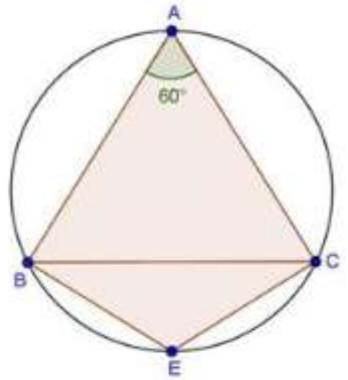
Exercise 15.5

Q1

In fig., ΔABC is an equilateral triangle. Find m∠BEC.



Solution

Since, $\triangle ABC$ is an equilateral triangle.

Then, $\angle BAC = 60^\circ$

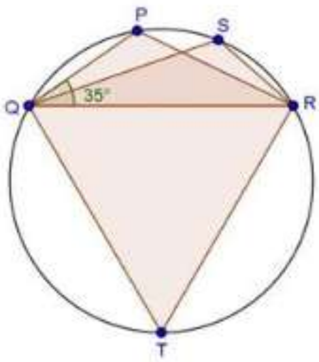
$\therefore \quad \angle BAC + \angle BEC = 180^\circ$ [Opposite angles of cyclic quad.]

$\Rightarrow \quad 60^\circ + \angle BEC = 180^\circ$

$\Rightarrow \quad \angle BEC = 180^\circ - 60^\circ = 120^\circ$

Q2

In fig., ΔPQR is an isosceles triangle with PQ = PR and m∠PQR = 35°. find m∠QSR and m∠QTR.



Solution

We have, $\angle PQR = 35^\circ$

Since, $\triangle PQR$ is an isosceles triangle with $PQ = PR$.
Then, $\angle PQR = \angle PRQ = 35^\circ$

In $\triangle PQR$, by angle sum property

$\angle P + \angle PQR + \angle PRQ = 180^\circ$

$\Rightarrow \quad \angle P + 35^\circ + 35^\circ = 180^\circ$
$\Rightarrow \quad \angle P = 180^\circ - 35^\circ - 35^\circ = 110^\circ$

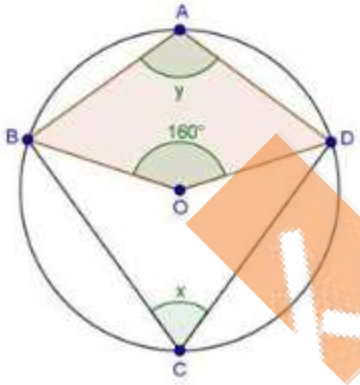
$\therefore \quad \angle QSR = \angle P = 110^\circ$ [Angles in same segment]

Now, $\angle QSR + \angle QTR = 180^\circ$ [Opposite angles of cyclic quad.]
$\Rightarrow \quad 110^\circ + \angle QTR = 180^\circ$

$\Rightarrow \quad \angle QTR = 180^\circ - 110^\circ = 70^\circ$

Q3

In fig., O is the centre of the circle. If ∠BOD = 160°, find the values of x and y.



Solution

We have, $\angle BOD = 160^\circ$

By degree measure theorem

$\angle BOD = 2\angle BCD$

$\Rightarrow \quad 160^\circ = 2 \times x$

$\Rightarrow \quad x = \frac{160^\circ}{2} = 80^\circ$

$\therefore \quad \angle BAD + \angle BCD = 180^\circ$ [Opposite angles of cyclic quad.]

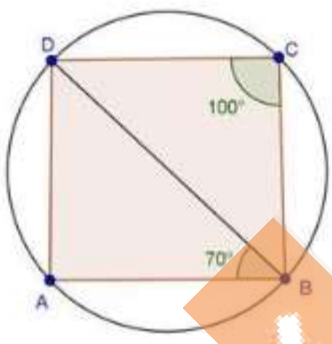
$\Rightarrow \quad y + x = 180^\circ$

$\Rightarrow \quad y + 80^\circ = 180$

$\Rightarrow \quad y = 180^\circ - 80^\circ = 100^\circ$

Q4

In fig., ABCD is a cyclic qudrilateral. If ∠BCD = 100° and ABD = 70°, find ∠ADB.



Solution

We have, $\angle BCD = 100^\circ$ *and* $\angle ABD = 70^\circ$

$\therefore \quad \angle DAB + \angle BCD = 180^\circ$ [Opposite angles of cyclic quad.]

