Exercise 14.1

1.

Sol:

(i) $P(5,0)$ lies on $x-axis$

(ii) $Q(0,-2)$ lies on $y-axis$

(iii) $R(-4,0)$ lies on $x-axis$

(iv) $S(0,5)$ lies on $y-axis$

2.

Sol:

(i) Coordinate of the vertices of the square of side $2a$ are:

$A(0,0), B(2a,0), C(2a,2a)$ *and* $D(0,2a)$

(ii) Coordinate of the vertices of the square of side $2a$ are:

$A(a,a), B(-a,a), C(-a,-a)$ *and* $(a,-a)$

3.

Sol:

We have two equilateral triangle PQR and PQR' with side $2a$.

O is the mid-point of PQ.

In ΔQOR, $\angle QOR = 90°$

Hence, by Pythagoras theorem

$$OR^2 + OQ^2 = QR^2$$

$$OR^2 = (2a)^2 - (a)^2$$

$$OR^2 = 3a^2$$

$$OR = \sqrt{(3)}a$$

Coordinates of vertex R is $(\sqrt{3}a, 0)$ and coordinate of vertex R' is $(-\sqrt{3}a, 0)$

Exercise 14.2

1.

Sol:

(i) We have $P(-6,7)$ and $Q(-1,-5)$

Here,

$x_1=-6, y_1=7$ and

$x_2=-1, y_2=-5$

$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$PQ=\sqrt{[-1-(-6)]^2+(-5-7)^2}$

$PQ=\sqrt{(-1+6)^2+(-5-7)^2}$

$PQ=\sqrt{(5)^2+(-12)^2}$

$PQ=\sqrt{25+144}$

$PQ=\sqrt{169}$

$PQ=13$

(ii) we have $P(a+b,b+c)$ and $Q(a-b,c-b)$ here,

$x_1=a+b, y_1=b+c$ and $x_2=a-b, y_2=c-b$

$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$PQ=\sqrt{[a-b-(a+b)]^2+(c-b-(b+c))^2}$

$PQ=\sqrt{(a-b-a-b)^2+(c-b-b-c)^2}$

$PQ=\sqrt{(-2b)^2+(-2b)^2}$

$PQ=\sqrt{4b^2+4b^2}$

$PQ=\sqrt{8b^2}$

$PQ=\sqrt{4\times 2b^2}$

$PQ=2\sqrt{2}b$

(iii) we have $P(a\sin\alpha,-b\cos\alpha)$ and $Q(-a\cos\alpha,b\sin\alpha)$ here

$x_1=a\sin\alpha, y_1=-b\cos\alpha$ and

$x_2=-a\cos\alpha, y_2=b\sin\alpha$

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$PQ=\sqrt{(-a\cos\alpha-a\sin\alpha)^2+[-b\sin\alpha-(-b\cos\alpha)]^2}$$

$$PQ=\sqrt{(-a\cos\alpha)^2+(-a\sin\alpha)^2+2(-a\cos\alpha)(-a\sin\alpha)+(b\sin\alpha)^2+(-b\cos\alpha)^2-2(b\sin\alpha)(-b\cos\alpha)}$$

$$PQ=\sqrt{a^2\cos^2\alpha+a^2\sin^2\alpha+2a^2\cos\alpha\sin\alpha+b^2\sin^2\alpha+b^2\cos^2\alpha+2b^2\sin\alpha\cos\alpha}$$

$$PQ=\sqrt{a^2(\cos^2\alpha+\sin^2\alpha)+2a^2\cos\alpha\sin\alpha+b^2(\sin^2\alpha+\cos^2\alpha)+2b^2\sin\alpha\cos\alpha}$$

$$PQ=\sqrt{a^2\times1+2a^2\cos\alpha\sin\alpha+b^2\times1+2b^2\sin\alpha\cos\alpha}\qquad[\because\sin^2\alpha+\cos^2\alpha=1]$$

$$PQ=\sqrt{a^2+b^2+2a^2\cos\alpha\sin\alpha+2b^2\sin\alpha\cos\alpha}$$

$$PQ=\sqrt{(a^2+b^2)+2\cos\alpha\sin\alpha(a^2+b^2)}$$

$$PQ=\sqrt{(a^2+b^2)(1+2\cos\alpha\sin\alpha)}$$

(iv) We have $P(a,0)$ and $Q(0,b)$

Here,

$x_1=a, y_1=0, x_2=0, y_2=b,$

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$PQ=\sqrt{(0-a)^2+(b-0)^2}$$

$$PQ=\sqrt{(-a)^2+(b)^2}$$

$$PQ=\sqrt{a^2+b^2}$$

2.

Sol:

We have $P(3,a)$ and $Q(4,1)$

Here,

$x_1=3, y_1=a$

$x_2=4, y_2=1$

$PQ=\sqrt{10}$

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\Rightarrow\sqrt{10}=\sqrt{(4-3)^2+(1-a)^2}$$

$$\Rightarrow\sqrt{10}=\sqrt{(1)^2+(1-a)^2}$$

$\Rightarrow \sqrt{10} = \sqrt{1+1+a^2-2a}$ $\left[\because (a-b)^2 = a^2+b^2-2ab\right]$

$\Rightarrow \sqrt{10} = \sqrt{2+a^2-2a}$

Squaring both sides

$\Rightarrow \left(\sqrt{10}\right)^2 = \left(\sqrt{2+a^2-2a}\right)^2$

$\Rightarrow 10 = 2+a^2-2a$

$\Rightarrow a^2-2a+2-10=0$

$\Rightarrow a^2-2a-8=0$

Splitting the middle team.

$\Rightarrow a^2-4a+2a-8=0$

$\Rightarrow a(a-4)+2(a-4)=0$

$\Rightarrow (a-4)(a+2)=0$

$\Rightarrow a=4, a=-2$

3.

Sol:

We have $P(2,1)$ and $Q(1,-2)$ and $R(X,Y)$

Also, $PR = QR$

$PR = \sqrt{(x-2)^2+(y-1)^2}$

$\Rightarrow PR = \sqrt{x^2+(2)^2-2xx\times 2+y^2+(1)^2-2\times y\times 1}$

$\Rightarrow PR = \sqrt{x^2+4-4x+y^2+1-2y}$

$\Rightarrow PR = \sqrt{x^2+5-4x+y^2-2y}$

$QR = \sqrt{(x-1)^2+(y+2)^2}$

$\Rightarrow PR = \sqrt{x^2+1-2x+y^2+4+4y}$

$\Rightarrow PR = \sqrt{x^2+5-2x+y^2+4y}$

$\because PR = QR$

$\Rightarrow \sqrt{x^2+5-4x+y^2-2y} = \sqrt{x^2+5-2x+y^2+4y}$

$\Rightarrow x^2+5-4x+y^2-2y = x^2+5-2x+y^2+4y$

$\Rightarrow x^2+5-4x+y^2-2y = x^2+5-2x+y^2+4y$

$\Rightarrow -4x+2x-2y-4y=0$

$\Rightarrow -2x-6y=0$

$\Rightarrow -2(x+3y)=0$

$\Rightarrow x+3y=\frac{0}{-2}$

$\Rightarrow x+3y=0$

Hence proved.

4.

Sol:

We have $P(x,y), Q(-3,0)$ and $R(3,0)$

$PQ=\sqrt{(x+3)^2+(y-0)^2}$

$\Rightarrow 4=\sqrt{x^2+9+6x+y^2}$

Squaring both sides

$\Rightarrow (4)^2=\left(\sqrt{x^2+9+6x+y^2}\right)^2$

$\Rightarrow 16=x^2+9+6x+y^2$

$\Rightarrow x^2+y^2=16-9-6x$

$\Rightarrow x^2+y^2=7-6x$ …………..(1)

$PR=\left(\sqrt{(x-3)^2+(y-0)^2}\right)$

$\Rightarrow 4=\sqrt{x^2+9-6x+y^2}$

Squaring both sides

$(4)^2=\left(\sqrt{x^2+9-6x+y^2}\right)^2$

$\Rightarrow 16=x^2+9-6x+y^2$

$\Rightarrow x^2+y^2=16-9+6x$

$\Rightarrow x^2+y^2=7+6x$ …………..(2)

Equating (1) and (2)

$7-6x=7+6x$

$\Rightarrow 7-7=6x+6x$

$\Rightarrow 0=12x$

$\Rightarrow x=12$

Equating (1) and (2)

$7-6x=7+6x$

$\Rightarrow 7-7=6x+6x$

$\Rightarrow 0=12x$

$\Rightarrow x = 12$

Substituting the value of $x = 0$ in (2)

$x^2 + y^2 = 7 + 6x$

$0 + y^2 = 7 + 6 \times 0$

$y^2 = 7$

$y = \pm\sqrt{7}$

5.

Sol:

Let two ordinate of the other end R be Y

$\therefore$ Coordinates of other end R are $(10, y)$ i.e., $R(10, y)$

Distance $PR = 10$ [given]

$PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow 10 = \sqrt{(10-2)^2 + (y+3)^2}$

$\Rightarrow 10 = \sqrt{8^2 + y^2 + 9 + 6y}$

$\Rightarrow 10 = \sqrt{64 + y^2 + 9 + 6y}$

$= 10 = \sqrt{73 + y^2 + 6y}$

Squaring both sides

$(10)^2 = \left(\sqrt{73 + y^2 + 6y}\right)^2$

$\Rightarrow 100 = 73 + y^2 + 6y$

$\Rightarrow y^2 + 6y + 73 - 100 = 0$

$\Rightarrow y^2 + 6y - 27 = 0$

Splitting the middle term

$y^2 + 9y - 3y - 27 = 0$

$\Rightarrow y^2 + 9y - 3y - 27 = 0$

$\Rightarrow y(y+9) - 3(y+9) = 0$

$\Rightarrow (y+9)(y-3) = 0$

$\Rightarrow y = -9, y = 3$

6.

Sol:

Let $A(-4,-1), B(-2,-4), C(4,0)$ *and* $D(2,3)$ be the given points

Now,

$AB=\sqrt{(-2+4)^2+(-4+1)^2}$

$\Rightarrow AB=\sqrt{(2)^2+(-3)^2}$

$\Rightarrow AB=\sqrt{4+9}$

$\Rightarrow AB=\sqrt{13}$

$CD=\sqrt{(4-2)^2+(0-3)^2}$

$\Rightarrow CD=\sqrt{(2)^2+(-3)^2}$

$\Rightarrow CD=\sqrt{4+9}$

$\Rightarrow CD=\sqrt{13}$

$BC=\sqrt{(4+2)^2+(0+4)^2}$

$\Rightarrow BC=\sqrt{(6)^2+(4)^2}$

$\Rightarrow BC=\sqrt{36+16}$

$\Rightarrow BC+\sqrt{52}$

$AD=\sqrt{(-4-2)^2+(-1-3)^2}$

$\Rightarrow AD=\sqrt{(-6)^2+(-4)^2}$

$\Rightarrow AD=\sqrt{36+16}$

$\Rightarrow AD=\sqrt{52}$

$\therefore\ AB=CD$ and $AD=BC\Rightarrow ABCD$ is a parallelogram

Now,

$AC=\sqrt{(4+4)^2+(0+1)^2}$

$\Rightarrow AC=\sqrt{(8)^2+(1)^2}$

$\Rightarrow AC=\sqrt{64+1}$

$\Rightarrow AC=\sqrt{65}$

$BD=\sqrt{(2+2)^2+(3+4)^2}$

$\Rightarrow BD=\sqrt{(4)^2+(7)^2}$

$\Rightarrow BD=\sqrt{16+49}$

$\Rightarrow BD=\sqrt{65}$

Since the diagonals of parallelogram $ABCD$ are equal i.e., $AC=BD$

Hence, $ABCD$ is a rectangle

7.

Sol:

Let $A(1,-2),B(3,6),C(5,10),D(3,2)$ be the given points

$AB=\sqrt{(3-1)^2+(6+2)^2}$

$\Rightarrow AB=\sqrt{(2)^2+(8)^2}$

$\Rightarrow AB=\sqrt{4+64}$

$\Rightarrow AB=\sqrt{68}$

$CD=\sqrt{(5-3)^2+(10-2)^2}$

$\Rightarrow CD=\sqrt{(2)^2+(8)^2}$

$\Rightarrow CD=\sqrt{4+64}$

$\Rightarrow CD=\sqrt{68}$

$AD=\sqrt{(3-1)^2+(2+2)^2}$

$\Rightarrow AD=\sqrt{(2)^2+(4)^2}$

$\Rightarrow AD=\sqrt{4+16}$

$\Rightarrow AD=\sqrt{20}$

$BC=\sqrt{(5-3)^2+(10-6)^2}$

$\Rightarrow BC=\sqrt{(2)^2+(4)^2}$

$\Rightarrow BC=\sqrt{4+16}$

$\Rightarrow BC=\sqrt{20}$

$\therefore \quad AB=CD$ and $AD=BC$

Since opposite sides of a parallelogram are equal

Hence, $ABCD$ is a parallelogram

8.

Sol:

Let $A(1,7), B(4,2), C(-1,-1)$ and $D(-4,4)$ be the given point. One way of showing that $ABCD$ is a square is to use the property that all its sides should be equal and both its diagonals should also be equal

Now,

$AB=\sqrt{(1-4)^2+(7-2)^2}=\sqrt{9+25}=\sqrt{34}$

$BC=\sqrt{(4+1)^2+(2+1)^2}=\sqrt{25+9}=\sqrt{34}$

$CD=\sqrt{(-1+4)^2+(-1-4)^2}=\sqrt{9+25}=\sqrt{34}$

$DA=\sqrt{(1+4)^2+(7-4)^2}=\sqrt{25+9}=\sqrt{34}$

$AC=\sqrt{(1+1)^2+(7+1)^2}=\sqrt{4+64}=\sqrt{68}$

$BD=\sqrt{(4+4)^2+(2-4)^2}=\sqrt{64+4}=\sqrt{68}$

Since, $AB=BC=CD=DA$ and $AC=BD$, all the four sides of the quadrilateral $ABCD$ are equal and its diagonals AC and BD are also equal. Therefore, $ABCD$ is a square

9.

Sol:

Let $A(3,0), B(6,4)$ and $C(-1,3)$ be the given points

$AB=\sqrt{(6-3)^2+(4-0)^2}$

$\Rightarrow AB=\sqrt{(3)^2+(4)^2}$

$\Rightarrow AB=\sqrt{9+16}$

$\Rightarrow AB=\sqrt{25}$

$BC=\sqrt{(-1-6)^2+(3-4)^2}$

$\Rightarrow BC=\sqrt{(-7)^2+(-1)^2}$

$\Rightarrow BC=\sqrt{49+1}$

$\Rightarrow BC=\sqrt{50}$

$AC=\sqrt{(-1-3)^2+(3-0)^2}$

$\Rightarrow AC=\sqrt{(-4)^2+(3)^2}$

$\Rightarrow A\sqrt{16+9}$

$\Rightarrow AC = \sqrt{25}$

$AB^2 = \left(\sqrt{25}\right)^2$

$\Rightarrow AB^2 = 25$

$AC^2 = 25$

$BC^2 = \left(\sqrt{50}\right)^2$

$BC^2 = 50$

Since $AB^2 + AC^2 = BC^2$ and $AB = AC$

$\therefore ABC$ is a right angled isosceles triangle

10.

Sol:

Let $A(2,-2), B(-2,1)$ and $C(5,2)$ be the given points

$AB = \sqrt{(-2-2)^2 + (1+2)^2}$

$\Rightarrow AB = \sqrt{(-4)^2 + (3)^2}$

$\Rightarrow AB + \sqrt{16-9}$

$\Rightarrow AB = \sqrt{25}$

$BC = \sqrt{(5+2)^3 + (2-1)^2}$

$\Rightarrow B\sqrt{(7)^2 + (1)^2}$

$\Rightarrow BC = \sqrt{49+1}$

$\Rightarrow BC = \sqrt{50}$

$AC = \sqrt{(5-2)^2 + (2+2)^2}$

$\Rightarrow AC = \sqrt{(3)^2 + (4)^2}$

$\Rightarrow AC = \sqrt{9+16}$

$\Rightarrow AC = \sqrt{25}$

$AB^2 = \left(\sqrt{25}\right)^2$

$\Rightarrow AB^2 = 25$

$BC^2 = \left(\sqrt{50}\right)^2$

$\Rightarrow BC^2 = 50$

Since, $AB^2 + AC^2 = BC^2$

$\therefore ABC$ is a right angled triangle.

Length of the hypotenuse $BC = \sqrt{50} = 5\sqrt{2}$

Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times \sqrt{25} \times \sqrt{25}$

$= \frac{25}{2}$ square units.

11.

Sol:

Let $A(2a, 4a), B(2a, 6a)$ and $C\left(2a+\sqrt{3}a, 5a\right)$ be the given points

$AB = \sqrt{(2a-2a)^2 + (6a-4a)^2}$

$\Rightarrow AB = \sqrt{(0)^2 + (2a)^2}$

$\Rightarrow AB = \sqrt{4a^2}$

$\Rightarrow AB = 2a$

$BC = \sqrt{\left(2a+\sqrt{3}a-2a\right)^2 + (5a-6a)^2}$

$\Rightarrow BC = \sqrt{\left(\sqrt{3}a\right)^2 + (-a)^2}$

$\Rightarrow BC = \sqrt{3a^2 + a^2}$

$\Rightarrow BC = \sqrt{4a^2}$

$\Rightarrow BC = 2a$

$AC = \sqrt{\left(2a+\sqrt{3}a-2a\right)^2 + (5a-4a)^2}$

$\Rightarrow AC = \sqrt{\left(\sqrt{3}a\right)^2 + (a)^2}$

$\Rightarrow AC = \sqrt{3a^2 + a^2}$

$\Rightarrow AC = \sqrt{4a^2}$

$\Rightarrow AC = 2a$

Since, $AB = BC = AC$

$\therefore$ ABC is an equilateral triangle

12.

Sol:

Let $A(2,3), B(-4,-6)$ and $C(1, 3/2)$ be the given points

$$AB=\sqrt{(-4-2)^2+(-6-3)^2}$$
$$\Rightarrow AB=\sqrt{(-6)^2+(-9)^2}$$
$$\Rightarrow AB=\sqrt{36+81}$$
$$\Rightarrow AB=\sqrt{117}$$
$$BC=\sqrt{(1+4)^2+\left(\frac{3}{2}+6\right)^2}$$
$$\Rightarrow BC=\sqrt{(5)^2+\left(\frac{15}{2}\right)^2}$$
$$\Rightarrow BC=\sqrt{25+\frac{225}{4}}$$
$$\Rightarrow BC=\sqrt{\frac{325}{4}}$$
$$\Rightarrow BC=\sqrt{8125}$$
$$AC=\sqrt{(2-1)^2+\left(3-\frac{3}{2}\right)^2}$$
$$\Rightarrow AC=\sqrt{(1)^2+\left(\frac{3}{2}\right)^2}$$
$$\Rightarrow AC=\sqrt{1+\frac{9}{4}}$$
$$\Rightarrow AC=\sqrt{\frac{13}{4}}$$
$$\Rightarrow AC=\sqrt{3.25}$$
We know that for a triangle sum of two sides is greater than the third side

Here $AC+BC$ is not greater than $AB.$

$\therefore ABC$ is not triangle

13.

