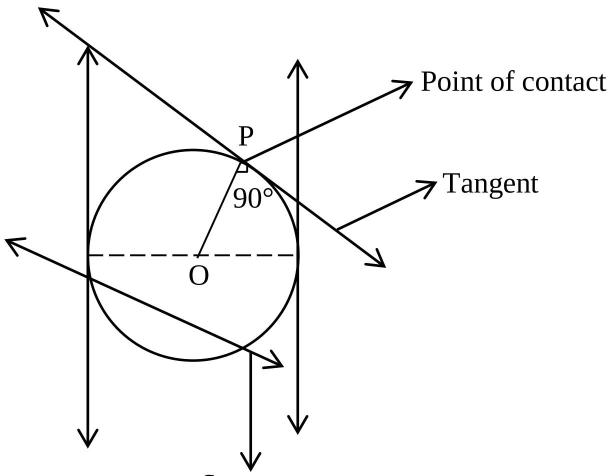
Exercise – 10.1

1.

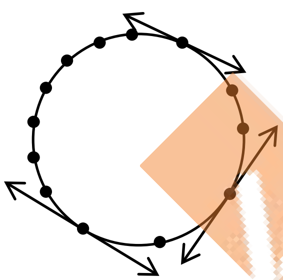
Sol:



2.

Sol:

Tangent: A line intersecting circle in one point is called a tangent.

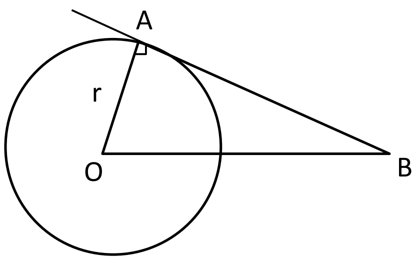


As there are infinite number of points on the circle a circle has many (infinite) tangents.

3.

Sol:

Consider a circle with center O and radius OA = 8cm = r, AB = 15 cm.



(AB) tangent is drawn at A (point of contact)

At point of contact, we know that radius and tangent are perpendicular.
In ΔOAB, ∠OAB = 90°, By Pythagoras theorem

$OB^2 = OA^2 + AB^2$

$OB = \sqrt{8^2 + 15^2}$

$= \sqrt{64 + 225} = \sqrt{229} = 17\ cm$

$\therefore OB = 17\ cm$

4.

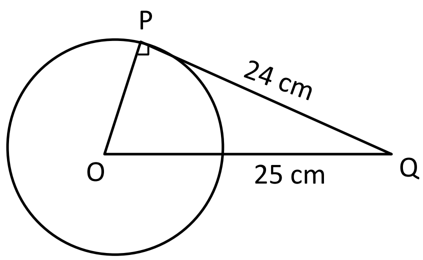
Sol:

Given,

PQ = 24 cm

OQ = 25 cm

OP = radius = ?



P is point of contact, At point of contact, tangent and radius are perpendicular to each other
∴ ΔPOQ is right angled triangle ∠OPQ = 90°
By Pythagoras theorem,

$PQ^2 + OP^2 = OQ^2$

$\Rightarrow 24^2 + OP^2 = 25^2$

$\Rightarrow OP = \sqrt{25^2 - 24^2} = \sqrt{625 - 576}$

$= \sqrt{49} = 7cm$

$\therefore OP = radius = 7cm$

Exercise – 10.2

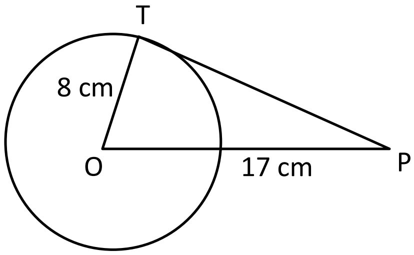
1.

Sol:

OT = radius = 8cm

OP = 17cm

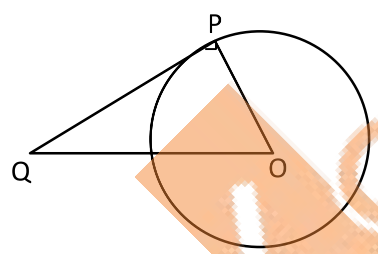
PT = length of tangent = ?