$\Rightarrow \quad \angle DAB + 100^\circ = 180^\circ$

$\Rightarrow \quad \angle DAB + 180^\circ - 100^\circ = 80^\circ$

In $\triangle DAB$, by angle sum property

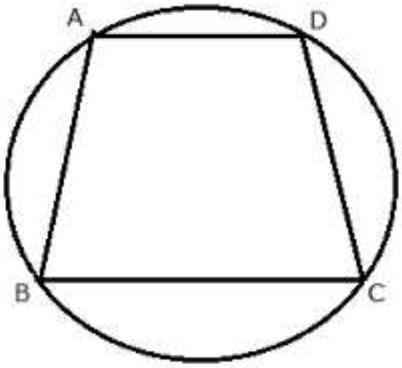
$\angle ADB + \angle DAB + \angle ABD = 180^\circ$

$\Rightarrow \quad \angle ADB + 80^\circ + 70^\circ = 180^\circ$

$\Rightarrow \quad \angle ADB = 180^\circ - 80^\circ - 70^\circ = 30^\circ$

Q5

If ABCD is a cyclic quadrilateral in which AD ∥ BC. Prove that ∠B = ∠C.



Solution

Since, $ABCD$ is a cyclic quadrilateral with $AD \parallel BC$

Then, $\angle A + \angle C = 180^\circ$ ---(1) [Opposite angles of cyclic quad.]

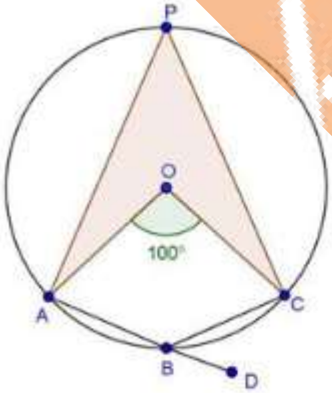
And, $\angle A + \angle B = 180^\circ$ ---(2) [Co-interior angles]

Compare equations (1) and (2)

$\angle B = \angle C$

Q6

In fig., O is the centre of the circle, find ∠CBD.



Solution

We have, $\angle AOC = 100^\circ$

By degree measure theorem

$\angle AOC = 2\angle APC$

$\Rightarrow \quad 100^\circ = 2\angle APC$

$\Rightarrow \quad \angle APC = \frac{100^\circ}{2} = 50^\circ$

$\therefore \quad \angle APC + \angle ABC = 180^\circ$ [Opposite angles of cyclic quad.]

$\Rightarrow \quad 50^\circ + \angle ABC = 180^\circ$

$\Rightarrow \quad \angle ABC = 180^\circ - 50^\circ = 130^\circ$

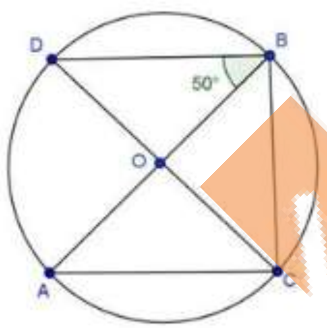
$\therefore \quad \angle ABC + \angle CBD = 180^\circ$ [Linear pair of angles]

$\Rightarrow \quad 130^\circ + \angle CBD = 180^\circ$

$\Rightarrow \quad \angle CBD = 180^\circ - 130^\circ = 50^\circ$

Q7

In fig., AB and CD are diameters of a circle with centre O. If ∠OBD = 50°, find ∠AOC.



Solution

We have, $\angle OBD = 50°$

Since, AB and CD are diameters of circle then O is the centre of the circle.

$\therefore \quad \angle DBC = 90°$ [Angle in semicircle]

$\Rightarrow \quad \angle DBO + \angle OBC = 90°$

$\Rightarrow \quad 50° + \angle OBC = 90°$

$\Rightarrow \quad \angle OBC = 90° - 50° = 40°$

By degree measure theorem

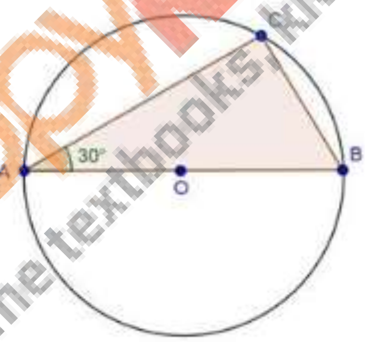
$\angle AOC = 2\angle ABC$

$\Rightarrow \quad \angle AOC = 2 \times 40° = 80°$

Q8

On a semi-circle with AB as diameter, a point C is taken, so that $m(\angle CAB) = 30°$. Find $m(\angle ACB)$ and $m(\angle ABC)$.

