P. is 2n + 1, then the sum of first n terms of the A.P. is
(a) n (n – 2)
(b) n (n + 2)
(c) n (n + 1)
(d) n (n – 1)
Solution:
(b)

$a_n = 2n + 1$

a or $a_1 = 2 \times 1 + 2 = 2 + 1 = 3$

$a_2 = 2 \times 2 + 1 = 4 + 1 = 5$

$\therefore d = a_2 - a_1 = 5 - 3 = 2$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 3 + (n-1) \times 2]$

$= \frac{n}{2}[6 + 2n - 2] = \frac{n}{2}[2n + 4]$

$= n(n + 2)$

Question 35.
If 18th and 11th term of an A.P. are in the ratio 3 : 2, then its 21st and 5th terms are in the ratio
(a) 3 : 2
(b) 3 : 1
(c) 1 : 3

(d) 2 : 3
Solution:
(b)

18th term : 11th term = 3 : 2

$$\Rightarrow \frac{a_{18}}{a_{11}} = \frac{3}{2} \Rightarrow \frac{a+17d}{a+10d} = \frac{3}{2}$$

$$\Rightarrow 2a + 34d = 3a + 30d$$

$$\Rightarrow 34d - 30d = 3a - 2a \Rightarrow a = 4d$$

$$\text{Now } \frac{a_{21}}{a_5} = \frac{a+20d}{a+4d} = \frac{4d+20d}{4d+4d}$$

$$= \frac{24d}{8d} = \frac{3}{1}$$

$$a_{21} : a_5 = 3 : 1$$

Question 36.
The sum of first 20 odd natural numbers is
(a) 100
(b) 210
(c) 400
(d) 420 **[CBSE 2012]**
Solution:
(c)

First 20 odd natural numbers are
1, 3, 5, 7, 9, 11, 13, 15, ..., 39
Here $a = 1$, $d = 2$, $n = 20$

$$\therefore S_{20} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{20}{2}[2 \times 1 + (20-1) \times 2]$$

$$= 10(2 + 38) = 10 \times 40 = 400$$

Question 37.

The common difference of the A.P. is $\frac{1}{2q}$,

$\frac{1-2q}{2q}$, $\frac{1-4q}{2q}$, ... is

(a) -1 (b) 1

(c) q (d) $2q$ [CBSE 2013]

Solution:

(a)

A.P. is $\frac{1}{2q}, \frac{1-2q}{2q}, \frac{1-4q}{2q}, \ldots$

$$\Rightarrow \frac{1}{2q}, \left(\frac{1}{2q}-1\right), \left(\frac{1}{2q}-2\right), \ldots$$

Clearly $d = \left(\frac{1}{2q}-1\right) - \frac{1}{2q}$

$$= \frac{1}{2q} - 1 - \frac{1}{2q} = -1$$

Question 38.

The common difference of the A.P. $\frac{1}{3}$,

$\frac{1-3b}{3}$, $\frac{1-6b}{3}$, ... is

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$

(c) $-b$ (d) b [CBSE 2013]

Solution:

(c)

$$\text{A.P. is } \frac{1}{3}, \frac{1-3b}{3}, \frac{1-6b}{3}, \ldots$$

$$\Rightarrow \frac{1}{3}, \frac{1}{3} - \frac{3b}{3}, \frac{1}{3} - \frac{6b}{3}, \ldots$$

$$\Rightarrow \frac{1}{3}, \frac{1}{3} - b, \frac{1}{3} - 2b, \ldots$$

$$\therefore d = \left(\frac{1}{3} - b\right) - \frac{1}{3} = \frac{1}{3} - b - \frac{1}{3} = -b$$

Question 39.

The common difference of the A.P. 12b ,

$$\frac{1-6b}{2b}, \frac{1-12b}{2b}, \ldots \text{ is}$$

(a) $2b$ **(b)** $-2b$

(c) 3 **(d)** -3 **[CBSE 2013]**

Solution:

(d)

$$\text{A.P. is } \frac{1}{2b}, \frac{1-6b}{2b}, \frac{1-12b}{2b}, \ldots$$

$$\Rightarrow \frac{1}{2b}, \frac{1}{2b} - \frac{6b}{2b}, \frac{1}{2b} - \frac{12b}{2b}, \ldots$$

$$\Rightarrow \frac{1}{2b}, \frac{1}{2b} - 3, \frac{1}{2b} - 6, \ldots$$

$$\therefore d = \frac{1}{2b} - 3 - \frac{1}{2b} = -3$$

Question 40.

If k, 2k – 1 and 2k + 1 are three consecutive terms of an AP, the value of k is
(a) -2
(b) 3
(c) -3
(d) 6 **[CBSE 2014]**

Solution:

(b) (2k – 1) – k = (2k + 1) – (2k- 1)
2k – 1 – k = 2
=> k = 3

Question 41.
The next term of the A.P. , √7 , √28, √63, …………
(a) √70
(b) √84
(c) √97
(d) √112 **[CBSE 2014]**
Solution:
(d)

AP is $\sqrt{7}, \sqrt{28}, \sqrt{63}, \ldots$

$\Rightarrow \sqrt{7}, \sqrt{4 \times 7}, \sqrt{9 \times 7}, \ldots$

$\Rightarrow \sqrt{7}, 2\sqrt{7}, 3\sqrt{7}, \ldots$

$\therefore$ Here $a = \sqrt{7}$

and $d = 2\sqrt{7} - \sqrt{7} = \sqrt{7}$

$\therefore$ Next term $= 4\sqrt{7}$

= √(l6 x 7)= √112

Question 42.
The first three terms of an A.P. respectively are 3y – 1, 3y + 5 and 5y + 1. Then, y equals
(a) -3
(b) 4
(c) 5
(d) 2 **[CBSE 2014]**
Solution:
(c) 2 (3y + 5) = 3y – 1 + 5y + 1
(If a, b, c are in A.P., b – a = c – b=> 2b = a + c)
=> 6y + 10 = 8y
=> 10 = 2y
=> y = 5