Sol:

Let $A(3,4), B(-2,3)$ and $C(x,y)$ be the three vertices of the equilateral triangle then,

$$AB^2=BC^2=CA^2$$
$$AB=\sqrt{(-2-3)^2+(3-4)^2}=\sqrt{(-5)^2+(-1)^2}=\sqrt{25+1}=\sqrt{26}$$
$$BC=\sqrt{(x+2)^2+(y-3)^2}=\sqrt{x^2+4+4x+y^2+9-6y}=\sqrt{x^2+y^2-6x-8y+25}$$

$CA=\sqrt{(x-3)^2+(y-4)^2}=\sqrt{x^2+9-6x+y^2+16-8y}=\sqrt{x^2+y^2-6x-8y+25}$

Now, $AB^2=BC^2$

$\Rightarrow x^2+y^2+4x-6y+13=26$

$\Rightarrow x^2+y^2+4x-6y-13=0$ …….(i)

$AB^2=CA^2$

$\Rightarrow 26-x^2+y^2-6x-8y+25$

$\Rightarrow x^2+y^2-6x-8y-1=0$ …….(ii)

Subtracting (ii) from (i) we get,

$10x+2y-12=0$

$\Rightarrow 5x+y=6$ …….(iii)

$\Rightarrow 5x=6-y$

$\Rightarrow x=\dfrac{6-y}{5}$

Subtracting $x=\dfrac{6-y}{5}$ in (i) we get

$$\left(\frac{6-y}{5}\right)^2+y^2+4\left(\frac{6-y}{5}\right)-6y-13=0$$

$$\Rightarrow \frac{(6-y)^2}{25}+y^2+\frac{24-4y}{5}-6y-13=0$$

$$\Rightarrow \frac{36+y^2-12y}{25}+y^2+\frac{24-4y}{5}-6y-13=0$$

$$\Rightarrow \frac{36+y^2-12y+25y^2+120-20y-150-13\times 25}{25}=0$$

$\Rightarrow 26y^2-32y+6-325=0$

$\Rightarrow 26y^2-32y-319=0$

$D=b^2-4ac$

$D=(-32)^2-4\times 26\times(-319)=1024+33176=34200$

$\therefore \quad y=\dfrac{-b\pm\sqrt{D}}{2a}=\dfrac{-(-32)\pm\sqrt{34200}}{2\times 26}$

$\therefore y=\dfrac{32+185}{52}=\dfrac{217}{52}$ or $y=\dfrac{32-185}{52}=\dfrac{-153}{52}$

Substituting $y=\dfrac{217}{52}$ in (iii)

$$5x+\frac{217}{52}=6$$

$$5x=6-\frac{217}{52}=\frac{95}{52}$$

$$x=\frac{19}{52}$$

Again substituting $y=\frac{-153}{52}$ in (iii)

$$5x-\frac{153}{52}=6$$

$$5x=6+\frac{153}{52}=\frac{465}{52}$$

$$x=\frac{93}{52}$$

Therefore, the coordinates of the third vertex are $\left(\frac{19}{52},\frac{217}{52}\right)$ or $\left(\frac{93}{52},\frac{-153}{52}\right)$

14.

Sol:

Let $A(2,-1), B(3,4), C(-2,3)$ and $D(-3,-2)$

$$AB=\sqrt{(3-2)^2+(4+1)^2}=\sqrt{(1)^2+(5)^2}=\sqrt{1+25}=\sqrt{26}$$

$$BC=\sqrt{(-2-3)^2+(3-4)^2}=\sqrt{(-5)^2+(-1)^2}=\sqrt{25+1}=\sqrt{26}$$

$$CD=\sqrt{(-3+2)^2+(-2-3)^2}=\sqrt{(-1)^2+(-5)^2}=\sqrt{1+25}=\sqrt{26}$$

$$AD=\sqrt{(-3-2)^2+(-2+1)^2}=\sqrt{(-5)^2+(-1)^2}=\sqrt{25+1}=\sqrt{26}$$

Since $AB=BC=CD=AD$

$\therefore$ $ABCD$ is a rhombus

15.

Sol:

Two vertices of an isosceles triangle are $A(2,0)$ and $B(2,5)$. Let $C(x,y)$ be the third vertex

$$AB=\sqrt{(2-2)^2+(5-0)^2}=\sqrt{(0)^2+(5)^2}=\sqrt{25}=5$$

$$BC=\sqrt{(x-2)^2+(y-5)^2}=\sqrt{x^2+4-4x+y^2+25-10y}=\sqrt{x^2-4x+y^2-10y+29}$$

$$AC=\sqrt{(x-2)^2+(y-0)^2}=\sqrt{x^2+4-4x+y^2}$$

Also we are given that

$AC - BC = 3$

$\Rightarrow AC^2 = BC^2 = 9$

$\Rightarrow x^2 + 4 - 4x + y^2 = x^2 - 4x + y^2 - 10y + 29$

$\Rightarrow 10y = 25$

$\Rightarrow y = \frac{25}{10} = \frac{5}{2}$

$AC^2 = 9$

$x^2 + 4 - 4x + y^2 = 9$

$x^2 + 4 - 4x + (2.5)^2 = 9$

$x^2 + 4 - 4x + 6.25 = 9$

$x^2 - 4x + 1.25 = 0$

$D = (-4)^2 - 4 \times 1 \times 1.25$

$D = 16 - 5$

$D = 11$

$x = \frac{-(-4) + \sqrt{11}}{2 \times 1} = \frac{4 + 3.31}{2} = \frac{7.31}{2} = 3.65$

Or $x = \frac{-(-4) - \sqrt{11}}{2} = \frac{4 - \sqrt{11}}{2} = \frac{4 - 3.31}{2} = 0.35$

The third vertex is $(3.65, 2.5)$ or $(0.35, 2.5)$

16.

Sol:

Let $A(5,9)$ and $B(-4,6)$ be the given points.

Let $C(x,0)$ be the point on $x-axis$

Now,

$AC = \sqrt{(x-5)^2 + (0-9)^2}$

$\Rightarrow AC = \sqrt{x^2 + 25 - 10x + (-9)^2}$

$\Rightarrow AC = \sqrt{x^2 - 10x + 25 + 81}$

$\Rightarrow AC = \sqrt{x^2 - 10x + 106}$

$BC = \sqrt{(x+4)^2 + (0-6)^2}$

$\Rightarrow BC = \sqrt{x^2 + 16 + 8d + (-6)^2}$

$\Rightarrow BC = \sqrt{x^2 + 8x + 16 + 36}$

$\Rightarrow BC = \sqrt{x^2 + 8x + 5x}$

Since $AC = BC$

Or, $AC^2 = BC^2$

$x^2 - 10x + 106 = x^2 + 8x + 52$

$\Rightarrow -10x + 106 = 8x + 52$

$\Rightarrow -10x - 8x = 52 - 106$

$\Rightarrow -18x = -54$

$\Rightarrow x = \frac{54}{18}$

$\Rightarrow x = 3$

Hence the points on x-axis is $(3, 0)$.

17.

Sol:

Let $A(-2,5), B(0,1)$ and $C(2,-3)$ be the given points

$AB = \sqrt{(0+2)^2 + (1-5)^2}$

$\Rightarrow AB = \sqrt{4 + (-4)^2}$

$\Rightarrow AB = \sqrt{4+16}$

$\Rightarrow AB = \sqrt{20}$

$\Rightarrow AB = 2\sqrt{5}$

$BC = \sqrt{(2-0)^2 + (-3-1)^2}$

$\Rightarrow BC = \sqrt{(2)^2 + (-4)^2}$

$\Rightarrow BC = \sqrt{4+16}$

$\Rightarrow BC = \sqrt{20}$

$\Rightarrow BX = 2\sqrt{5}$

$AC = \sqrt{(2+2)^2 + (-3-5)^2}$

$\Rightarrow AC = \sqrt{(4)^2 + (-8)^2}$

$\Rightarrow AC = \sqrt{16+64}$

$\Rightarrow AC = \sqrt{80}$

$\Rightarrow AC = 4\sqrt{5}$

Since $AB + BC = AC$

Hence $A(-2,5), B(0,1)$, and $C(2,-3)$ are collinear

18.

Sol:

Let the coordinates of Q be (x, y)

Since Q lies on the line joining P and O (origin) and $OP = OQ$

By mid-point theorem

$$\frac{(x-3)}{2} = 0 \text{ and } \frac{(y+2)}{2} = 0$$

$\therefore x = 3, y = -2$

Hence coordinates of points Q are $(3,-2)$

19.

Sol:

$A(2,3)$ and $B(-4,1)$ are the given points.

Let $C(0, y)$ be the points are $y - axis$

$AC = \sqrt{(0-2)^2 + (y-3)^2}$

$\Rightarrow AC = \sqrt{4 + y^2 + 9 - 6y}$

$\Rightarrow AC = \sqrt{y^2 - 6y + 13}$

$BC = \sqrt{(0+4)^2 + (y-1)^2}$

$\Rightarrow BC = \sqrt{16 + y^2 + 1 - 2y}$

$\Rightarrow BC = \sqrt{y^2 - 2y + 17}$

Since $AC = BC$

$AC^2 = BC^2$

$y^2 - 6y + 13 = y^2 - 2y + 17$

$\Rightarrow -6y + 2y = 17 - 13$

$\Rightarrow -4y = 4$

$\Rightarrow y = -1$

$\therefore$ The point on $y - axis$ is $(0,-1)$

20.

Sol:

Let $A(3,4), B(3,8)$ and $C(9,8)$ be the given points

Let the forth vertex be $D(x, y)$

$AB = \sqrt{(3-3)^2 + (8-4)^2}$

$\Rightarrow AB = \sqrt{0 + (4)^2}$

$\Rightarrow AB = \sqrt{16}$

$\Rightarrow AB = 4$

$BC = \sqrt{(9-3)^2 + (8-8)^2}$

$\Rightarrow BC = \sqrt{(6)^2 + 0}$

$\Rightarrow BC = \sqrt{36}$

$\Rightarrow BC = B$

$CD = \sqrt{(x-9)^2 + (y-8)^2}$

$\Rightarrow CD = \sqrt{x^2 + (9^2) - 18x + y^2 + (8^2) - 16y}$

$\Rightarrow CD = \sqrt{x^2 + 81 - 18x + y^2 + 64 - 16y}$

$\Rightarrow CD = \sqrt{x^2 - 18x + y^2 - 16y + 145}$

$AD = \sqrt{(x-3)^2 + (y-4)^2}$

$\Rightarrow AD = \sqrt{x^2 + 9 - 6x + y^2 + 16 - 8y}$

$\Rightarrow AD = \sqrt{x^2 - 6x + y^2 - 8y + 25}$

Since ABCD is a parallelogram and opposite sides of a parallelogram are equal

$AB = CD$ and $AD = BC$

$AB = CD$

$AB^2 = CD^2$

$\Rightarrow x^2 - 18x + y^2 - 16y + 145 = 16$

$\Rightarrow x^2 - 18x + y^2 - 16y + 145 - 16 = 0$

$\Rightarrow x^2 - 18x + y^2 - 16y + 129 = 0$ ………..(1)

$BC = AD$

$BC^2 = AD^2$

$x^2 - 6x + y^2 - 8y + 25 = 36$

$\Rightarrow x^2 - 6x + y^2 - 8y + 25 - 36 = 0$

$\Rightarrow x^2 - 6x + y^2 - 8y - 11 = 0$ ....(2)

$x = 9, y = 4$

The fourth vertex is $D(9,4)$

21.

Sol:

Circumcenter of a triangle is the point of intersection of all the three perpendicular bisectors of the sides of triangle. So, the vertices of the triangle lie on the circumference of the circle.

Let the coordinates of the circumcenter of the triangle be (x, y)

$\therefore (x, y)$ will the equidistant from the vertices of the triangle.

Using distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, it is obtained:

$D_1 = \sqrt{(x+2)^2 + (y+3)^2}$

$\Rightarrow D_1 = \sqrt{x^2 + 4 + 4x + y^2 + 9 + 6y}$ (Taking points (x, y) and $(-2,-3)$)

$\Rightarrow D_1 = \sqrt{x^2 + y^2 + 4x + 6y + 13}$

$D^2 = \sqrt{(x+1)^2 + (y-0)^2}$ (Taking points (x, y) and $(-1,0)$)

$\Rightarrow D_2 = \sqrt{x^2 + 1 + 2x + y^2}$

$D_3 = \sqrt{(x-7)^2 + (y+6)^2}$ (Taking points (x, y) and $(7,-6)$)

$\Rightarrow D_3 = \sqrt{x^2 + 49 - 14x + y^2 + 36 + 12y}$

$\Rightarrow D_3 = \sqrt{x^2 + y^2 - 14x + 12y + 85}$

As (x, y) is equidistant from all the three vertices

So, $D_1 = D_2 = D_3$

$D_1 = D_2$

$\therefore \sqrt{x^2 + y^2 + 4x + 6y + 13} = \sqrt{x^2 + 1 + 2x + y^2}$

$\Rightarrow x^2 + y^2 + 4x + 6y + 13 = x^2 + 1 + 2x + y^2$

$\Rightarrow 4x + 6y - 2x = 1 - 13$

$\Rightarrow 2x + 6y = -12$

$\Rightarrow x + 3y = -6$ ....(1)

$D_2 = D_3$

$\therefore \sqrt{x^2 + 1 + 2x + y^2} = \sqrt{x^2 + y^2 - 14x + 12y + 85}$

$\Rightarrow x^2+1+2x+y^2=x^2+y^2-14x+12y+85$

$\Rightarrow 2x+14x-12y=85-1$

$\Rightarrow 16x-12y=84$

$\Rightarrow 4x-3y=21$ ....(2)

Adding equations (1) and (2):

$x+3y+4x-3y=-6+21$

$\therefore 5x=15$

$\Rightarrow x=\frac{15}{3}$

$\Rightarrow x=3$

When $x=3,$ we get

$y=\frac{4(3)-21}{3}$ [Using (2)]

$\Rightarrow y=\frac{12-21}{3}$

$\Rightarrow y=-\frac{9}{3}$

$\Rightarrow y=-3$

$\therefore (3,-3)$ are the coordinates of the circumcenter of the triangle

22.

Sol:

Let the point $P(0,100)$ and $Q(10,0)$ be the given points.

$\therefore$ The angle subtended by the line segment PQ at the origin O is $90°$.

23.

Sol:

Let the center of the circle be $O(x,y)$

Since radii of the circle is constant

Hence, distance of O from $A(5,-8), B(2,-9)$ and $C\ (2,1)$ will be constant and equal

$\therefore OA^2=OB^2=OC^2$

$(x-5)^2+(y+B)^2=(x-2)^2+(y+9)^2$

$x^2+25-10x+y^2+64+16y=x^2+4-4x+y^2+81+18y$

$-6x-2y+4=0$

$3x+y-2=0$

$y = 2 - 3x$ …………(i)

Also, $OB^2 = OC^2$

$(x-2)^2 + (y+9)^2 + (y-1)^2$

$y^2 + 81 + 18y = y^2 + 1 - 2y$

$80 + 20y = 0$

$y = -4$

Substituting y in (i)

$-4 = 2 - 3x$

$3x = 6$

$x = 2$

Hence center of circle $(2, -4)$

24.

Sol:

Let the point $P(0,2)$ is equidistant from $A(3,k)$ and $(k,5)$

$PA = PB$

$PA^2 = PB^2$

$(3-0)^2 + (k-2)^2 = (k-0)^2 + (5-2)^2$

$\Rightarrow 9 + k^2 + 4 - 4k = k^2 + 9.$

$\Rightarrow 9 + k^2 + 4 - 4k - k^2 - 9 = 0$

$\Rightarrow 4 - 4k = 0$

$\Rightarrow -4k = -4$

$\Rightarrow k = 1$

25.

Sol:

Let $ABCD$ be a square and let $A(5,4)$ and $C(1,-6)$ be the given points.

Let (x, y) be the coordinates of $B.$

$AB = BC$

$AB^2 = BC^2$

$(x-5)^2 + (y-4)^2 = (x-1)^2 + (y+6)^2$

$\Rightarrow x^2 + 25 - 10x + y^2 + 16 - 8y = x^2 + 1 - 2x + y^2 + 36 + 12y$

$\Rightarrow x^2 - 10x + y^2 - 8y - x^2 + 2x - y^2 - 12y = 1 + 36 - 25 - 16$

$\Rightarrow -8x - 20y = -4$

$\Rightarrow -8x = 20y - 4$

$$\Rightarrow x = \frac{20y-4}{-8}$$

$$\Rightarrow x = \frac{4(5y-1)}{-8}$$

$$\Rightarrow x = \frac{5y-1}{-2}$$

$$\Rightarrow x = \frac{1-5y}{2} \qquad \ldots\ldots\ldots..(1)$$

In right triangle ABC

$$AB^2 + BC^2 = AC^2$$

$$(x-5)^2 + (y-4)^2 + (x-1)^2 + (y+6)^2 = (5-1)^2 + (4+6)^2$$

$$\Rightarrow x^2 + 25 - 10x + y^2 + 16 - 8y + x^2 + 1 - 2x + y^2 + 36 + 12y = 16 + 100$$

$$\Rightarrow 2x^2 + 2y^2 - 12x + 4y = 116 - 78$$

$$\Rightarrow 2x^2 + 2y^2 - 12 + 4y = 38$$

$$\Rightarrow x^2 + y^2 - 6x + 2y = 19$$

$$\Rightarrow x^2 + y^2 - 6x + 2y - 19 = 0 \qquad \ldots\ldots\ldots..(2)$$

Substituting the value of x form (1) in (2), we get

$$\left(\frac{1-5y}{2}\right)^2 + y^2 - 6\left(\frac{1-5y}{2}\right) + 2y - 19 = 0$$

$$\Rightarrow \frac{(1-5y)^2}{4} + y^2 - 3(1-5y) + 2y - 19 = 0$$

$$\Rightarrow \frac{1 + 25y^2 - 10y}{4} + y^2 - 3 + 15y + 2y - 19 = 0$$

$$\Rightarrow \frac{1 + 25y^2 - 10y + 4y^2 - 12 + 60y + 8y - 76}{4} = 0$$

$$\Rightarrow 29y^2 + 58y - 87 = 0$$

$$\Rightarrow y^2 + 2y - 3 = 0$$

$$\Rightarrow y^2 + 3y - y - 3 = 0$$

$$\Rightarrow y(y+3) - 1(y+3) = 0$$

$$\Rightarrow (y+3)(y-1) = 0$$

$$\Rightarrow y = -3, y = 1$$

Substituting $y = -3$ and $y = 1$ in equation (1), we get

$$x = \frac{1-5(-3)}{2}$$

$\Rightarrow x = \frac{1+15}{2}$

$\Rightarrow x = 8$

$x = \frac{1-5(1)}{2}$

$\Rightarrow x = \frac{1-5}{2}$

$\Rightarrow x = \frac{-4}{2}$

$\Rightarrow x = -2$

Hence, the required vertices of the square are $(-2,1)$ and $(8,-3)$.