Solution



We have, $\angle CAB = 30°$

$\therefore \quad \angle ACB = 90°$ [Angle in semicircle]

In $\triangle ABC$, by angle sum property

$\angle CAB + \angle ACB + \angle ABC = 180°$

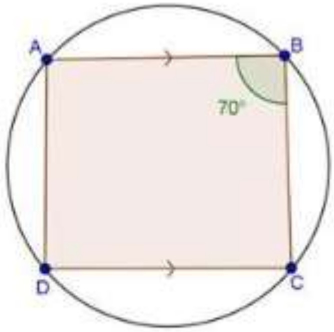
$\Rightarrow \quad 30° + 90° + \angle ABC = 180°$

$\Rightarrow \quad \angle ABC = 180° - 90° - 30° = 60°$

Q9

In a cyclic quadrilateral $ABCD$ if $AB \parallel CD$ and $\angle B = 70°$, find the remaining angles.

Solution



We have, $\angle B = 70^\circ$

Since, $ABCD$ is a cyclic quadrilateral

Then, $\angle B + \angle D = 180^\circ$

$\Rightarrow \quad 70^\circ + \angle D = 180^\circ$

$\Rightarrow \quad \angle D = 180^\circ - 70^\circ = 110^\circ$

Since, $AB \parallel DC$

Then, $\angle B + \angle C = 180^\circ$ [Co-interior angles]

$\Rightarrow \quad 70^\circ + \angle C = 180^\circ$

$\Rightarrow \quad \angle C = 180^\circ - 70^\circ = 110^\circ$

Now, $\angle A + \angle C = 180^\circ$ [Opposite angles of cyclic quad.]

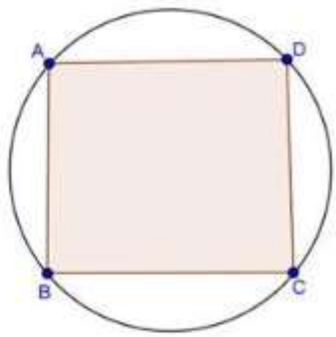
$\Rightarrow \quad \angle A + 110^\circ = 180^\circ$

$\Rightarrow \quad \angle A = 180^\circ - 110^\circ = 70^\circ$

Q10

In a cyclic quadrilateral $ABCD$, if $m\angle A = 3(m\angle C)$. find $m\angle A$.

Solution



We have, $\angle A = 3\angle C$

Let $\angle C = x$
Then, $\angle A = 3x$

$\therefore \quad \angle A + \angle C = 180^\circ$ [Opposite angles of cyclic quad.]

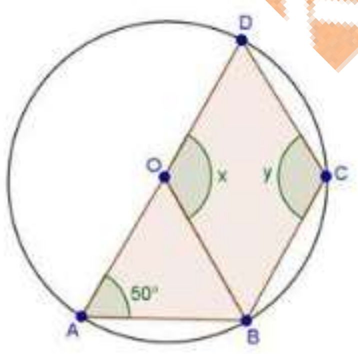
$\Rightarrow \quad 3x + x = 180^\circ$
$\Rightarrow \quad 4x = 180^\circ$
$\Rightarrow \quad x = \frac{180^\circ}{4} = 45^\circ$

$\therefore \quad \angle A = 3x$
$= 3 \times 45^\circ$

$= 135^\circ$

Q11

In fig., O is the centre of the circle and DAB = 50. calculate the values of x and y.



Solution

We have, $\angle DAB = 50^\circ$

By degree measure theorem

$\angle BOD = 2\angle BAD$

$\Rightarrow \quad x = 2 \times 50^\circ = 100^\circ$

Since, $ABCD$ is a cyclic quadrilateral

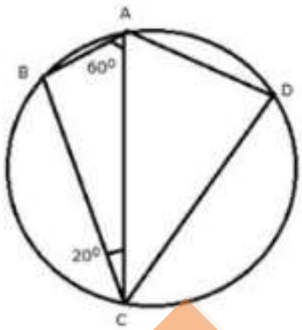
Then, $\angle A + \angle C = 180^\circ$

$\Rightarrow \quad 50^\circ + y = 180^\circ$

$\Rightarrow \quad y = 180^\circ - 50^\circ = 130^\circ$

Q12

In fig., if ∠BAC = 60°, and ∠BCA = 20°, find ∠ADC.