T is point of contact. We know that at point of contact tangent and radius are perpendicular.
∴ OTP is right angled triangle ∠OTP = 90°, from Pythagoras theorem $OT^2 + PT^2 = OP^2$
$8^2 + PT^2 = 17^2$
PT $\sqrt{17^2 - 8^2} = \sqrt{289 - 64}$
= $\sqrt{225} = 15cm$
∴ PT = length of tangent = 15 cm.

2.

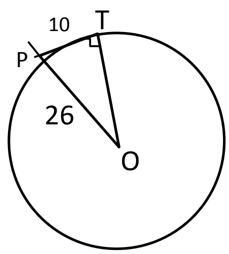
Sol:
Consider a circle with center O.
OP = radius = 5 cm.
A tangent is drawn at point P, such that line through O intersects it at Q, OB = 13cm.
Length of tangent PQ = ?



A + P, we know that tangent and radius are perpendicular.
ΔOPQ is right angled triangle, ∠OPQ = 90°
$By\ pythagoras\ theorem, OQ^2 = OP^2 + PQ^2$
$\Rightarrow 13^2 = 5^2 + PQ^2$
$\Rightarrow PQ^2 = 169 - 25 = 144$
⇒ PQ = $\sqrt{144} = 12cm$
Length of tangent = 12 cm

3.

Sol:
Given OP = 26 cm
PT = length of tangent = 10cm
radius = OT = ?



At point of contact, radius and tangent are perpendicular ∠OTP = 90°, ΔOTP is right angled triangle.

By Pythagoras theorem, $OP^2 = OT^2 + PT^2$

$26^2 = OT^2 + 10^2$

$OT^k = \left(\sqrt{676 - 100}\right)^k$

$OT = \sqrt{576}$

= 24 cm

OT = length of tangent = 24 cm

4.

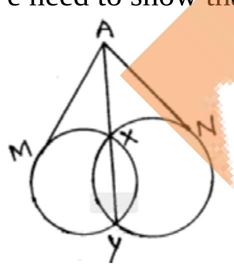
Sol:

Let the two circles intersect at points X and Y.

XY is the common chord.

Suppose ‘A’ is a point on the common chord and AM and AN be the tangents drawn A to the circle

We need to show that AM = AN.



In order to prove the above relation, following property will be used.

“Let PT be a tangent to the circle from an external point P and a secant to the circle through P intersects the circle at points A and B, then $PT^2 = PA \times PB$"

Now AM is the tangent and AXY is a secant ∴ $AM^2 = AX \times AY \ldots..(i)$

AN is a tangent and AXY is a secant ∴ $AN^2 = AX \times AY \ldots..(ii)$

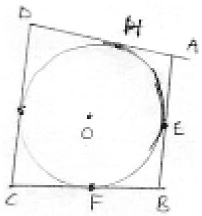
From (i) & (ii), we have $AM^2 = AN^2$

∴ AM = AN

5.

Sol:

Consider a quadrilateral ABCD touching circle with center O at points E, F, G and H as in figure.



We know that

The tangents drawn from same external points to the circle are equal in length.

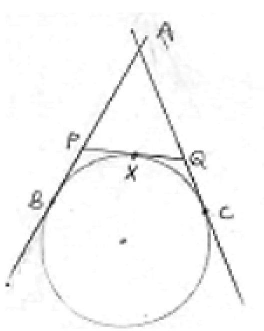
1. Consider tangents from point A [AM ⊥ AE]
   AH = AE …. (i)
2. From point B [EB & BF]
   BF = EB …. (ii)
3. From point C [CF & GC]
   FC = CG …. (iii)
4. From point D [DG & DH]
   DH = DG …. (iv)
   Adding (i), (ii), (iii), & (iv)
   (AH + BF + FC + DH) = [(AC + CB) + (CG + DG)]
   ⇒ (AH + DH) + (BF + FC) = (AE + EB) + (CG + DG)
   ⇒ AD + BC = AB + DC [from fig.]
   Sum of one pair of opposite sides is equal to other.

6.

Sol:

Perimeter of ΔAPQ, (P) = AP + AQ + PQ

= AP + AQ + (PX + QX)



We know that

The two tangents drawn from external point to the circle are equal in length from point A,

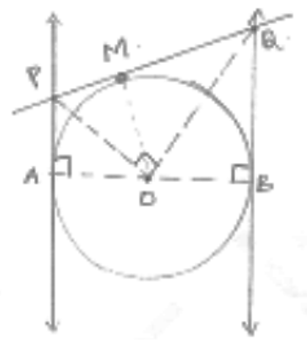
AB = AC = 5 cm

From point P, PX = PB
From point Q, QX = QC
Perimeter (P) = AP + AQ + (PB + QC)
= (AP + PB) + (AQ + QC)
= AB + AC = 5 + 5
= 10 cms.