26.

Sol:

$A(-3,2), B(-5,-5), C(2,-3)$ and $D(4,4)$ be the given points.

$AB = \sqrt{(-5+3)^2 + (-5-2)^2}$

$\Rightarrow AB = \sqrt{(2)^2 + (-7)^2}$

$\Rightarrow AB = \sqrt{4+49}$

$\Rightarrow AB = \sqrt{53}$

$BC = \sqrt{(2+5)^2 + (-5-2)^2}$

$\Rightarrow BC = \sqrt{(7)^2 + (2)^2}$

$\Rightarrow BC = \sqrt{49+4}$

$\Rightarrow BC = \sqrt{53}$

$CD = \sqrt{(4-2)^2 + (4+3)^2}$

$\Rightarrow CD = \sqrt{(2)^2 + (7)^2}$

$\Rightarrow CD = \sqrt{4+49}$

$\Rightarrow CD = \sqrt{53}$

$AD = \sqrt{(4+3)^2 + (4-2)^2}$

$\Rightarrow AD = \sqrt{(7)^2 + (2)^2}$

$\Rightarrow AD = \sqrt{49+4}$

$\Rightarrow AD = \sqrt{53}$

$AC = \sqrt{(2+3)^2 + (-3-2)^2}$

$\Rightarrow AC = \sqrt{(5)^2 + (-5)^2}$

$\Rightarrow AC = \sqrt{25+25}$

$\Rightarrow AC = \sqrt{50}$

$BD = \sqrt{(4+5)^2 + (4+5)^2}$

$\Rightarrow BD = \sqrt{(9)^2 + (9)^2}$

$\Rightarrow BD = \sqrt{81+81}$

$\Rightarrow BD = \sqrt{162}$

Since $AB = BC = CD = AD$ and diagonals $AC \neq BD$

$\therefore ABCD$ is a rhombus

Area of rhombus $ABCD = \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times \sqrt{50} \times \sqrt{162}$

$= \frac{1}{2} \times 90$

$= 45$ sq. units

27.

Sol:

Let $A(3,0), B(-1,-6)$ and $C(4,-1)$ be the given points.

Let $O(x, y)$ be the circumcenter of the triangle

$OA = OB = OC$

$OA^2 = OB^2$

$(x-3)^2 + (y-0)^2 = (x+1)^2 + (y+6)^2$

$\Rightarrow x^2 + 9 - 6x + y^2 = x^2 + 1 + 2x + y^2 + 36 + 12y$

$\Rightarrow x^2 - 6x + y^2 - x^2 - 2x - y^2 - 12y = 1 + 36 - 9$

$\Rightarrow -8x - 12y = 28$

$\Rightarrow -2x - 3y = 7$

$\Rightarrow 2x + 3y = -7$ ....(1)

Again

$OB^2 = OC^2$

$(x+1)^2 + (y+6)^2 = (x-4)^2 + (y+1)^2$

$\Rightarrow x^2 + 1 + 2x + y^2 + 36 + 12y = x^2 + 16 - 8x + y^2 + 1 + 2y$

$\Rightarrow x^2 + 2x + y^2 + 12y - x^2 + 8x - y^2 - 2y = 16 + 1 - 1 - 36$

$\Rightarrow 10x + 10y = -20$

$\Rightarrow x + y = -2$ ....(2)

Solving (1) and (2), we get

$x = 1, y = -3$

Hence circumcenter of the triangle is $(1, -3)$

Circum radius $= \sqrt{(1+1)^2 + (-3+6)^2}$

$= \sqrt{(2)^2 + (3)^2}$

$= \sqrt{4+9}$

$\sqrt{13}$ units

28.

Sol:

Let $A(7,6)$ and $B(-3,4)$ be the given points.

Let $P(x.0)$ be the point on $x-axis$ such that $PA = PB$

$PA = PB$

$PA^2 = PB^2$

$(x-7)^2 + (0-6)^2 = (x+3)^2 + (0-4)^2$

$\Rightarrow x^2 + 49 - 14x + 36 = x^2 + 9 + 6x + 16$

$\Rightarrow x^2 - 14 - x^2 - 6x = 9 + 16 - 36 - 49$

$\Rightarrow -20x = -60$

$\Rightarrow x = 3$

$\therefore$ The point on $x-axis$ is $(3,0)$.

29.

Sol:

$A(5,6), B(1,5), C(2,1)$ and $D(6,2)$ are the given points

$AB = \sqrt{(5-1)^2 + (6-5)^2}$

$\Rightarrow AB = \sqrt{(4)^2 + (1)^2}$

$\Rightarrow AB=\sqrt{16+1}$

$\Rightarrow AB=\sqrt{17}$

$BC=\sqrt{(1-2)^2+(5-1)^2}$

$\Rightarrow BC=\sqrt{(-1)^2+(4)^2}$

$\Rightarrow BC=\sqrt{1+16}$

$\Rightarrow BC=\sqrt{17}$

$CD=\sqrt{(6-2)^2+(2-1)^2}$

$\Rightarrow CD=\sqrt{(4)^2+(1)^2}$

$\Rightarrow CD=\sqrt{16+1}$

$\Rightarrow CD=\sqrt{17}$

$AD=\sqrt{(6-5)^2+(2-6)^2}$

$\Rightarrow AD=\sqrt{(1)^2+(-4)^2}$

$\Rightarrow AD=\sqrt{1+16}$

$\Rightarrow AD=\sqrt{17}$

$AC=\sqrt{(5-2)^2+(6-1)^2}$

$\Rightarrow AC=\sqrt{(3)^2+(5)^2}$

$\Rightarrow AC=\sqrt{9+25}$

$\Rightarrow AC=\sqrt{34}$

$BD=\sqrt{(6-1)^2+(2-5)^2}$

$\Rightarrow BD=\sqrt{(5)^2+(-3)^2}$

$\Rightarrow BD=\sqrt{25+9}$

$\Rightarrow BD=\sqrt{34}$

Since $AB=BC=CD=AD$ and diagonals $AC=BD$

$\therefore ABCD$ is a square

30.

Sol:

Let $A(-2,5)$ and $(2,-3)$ be the given points.

Let $(x,0)$ be the point on $x-axis$

Such that $PA = PB$

$PA = PB$

$PA^2 = PB^2$

$(x+2)^2 + (0-5)^2 = (x-2)^2 + (0+3)^2$

$\Rightarrow x^2 + 4 + 4x + 25 = x^2 + 4 - 4x + 9$

$\Rightarrow x^2 + 4x + x^2 + 4x = 4 + 9 - 4 - 25$

$\Rightarrow 8x = -16$

$\Rightarrow x = -2$

$\therefore$ The point on $x - axis$ is $(-2, 0)$

31.

Sol:

$P(6,-1), Q(1,3)$ and $R(x,8)$ are the given points.

$PQ = QR$

$PQ^2 = QR^2$

$\Rightarrow (6-1)^2 + (-1-3)^2 = (x-1)^2 + (8-3)^2$

$\Rightarrow (5)^2 + (-4)^2 = x^2 + 1 - 2x + (5)^2$

$\Rightarrow 25 + 16 = x^2 + 1 - 2x + 25$

$\Rightarrow 41 = x^2 - 2x + 26$

$\Rightarrow x^2 - 2x + 26 - 41 = 0$

$\Rightarrow x^2 - 2x - 15 = 0$

$\Rightarrow x^2 - 5x + 3x - 15 = 0$

$\Rightarrow x(x-5) + 3(x-5) = 0$

$\Rightarrow (x+3)(x-5) = 0$

$\Rightarrow x = -3$ or $x = 5$

32.

Sol:

Let $A(0,0), B(5,5)$ and $C(-5,5)$ be the given points

$AB = \sqrt{(5-0)^2 + (5-0)^2}$

$\Rightarrow AB = \sqrt{25+25}$

$\Rightarrow AB = \sqrt{50}$

$BC = \sqrt{(5+5)^2 + (5-5)^2}$

$\Rightarrow BC = \sqrt{(10)^2 + 0}$

$\Rightarrow BC = \sqrt{100}$

$AC = \sqrt{(0+5)^2 + (0-5)^2}$

$\Rightarrow AC = \sqrt{25+25}$

$\Rightarrow AC = \sqrt{50}$

$AB^2 = 50$

$BC^2 = 100$

$AC^2 = 50$

$\Rightarrow AB^2 + AC^2 = BC^2$

Since, $AB = AC$ and $AB^2 + AC^2 = BC^2$

$\therefore$ ABC is a right isosceles triangle

33.

Sol:

Since $P(x, y)$ is equidistant from $A(5,1)$ and $B(1,5)$

$AP = BP$

Hence, $AP^2 = BP^2$

$(x-5)^2 + (y-1)^2 = (x-1)^2 + (y-5)^2$

$x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 1 - 2x + y^2 + 25 - 10y$

$-10x + 2x = -10y + 2y$

$-8x = -8y$

$x = y$

Hence, proved.

34.

Sol:

Given $Q(0,1)$ is equidistant from $P(-5,-3)$ and $R(x,6)$ so $PQ = QR$

$\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(0-x)^2 + (1-6)^2}$

$\sqrt{(5)^2 + (-4)^2} = \sqrt{(-x)^2 + (-5)^2}$

$\sqrt{25+16} = \sqrt{x^2 + 25}$

$41 = x^2 + 25$

$16 = x^2$

$x = \pm 4$

So, point R is $(4,6)$ or $(-4,6)$

When point R is $(4,6)$

$$PR=\sqrt{(5-4)^2+(-3-6)^2}=\sqrt{1^2+(-9)}=\sqrt{1+81}=\sqrt{82}$$

$$QR=\sqrt{(0-4)^2+(1-6)^2}=\sqrt{(-4)^2+(-5)^2}=\sqrt{16+25}=\sqrt{41}$$

When point R is $(-4,6)$

$$PR=\sqrt{(5-(-4))^2+(-3-6)^2}=\sqrt{(9)^2+(-9)^2}=\sqrt{81+81}=9\sqrt{2}$$

$$QR=\sqrt{(0(-4))^2+(1-6)^2}=\sqrt{(4)^2+(-5)^2}=\sqrt{16+25}=\sqrt{41}$$

35.

Sol:

Given that distance between $(2,-3)$ and $(10,y)$ is 10

Therefore using distance formula $\sqrt{(2-10)^2+(-3-y)^2}=10$

$$\sqrt{(-8)^2+(3+y)^2}=10$$

$$64+(y-3)^2=100$$

$$(y+3)^2=36$$

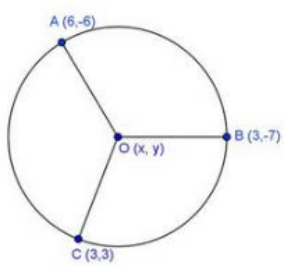
$$y+3=\pm 6$$

$y+3=6$ or $y+3=-6$

Therefore $y=3$ or -9

36.

Sol:



Let $O(x,y)$ be the center of the circle passing through $A(6,-6)$, $B(3,-7)$ and $C(3,3)$

$$OA=OB=OC$$

$$OA^2=OB^2$$

$$(x-6)^2+(y+6)^2=(x-3)^2+(y-7)^2$$

$\Rightarrow x^2+36-12x+y^2+36+12y=x^2+9-6x+y^2+49+14y$

$\Rightarrow x^2+36-12x+y^2+36+12y-x^2-9+6x-y^2-49-14y=0$

$\Rightarrow -6x-2y=-36-36+9+49$

$\Rightarrow -6x-2y=-14$ ....(1)

$OB^2=OC^2$

$(x-3)^2+(y+7)^2=(x-3)^2+(y-3)^2$

$\Rightarrow x^2+9-6x+y^2+49+14y=x^2+9-6x+y^2+9-6y$

$\Rightarrow x^2-6x+y^2+14y-x^2+6x-y^2+6y=9+9-9-49$

$\Rightarrow 20y=-40$

$\Rightarrow y=-2$

Substituting $y=-2$ in (1)

$-6x-2(-2)=-14$

$\Rightarrow -6x+4=-14$

$\Rightarrow -6x=-14-4$

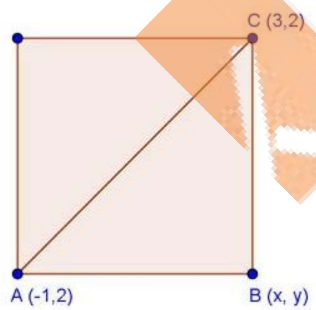
$\Rightarrow -6x=-18$

$\Rightarrow x=3$

$\therefore$ The centre of the circle is $(3,-2)$

37.

Sol:



Let $ABCD$ be a square and let $A(-1,2)$ and $(3,2)$ be the opposite vertices and let $B(x,y)$ be the unknown vertex.

$AB=BC$

$AB^2=BC^2$

$(x+1)^2+(y-2)^2=(x-3)^2+(y-2)^2$

$\Rightarrow x^2+1+2x+y^2+4-4y=x^2+9-6x+y^2+4-4y$

$\Rightarrow x^2+2x+y^2-4y-x^2+6x-y^2+4y=9+4-1-4$

$\Rightarrow 8x = 8$

$\Rightarrow x = 1$ ………(1)

In right triangle ABC

$AB^2 + BC^2 = AC^2$

$\Rightarrow (x+1)^2 + (y-2)^2 + (x-3)^2 + (y-2)^2 = (3+1)^2 + (2-2)^2$

$\Rightarrow x^2 + 1 + 2x + y^2 + 4 - 4y + x^2 + 9 - 6x + y^2 + 4 - 4y = 16$

$\Rightarrow 2x^2 + 2y^2 - 4x - 8y = 16 - 1 - 4 - 9 - 4$

$\Rightarrow 2x^2 + 2y^2 - 4x - 8y = -2$ ………(2)

Substituting $x = 1$ from (1) and (2)

$2(1)^2 + 2y^2 - 4(1) - 8y = -2$

$\Rightarrow 2 + 2y^2 - 4 - 8y = -2$

$\Rightarrow 2y^2 - 8y - 2 + 2 = 0$

$\Rightarrow 2y^2 - 8y = 0$

$\Rightarrow 2y(y-4) = 0$

$\Rightarrow y = 0$, or $y = 4$

Hence the required vertices of the square are $(1,0)$ and $(1,4)$

38.

Sol:

(i) Let, $A = (-1,-2), B = (1,0), C = (-1,2,), D(-3,0)$

$AB = \sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$BC = \sqrt{(1-(-1))^2 + (0-2)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$CD = \sqrt{(-1-(-3))^2 + (2-0)^2} = \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$AD = \sqrt{(-1-(-3))^2 + (-2-0)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

Diagonal $AC - \sqrt{(-1-(-1))^2 + (-2-2)^2} = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$

Diagonal $BD - \sqrt{(1-(-3))^2 + (0-0)^2} = \sqrt{(4)^2 + 0^2} = \sqrt{16} = 4$

Here, all sides of this quadrilateral are of same length and also diagonals are of same length. So, given points are vertices of a square

(ii) Let, $A = (-3,5), B = (3,1), C = (0,3), D = (-1,-4)$

$$AB=\sqrt{(-3-3)^2+(5-1)^2}=\sqrt{(-6)^2+(4)^2}=\sqrt{36+16}=\sqrt{52}=2\sqrt{13}$$

$$BC=\sqrt{(3-0)^2+(1-3)^2}=\sqrt{(3)^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$$

$$CD=\sqrt{(0-(-1))^2+(3-(-4))^2}=\sqrt{(1)^2+(7)^2}=\sqrt{1+49}=\sqrt{50}=5\sqrt{2}$$

$$AD=\sqrt{(-3-(-1))^2+(5-(-4))^2}=\sqrt{(-2)^2+(9)^2}=\sqrt{4+81}=\sqrt{85}$$

Here, all sides of this quadrilateral are of different length . So, we can say that it is only a general quadrilateral not specific like square, rectangle etc.

(iii) Let, $A=(4,5), B=(7,6), C=(4,3), D=(1,4)$

$$AB=\sqrt{(4-7)^2+(5-6)^2}=\sqrt{(-3)^2+(-1)^2}=\sqrt{9+1}=\sqrt{10}$$

$$BC=\sqrt{(7-4)^2+(6-3)^2}=\sqrt{(3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}$$

$$CD=\sqrt{(4-1)^2+(3-2)^2}=\sqrt{(3)^2+(1)^2}=\sqrt{9+1}=\sqrt{10}$$

$$AD=\sqrt{(4-1)^2+(5-2)^2}=\sqrt{(3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}$$

Diagonal $AC-\sqrt{(4-4)^2+(5-3)^2}=\sqrt{(0)^2+(2)^2}=\sqrt{0+4}=2$

Diagonal $BD-\sqrt{(7-1)^2+(6-2)^2}=\sqrt{(6)^2+(4)^2}=\sqrt{36+16}=\sqrt{52}=13\sqrt{2}$

Here, opposite sides of this quadrilateral are of same length but diagonals are different length . So, given points are vertices of a parallelogram.

39.

Sol:

Bisector passes through midpoint

Midpoint of $(7,1)$ and $(3,5)=\left[\frac{(7+3)}{2},\frac{(1+5)}{2}\right]=(5,3)$

Perpendicular bisector has slope that is negative reciprocal of line segment joining points $(7,1)$ and $(3,5)$

Slope of line segment $=\left(\frac{5-1}{3-7}\right)=\frac{4}{(-4)}=-1$

Perpendicular bisector has slope = 1 and passes through point $(4,4)$

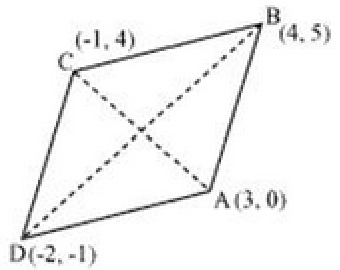
Use point slope form

$y-3=1(x-5)$

$y=x\cdot 2$

40.