Solution

Using angle sum property in ΔABC,

$\angle B = 180^\circ - (60^\circ + 20^\circ) = 100^\circ$

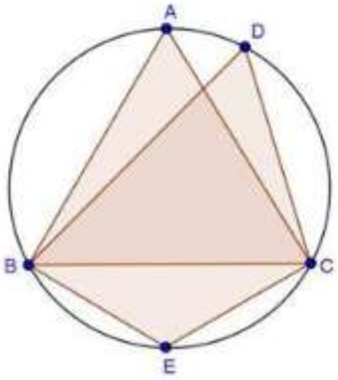
In cyclic quadrilateral ABCD, we have:

$\angle B + \angle D = 180^\circ$

$\angle D = 180^\circ - 100^\circ = 80^\circ$

Q13

In fig., if ABC is an equilateral triangle. Find ∠BDC and ∠BEC.



Solution

Since, $\triangle ABC$ is an equilateral triangle

Then, $\angle BAC = 60^\circ$

$\therefore \quad \angle BDC = \angle BAC = 60^\circ$ [Angles in same segment]

Since, quad. $ABEC$ is a cyclic quadrilateral.

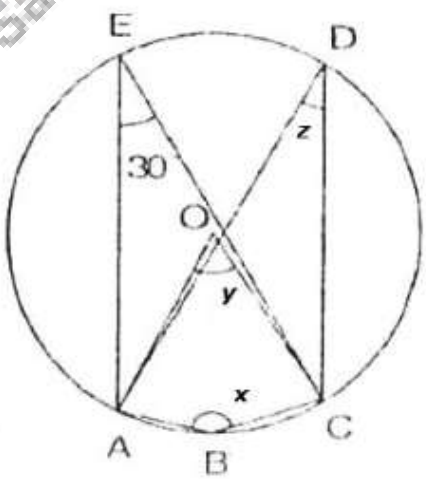
Then, $\angle BAC + \angle BEC = 180^\circ$

$\Rightarrow \quad 60^\circ + \angle BEC = 180^\circ$

$\Rightarrow \quad \angle BEC = 180^\circ - 60^\circ = 120^\circ$

Q14

In fig., O is the centre of the circle. If ∠CEA = 30°, find the values of x, y and z.



Solution

We have, $\angle AEC = 30^\circ$

Since, quad. $ABCE$ is a cyclic quadrilateral.

Then, $\angle ABC + \angle AEC = 180^\circ$

$\Rightarrow \quad x + 30^\circ = 180^\circ$

$\Rightarrow \quad x = 180^\circ - 30^\circ = 150^\circ$

By degree measure theorem

$\angle AOC = 2\angle AEC$

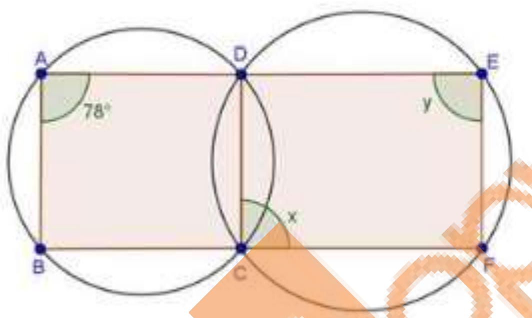
$\Rightarrow \quad y = 2 \times 30^\circ = 60^\circ$

$\therefore \quad \angle ADC = \angle AEC$ [Angles in same segment]

$\Rightarrow \quad z = 30^\circ$

Q15

In fig., $\angle BAD = 78^\circ$, $\angle DCF = x^\circ$ and $DEF = y^\circ$ find the values of x and y.



Solution

We have, $\angle BAD = 78^\circ$, $\angle DCF = x^\circ$ and $\angle DEF = y^\circ$

Since, $ABCD$ is a cyclic quadrilateral.