7.

Sol:

Consider circle with center 'O' and has two parallel tangents through A & B at ends of diameter.



Let tangents through M intersects the tangents parallel at P and Q required to prove is that ∠POQ = 90°.

From fig. it is clear that ABQP is a quadrilateral

∠A + ∠B = 90° + 90° = 180° [At point of contact tangent & radius are perpendicular]

∠A + ∠B + ∠P + ∠Q = 360° [Angle sum property]

∠P + ∠Q = 360° − 180° = 180° …..(i)

At P & Q ∠APO = ∠OPQ = $\frac{1}{2}\angle P$

∠BQO = ∠PQO = $\frac{1}{2}\angle Q$ in (i)

2∠OPQ + 2 ∠PQO = 180°

∠OPQ + ∠PQO = 90° …. (ii)

In ΔOPQ, ∠OPQ + ∠PQO + ∠POQ = 180° [Angle sum property]

90° + ∠POQ = 180° [from (ii)]

∠POQ = 180° − 90° = 90°

∴ ∠POQ = 90°

8.

Sol:

Given ∠TRQ = 30°.

At point R, OR ⊥ RQ.

∠ORQ = 90°

⇒ ∠TRQ + ∠ORT = 90°

⇒ ∠ORT = 90° − 30° = 60°

ST is diameter, ∠SRT = 90° [∵ Angle in semicircle = 90°]

∠ORT + ∠ SRO = 90°

∠SRO + ∠PRS = 90°

∠PRS = 90° − 30° = 60°

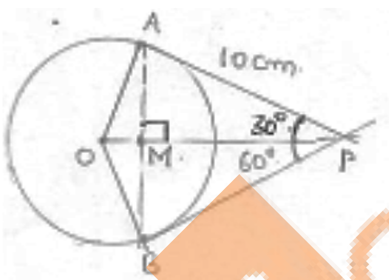
9.

Sol:

AP = 10 cm ∠APB = 60°

Represented in the figure

We know that



A line drawn from center to point from where external tangents are drawn divides or bisects the angle made by tangents at that point ∠APO = ∠OPB = $\frac{1}{2} \times 60° = 30°$

The chord AB will be bisected perpendicularly

∴ AB = 2AM

In ΔAMP,

$$\sin 30° = \frac{opp.side}{hypotenuse} = \frac{AM}{AP}$$

AM = AP sin 30°

$= \frac{AP}{2} = \frac{10}{2} = 5cm$

AP = 2 AM = 10 cm ---- Method (i)

In ΔAMP, ∠AMP = 90°, ∠APM = 30°

$\angle AMP + \angle APM + \angle MAP = 180°$

$90° + 30° + \angle MAP = 180°$

$\angle MAP = 180°$

In $\Delta PAB, \angle MAP = \angle BAP = 60°, \angle APB = 60°$

We also get, $\angle PBA = 60°$

$\therefore \Delta PAB$ is equilateral triangle

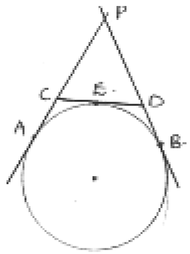
$AB = AP = 10$ cm. -----Method (ii)

10.

Sol:

PA = 14 cm

Perimeter of ΔPCD = PC + PD + CD = PC + PD + CE + ED



We know that

The two tangents drawn from external point to the circle are equal in length.

From point P, PA = PB = 14cm

From point C, CE = CA

From point D, DB = ED

Perimeter = PC + PD + CA +DB

= (PC + CA) + (PD + DB)

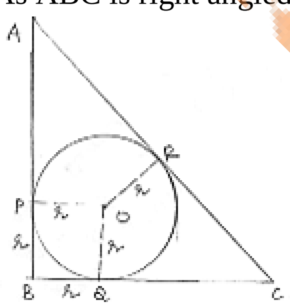
= PA + PB = 14 + 14 = 28 cm.

11.