Sol:



Let the given vertices be $A(3,0), B(4,5), C(-1,4)$ and $D(-2,-1)$

Length of $AB = \sqrt{(4-3)^2+(5-0)^2} = \sqrt{1+25} = \sqrt{26}$

Length of $BC = \sqrt{(-1-4)^2+(4-5)^2} = \sqrt{25+1} = \sqrt{26}$

Length of $CD = \sqrt{(-2+1)^2+(-1-4)^2} = \sqrt{1+25} = \sqrt{26}$

Length of $DA = \sqrt{(3+2)^2+(0+1)^2} = \sqrt{25+1} = \sqrt{26}$

Length of diagonal $AC = \sqrt{\left[3-(-1)^2+(0-4)^2\right]}$

$= \sqrt{16+16} = 4\sqrt{2}$

Length of diagonal $BD = \sqrt{\left[4-(-2)^2\right]+\left[5-(-1)\right]^2}$

$= \sqrt{36+36} = 6\sqrt{2}$

Here all sides of the quadrilateral ABCD are off same lengths but the diagonals are of different lengths

So, ABCD is a rhombus.

Therefore area of rhombus $ABCD = \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

$= 24$ square units

41.

Sol:

Let $A(3,1), B(6,4)$ and $C(8,6)$ be thee given points

$AB = \sqrt{(6-3)^2+(4-1)^2}$

$\Rightarrow AB = \sqrt{(3)^2+(3)^2}$

$\Rightarrow AB = \sqrt{9+9}$

$\Rightarrow AB = \sqrt{18}$

$\Rightarrow AB = 3\sqrt{2}$

$BC = \sqrt{(8-6)^2 + (6-4)^2}$

$\Rightarrow BC = \sqrt{(2)^2 + (2)^2}$

$\Rightarrow BC = \sqrt{4+4}$

$\Rightarrow BC = \sqrt{8}$

$\Rightarrow BC = 2\sqrt{2}$

$AC = \sqrt{(8-3)^2 + (6-1)^2}$

$\Rightarrow AC = \sqrt{(5)^2 + (5)^2}$

$\Rightarrow AC = \sqrt{25+25}$

$\Rightarrow AC = \sqrt{50}$

$\Rightarrow AC = 5\sqrt{2}$

Since, $AB + BC = AC$

Points A, B, C are collinear

Hence, Rohini, Sandhya and Bina are seated in a line

42.

Sol:

Let $A(5,-2)$ and $B(-3,2)$ be the given points,

Let $P(0, y)$ be the point on $y-axis$

$PA = PB$

$PA^2 = PB^2$

$(0-5)^2 + (y+2)^2 = (0+3)^2 + (y-2)^2$

$\Rightarrow 25 + y^2 + 4 + 4y = 9 + y^2 + 4 - 4y$

$\Rightarrow y^2 + 4y - y^2 + 4y = 9 + 4 - 4 - 25$

$\Rightarrow 8y = -16$

$\Rightarrow y = -2$

43.

Sol:

Point (x, y) is equidistant form $(3,6)$ and $(-3,4)$

Therefore $\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x-(-3))^2 + (y-4)^2}$

$$\sqrt{(x-3)^2+(y-6)^2}=\sqrt{(x+3)^2+(y-4)^2}$$

$$(x-3)^2+(y-6)^2=(x+3)^2+(y-4)^2$$

$$x^2+9-6x+y^2+36-12y=x^2+9+6x+y^2+16-8y$$

$$36-16=6x+6x+12y-8y$$

$$20=12x+4y$$

$$3x+y=5$$

44.

Sol:

$A(0,2), B(3,P)$ and $C(p,5)$ are given points

It is given that $AB=AC$

$$\therefore AB^2=AC^2$$

$$(3-0)^2+(p-2)^2=(p-0)^2+(5-2)^2$$

$$9+p^2+4-4p=p^2+9$$

$$4-4p=0$$

$$p=1$$

Exercise 14.3

1.

Sol:

Let $P(x,y)$ be the required point.

$$x=\frac{mx_2+nx_1}{m+n}$$

$$y=\frac{my_2+ny_1}{m+n}$$

Here, $x_1=-1$

$$y_1=3$$

$$x_2=4$$

$$y_2=-7$$

$$m:n=3:4$$

$$x=\frac{3\times4+4\times(-1)}{3+4}3$$

$$x=\frac{12-4}{7}$$

$$x = \frac{8}{7}$$

$$y = \frac{3\times(-7)+4\times3}{3+4}$$

$$y = \frac{-21+12}{7}$$

$$y = \frac{-9}{7}$$

$\therefore$ The coordinates of P are $\left(\frac{8}{7}, \frac{-9}{7}\right)$

2.

Sol:

(i) Let P and Q be the point of trisection of AB i.e., $AP = PQ = QB$

$(5,-6)$ $(-7,5)$

Therefore, P divides AB internally in the ratio of 1:2, thereby applying section formula, the coordinates of P will be

$$\left(\frac{1(-7)+2(5)}{1+2}\right), \left(\frac{1(5)+2(-6)}{1+2}\right) i.e., \left(1, \frac{-7}{3}\right)$$

Now, Q also divides AB internally in the ratio of 2:1 there its coordinates are

$$\left(\frac{2(-7)+1(5)}{2+1}\right), \frac{2(5)+1(-6)}{2+1} i.e., \left(-3, \frac{4}{3}\right)$$

(ii)

Let P, Q be the point of tri section of AB i.e.,

$AP = PQ = QB$

$(3,-2)$ $(-3,-4)$

Therefore, P divides AB internally in the ratio of 1:2

Hence by applying section formula, Coordinates of P are

$$\left(\left(\frac{1(-3)+2(3)}{1+2}\right), \frac{1(-4)+1(-2)}{1+2}\right) i.e., \left(1, \frac{-8}{3}\right)$$

Now, Q also divides as internally in the ratio of 2:1

So, the coordinates of Q are

$$\left(\left(\frac{2(-3)+1(3)}{2+1}\right),\frac{2(-4)+1(-2)}{2+1}\right) i.e., \left(-1,\frac{-10}{3}\right)$$

Let P and Q be the points of trisection of AB i.e., $AP = PQ = OQ$

Therefore, P divides AB internally in the ratio 1 : 2. Therefore, the coordinates of P, by applying the section formula, are

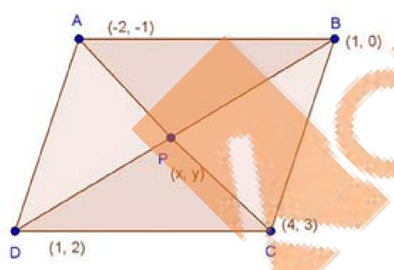
$$\left(\left(\frac{1(-7)+2(2)}{(1+2)}\right),\left(\frac{1(4)+2(-2)}{(1+2)}\right)\right), i.e., (-1,0)$$

Now, Q also divides AB internally in the ration 2 : 1. So, the coordinates of Q are

$$\left(\frac{2(-7)+1(2)}{2+1},\frac{2(4)+1(2)}{2+1}\right), i.e., (-4,2)$$

3.

Sol:



Let $P(x,y)$ be the given points.

We know that diagonals of a parallelogram bisect each other.

$$x = \frac{-2+4}{2}$$

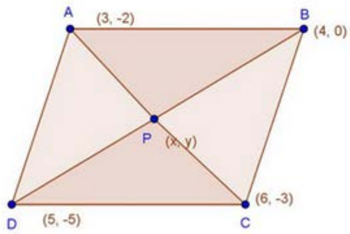
$$\Rightarrow x = \frac{2}{2} = 1$$

$$y = \frac{-1+3}{2} = \frac{2}{2} = 1$$

$\therefore$ Coordinates of P are $(1,1)$

4.

Sol:



Let $P(x, y)$ be the point of intersection of diagonals AC and 80 of $ABCD$.

$x = \frac{3+6}{2} = \frac{9}{2}$

$y = \frac{-2-3}{2} = \frac{-5}{2}$

Mid – point of $AC = \left(\frac{9}{5}, \frac{-5}{2}\right)$

Again,

$x = \frac{5+4}{2} = \frac{9}{2}$

$y = \frac{-5+0}{2} = \frac{-5}{2}$

Mid – point of $BD = \left(\frac{9}{2}, -\frac{5}{2}\right)$

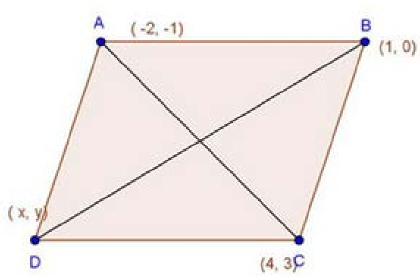
Here mid-point of AC – Mid - point of BD i.e, diagonals AC and BD bisect each other.

We know that diagonals of a parallelogram bisect each other

$\therefore$ $ABCD$ is a parallelogram.

5.

Sol:



Let $A(-2,-1), B(1,0), C(4,3)$ and $D(x, y)$ be the vertices of a parallelogram *ABCD* taken in order.

Since the diagonals of a parallelogram bisect each other.

$\therefore$ Coordinates of the mid - point of *AC* = Coordinates of the mid-point of *BD*.

$$\Rightarrow \frac{-2+4}{2} = \frac{1+x}{2}$$
$$\Rightarrow \frac{2}{2} = \frac{x+1}{2}$$
$$\Rightarrow 1 = \frac{x+1}{2}$$
$$\Rightarrow x+1 = 2$$
$$\Rightarrow x = 1$$

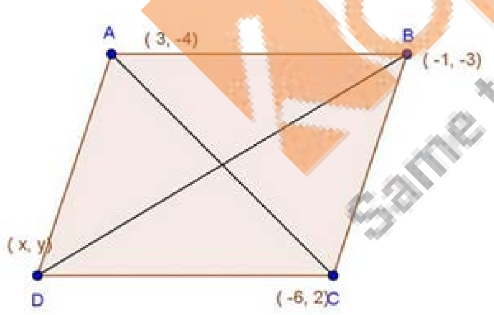
And, $\frac{-1+3}{2} = \frac{y+0}{2}$

$$\Rightarrow \frac{2}{2} = \frac{y}{2}$$
$$\Rightarrow y = 2$$

Hence, fourth vertex of the parallelogram is $(1, 2)$

6.

Sol:



Let $A(3,-4)$ and $C(-6,-2)$ be the extremities of diagonal *AC* and $B(-1,-3)$, $D(x, y)$ be the extremities of diagonal *BD*.

Since the diagonals of a parallelogram bisect each other.

$\therefore$ Coordinates of mid-point of *AC* = Coordinates of mid point of *BD*.

$\Rightarrow \frac{3-6}{2} = \frac{x-1}{2}$

$\Rightarrow \frac{-3}{2} = \frac{x-1}{2}$

$\Rightarrow x = -2$

And, $\frac{-4+2}{2} = \frac{y-3}{2}$

$\Rightarrow \frac{-2}{2} = \frac{y-3}{2}$

$\Rightarrow y = 1$

Hence, fourth vertex of parallelogram is $(-2, 1)$

7.

Sol:

Let the point $P(2, y)$ divide the line segment joining the points $A(-2, 2)$ and $B(3, 7)$ in the ratio $K:1$

Then, the coordinates of P are

$\left[\frac{3k+(-2)\times 1}{k+1}, \frac{7k+2\times 1}{k1}\right]$

$= \left[\frac{3k-2}{k+1}, \frac{7k+2}{k+1}\right]$

But the coordinates of P are given as $(2, y)$

$\therefore \frac{3k-2}{k+1} = 2$

$\Rightarrow 3k - 2 = 2k + 2$

$\Rightarrow 3k - 2k = 2 + 2$

$\Rightarrow k = 4$

$\frac{7k+2}{k+1} = y$

Putting the value of k, we get

$\frac{7\times 4+2}{4+1} = y$

$\frac{30}{5} = y$

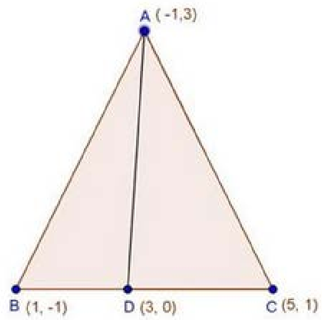
$6 = y$

i.e., $y = 6$

Hence the ratio is $4:1$ and $y=6$.

8.

Sol:



Let $A(1,3), B(1,-1)$ and $C(5,1)$ be the vertices of triangle ABC and let AD be the median through A.

Since, AD is the median, D is the mid-point of BC

$\therefore$ Coordinates of D are $\left(\frac{1+5}{2}, \frac{-1+1}{2}\right) = (3,0)$

Length of median $AD = \sqrt{(3+1)^2 + (0-3)^2}$

$= \sqrt{(4)^2 + (-3)^2}$

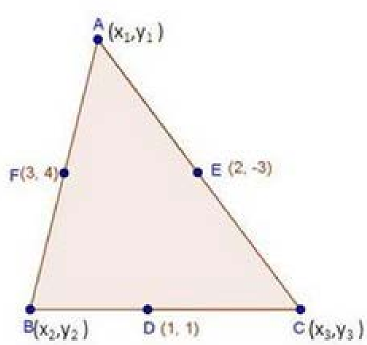
$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$ units.

9.

Sol:



Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC.

Let $D(1,1), E(2,-3)$ and $F(3,4)$ be the mid-points of sides BC, CA and AB respectively.

Since, D is the mid-point of BC.

$\therefore \frac{x_2 + x_3}{2} = 1$ and $\frac{y_2 + y_3}{2} = 1$

$\Rightarrow x_2 + x_3 = 2$ and $y_2 + y_3 = 2$ ....(i)

Similarly E and F are the mid-points of CA and AB respectively.

$\therefore \frac{x_1 + x_3}{2} = 2$ and $\frac{y_1 + y_3}{2} = -3$

$\Rightarrow x_1 + x_3 = 4$ and $y_1 + y_3 = 6$ ....(ii)

And, $\frac{x_1 + x_2}{2} = 3$ and $\frac{y_1 + y_2}{2} = 4$

$\Rightarrow x_1 + x_2 = 6$ and $y_1 + y_2 = 8$ ....(iii)

From (i), (ii) and (iii) we get

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = 2 + 4 + 6$ and

$y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 2 + (-6) + 8$

$\Rightarrow 2(x_1 + x_2 + x_3) = 12$ and $2(y_1 + y_2 + y_3) = 4$

$x_1 + x_2 + x_3 = 6$ and $y_1 + y_2 + y_3 = 2$ ....(iv)

From (i) and (iv) we get

$x_1 + 2 = 6$ and $y_1 + 2 = 2$

$\Rightarrow x_1 = 6 - 2$ and $\Rightarrow y_2 = 2 - 2\,2$

$\Rightarrow x_1 = 4 \Rightarrow y_1 = 0$

So the coordinates of A are $(4,0)$

From (ii) and (iv) we get

$x_2 + 4 = 6$ and $y_2 + (-6) = 2$

$\Rightarrow x_2 = 2 \Rightarrow y_2 - 6 = 2 \Rightarrow y_2 = 8$

So the coordinates of B are $(2,8)$

From (iii) and (iv) we get

$6 + x_3 = 6$ and $8 + y_3 = 2$

$\Rightarrow x_3 = 6 - 6 \Rightarrow y_3 = 2 - 8$

$\Rightarrow x_3 = 0$ and $y_3 = -6$

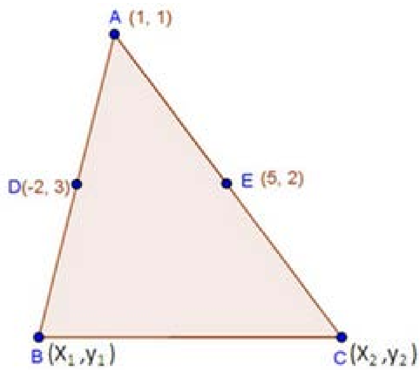
So the coordinates of C are $(0,-6)$

Hence, the vertices of triangle ABC are:

$A(4,0), B(2,8)$ and $C(0,-6)$

10.

Sol:



Let $A(1,1)$, be the given vertex

And, $D(-2,3), E(5,2)$ be the mid-point of AB and AC respectively,

Now, since D and E are the midpoints of AB and AC

$\frac{x_1+1}{2}=-2, \frac{y_1+1}{2}=3$

$\Rightarrow x_1+1=-4 \Rightarrow y_1+1=6$

$\Rightarrow x_1=-5 \Rightarrow y_1=5$

So, the coordinates of B are $(-5,5)$

And, $\frac{x_2+1}{2}=5, \quad \frac{y_2+1}{2}=2$

$\Rightarrow x_2+1=10 \Rightarrow y_2+1=4$

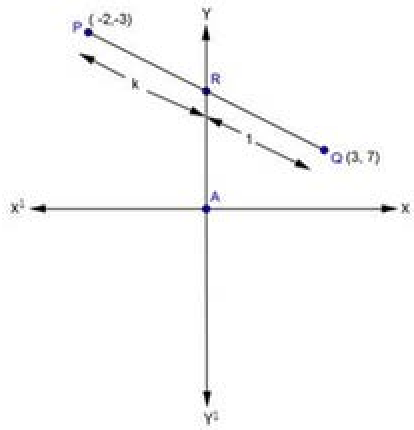
$\Rightarrow x_2=9 \Rightarrow y_2=3$

So the coordinates of C are $(9,3)$

Hence, the over vertices are $B(-5,5)$ and $C(9,3)$

11.

Sol:



Suppose $y-axis$ divides PQ in the ration $K:1$ at R

Then, the coordinates of the point of division are:

$$R\left[\frac{3k+(-2)\times 1}{k+1},\frac{7k+(-3)\times 1}{k+1}\right]$$

$$=R\left[\frac{3k-2}{k+1},\frac{7k-3}{k+1}\right]$$

Since, R lies on $y-axis$ and $x-$ coordinate of every point on y-axis is zero

$$\therefore \frac{3k-2}{k+1}=0$$

$$\Rightarrow 3k-2=0$$

$$\Rightarrow 3k=2$$

$$\Rightarrow k=\frac{2}{3}$$

Hence, the required ratio is $\frac{2}{3}:1$

i.e., 2 : 3

Putting $k=\frac{2}{3}$ in the coordinates of R

We get, $(0,1)$

Let the point P divide AB in the ratio $K:1$

Then, the coordinates of P are $\left[\frac{-8k-3}{k+1},\frac{-9k-1}{k+1}\right]$

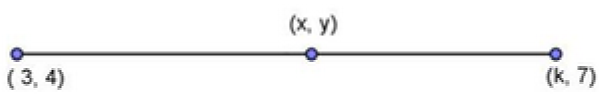
But the coordinates of P are given as $\left(-5, \frac{-21}{5}\right)$

$$\therefore \frac{-8k-3}{k+1} = -5$$
$$\Rightarrow -8k-3 = -5k-5$$
$$\Rightarrow -8k+5k = -5+3$$
$$\Rightarrow -3k = -2$$
$$\Rightarrow k = \frac{2}{3}$$

Hence, the point P divides AB in the ratio $\frac{2}{3}:1 \Rightarrow 2:3$

12.