Then, $\angle BAD + \angle BCD = 180^\circ$

$\Rightarrow \quad 78^\circ + \angle BCD = 180^\circ$

$\Rightarrow \quad \angle BCD = 180^\circ - 78^\circ = 102^\circ$

Now, $\angle BCD + \angle DCF = 180^\circ$ [Linear pair of angles]

$\Rightarrow \quad 102^\circ = x^\circ = 180^\circ$

$\Rightarrow \quad x = 180^\circ - 102^\circ = 78^\circ$

Since, $DCFE$ is a cyclic quadrilateral

Then, $x + y = 180^\circ$

$\Rightarrow \quad 78^\circ + y = 180^\circ$

$\Rightarrow \quad y = 180^\circ - 78^\circ = 102^\circ$

Q16

In a cyclic quadrilateral $ABCD$, if $\angle A - \angle C = 60^\circ$, prove that the smaller of two is 60°.

Solution

We have

$\angle A - \angle C = 60^\circ$ ---(1)

Since, $ABCD$ is a cyclic quadrilateral

Then $\angle A + \angle C = 180^\circ$ ---(2)

Add equations (1) and (2)

$\angle A - \angle C + \angle A + \angle C = 60^\circ + 180^\circ$

$\Rightarrow \quad 2\angle A = 240^\circ$

$\Rightarrow \quad \angle A = \frac{240}{2} = 120^\circ$

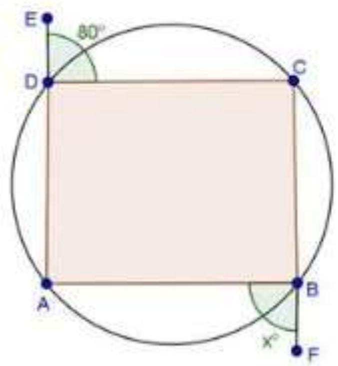
Put value of $\angle A$ in equation (2)

$120^\circ + \angle C = 180^\circ$

$\Rightarrow \quad \angle C = 180^\circ - 120^\circ = 60^\circ$

Q17

In fig., ABCD is cyclic qudrilateral. Find the value of x.



Solution

$\angle EDC + \angle CDA = 180^\circ$ [Linear pair of angles]

$\Rightarrow \quad 80^\circ + \angle CDA = 180^\circ$

$\Rightarrow \quad \angle CDA = 180^\circ - 80^\circ = 100^\circ$

Since, $ABCD$ is a cyclic quadrilateral.

$\angle ADC + \angle ABC = 180^\circ$

$\Rightarrow \quad 100^\circ + \angle ABC = 180^\circ$

$\Rightarrow \quad \angle ABC = 180^\circ - 100^\circ = 80^\circ$

Now, $\angle ABC + \angle ABF = 180^\circ$ [Linear pair of angles]

$\Rightarrow \quad 80^\circ + x^\circ = 180^\circ$

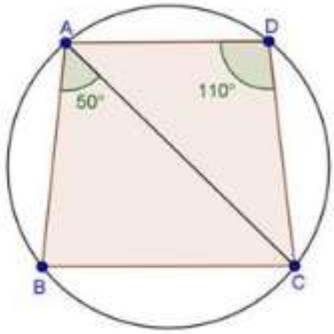
$\Rightarrow \quad x = 180^\circ - 80^\circ = 100^\circ$

Q18

$ABCD$ is a cyclic quadrilateral in which:

(i) $BC \parallel AD$, $\angle ADC = 110^\circ$ and $\angle BAC = 50^\circ$. Find $\angle DAC$.

Solution



Since, $ABCD$ is a cyclic quadrilateral.

Then, $\angle ABC + \angle ADC = 180°$

$\Rightarrow \quad \angle ABC + 110° = 180°$

$\Rightarrow \quad \angle ABC = 180° - 110° = 70°$

Since, $AD \parallel BC$

Then, $\angle DAB + \angle ABC = 180°$ [Co-interior angles]

$\Rightarrow \quad \angle DAC + 50° + 70° = 180°$

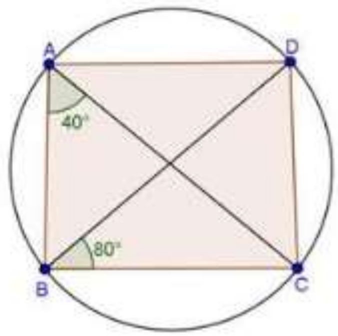
$\Rightarrow \quad \angle DAC = 180° - 50° - 70° = 60°$

Q19

$ABCD$ is a cyclic quadrilateral in which:

(i) $\angle DBC = 80°$ and $\angle BAC = 40°$. Find $\angle BCD$.