Sol:

BC = 6cm AB = 8cm

As ABC is right angled triangle



By Pythagoras theorem

$AC^2 = AB^2 + BC^2 = 6^2 + 8^2 = 100$

$AC = 10\ cm$

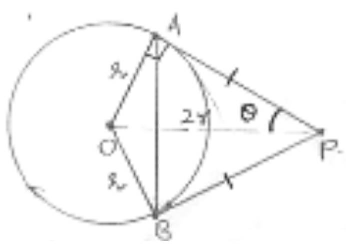
Consider BQOP ∠B = 90°,

∠BPO = ∠OQB = 90° [At point of contact, radius is perpendicular to tangent]

All the angles = 90° & adjacent sides are equal

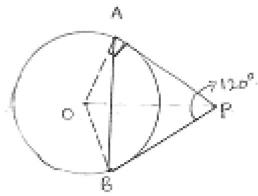
∴ BQOP is square BP = BQ = r
We know that
The tangents drawn from any external point are equal in length.
AP = AR = AB – PB = 8 – r
QC = RC = BC – BQ = 6 – r
AC = AR + RC ⇒ 10 = 8 – r + 6 – r
⇒ 10 = 14 – 2r
⇒ 2r = 4
⇒ Radius = 2cm

12.
Sol:
OP = 2r
Tangents drawn from external point to the circle are equal in length
PA = PB



At point of contact, tangent is perpendicular to radius.
In ΔAOP, sin $\theta = \frac{opp.side}{hypotenuse} = \frac{r}{2r} = \frac{1}{2}$
θ = 30°
∠APB = 20 = 60°, as PA = PB ∠BAP = ∠ABP = x.
In ΔPAB, by angle sum property
∠APB + ∠BAP + ∠ABP = 180°
2x = 120° ⇒ x = 60°
In this triangle all angles are equal to 60°
∴ ΔAPB is equilateral.

13.
Sol:
A + P
OP bisects ∠APB
∠APO = ∠OPB = $\frac{1}{2}\angle APB = \frac{1}{2} \times 120° = 60°$



At point A

OA ⊥ AP, ∠OAP = 90°

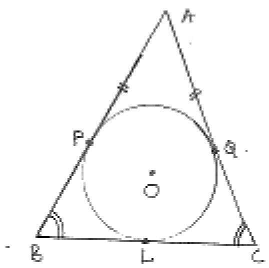
In ΔPDA, cos 60° = $\frac{AP}{DP}$

$\frac{1}{2} = \frac{AP}{DP} \Rightarrow DP = 2AP$

14.

Sol:

Given ΔABC is isosceles AB = AC



We know that

The tangents from external point to circle are equal in length

From point A, AP = AQ

But AB = AC ⇒ AP + PB = AQ + QC

⇒ PB = PC …. (i)

From B, PB = BL; ….(ii) from C, CL = CQ …..(iii)

From (i), (ii) & (iii)

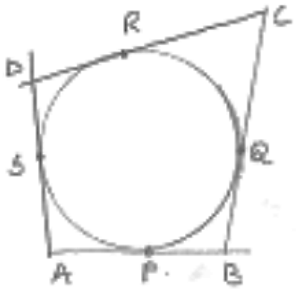
BL = CL

∴ L bisects BC.

15.

Sol:

We know that the tangents drawn from any external point to circle are equal in length.



From A ⟶ AS = AP ….(i)

From B ⟶ QB = BP …. (ii)

From C ⟶ QC = RC …..(iii)

From D ⟶ DS = DR …. (iv)

Adding (i), (ii), (iii) & (iv)

(AS + QB + QC + DS) = (AB + BP + RC + OR)

(AS + DS) + (QB + QC) = (AP + BP) + (RC + DR)

AD + BC = AB + CD

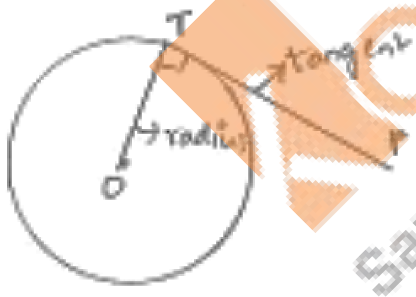
⇒ AD + 7 = 6 + 4 AD = 3cm

⇒ AD = 10 – 7 = 3cm

16.

Sol:

We know that



The at point of contact, the tangent is perpendicular to the radius. Radius is line from center to point on circle. Therefore, perpendicular to tangent will pass through center of circle.