Sol:



Since, (x, y) is the mid-point

$$x = \frac{3+k}{2}, y = \frac{4+7}{2} = \frac{11}{2}$$

Again,

$$2x+2y+1=0$$
$$\Rightarrow 2\times\frac{(3\times k)}{2} + 2\times\frac{11}{2} + 1 = 0$$
$$\Rightarrow 3+k+11+1=0$$
$$\Rightarrow 3+k+12=0$$
$$\Rightarrow k+15=0$$
$$\Rightarrow k=-15$$

13.

Sol:

Suppose the line $x-y-2=0$ divides the line segment joining $A(3,-1)$ and $B(8,9)$ in the ratio $K:1$ at point P. Then the coordinates of P are

$$\left(\frac{8k+3}{k+1}, \frac{9k-1}{k+1}\right)$$

But P lies on $x-y-2=0$

$$\therefore \frac{8k+3}{k+1} - \frac{9k-1}{k+1} - 2 = 0$$

$\Rightarrow \frac{8k+3}{k+1} - \frac{9k-1}{k+1} = 2$

$\Rightarrow \frac{8k+3-9k+1}{k+1} = 2$

$\Rightarrow -k+4 = 2k+2$

$\Rightarrow -k-2k = 2-4$

$\Rightarrow -3k = -2 \Rightarrow k = \frac{2}{3}$

So, the required ratio is $2:3$

14.

Sol:

(i) Suppose $x-axis$ divides AB in the ratio $K:1$ at point P

Then, the coordinates of the point of division of division are

$P\left[\frac{5k-2}{k+1}, \frac{6k-3}{k+1}\right]$

Since, P lies on x-axis, and y-coordinates of every point on x-axis is zero.

$\therefore \frac{6k-3}{k+1} = 0$

$\Rightarrow 6k-3 = 0$

$\Rightarrow 6k = 3$

$\Rightarrow k = \frac{3}{6} \Rightarrow k = \frac{1}{2}$

Hence, the required ratio is $1:2$

Putting $k = \frac{1}{2}$ in the coordinates of P

We find that its coordinates are $\left(\frac{1}{3}, 0\right)$.

(ii) Suppose y-axis divides AB in the ratio $k:1$ at point Q.

Then, the coordinates of the point of division are

$Q\left[\frac{5k-2}{k+1}, \frac{6k-3}{k+1}\right]$

Since, Q lies on $y-axis$ and x-coordinates of every point on y-axis is zero.

$\therefore \frac{5k-2}{k+1} = 0$

$\Rightarrow 5k-2 = 0$

$\Rightarrow k = \frac{2}{5}$

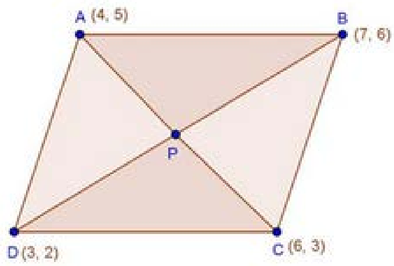
Hence, the required ratio is $\frac{2}{5}:1 = 2:5$

Putting $k = \frac{2}{5}$ in thee coordinates of Q.

We find that the coordinates are $\left(0, \frac{-3}{7}\right)$

15.

Sol:



Let $A(4,5), B(7,6), C(6,3)$ and $D(3,2)$ be the given points.

And, P the points of intersection of AC and BD.

Coordinates of the mid-point of AC are $\left(\frac{4+6}{2}, \frac{5+3}{2}\right) = (5,4)$

Coordinates of the mid-point of BD are $\left(\frac{7+3}{2}, \frac{6+2}{2}\right) = (5,4)$

Thus, AC and BD have the same mid-point.
Hence, $ABCD$ is a parallelogram
Now, we shall see whether $ABCD$ is a rectangle.
We have,

$AC = \sqrt{(6-4)^2 + (3-5)^2}$

$\Rightarrow AC = \sqrt{4+4}$

$\Rightarrow AC = \sqrt{8}$

And, $BD + \sqrt{(7-3)^2 + (6-2)^2}$

$\Rightarrow BD = \sqrt{16+16}$

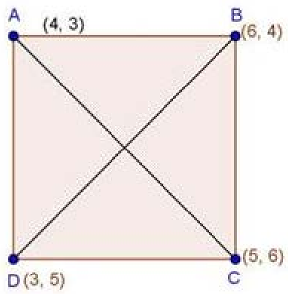
$\Rightarrow BD = \sqrt{32}$

Since, $AC \neq BD$

So, $ABCD$ is not a rectangle

16.

Sol:



Let $A(4,3), B(6,4), C(5,6)$ and $D(3,5)$ be the given points.

Coordinates of the mid-point of AC are $\left(\frac{4+5}{2}, \frac{3+6}{2}\right) = \left(\frac{9}{2}, \frac{9}{2}\right)$

Coordinates of the mid-point of BD are $\left(\frac{6+3}{2}, \frac{4+5}{2}\right) = \left(\frac{9}{2}, \frac{9}{2}\right)$

AC and BD have the same mid-point

$\therefore$ $ABCD$ is a parallelogram

Now,

$AB = \sqrt{(6-4)^2 + (4-3)^2}$

$\Rightarrow AB = \sqrt{4+1}$

$\Rightarrow AB = \sqrt{5}$

And, $BC = \sqrt{(6-5)^2 + (4-6)^2}$

$\Rightarrow BC = \sqrt{1+4}$

$\Rightarrow BC = \sqrt{5}$

$\therefore$ $AB = BC$

So, ABCD is a parallelogram whose adjacent sides are equal

$\therefore$ $ABCD$ is a rhombus

We have,

$AC = \sqrt{(5-4)^2 + (6-3)^2}$

$\Rightarrow AC = \sqrt{10}$

$BD = \sqrt{(6-3)^2 + (4-5)^2}$

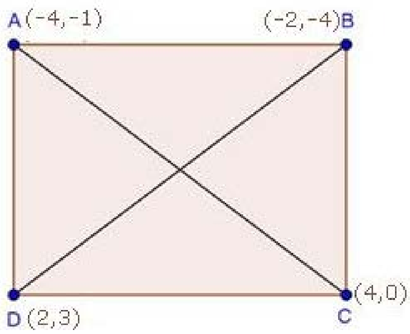
$\Rightarrow BD = \sqrt{10}$

$AC = BD$

Hence, $ABCD$ is a square

17.

Sol:



Let $A(-4,-1), B(-2,-4), C(4,0)$ and $D(2,3)$ be the given points

Coordinates of the mid-point of AC are $\left(\frac{-4+4}{2}, \frac{-1+0}{2}\right) = \left(0, \frac{-1}{2}\right)$

Coordinates of the mid-point of BD are $\left(\frac{-2+2}{2}, \frac{-4+3}{2}\right) = \left(0, \frac{-1}{2}\right)$

Thus AC and BD have the same mid-point

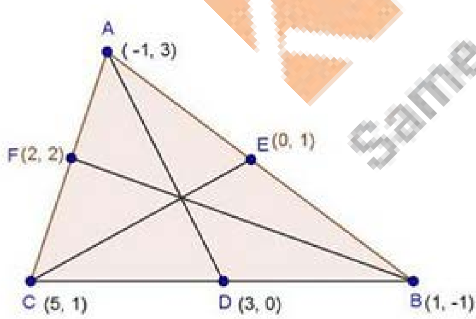
$AC = \sqrt{(4+4)^2 + (0+1)^2} = \sqrt{65}$

$BD = \sqrt{(-2-2)^2 + (-4-3)^2} = \sqrt{65}$

Hence $ABCD$ is a rectangle

18.

Sol:



Let AD, BF and CE be the medians of $\Delta ABCD$

Coordinates of D are $\left(\frac{5+1}{2}, \frac{1-1}{2}\right) = (3,0)$

Coordinates of E are $\left(\frac{-1+1}{2}, \frac{3-1}{2}\right) = (0,1)$

Coordinates of F are $\left(\frac{5-1}{2},\frac{1+3}{2}\right)=(2,2)$

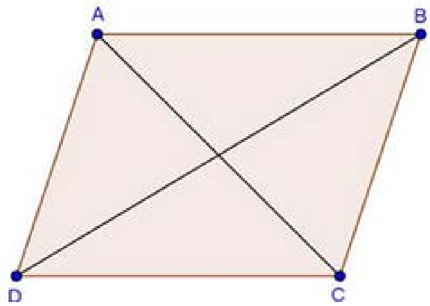
Length of $AD=\sqrt{(-1-3)^2+(3-0)^2}=5$ units

Length of $BF=\sqrt{(2-1)^2+(2+1)^2}=\sqrt{10}$ units

Length of $CE=\sqrt{(5-0)^2+(1-1)^2}=5$ units

19.

Sol:



Let $A(a+b,a-b),B(2a+b,2a-b),C(a-b,a+b)$ and (x,y) be the given points

Since, the diagonals of a parallelogram bisect each other

$\therefore$ Coordinates of the midpoint of $AC=$ Coordinates of the midpoint of BD

$$\left(\frac{a+b+a-b}{2},\frac{a-b+a+b}{2}\right)=\left(\frac{2a+b+x}{2},\frac{2a-b+y}{2}\right)$$

$$\Rightarrow (a,a)=\left(\frac{2a+b+x}{2},\frac{2a-b+y}{2}\right)$$

$$\Rightarrow \frac{2a+b+x}{2}=a \text{ and } \frac{2a-b+y}{2}=a$$

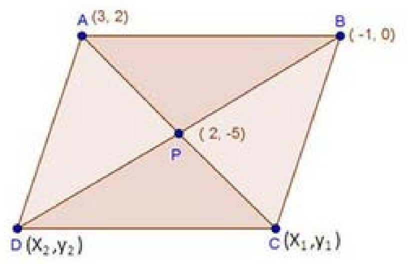
$$\Rightarrow 2a+b+x=2a \Rightarrow 2a-b+y=2a$$

$$\Rightarrow x=-b \Rightarrow y=b$$

Hence, the fourth vertex is $(-b,b)$.

20.

Sol:



Let $A(3,2), B(-1,0), C(x_1, y_1)$ and $D(x_2, y_2)$ be the given points.

Since, the diagonals of parallelogram bisect each other.

Coordinates of the midpoint of AC = Coordinates of the midpoint of BD

$$\left(\frac{x_1+3}{2}, \frac{y_1+2}{2}\right) = \left(\frac{x_2-1}{2}, \frac{y_2+0}{2}\right)$$

But $\frac{x_1+3}{2} = 2, \frac{y_1+2}{2} = -5$

$\Rightarrow x_1 + 3 = 4 \Rightarrow y_1 = -10 - 2$

$\Rightarrow x_1 = 1 \Rightarrow y_1 = -12$

And, $\frac{x_2-1}{2} = 2$

$\Rightarrow x_2 - 1 = 4$

$\Rightarrow x_2 = 5$

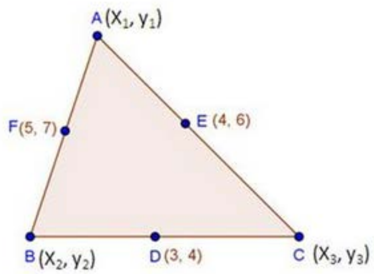
$\frac{y_2+0}{2} = -5$

$y_2 = -10$

Hence, the other vertices of parallelogram are $(1,-12)$ and $(5,-10)$.

21.

Sol:



Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC

Let $D(3,4), E(4,6)$ and $F(5,7)$ be the midpoints of BC, CA and AB.

Since, D is the midpoint of BC

$\therefore \frac{x_2 + x_3}{2} = 3$ and $\frac{y_2 + y_3}{2} = 4$

$\Rightarrow x_2 + x_3 = 6$ and $y_2 + y_3 = 8$ ……….(i)

Since, E is the midpoint of CA

$\therefore \frac{x_1 + x_3}{2} = 4$ and $\frac{y_1 + y_3}{2} = 6$

$\therefore x_1 + x_3 = 8$ and $y_1 + y_3 = 12$ ……….(ii)

Since F is the mid-point of AB

$\frac{x_1 + x_2}{2} = 5$ and $\frac{y_1 + y_2}{2} = 7$

$\Rightarrow x_1 + x_2 = 10$ and $y_1 + y_3 = 14$ ……….(iii)

From (i), (ii) and (iii), we get

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = 6 + 8 + 10$

$x_1 + x_2 + x_3 = 12$ ……….(iv)

And $y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 8 + 12 + 14$

$y_1 + y_2 + y_3 = 17$ ……….(iv)

From (i) and (iv)

$x_1 + 6 = 12, y_1 + 8 = 17$

$x_1 = 6, y_1 = 9$

From (ii) and (iv)

$x_2 + 8 = 12, y_2 + 12 = 17$

$x_2 = 4, y_2 = 5$

From (iii) and (iv)

$x_3 + 10 = 12, y_3 + 14 = 17$

$x_3 = 2, y_3 = 3$

Hence the vertices of triangle ABC are $(6,9);(4,5);(2,3)$

22.

Sol:

We are given PQ is the line segment, A and B are the points of trisection of PQ.

We know that $PA : QA = 1:2$

So, the coordinates of A are

$$\left(\frac{6\times1+3\times2}{2+1},\frac{-6\times1+3\times2}{2+1}\right)$$

$$=\frac{12}{3},0$$

$$=(4,0)$$

Since, A lies on the line

$$2x+y+k=0$$

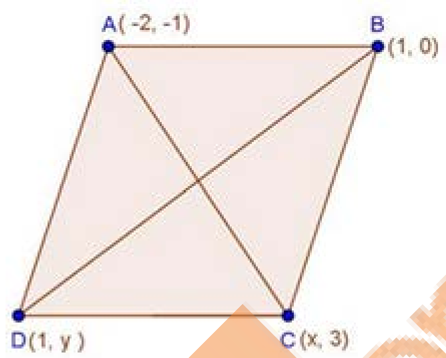
$$\Rightarrow 2\times4+0+k=0$$

$$\Rightarrow 8+k=0$$

$$\Rightarrow 8+k=-8$$

23.

Sol:



Let $A(-2,-1), B(1,0), C(x,3)$ and $D(1,y)$ be the given points.

We know that diagonals of a parallelogram bisect each other

$\therefore$ Coordinates of the mid-point of AC = Coordinates of the mid-point of BD

$$\left(\frac{x-2}{2},\frac{3-1}{2}\right)=\left(\frac{1+1}{2},\frac{y+0}{2}\right)$$

$$\Rightarrow\left(\frac{x-2}{2},1\right)=\left(1,\frac{y}{2}\right)$$

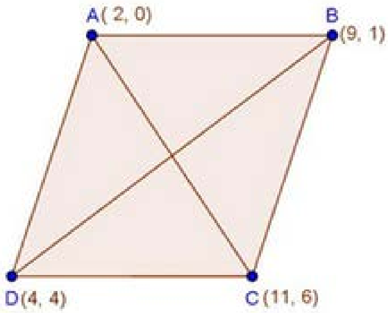
$$\Rightarrow\frac{x-2}{2}=1 \text{ and } \frac{y}{2}=1$$

$$\Rightarrow x-2=2\Rightarrow y=2$$

$$\Rightarrow x=4\Rightarrow y=2$$

24.

Sol:



Let $A(2,0), B(9,1), C(11,6)$ and $D(4,4)$ be the given points.

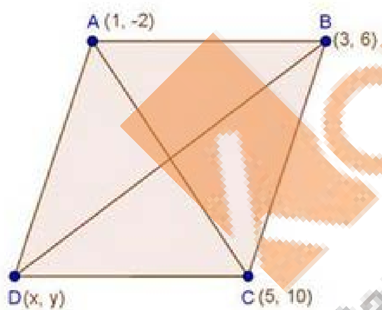
Coordinates of midpoint AC are $\left(\frac{11+2}{2}, \frac{6+0}{2}\right) = \left(\frac{13}{2}, 3\right)$

Coordinates of midpoint BD are $\left(\frac{9+4}{2}, \frac{1+4}{2}\right) = \left(\frac{13}{2}, \frac{5}{2}\right)$

Since, coordinates of mid-point of $AC \neq$ coordinates of mid-point of BD.
So, $ABCD$, is not a parallelogram. Hence, it is not a rhombus.

25.

Sol:



Let $A(1,-2), B(3,6), C(5,10)$ and $D(x,y)$ be the given points taken in order.
Since, diagonals of parallelogram bisect each other
Coordinates of mid-point of $AC =$ Coordinates of midpoint of BD

$$\left(\frac{5+1}{2}, \frac{10-2}{2}\right) = \left(\frac{x+3}{2}, \frac{y+6}{2}\right)$$

$$\Rightarrow (3,4) = \frac{x+3}{2}, \frac{y+6}{2}$$

$$\Rightarrow \frac{x+3}{2} = 3 \text{ and } \frac{y+6}{2} = 4$$

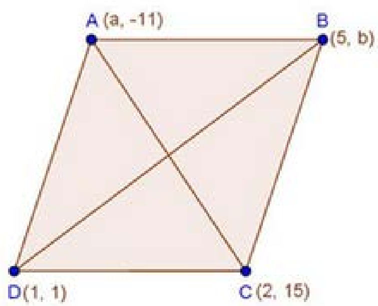
$$\Rightarrow x+3=6 \qquad \Rightarrow y+6=8$$

$\Rightarrow x = 3 \qquad \Rightarrow y = 2$

Hence, the fourth vertex is $(3, 2)$.

26.

Sol:



Let $A(a,-11), B(5,b), C(2,15)$ and $D(1,1)$ be the given points.

We know that diagonals of parallelogram bisect each other.

$\therefore$ Coordinates of mid-point of $AC =$ Coordinates of mid-point of BD

$$\left(\frac{a+2}{2}, \frac{15-11}{2}\right) = \left(\frac{5+1}{2}, \frac{b+1}{2}\right)$$

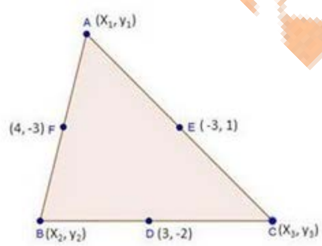
$$\Rightarrow \frac{a+2}{2} = 3 \text{ and } \frac{b+1}{2} = 2$$

$\Rightarrow a + 2 = 6 \qquad \Rightarrow b + 1 = 4$

$\Rightarrow a = 4 \qquad \Rightarrow b = 3$

27.

Sol:



Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC

Let $D(3,-2), E(-3,1)$ and $F(4,-3)$ be the midpoint of sides BC, CA and AB respectively

Since, D is the midpoint of BC

$$\therefore \frac{x_2 + x_3}{2} = 3 \text{ and } \frac{y_2 + y_3}{2} = -2$$

$\Rightarrow x_2 + x_3 = 6$ and $y_2 + y_3 = -4$ …….(i)

Similarly, E and F are the midpoint of CA and AB respectively.