Solution



$\angle BAC = \angle BDC = 40^\circ$ [Angles in same segment]

In $\triangle BDC$, by angle sum property

$\angle DBC + \angle BCD + \angle BDC = 180^\circ$

$\Rightarrow \quad 80^\circ + \angle BCD + 40^\circ = 180^\circ$

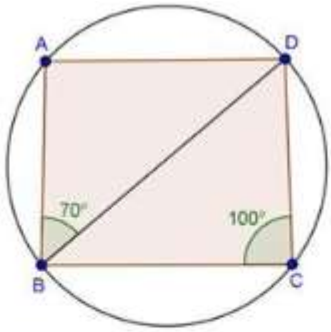
$\Rightarrow \quad \angle BCD = 180^\circ - 80^\circ - 40^\circ = 60^\circ$

Q20

ABCD is a cyclic quadrilateral in which:

(i) $\angle BCD = 100^\circ$ and $\angle ABD = 70^\circ$. Find $\angle ADB$.

Solution



Since, $ABCD$ is a cyclic quadrilateral.

Then, $\angle BAD + \angle BCD = 180°$

$\Rightarrow \quad \angle BAD + 100° = 180°$

$\Rightarrow \quad \angle BAD = 180° - 100° = 80°$

In $\triangle ABD$, by angle sum property

$\angle ABD + \angle ADB + \angle BAD = 180°$

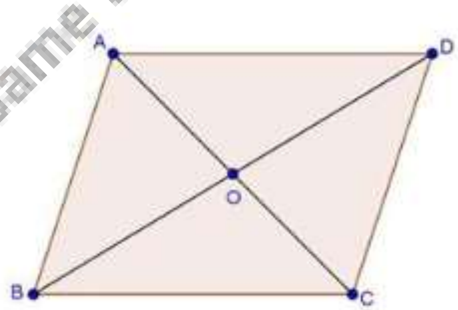
$\Rightarrow \quad 70° + \angle ADB + 80° = 180°$

$\Rightarrow \quad \angle ADB = 180° - 70° - 80° = 30°$

Q21

Prove that the circles described on the four sides of a rhombus as diameters, pass through the point of intersection of its diagonals.

Solution



Let $ABCD$ be a rhombus such that its diagonals AC and BD intersect at O.
Since, the diagonals of a rhombus intersect at right angle.

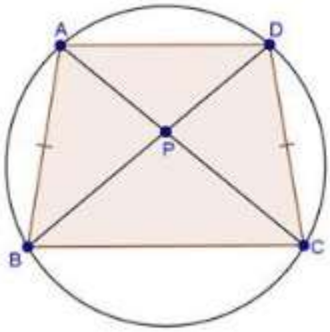
$\therefore \quad \angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$.

Now, $\angle AOB = 90° \Rightarrow$ circle described on AB as diameter will pass through O.
Similarly, all the circles described on BC, AD and CD as diameters pass through O.

Q22

If the two sides of a pair of opposite sides of a cyclic quadrilateral are equal, prove that its diagonals are equal.

Solution



Given $ABCD$ is a cyclic quadrilateral in which $AB = DC$

To prove $AC = BD$

Proof In $\triangle PAB$ and $\triangle PDC$

$AB = DC$ [Given]

$\angle BAP = \angle CDP$ [Angles in the same segment]

$\angle PBA = \angle PCD$ [Angles in same segment]

Then, $\triangle PAB \cong \triangle PDC$ [By ASA condition]

$\therefore$ $PA = PD$ ---(1) [c.p.c.t]

and $PC = PB$ ---(2) [c.p.c.t]

Add equation (1) and (2)

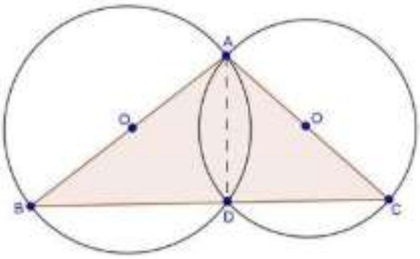
$PA + PC = PD + PB$

$\Rightarrow$ $AC = BD$

Q23

Circles are described on the sides of a triangle as diameters. prove that the circles on any two sides intersect each other on the third side(or third side produced).