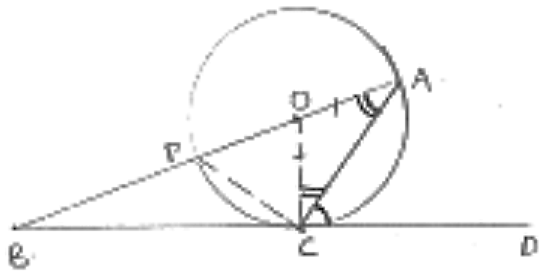
17.

Sol:

Given

O is center of circle

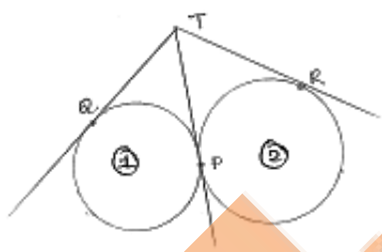
BCD is tangent.



Required to prove: ∠BAC + ∠ACD = 90°
Proof: OA = OC [radius]
In ΔOAC, angles opposite to equal sides are equal.
∠OAC = ∠OCA …. (i)
∠OCD = 90° [tangent is radius are perpendicular at point of contact]
∠ACD + ∠OCA = 90°
∠ACD + ∠OAC = 90° [∵ ∠OAC = ∠BAC]
∠ACD + ∠BAC = 90° ⟶ Hence proved

18.

Sol:

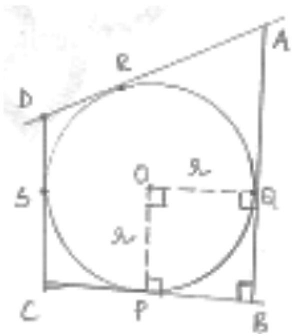


Let the circles be represented by (i) & (ii) respectively
TQ, TP are tangents to (i)
TP, TR are tangents to (ii)
We know that
The tangents drawn from external point to the circle will be equal in length.
For circle (i), TQ = TP …. (i)
For circle (ii), TP = TR …. (ii)
From (i) & (ii) TQ = TR

19.

Sol:

Given AD = 23 cm
AB = 29 cm
∠B = 90°
DS = 5cm



From fig in quadrilateral POQB

∠OPB = ∠OQB = 90° = ∠B = ∠POQ

and PO = OQ. ∴ POQB is a square PB = BQ = r

We know that

Tangents drawn from external point to circle are equal in length.

We know that

Tangents drawn from external point to circle are equal in length.

From A, AR = AQ …. (i)

From B, PB = QB …. (ii)

From C, PC = CS …. (iii)

From D, DR = DS …. (iv)

(i) + (ii) + (iv) ⇒ AR + DB + DR = AQ + QB + DS

$\Rightarrow (AR + DR) + r = (AQ + QB) + DS$

$AD + r = AB + DS$

$\Rightarrow 23 + r = 29 + 5$

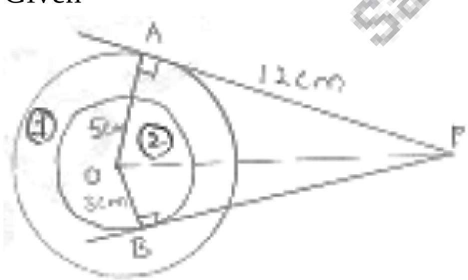
$\Rightarrow r = 34 - 23 = 11$ cm

∴ radius = 11 cm

20.

Sol:

Given



OA = 5 cm

OB = 3 cm

AP = 12 cm

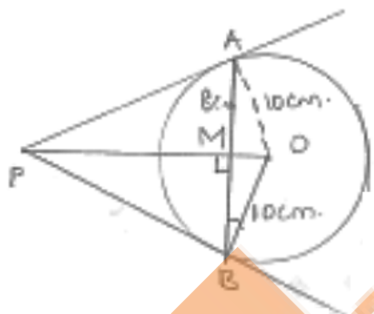
BP = ?

We know that

At the point of contact, radius is perpendicular to tangent.
For circle 1, ΔOAP is right triangle
By Pythagoras theorem, $OP^2 = OA^2 + AP^2$
$\Rightarrow OP^2 = 5^2 + 12^2 = 25 + 144$
= 169
⇒ OP = $\sqrt{169} = 13\ cm$
For circle 2, ΔOBP is right triangle by Pythagoras theorem,
$OP^2 = OB^2 + BP^2$
$13^2 = 3^2 + BP^2$
$BP^2 = 169 - 9 = 160$
$BP = \sqrt{160} = 4\sqrt{10}\ cm$

21.