$\therefore \frac{x_1 + x_3}{2} = -3$ and $\frac{y_1 + y_3}{2} = 1$

$\Rightarrow x_1 + x_3 = -6$ and $y_1 + y_3 = 2$ …….(ii)

And,

$\therefore \frac{x_1 + x_2}{2} = 4$ and $\frac{y_1 + y_2}{2} = -3$

$\Rightarrow x_1 + x_2 = 8$ and $y_1 + y_2 = -6$ …….(iii)

From (i), (ii) and (iii), we have

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = 6 + (-6) + 8$ and

$y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = -4 + 2 - 6$

$\Rightarrow 2(x_1 + x_2 + x_3) = 8$ and $2(y_1 + y_2 + y_3) = -8$

$\Rightarrow x_1 + x_2 + x_3 = 4$ and $y_1 + y_2 + y_3 = -4$ …….(iv)

From (i) and (iv)

$x_1 + 6 = 4$ and $y_1 - 4 = -4$

$\Rightarrow x_1 = -2$ $\Rightarrow y_1 = 0$

So, the coordinates of A are $(-2, 0)$

From (ii) and (iv)

$x_2 - 6 = 4$ and $y_2 + 2 = -4$

$\Rightarrow x_2 = 10 \Rightarrow y_2 = -6$

So, the coordinates of B are $(10, -6)$

From (iii) and (iv)

$x_3 + 8 = 4$ and $y_3 - 6 = -4$

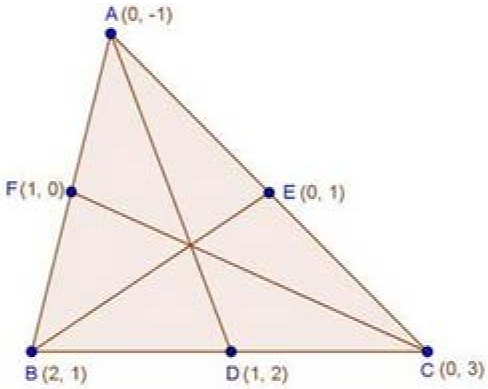
$\Rightarrow x_3 = -4$ $\Rightarrow y_3 = 2$

So, the coordinates of C are $(-4, 2)$

Hence, the vertices of ΔABC are $A(-2, 0), B(10, 6)$ and $C(-4, 2)$.

28.

Sol:



Let $A(0,1), B(2,1)$ and $C(0,3)$ be the given points

Let AD, BE and CF be the medians

Coordinates of D are $\left(\frac{2+0}{2},\frac{1+3}{2}\right)=(1,2)$

Coordinates of E are $\left(\frac{0}{2},\frac{3-1}{2}\right)=(0,1)$

Coordinates of F are $\left(\frac{2+0}{2},\frac{1-1}{2}\right)=(1,0)$

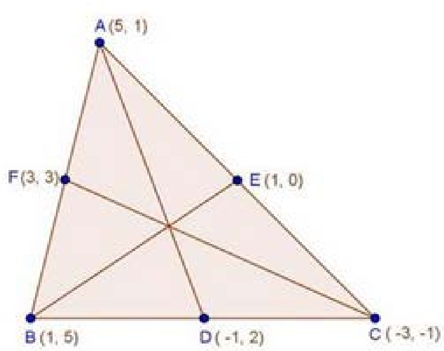
Length of median $AD=\sqrt{(1-0)^2+(2+1)^2}=\sqrt{10}$ units

Length of median $BE=\sqrt{(2-0)^2+(1-1)^2}=2$ units

Length of median $CF=\sqrt{(1-0)^2+(0-3)^2}=\sqrt{10}$ units

29.

Sol:



Let $A(5,1), B(1,5)$ and $C(-3,-1)$ be vertices of ΔABC

Let AD, BE and CF be the medians

Coordinates of D are $\left(\frac{1-3}{2}, \frac{5-1}{2}\right) = (-1, 2)$

Coordinates of E are $\left(\frac{5-3}{2}, \frac{1-1}{2}\right) = (1, 0)$

Coordinates of F are $\left(\frac{5+1}{2}, \frac{1+5}{2}\right) = (3, 3)$

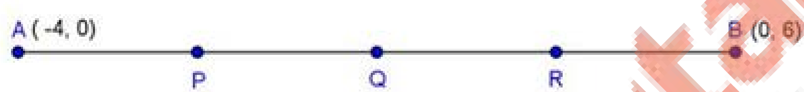
Length of median $AD = \sqrt{(5+1)^2 + (1-2)^2} = \sqrt{37}$ units

Length of median $BE = \sqrt{(1-1)^2 + (5-0)^2} = 5$ units

Length of median $CF = \sqrt{(3+3)^2 + (3+1)^2} = 2\sqrt{13} = \sqrt{52}$ units

30.

Sol:



Let $A(-4, 0)$ and $B(0, 6)$ be the given points.

And, Let P, Q, R be the points which divide AB in four equal points..

We know $AP : PB = 1 : 3$

$\therefore$ Coordinates of P are

$\left(\frac{1\times 0+3(-4)}{1+3}, \frac{1\times 6+3\times 0}{1+3}\right)$

$= \left(-3, \frac{3}{2}\right)$

We know that Q is midpoint of AB

$\therefore$ Coordinates of Q are

$\left(\frac{3\times 0+1\times(-4)}{3+1}, \frac{3\times 6+1\times 0}{3+1}\right)$

$= \left(-1, \frac{9}{2}\right)$

31.

Sol:

Let $A(5, 7), B(3, 9), C(8, 6)$ and $D(0, ,10)$ be the given points

Coordinates of the mid-point of AB are $\left(\frac{5+3}{2}, \frac{7+9}{2}\right) = (4,8)$

Coordinates of the mid-point of CD are $\left(\frac{8+0}{2}, \frac{6+10}{2}\right) = (4,8)$

Hence, the midpoints of AB = midpoint of CD.

32.

Sol:

Let $P(1,2), A(6,8)$ and $B(2,4)$ be thee given points.

Coordinates of midpoint of the line segment joining $A(6,8)$ and $B(2,4)$ are

$Q\left(\frac{6+2}{2}, \frac{8+4}{2}\right) = Q(4,6)$

Now, distance $PQ = \sqrt{(4-1)^2 + (6-2)^2}$

$\Rightarrow PQ = \sqrt{9+16}$

$\Rightarrow PQ = \sqrt{25}$

$\Rightarrow PQ = 5$

Hence, the distant $= 5$ units

33.

Sol:

Let $A(1,4)$ and $B(5,2)$ be the given points.

We know that $\frac{AP}{BP} = \frac{3}{4}$

Or, $AP : BP = 3:4$

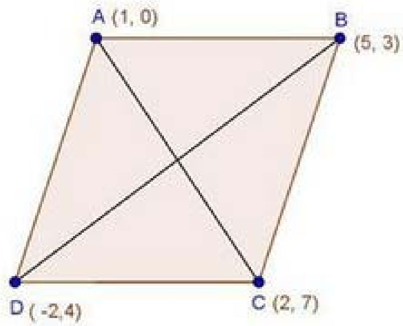
Coordinates of P are

$\left(\frac{3\times5+4\times1}{3+4}, \frac{3\times2+4\times4}{3+4}\right)$

$= \left(\frac{19}{7}, \frac{22}{7}\right)$

34.

Sol:



Let $A(1,0), B(5,3), C(2,7)$ and $D(-2,4)$ be the given points

Coordinates of the midpoint of AC are $\left(\frac{1+2}{2}, \frac{0+7}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$

Coordinates of the midpoint of BD are $\left(\frac{5-2}{2}, \frac{3+4}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$

Since, coordinates of midpoint of $AC =$ coordinates off midpoint of BD

$\therefore ABCD$ is a parallelogram as we know diagonals of parallelogram bisect each other.

35.

Sol:

Let $P(m,6)$ divides the join of $A(-4,3)$ and $B(2,8)$ in the ratio $K:1$

Then, the coordinates of P are

$\left(\frac{2k+1\times(-4)}{k+1}, \frac{8k+1\times 3}{k+1}\right)$

$= \left(\frac{2k-4}{k+1}, \frac{8k+3}{k+1}\right)$

But, $\frac{8k+3}{k+1} = 6$

$\Rightarrow 8k+3 = 6k+6$

$\Rightarrow 8k-6k = 3$

$\Rightarrow k = \frac{3}{2}$

Hence, P divides AB in the ratio $3:2$

Again,

$$\frac{2k-4}{k+1}=m$$

Substituting $k=\frac{3}{2}$, we get

$$\frac{2\times\frac{3}{2}-4}{\frac{3}{2}+1}=m$$

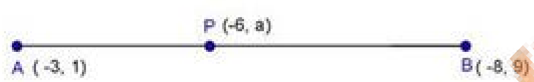
$$\Rightarrow \frac{-1}{\frac{5}{2}}=m$$

$$\Rightarrow \frac{-2}{5}=m$$

$$\therefore m=\frac{-2}{5}$$

36.

Sol:



Let $P(-6,a)$ divides the join of $A(-3,1)$ and $B(-8,9)$ in the ratio $k:1$

Then, the coordinates of P are

$$\left(\frac{-8k-3}{k+1},\frac{9k+1}{k+1}\right)$$

But, $\frac{-8k-3}{k+1}=-6$

$\Rightarrow -8k-3=-6k-6$

$\Rightarrow -8k+6k=-6+3$

$\Rightarrow -2k=-3$

$\Rightarrow k=\frac{3}{2}$

Hence, P divides AB in the ratio $3:2$

Again

$$\frac{9k+1}{k+1}=a$$

Substituting $k=\frac{3}{2}$

We get,

$$\frac{9\times\frac{3}{2}+1}{\frac{3}{2}+1}=a$$

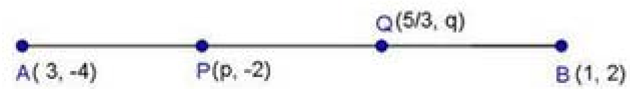
$$\Rightarrow\frac{\frac{29}{2}}{\frac{5}{2}}=a$$

$$\Rightarrow\frac{29}{5}=a$$

$$\therefore a=\frac{29}{5}$$

37.

Sol:



We have $P(p,-2)$ and $Q\left(\frac{5}{3},q\right)$ are the points of trisection of the line segment joining

$A(3,-4)$ *and* $B(1,2)$

We know $AP:PB=1:2$

$\therefore$ Coordinates of P are

$$\left(\frac{1\times1+2\times3}{1+2},\frac{1\times2+2\times(-4)}{1+2}\right)$$

$$=\left(\frac{7}{3},-2\right)$$

Hence, $P=\frac{7}{3}$

Again we know that $AQ:QB=2:1$

$\therefore$ Coordinates of Q are

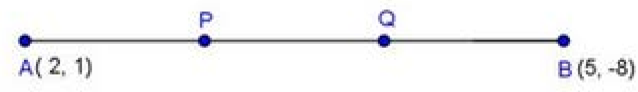
$$\left(\frac{2\times1+1\times3}{2+1},\frac{2\times2+1\times(-4)}{2+1}\right)$$

$$=\left(\frac{5}{3},0\right)$$

Hence, $q=0$

38.

Sol:



Since, P is the point of trisection of the line segment joining the point $A(2,1)$ and $B(5,-8)$

We have $AP:PB=1:2$

$\therefore$ Coordinates of the point P are

$$\left(\frac{1\times5+2\times2}{1+2},\frac{1\times(-8)+2\times1}{1+2}\right)$$

$=(3,-2)$

But, P lies on the line

$2x-y+k=0$

$\Rightarrow 2\times3-(-2)+k=0$

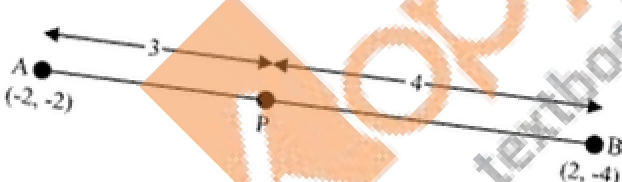
$\Rightarrow 6+2+k=0$

$\Rightarrow 8+k=0$

$\Rightarrow k=-8$

39.

Sol:



The Coordinates of point A and B are $(-2,-2)$ and $(2,-4)$ respectively

Since $AP=\frac{3}{7}AB$

Therefore $AP:PB=3:4$

So, point P divides the line segment AB in a ratio $3:4$.

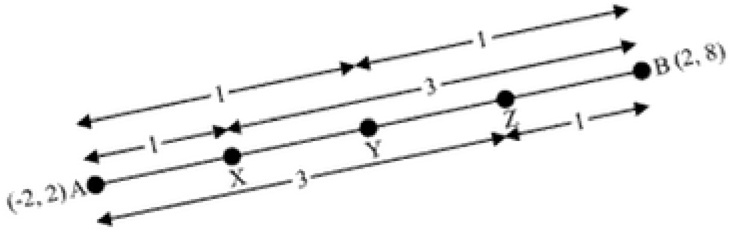
Coordinates of $P=\left(\frac{3\times2+4\times(-2)}{3+4},\frac{3\times(-4)+4\times(-2)}{3+4}\right)$

$=\left(\frac{6-8}{7},\frac{-12-8}{7}\right)$

$=\left(\frac{-2}{7},\frac{20}{7}\right)$

40.

Sol:



From the figure we have points X, Y, Z are dividing the line segment in a ratio $1:3, 1:1, 3:1$ respectively.

Coordinates of $X = \left(\frac{1+2+3\times(-2)}{1+3}, \frac{1\times 8+3\times 2}{1+3} \right)$

$= \left(-1, \frac{7}{2} \right)$

Coordinates of $Y = \left(\frac{2+(-2)}{2}, \frac{2+8}{2} \right)$

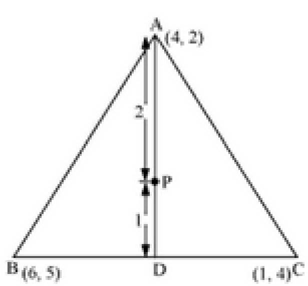
$= (0, 5)$

Coordinates of $Z = \left(\frac{3\times 2+1\times(-2)}{3+1}, \frac{3\times 8+1\times 2}{3+1} \right)$

$= \left(1, \frac{13}{2} \right)$

41.

Sol:



(i) Median AD of the triangle will divide the side BC in two equals parts. So D is the midpoint of side BC.

Coordinates of D $=\left(\frac{6+1}{2},\frac{5+4}{2}\right)=\left(\frac{7}{2},\frac{9}{2}\right)$

(ii) Point P divides the side AD in a ratio 2:1

Coordinates of P $=\left(\frac{2\times\frac{7}{6}+1\times4}{2+1},\frac{2\times\frac{9}{2}+1\times2}{2+1}\right)$

$=\left(\frac{11}{3},\frac{11}{3}\right)$

(iii) Median BE of the triangle will divide the side AC in two equal parts. So E is the midpoint of side AC.

Coordinates of E $=\left(\frac{4+1}{2},\frac{2+4}{2}\right)=\left(\frac{5}{2},3\right)$

Point Q divides the side BE in a ratio 2:1

Coordinates of Q $=\left(\frac{2\times\frac{5}{2}+1\times6}{2+1},\frac{2\times3+1\times5}{2+1}\right)=\left(\frac{11}{3},\frac{11}{3}\right)$

Median CF of the triangle will divide the side AB in two equal parts. So F is the midpoint of side AB.

Coordinates of F $=\left(\frac{4+6}{2},\frac{2+1}{2}\right)=\left(5,\frac{7}{2}\right)$

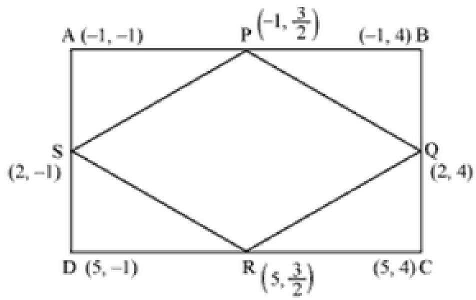
Point R divides the side CF in a ratio 2:1.

Coordinates of R $=\left(\frac{2\times5+1\times1}{2+1},\frac{2\times\frac{7}{2}+1\times4}{2+1}\right)=\left(\frac{11}{3},\frac{11}{3}\right)$

(iv) Now we may observe that coordinates of point P, Q, R are same. So, all these are representing same point on the plane i.e. centroid of the triangle.

42.

Sol:



Length of $PQ = \sqrt{(-1-2)^2 + \left(\frac{3}{2} - 4\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$

Length of $QR = \sqrt{(2-5)^2 + \left(4 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$

Length of $RS = \sqrt{(5-2)^2 + \left(\frac{3}{2} + 1\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$

Length of $SP = \sqrt{(2+1)^2 + \left(-1 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$

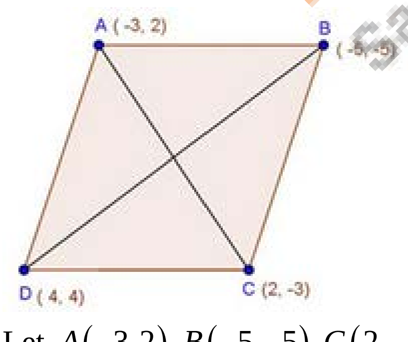
Length of $PR\sqrt{(-1-5)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2} = 6$

Length of $QS\sqrt{(2-2)^2 + (4+1)^2} = 5$

Here all sides of given quadrilateral is of same measure but the diagonals are of different lengths. So, PQRS is a rhombus.

43.

Sol:



Let $A(-3,2), B(-5,-5), C(2,-3)$ and $D(4,4)$ be the given points

Coordinates of the midpoint of AC *are* $\left(\frac{-3+2}{2}, \frac{2-3}{2}\right) = \left(\frac{-1}{2}, \frac{-1}{2}\right)$

Coordinates of the midpoint of BD are $\left(\frac{-5+4}{2},\frac{-5+4}{2}\right)=\left(\frac{-1}{2},\frac{-1}{2}\right)$

Thus, AC and BD have the same midpoint

Hence, $ABCD$ is a parallelogram

Now, $AB=\sqrt{(-5+3)^2+(-5-2)^2}$

$\Rightarrow AB=\sqrt{4+49}$

$\Rightarrow AB=\sqrt{53}$

Now, $BC=\sqrt{(-5-2)^2+(-5+3)^2}$

$\Rightarrow BC=\sqrt{49+4}$

$\Rightarrow BC=\sqrt{53}$

$\therefore AB=BC$

So, $ABCD$ is a parallelogram whose adjacent sides are equal.

Hence, $ABCD$ is a rhombus.

44.

Sol:

Let $P(5,-6)$ and $Q(-1,-4)$ be the given points.

Let y-axis divide PQ in the ratio $k:1$

Then, the coordinates of the point of division are

$R\left[\frac{-k+5}{k+1},\frac{-4k-6}{k+1}\right]$

Since, R lies on y-axis and x-coordinates of every point on y-axis is zero

$\therefore \frac{-k+5}{k+1}=0$

$\Rightarrow -k+5=0$

$\Rightarrow k=5$

Hence, the required ratio is $5:1$

Putting $k=5$ in the coordinates of R, we get

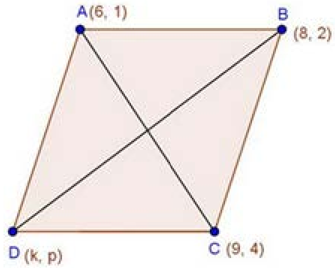
$\left(\frac{-5+5}{5+1},\frac{-4\times5-6}{5+1}\right)$

$=\left(0,\frac{-13}{3}\right)$

Hence, the coordinates of the point of division are $\left(0,-\frac{13}{3}\right)$.

45.

Sol:



Let $A(6,1), B(8,2), C(9,4)$ and $D(k,p)$ be the given points.

Since, $ABCD$ is a parallelogram

Coordinates of midpoint of $AC =$ Coordinates of the midpoints of BD

$\Rightarrow \left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+k}{2}, \frac{2+p}{2}\right)$

$\Rightarrow \left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+k}{2}, \frac{2+p}{2}\right)$

$\Rightarrow \frac{8+k}{2} = \frac{15}{2}$ and $\frac{2+p}{2} = \frac{5}{2}$

$\Rightarrow 8+k = 15 \quad \Rightarrow 2+p = 5$

$\Rightarrow k = 7 \quad \Rightarrow p = 3$

46.

Sol:

Let $(-4,6)$

Divide AB internally in the ratio $k:1$ using thee section formula, we get

$(-4,6) = \left(\frac{3k-6}{k+1}, \frac{-8k+10}{k+1}\right)$ ……..(2)

So, $-4 = \frac{3k-6}{k+1}$

i.e., $-4k-4 = 3k-6$

i.e., $7k = 2$

i.e., $k:1 = 2:7$

You c check for the y-coordinate also. So, the point $(-4,6)$ divides the line segment j joining the points $A(-6,10)$ and $B(3,-8)$ in the ratio 2: 7

47.

Sol:

Let coordinates of point A be (x, y)

Mid-point of diameter AB is center of circle $(2, -3)$

$$(2,-3)=\left(\frac{x+1}{2},\frac{y+4}{2}\right)$$

$$\frac{x+1}{2}=2 \ and \ \frac{y+4}{2}=-3$$

$x+1=4$ and $y+4=-6$

$x=3$ and $y=-10$

Therefore coordinates of A are $(3,10)$

48.

Sol:

Given points are $A(3,-5)$ and $B(-4,8)$

P divides AB in the ratio $k:1$,

Using the section formula, we have:

Coordinate of point P are $\{(-4k+3/k+1)(8k-5/k+1)\}$

Now it is given, that P lies on the line $x+y=0$

Therefore

$-4k+3/k+1+8k-5/k+1=0$

$\Rightarrow -4k+3+8k-5=0$

$\Rightarrow 4k-2=0$

$\Rightarrow k=2/4$

$\Rightarrow k=1/2$

Thus, the value of k is $\frac{1}{2}$.

Exercise 14.4

1.

Sol:

We know that the coordinates of the centroid of a triangle whose vertices are $(x_1, y_1),(x_2, y_2),(x_3, y_3)$ are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

So, the coordinates of the centroid of a triangle whose vertices are

$(1,4),(-1,-1)$ *and* $(3,-2)$ are $\left(\frac{1-1+3}{3}, \frac{4-1-2}{3}\right)$

$$=\left(1, \frac{1}{3}\right)$$

2.

Sol:

Let the coordinates of the third vertex bee (x, y), Then

Coordinates of centroid of triangle are

$$\left(\frac{x+1+3}{3}, \frac{y+2+5}{3}\right)$$

We have centroid is at origin $(0,0)$

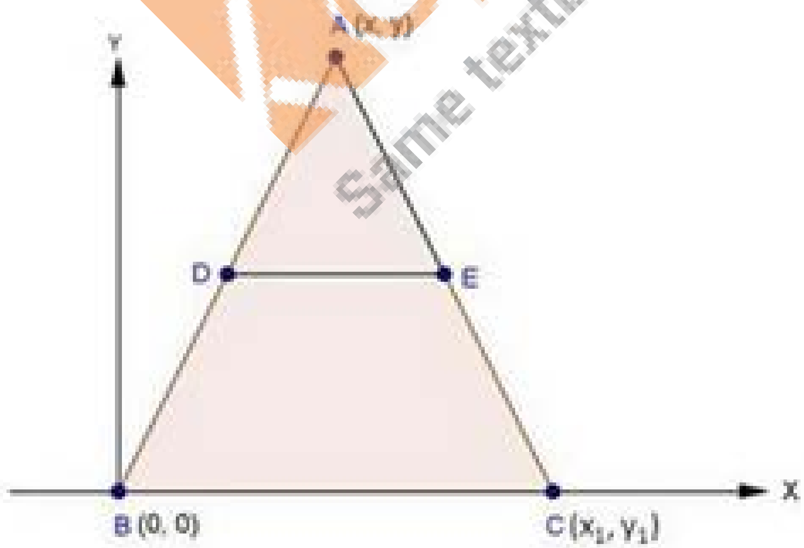
$$\therefore \frac{x+1+3}{3}=0 \text{ and } \frac{y+2+5}{3}=0$$

$\Rightarrow x+4=0 \qquad \Rightarrow y+7=0$

$\Rightarrow x=-4 \qquad \Rightarrow y=-7$

3.

Sol:



Let ABC be a triangle such that BC is along x-axis

Coordinates of A, B and C are $(x, y),(0,0)$ and (x_1, y_1)

D and E are the mid-points of AB and AC respectively

Coordinates of D are $\left(\frac{x+0}{2}, \frac{y+0}{2}\right)$

$=\left(\frac{x}{2}, \frac{y}{2}\right)$

Coordinates of E are $\left(\frac{x+x_1}{2}, \frac{y+y_1}{2}\right)$

Length of $BC=\sqrt{x_1^2+y_1^2}$

Length of DE $=\sqrt{\left(\frac{x+x_1}{2}-\frac{x}{2}\right)^2+\left(\frac{x+y_1}{2}-\frac{y}{2}\right)^2}$

$=\sqrt{\left(\frac{x_1}{2}\right)^2+\left(\frac{y_1}{2}\right)^2}$

$=\sqrt{\frac{x_1^2}{4}+\frac{y_1^2}{4}}$

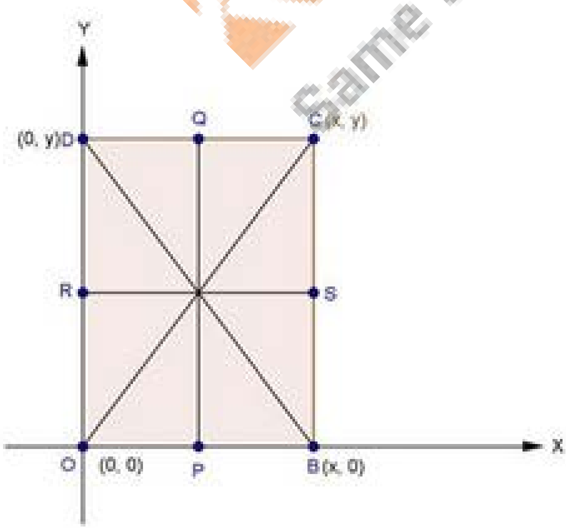
$=\sqrt{\frac{1}{4}\left(x_1^2+y_1^2\right)}$

$=\frac{1}{2}\sqrt{x_1^2+y_1^2}$

$=\frac{1}{2}BC$

4.

Sol:



Let $OBCD$ be the quadrilateral P, Q, R, S be the midpoint off OB, CD, OD and BC.

Let the coordinates of O, B, C, D are $(0,0),(x,0),(x,y)$ and $(0,y)$

Coordinates of P are $\left(\frac{x}{2},0\right)$

Coordinates of Q are $\left(\frac{x}{2},y\right)$

Coordinates of R are $\left(0,\frac{y}{2}\right)$

Coordinates of S are $\left(x,\frac{y}{2}\right)$

Coordinates of midpoint of PQ are

$$\left[\frac{\frac{x}{2}+\frac{x}{2}}{2},\frac{0+y}{2}\right]=\left(\frac{x}{2},\frac{y}{2}\right)$$

Coordinates of midpoint of RS are $\left[\frac{(0+x)}{2},\frac{\frac{y}{2}+\frac{y}{2}}{2}\right]=\left[\frac{x}{2},\frac{y}{2}\right]$

Since, the coordinates of the mid-point of $PQ=$ coordinates of mid-point of RS

$\therefore PQ$ and RS bisect each other

5.

Sol:

Let $A(0,0), B(a,0)$, and $C(c,d)$ are the o-ordinates of triangle ABC

Hence, $G\left[\frac{c+0+a}{3},\frac{d}{3}\right]$

i.e., $G\left[\frac{a+c}{3},\frac{d}{3}\right]$

let $P(x,y)$

To prove:

$PA^2+PB^2+PC^2=GA^2+GB^2+GC^2+3GP^2$

Or, $PA^2+PB^2+PC^2=GA^2+GB^2+GC^2+GP^2+GP^2+GP^2$

Or, $PA^2-GP^2+PB^2-GP^2+PC^2+GP^2=GA^2+GB^2+GC^2$

Proof:

$PA^2=x^2+y^2$

$$GP^2 = \left(x - \frac{a+c}{3}\right)^2 + \left(y - \frac{d}{3}\right)^2$$

$$PB^2 = (x-a)^2 + y^2$$

$$PC^2 = (x-c)^2 + (y-d)^2$$

L.H.S

$$= x^2 + y^2 - \left[x^2 + \left(\frac{a+c}{3}\right)^2 + 2x\frac{(a+c)}{3} + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right] + (x-a)^2 + y^2$$

$$-\left[x^2 + \left(\frac{a+c}{3}\right)^2 - 2x\left(\frac{a+c}{3}\right) + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right] + (x-c)^2 + (y-d)^2$$

$$-\left[x^2 + \left(\frac{a+c}{3}\right)^2 - 2x\left(\frac{a+c}{3}\right) + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right]$$

$$= x^2 + y + x^2 + x^2 + a^2 - 2ax + y^2 + x^2 + c^2 - 2xc + y^2 + d^2 - 2yd - 3$$

$$\left[x^2 + \left(\frac{a+c}{3}\right)^2 - 2x\left(\frac{a+c}{3}\right) + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right]$$

$$= \cancel{3x^2} + \cancel{3y^2} + a^2 + c^2 + d^2 - 2ax - 2xc - 2yd - \cancel{3x^2} - \frac{(a+c)^2}{3} + 2x(a+c) - \cancel{3y^2} - \frac{d^2}{3} + 2yd$$

$$= a^2 + c^2 + d^2 - \cancel{2ax} - \cancel{2xc} - \cancel{2yd} - \frac{a^2 + c^2 + 2ac}{3} + \cancel{2ax} + \cancel{2cx} - \frac{d^2}{3} + \cancel{2yd}$$

$$= \frac{3a^2 + 3c^2 + 3d^2 - a^2 - c^2 - 2ac - d^2}{3} = \frac{2a^2 + 2c^2 + 2d^2 - 2ac}{3} = L.H.S$$

Solving R.H.S

$$GA^2 + GB^2 + GC^2$$

$$GA^2 = \left(\frac{a+c}{3}\right)^2 + \left(\frac{d}{3}\right)^2 = \frac{a^2 + c^2 + 2ac}{9} + \frac{d^2}{9}$$

$$GC^2 = \left(\frac{a+c}{3} = a\right)^2 + \left(\frac{d}{3}\right)^2 = \left(\frac{c-2a}{3}\right)^2 + \left(\frac{d}{3}\right)^2$$

$$= \frac{a^2 + 4c^2 - 4ca}{9} + \frac{4d^2}{9}$$

$$GB^2 = \left(\frac{a+c}{3} - a\right)^2 + \left(\frac{d}{3}\right)^2 = \left(\frac{c-2a}{3}\right)^2 + \left(\frac{d}{3}\right)^2$$

$$= \frac{c^2 + 4a^2 - 4ac}{9} + \frac{d^2}{9}$$

$$GA^2+GB^2+GC^2=\frac{a^2+c^2+2ac}{9}+\frac{d^2}{9}+\frac{a^2+4c^2-4ac}{9}+\frac{4d^2}{9}+\frac{c^2+4a^2-4ac}{9}+\frac{d^2}{9}$$

$$=\frac{a^2+c^2+2ac+d^2+a^2+4c^2-4ac+4d^2+c^2+4a^2-4ac+d^2}{9}$$

$$=\frac{6a^2+6c^2+6d^2+6ac}{9}=\frac{2a^2+2c^2+2d^2+2ac}{3}$$

$\therefore L.H.S=R.H.S$

6.

Sol:

Let $A(b,c), B(0,0)$ and $C(a,0)$ be the coordinates of ΔABC

Then coordinates of centroid are $G\left[\frac{a+b}{3},\frac{c}{3}\right]$

To prove:

$$AB^2+BC^2+CA^2=3\left(GA^2+GB^2+GC^2\right)$$

Solving L.H.S

$$AB^2+BC^2+CA^2$$

$$=b^2+c^2+a^2\left(a-b\right)^2+c^2$$

$$=b^2+c^2+a^2+a^2+b^2-2ab+c^2$$

$$=2a^2+2b^2+2c^2-2ab$$

Solving R.H.S

$$3\left[\left(\frac{a+b}{3}-b\right)^2+\left(c-\frac{c}{3}\right)^2+\left(\frac{a+b}{3}\right)^2+\left(\frac{c}{3}\right)^2+\left(\frac{a+b}{2}-a\right)^2+\left(\frac{c}{3}\right)^2\right]$$

$$=3\left[\left(\frac{a-2b}{3}\right)^2+\left(\frac{2c}{3}\right)^2+\left(\frac{a+b}{3}\right)^2+\left(\frac{c}{3}\right)^2+\left(\frac{b-2a}{3}\right)^2+\left(\frac{c}{3}\right)^2\right]$$

$$=3\left[\frac{a^2+4b^2-4ab}{9}+\frac{4c^2}{9}+\frac{a^2+b^2+2ab}{9}+\frac{c^2}{9}+\frac{b^2+4a^2-4ab}{9}+\frac{c^2}{9}\right]$$

$$=3\left[\frac{a^2+4b^2-4ab+4c^2+a^2+b^2+2ab+c^2+b^2+4a^2-4ab+c^2}{9}\right]$$

$$=3\left[\frac{6a^2+6b^2+6c^2-6ab}{9}\right]$$

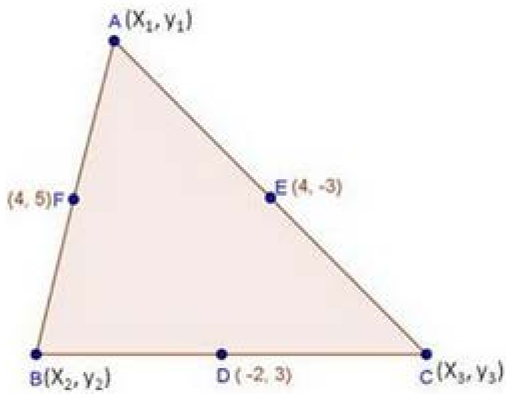
$$=\cancel{3}\times 3\left[\frac{2a^2+2b^2+2c^2-2ab}{\cancel{9}}\right]$$

$= 2a^2 + 2b^2 + 2c^2 - 2ab$

$\therefore$ L.H.S = R.H.S proved

7.

Sol:



Let $A(x_1, y_2), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC

Let $D(-2,3), E(4,-3)$ and $F(4,5)$ be the midpoints of sides BC, CA and AB respectively

Since, D is the midpoint of BC

$\frac{x_2 + x_3}{2} = -2$ and $\frac{y_2 + y_3}{2} = 3$

$\Rightarrow x_2 + x_3 = -4$ and $y_2 + y_3 = 6$ ....(i)

And, $\frac{x_1 + x_3}{2} = 4$ and $\frac{y_1 + y_3}{2} = -3$

$\Rightarrow x_1 + x_2 = 8$ and $y_1 + y_3 = -6$ ....(ii)

And, $\frac{x_1 + x_2}{2} = 4$ and $\frac{y_1 + y_2}{2} = 5$

$\Rightarrow x_1 + x_2 = 8$ and $y_1 + y_3 = 10$ ....(iii)

From (i), (ii) and (iii), we get

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = -4 + 8 + 8$ and

$y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 6 - 6 + 10$

$\Rightarrow 2(x_1 + x_2 + x_3) = 12$ and $2(y_1 + y_2 + y_3) = 10$

$\Rightarrow x_1 + x_2 + x_3 = 6$ and $y_1 + y_2 + y_3 = 5$ ....(iv)

From (i) and (iv), we get

$x_1 - 4 = 6$ and $y_1 + 6 = 5$

$\Rightarrow x_1 = 10$ $\Rightarrow y_1 = -1$

So, the coordinates of A are $(10,-1)$

From (ii) and (iv)

$x_2+8=6$ *and* $y_2-6=5$

$\Rightarrow x_2=-2 \qquad \Rightarrow y_2=11$

So, the coordinates of B are $(-2,11)$

From (iii) and (iv)

$x_3+8=6$ *and* $y_3+10=5$

$\Rightarrow x_3=-2 \qquad \Rightarrow y_3=-5$

So, the coordinates of C are $(-2,-5)$

$\therefore$ The vertices of ΔABC are $A(10,-1), B(-2,11)$ and $C(-2,-5)$

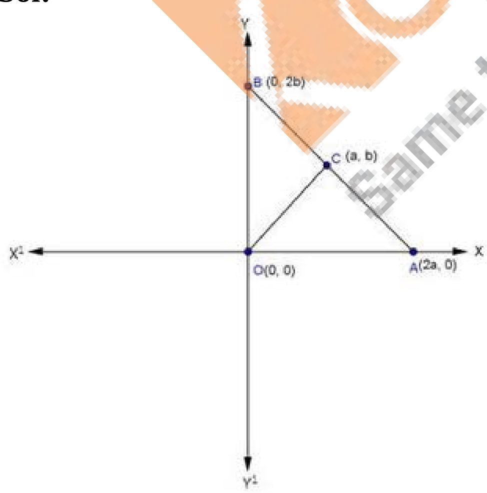
Hence, coordinates of the centroid of ΔABC are

$$\left(\frac{10-2-2}{3}, \frac{-1+11-5}{3}\right)$$

$$=\left(2, \frac{5}{3}\right)$$

8.

Sol:



Given a right triangle BOA with vertices $B(0,2b), 0(0,0)$ and $A(2a,0)$

Since, C is the midpoint of AB

$\therefore$ coordinates of C are $\left(\frac{2a+0}{2}, \frac{0+2b}{2}\right)$

$= (a, b)$

Now, $CO = \sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2+b^2}$

$CA = \sqrt{(2a-a)^2 + (0-b)^2} = \sqrt{a^2+b^2}$

$CB = \sqrt{(a-0)^2 + (b-2b)^2} = \sqrt{a^2+b^2}$

Since, $CO = CA = CB.$

$\therefore C$ is equidistant from O, A *and* $B.$

9.