Sol:
Given length of chord AB = 16cm.
Radius OB = OA = 10 cm.

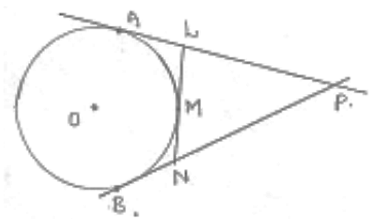


Let line through Centre to point from where tangents are drawn be intersecting chord AB at M. we know that the line joining Centre to point from where tangents are drawn be intersecting chord AB at M. we know that
The line joining Centre to point from where tangents are drawn bisects the chord joining the points on the circle where tangents intersects the circle.
AM = MB = $\frac{1}{2}(AB) = \frac{1}{2} \times 16 = 8cm$
Consider ΔOAM from fig. ∠AMO = 90°
By Pythagoras theorem, $OA^2 = AM^2 + OM^2$
$10^2 = 8^2 + OM^2$
$OM = \sqrt{100 - 64} = \sqrt{36} = 6cm$
In ΔAMP, ∠AMP = 90° by Pythagoras theorem $AP^2 = AM^2 + PM^2$
$AP^2 = 8^2 + (OP - OM)^2$
$PA^2 = 64 + (OP - 6)^2$
$(OP - 6)^2 = -64 + PA^2$ ….(i)
$In\ \Delta APO, \angle PAO = 90°$ [At point of contact, radius is perpendicular to tangent]

$OP^2 = OA^2 + PA^2$ [Pythagoras theorem]
$PA^2 = OP^2 - 10^2$
$= OP^2 - 100$ ….. (ii)

22.

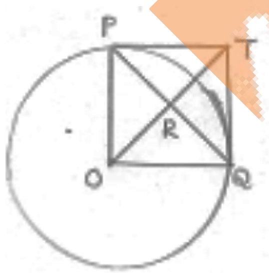
Sol:



Given
O is Centre of circle
PA and PB are tangents
We know that
The tangents drawn from external point to the circle are equal in length.
From point P, PA = PB
⇒ PL + AL = PN + NB …. (i)
From point L & N, AL = LM and MN = NB } …. Substitute in (i)
PL + Lm = PN + MN
⇒ Hence proved.

23.

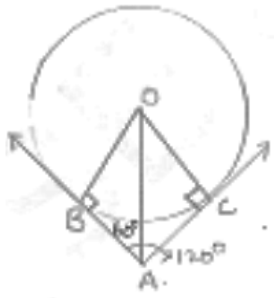
Sol:



Given
PO ⊥ OQ
Consider quadrilateral OQTP.
∠POQ = 90°
∠OPT = ∠OQT = 90° [At point of contact, tangent and radius are perpendicular]
∴ ∠PTO = 90°
OP = OQ = radius
In this quadrilateral, all the angles are equal and pair of adjacent sides are equal.
∴ OQTP is a square.

24.

Sol:



Consider Centre O for given circle

∠BAC = 120°

AB and AC are tangents

From the fig.

In ΔOBA, ∠OBA = 90° [radius perpendicular to tangent at point of contact]

∠OAB = ∠OAC = $\frac{1}{2}\angle BAC = \frac{1}{2} \times 120° = 60°$

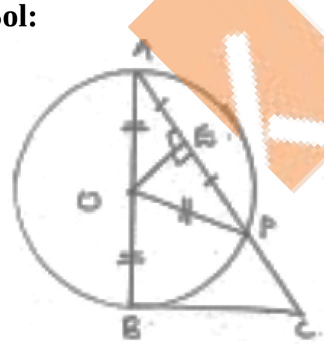
[Line joining Centre to external point from where tangents are drawn bisects angle formed by tangents at that external point1]

In ΔOBA, cos 60° = $\frac{AB}{OA}$

$\frac{1}{2} = \frac{AB}{OA} \Rightarrow OA = 2AB$

25.

Sol:



Given

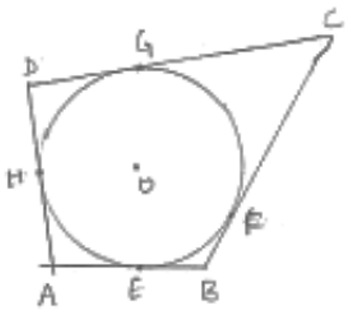
BC is tangent to circle

OE bisects AP, AE = EP

Consider ΔAOP

26.