Sol:

Let the coordinates of the third vertex be (x, y), Then

Coordinates of centroid of triangle are

$$\left(\frac{x-3+0}{3}, \frac{y+1-2}{3}\right) = \left(\frac{x-3}{2}, \frac{y-1}{3}\right)$$

We have centroid is at origin $(0,0)$

$\therefore \frac{x-3}{3} = 0$ *and* $\frac{y-1}{3} = 0$

$\Rightarrow x-3=0 \qquad \Rightarrow y-1=0$

$\Rightarrow x=3 \qquad \Rightarrow y=1$

Hence, the coordinates of the third vertex are $(3,1)$.

10.

Sol:

Let the third vertex be $C(x, y)$

Two vertices $A(3,2)$ and $B(-2,1)$

Coordinates of centroid of triangle are

$$\left(\frac{x+3-2}{3}, \frac{y+2+1}{3}\right)$$

But the centroid of the triangle are $\left(\frac{5}{3}, -\frac{1}{3}\right)$

$\therefore \frac{x+3-2}{3} = \frac{5}{3}$ and $\frac{y+2+1}{3} = -\frac{1}{3}$

$\Rightarrow \frac{x+1}{3} = \frac{5}{3} \qquad \Rightarrow \frac{y+3}{3} = -\frac{1}{3}$

$\Rightarrow x+1=5 \qquad \Rightarrow y+3=-1$

$\Rightarrow x=4 \qquad \Rightarrow y=-4$

Hence, the third vertex of the triangle is $C(4,-4)$

Exercise 14.5

1.

Sol:

(i) Area of a triangle is given by

$\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1+y_2)\right]$

Here, $x_1=6, y_1=3, x_2=-3, y_2=5, x_3=4, y_3=-2$

Let $A(6,3), B(-3,5)$ and $C(4,-2)$ be the given points

Area of $\Delta ABC = \frac{1}{2}\left[6(5+2)+(-3)(-2-3)+4(3-5)\right]$

$= \frac{1}{2}\left[6\times 7-3\times(-5)+4(-2)\right]$

$= \frac{1}{2}[42+15-8]$

$= \frac{49}{2}\, sq.units$

(ii) Let $A=(x_1,y_1)=(at_1^2, 2at_1)$

$B=(x_2,y_2)=(at_2^2, 2at_2)$

$=(x_3,y_3)=(at_3^2, 2at_3)$ be the given points.

The area of ΔABC

$= \frac{1}{2}\left[at_1^2(2at_2-2at_3)+at_2^2(2at_3-2at_1)+at_3^2(2at_1-2at_2)\right]$

$= \frac{1}{2}\left[2a^2t_1^2t_2-2a^2t_1^2t_3+2a^2t_2^2t_3-2a^2t_2^2t_1+2a^2t_3^2t_1-2a^2t_3^2t_2\right]$

$= \frac{1}{2}\times 2\left[a^2t_1^2(t_2-t_3)+a^2t_2^2(t_3-t_1)+a^2+t_3^2(t_1-t_2)\right]$

$= a^2\left[t_1^2(t_2-t_3)+t_2^2(t_3-t_1)+t_3^2(t_1-t_2)\right]$

(iii) Let $A=(x_1,y_1)=(a,c+a)$

$B = (x_2, y_2) = (a, c)$

$C = (x_3, y_3) = (-a, c-a)$ be the given points

The area of ΔABC

$$= \frac{1}{2}\left[a\left(c-\{c-a\}\right)+a\left(c-a-(c+a)\right)+(-a)(c+a-a)\right]$$

$$= \frac{1}{2}\left[a(c-c+a)+a(c-a-c-a)-a(c+a-c)\right]$$

$$= \frac{1}{2}\left[a\times a+ax(-2a)-a\times a\right]$$

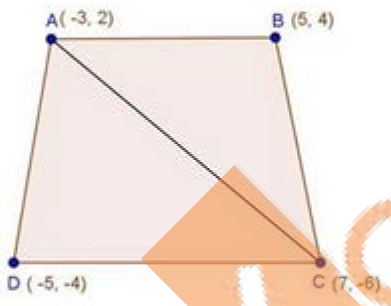
$$= \frac{1}{2}\left[a^2-2a^2-a^2\right]$$

$$= \frac{1}{2}\times(-2a)^2$$

$$= -a^2$$

2.

Sol:



Let $A(-3,2), B(5,4), C(7,-6)$ and $D(-5,-4)$ be the given points.

Area of ΔABC

$$= \frac{1}{2}\left[-3(4+6)+5(-6-2)+7(2-4)\right]$$

$$= \frac{1}{2}\left[-3\times10+5\times(-8)+7(-2)\right]$$

$$= \frac{1}{2}\left[-30-40-14\right]$$

$$= -42$$

But area cannot be negative

$\therefore$ Area of $\Delta ADC = 42$ square units

Area of ΔADC

$$= \frac{1}{2}\left[-3(-6+4)+7(-4-2)+(-5)(2+6)\right]$$

$=\frac{1}{2}\left[-3(-2)+7(-6)-5\times 8\right]$

$=\frac{1}{2}[6-42-40]$

$=\frac{1}{2}\times -76$

$=-38$

But area cannot be negative

$\therefore$ Area of $\Delta ADC = 38$ square units

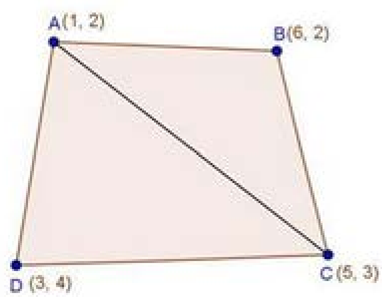
Now, area of quadrilateral $ABCD$

$= Ar.\ of\ ABC + Ar\ of\ ADC$

$=(42+38)$

$=80$ square. units

(i)



Let $A(1,2), B(6,2), C(5,3)$ and $(3,4)$ be the given points

Area of ΔABC

$=\frac{1}{2}\left[1(2-3)+6(3-2)+5(2-2)\right]$

$=\frac{1}{2}\left[-1+6\times(1)+0\right]$

$=\frac{1}{2}[-1+6]$

$=\frac{5}{2}$

Area of ΔADC

$=\frac{1}{2}\left[1(3-4)+5(4-2)+3(2-3)\right]$

$=\frac{1}{2}\left[-1\times 5\times 2+3(-1)\right]$

$=\frac{1}{2}[-1+10-3]$

$= \frac{1}{2}[6]$

$= 3$

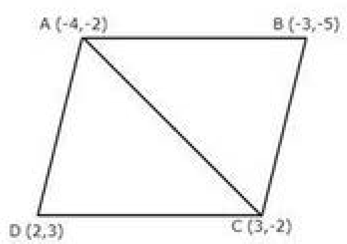
Now, Area of quadrilateral $ABCD$

$=$ Area of $ABC +$ Area of ADC

$= \left(\frac{5}{2} + 3\right) sq.\ units$

$= \frac{11}{2}\ sq.\ units$

(ii)



Let $A(-4,-2), B(-3,-5), C(3,-2)$ and $D(2,3)$ be the given points

Area of $\Delta ABC = \frac{1}{2}\left|(-4)(-5+2) - 3(-2+2) + 3(-2+5)\right|$

$= \frac{1}{2}\left|(-4)(-3) - 3(0) + 3(3)\right|$

$= \frac{21}{2}$

Area of $\Delta ACD = \frac{1}{2}\left|(-4)(3+2) + 2(-2+2) + 3(-2-3)\right|$

$= \frac{1}{2}\left|-4(5) + 2(0) + 3(-5)\right| = \frac{-35}{2}$

But area can't be negative, hence area of $\Delta ADC = \frac{35}{2}$

Now, area of quadrilateral $(ABCD) = ar(\Delta ABC) + ar(\Delta ADC)$

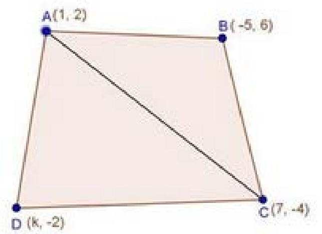
Area (quadrilateral $ABCD$) $= \frac{21}{2} + \frac{35}{2}$

Area (quadrilateral $ABCD$) $= \frac{56}{2}$

Area (quadrilateral $ABCD$) $= 28$ square. Units

3.

Sol:



Let $A(1,2), B(-5,6), C(7,-4)$ and $(k,-2)$ be the given points.

Area of ΔABC

$=\frac{1}{2}[1(6+4)+(-5)(-4-2)+7(2-6)]$

$=\frac{1}{2}[10+30-28]$

$=\frac{1}{2}\times 12$

$=6$

Area of ΔADC

$=\frac{1}{2}[1(-4+2)+7(-2-2)+k(2+4)]$

$=\frac{1}{2}[-2+7\times(-4)+k\times 6]$

$=\frac{1}{2}[-2-28+6k]$

$=\frac{1}{2}[-30+6k]$

$=-15+3k$

$=3k-15$

Area of quadrilateral $ABCD$

= Area of ABC + Area of ADC

$=(6+3k-15)$

But are of quadrilateral $=0$ (given)

$\therefore 6+3k-15=0$

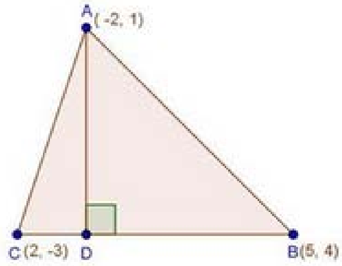
$\Rightarrow 3k=15-6$

$\Rightarrow 3k=9$

$\Rightarrow k=3$

4.

Sol:



Let $A(-2,1), B(5,4)$ and $C(2,-3)$ be the vertices of ΔABC.

Let AD be the altitude through A.

Area of ΔABC

$$=\frac{1}{2}\left[-2(4+3)+5(-3-1)+2(1-4)\right]$$

$$=\frac{1}{2}[-14-20-6]$$

$$=\frac{1}{2}\times -40$$

$$=-20$$

But area cannot be negative

$\therefore$ Area of $\Delta ABC = 20$ square units

Now, $BC=\sqrt{(5-2)^2+(4+3)^2}$

$$\Rightarrow BC=\sqrt{(3)^2+(7)^2}$$

$$\Rightarrow BC=\sqrt{58}$$

We know that area of Δ

$$=\frac{1}{2}\times Base\times Altitude$$

$$\therefore 20=\frac{1}{2}\times\sqrt{58}\times AD$$

$$\Rightarrow AD=\frac{40}{\sqrt{58}}$$

$\therefore$ Length of the altitude $AD=\dfrac{40}{\sqrt{58}}$

5.

Sol:

(a) Let $A(2,5), B(4,6)$ and $C(8,8)$ be the given points

Area of ΔABC

$=\frac{1}{2}[2(6-8)+4(8-5)+8(5-6)]$

$=\frac{1}{2}[2\times(-2)+4\times3+8\times(-1)]$

$=\frac{1}{2}[-4+12-8]$

$=\frac{1}{2}\times0$

$=0$

Since, area of $\Delta ABC=0$

$\therefore (2,5),(4,6)$ and $(8,8)$ are collinear.

(b) Let $A(1,-1), B(2,1)$ and $C(4,5)$ be the given points

Area of ΔABC

$=\frac{1}{2}[1(1-5)+2(5+1)+4(-1-1)]$

$=\frac{1}{2}[-4+12-8]$

$=\frac{1}{2}\times0$

$=0$

Since, area of $\Delta ABC=0$

$\therefore$ The points $(1,-1),(2,1)$ and $(4,5)$ are collinear

6.

Sol:

Let $A(a,0), B(0,b)$ and $C(1,1)$ be the given points

Area of ΔABC

$=\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$

$=\frac{1}{2}\{a(b-1)+0(1-0)+1(0-b)\}$

$=\frac{1}{2}\{ab-a+0-b\}$

$$= \frac{1}{2}\{ab - a - b\}$$

$$= \frac{1}{2}\{ab - (a+b)\}$$

$$= \frac{1}{2}\{ab - ab\} \qquad \left[\because \frac{1}{a} + \frac{1}{b} = 1\right]$$

$$\Rightarrow \frac{a+b}{ab} = 1$$

$$\Rightarrow a + b = ab$$

$$= \frac{1}{2} \times 0$$

$$= 0$$

Hence, $A(a,0), B(0,b)$ and $(1,1)$ are collinear if $\frac{1}{a} + \frac{1}{b} = 1$.

7.

Sol:

Let $A(x, y)$ divides the join of $P(-5,1)$ and $(3,5)$ in the ratio $k:1$

$$x = \frac{3k-5}{k+1}, y = \frac{5k+1}{k+1}$$

Area of ΔABC with $A\left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right) B(1,5)$ and $C(7,-2)$

$$= \frac{1}{2}\left\{\frac{3k-5}{k+1}(5-2) + 1\left(2 - \frac{5k+1}{k+1}\right) + 7\left(\frac{5k+1}{k+1} = 5\right)\right\}$$

$$= \frac{1}{2}\left\{\frac{3k-5}{k+1} \times 7 + \frac{-7k-3}{k+1} + \frac{-4}{k+1}\right\}$$

$$= \frac{1}{2}\left\{\frac{21k-35}{k+1} + \frac{-7k-3}{k+1} + \frac{-4}{k+1}\right\}$$

$$= \frac{1}{2}\left\{\frac{21k-35-7k-3-4}{k+1}\right\}$$

$$= \frac{1}{2}\left\{\frac{14k-42}{k+1}\right\}$$

$$= \frac{14k-42}{2(k+1)}$$

But area of $\Delta ABC = 2$ given,

$\Rightarrow \frac{14k-42}{2(k+1)} = 2$

$\Rightarrow 14k-42 = 4(k+1)$

$\Rightarrow 14k-42 = 4k+4$

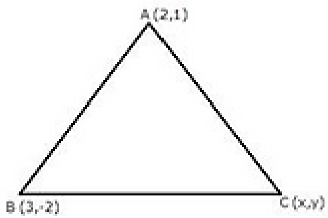
$\Rightarrow 14k-4k = 4+42$

$\Rightarrow 10k = 46$

$\Rightarrow k = \frac{46}{10} = \frac{23}{5}$

8.

Sol:



Let $A(2,1), B(3,-2)$ be the vertices of Δ

And $C(x,y)$ be the third vertex

Area of $\Delta ABC = \frac{1}{2}|2(-2-y)+3(y-1)+x(1+2)|$

$= \frac{1}{2}|-4-2y+3y-3+3x|$

$= \frac{1}{2}|3x+y-7|$

But it is given that area of $\Delta ABC = 5$

$\therefore 5 = \frac{\pm 1}{2}[3x+y-7]$

$\pm 10 = 3x+y-7$

$3x+y=17$ or $3x+y=-3$ (i)

But it is given that third vertices lies on $y=x+3$

Hence subsisting value of y in (i)

$3x+x+3=17$ *or* $3x+x+3=-3$

$4x=14$ *or* $4x=-6$

$x=\frac{7}{2}$ *or* $x=\frac{-3}{2}$

$y = \frac{7}{2} + 3$ or $y = \frac{-3}{2} + 3$

$y = \frac{13}{2}$ or $y = \frac{3}{2}$

Hence coordinates of c will be $\left(\frac{7}{2}, \frac{13}{2}\right)$ *or* $\left(\frac{-3}{2}, \frac{3}{2}\right)$

9.

Sol:

Let $A\left(a, a^2\right) B\left(b, b^2\right)$ *and* $\left(c, c^2\right)$ be the given points.

$\therefore$ Area of ΔABC

$= \frac{1}{2}\left\{a\left(b^2 - c^2\right) + b\left(c^2 - a^2\right) + c\left(a^2 - b^2\right)\right\}$

$= \frac{1}{2}\left\{ab^2 - ac^2 + bc^2 - ba^2 + ca^2 - cb^2\right\}$

$= \frac{1}{2} \times 0$

$= 0 \quad [if \ a = b = c]$

i.e., the points are collinear if $a = b = c$

Hence, the points can never be collinear if $a \neq b \neq c$.

10.

Sol:

Area of $\Delta DBC = \frac{1}{2}\left\{x(5+2) + (-3)(-2-3x) + 4(3x-5)\right\}$

$= \frac{1}{2}\left\{7\times + (6+9x) + 12x - 20\right\}$

$= \frac{1}{2}\left\{28x - 14\right\}$

Area of $\Delta ABC = \frac{1}{2}\left\{6(5+2) + (-3)(-2-3) + 4(3-5)\right\}$

$= \frac{1}{2}\left\{42 + 15 - 8\right\}$

$= \frac{1}{2} \times 49$

Given

$\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$

$$\Rightarrow \frac{\frac{1}{2}(28x-14)}{\frac{1}{2}\times 49}=\frac{1}{2}$$

$$\Rightarrow \frac{28x-14}{49}=\frac{1}{2}$$

$$\Rightarrow 2(28x-14)=49$$

$$\Rightarrow 56x-28=49$$

$$\Rightarrow 56x=77$$

$$\Rightarrow x=\frac{77}{56}$$

$$\Rightarrow x=\frac{11}{8}$$

11.

Sol:

Let $A(a,1), B(1,-1)$ and $C(11,4)$ be the given points

Area of ΔABC

$$=\frac{1}{2}\{a(-1-4)+1(4-1)+11(1+1)\}$$

$$=\frac{1}{2}\{-5+3+22\}$$

$$=\frac{1}{2}\{-5a+25\}$$

For the points to be collinear

Area of $\Delta ABC=0$

$$=\frac{1}{2}\{-5a+25\}=0$$

$$\Rightarrow -5a+25=0$$

$$\Rightarrow -5a=-25$$

$$\Rightarrow a=5$$

12.

Sol:

Let $A(a,b), B(a_1,b_1)$ and $C(a-a_1,b-b_1)$ be the given points.

Area of ΔABC

$$=\frac{1}{2}\left\{a\left[b_1-\left(b-b_1\right)+a_1\left(b-b_1-b\right)+\left(a-a_1\right)\left(b-b_1\right)\right]\right\}$$

$$=\frac{1}{2}\left\{a\left(b_1-b+b_1\right)+a_1\left(-b\right)_1+ab-ab_1-a_1b+a_1b_1\right\}$$

$$=\frac{1}{2}\left\{ab_1-ab+ab_1-a_1b_1+ab-ab_1-a_1b+a_1b_1\right\}$$

$$=\frac{1}{2}\left\{ab_1=a_1b\right\}$$

$$=\frac{1}{2}\times 0=0 \qquad \left[if\ \ ab_1=a_1b\right]$$

Hence, the points are collinear if $ab_1 = a_1b$.