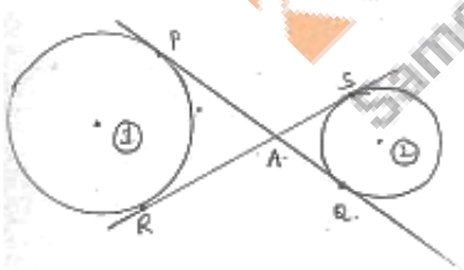
Sol:



Let us consider a quadrilateral ABCD, AB = 4cm, BC = 5 cm, CD = 7cm, CD as sides circumscribing circle with centre O. and intersecting at points E, F, G, H. as in fig.
We know that the tangents drawn from external point to the circle are equal in length.
From point A, AE = AH …. (i)
From point B, BE = BF ….. (ii)
From point C, GC = CE ….(iii)
From point D, GD = DH ….(iv)
(i) + (ii) + (iii) + (iv) ⇒ (AE + BE + GC + GD) = (AH + BF + CF + DH)
⇒ (AE + BE) + (GC + GD) = (AH + DH) + (BF + CF)
⇒ AB + CD = AD + BC
⇒ 4 + 7 = 5 + AD
⇒ AD = 11 – 5 = 6 cm
Fourth side = 6 cm

27.

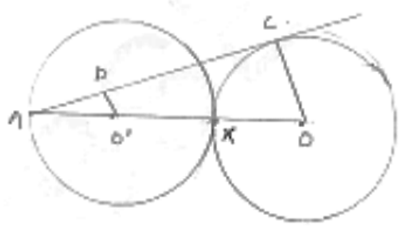
Sol:
Consider



Two circles namely (i) & (ii) as shown with common tangents as PQ and RS.
We know that
The tangents from external point to the circle are equal in length.
From A to circle (i) AP = AR … (i)
From A to circle (ii), AQ = AS …. (ii)
(i) + (ii) ⇒ AP + AQ = AR + RS
⇒ PQ = RS

28.

Sol:



Given circles with centers O and O'

O'D $\perp$ AC. Let radius = r

$O'A = O'X = OX = r$

In triangles, $\Delta AO'D$ and ΔAOC

$\angle A = \angle A$ [Common angle]

$\angle ADO' = \angle ACO = 90°$ [O'D $\perp$ AC and at point of contact C, radius $\perp$ tangent]

By A · A similarity $\Delta AO'D \sim \Delta AOC$.

when two triangles are similar then their corresponding sides will be in proportion

By A.A similarity $\Delta AO'D \sim \Delta AOC$

When two triangles are similar then their corresponding sides will be in proportion

$\frac{AO'}{AO} = \frac{DO'}{CO}$

$\Rightarrow \frac{DO'}{CO} = \frac{r}{r+r+r} = \frac{r}{3r} = \frac{1}{3}$

$\Rightarrow \frac{DO'}{CO} = \frac{1}{3}$

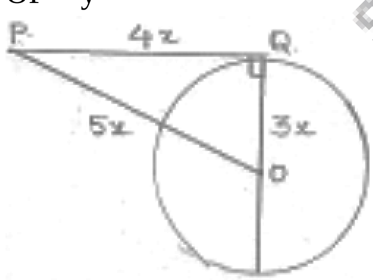
29.

Sol:

Given OQ: PQ = 3 : 4

Let OQ = 3x PQ = 4x

OP = y



$\angle OQP = 90°$ [since at point of contact, tangent is perpendicular to radius]

In ΔOQP, by Pythagoras theorem

$OP^2 = OQ^2 + QP^2$

$\Rightarrow y^2 = (3x)^2 + (4x)^2$

$\Rightarrow y^2 = 9x^2 + 16x^2 = 25x^2$

$\Rightarrow y^2 = \sqrt{25x^2} = 5x$

Perimeter = OQ + PQ + OP = 3x + 4x + 5x = 12x

According to problem perimeter = 60

$\therefore$ 12x = 60

x = $\frac{60}{12} = 5cm$

OQ = $3 \times 5 = 15cm$

PQ = $4 \times 5 = 20\ cm$

OP = $5\ \times 5 = 25cm$