Solution



Since, AB is a diameter

Then, $\angle ADB = 90^\circ$ ---(1) [Angle in semicircle]

Since, AC is a diameter

Then, $\angle ADC = 90^\circ$ ---(2) [Angle in semicircle]

Add equations (1) *and* (2)

$\angle ADB + \angle ADC = 90^\circ + 90^\circ$

$\Rightarrow \quad \angle BDC = 180^\circ$

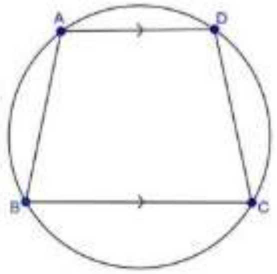
Then, BDC is a line

Hence, the circles on any two sides intersect each other on the third side.

Q24

$ABCD$ is a cyclic trapezium with $AD \parallel BC$. If $\angle B = 70^\circ$, determine other three angles of the trapezium.

Solution



We have

$ABCD$ is a cyclic trapezium with $AD \parallel BC$ and $\angle B = 70°$.

Since, $ABCD$ is a cyclic quadrilateral

Then, $\angle B + \angle D = 180°$

$\Rightarrow \quad 70° + \angle D = 180°$

$\Rightarrow \quad \angle D = 180° - 70° = 110°$

Since, $AD \parallel BC$

Then, $\angle A + \angle B = 180°$ [Co-interior angles]

$\Rightarrow \quad \angle A + 70° = 180°$

$\Rightarrow \quad \angle A = 180° - 70° = 110°$

Since, $ABCD$ is a cyclic quadrilateral

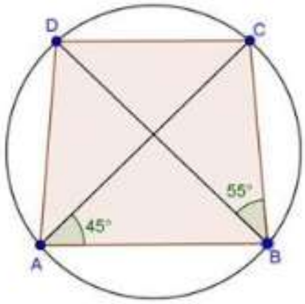
Then, $\angle A + \angle C = 180°$

$\Rightarrow \quad 110° + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 110° = 70°$

Q25

In fig., ABCD is cyclic quadrilaterial in which AC an BD are its diagonals. If ∠DBC = 55° and ∠BAC = 45°, find ∠BCD.



Solution

Since angles in the same segment of a circle are equal.

$\therefore \quad \angle CAD = \angle DBC = 55^\circ$

$\therefore \quad \angle DAB = \angle CAD + \angle BAC = 55^\circ + 45^\circ = 100^\circ$

But, $\angle DAB + \angle BCD = 180^\circ$ [Opposite angles of a cyclic quadrilateral]

$\therefore \quad \angle BCD = 180^\circ - 100^\circ = 80^\circ$

Q26

Prove that the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.

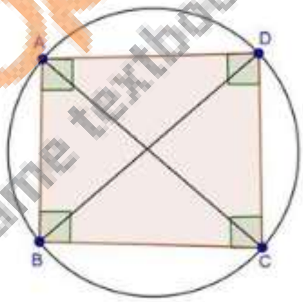
Solution

Let $ABCD$ be a cyclic quadrilateral, and let O be the centre of the corresponding circle.
Then, each side of quadrilateral $ABCD$ is a chord of the circle and the perpendicular bisector of a chord always passes through the centre of the circle.
So, right bisectors of the sides of quadrilateral $ABCD$ will pass through the centre O of the corresponding circle.

Q27

Prove that the centre of the circle circumscribing the cyclic rectangle ABCD is the point of intersection of its diagonals.

Solution



Let O be the centre of the circle circumscribing the cyclic rectangle ABCD. Since $\angle ABC = 90^\circ$ and AC is a chord of the circle, so, AC is a diameter of the circle. Similarly, BD is a diameter.

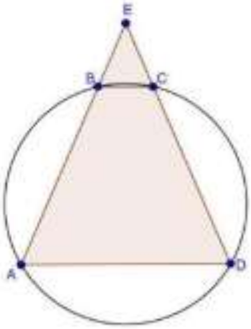
Hence, point of intersection of AC and BD is the centre of the circle.

Q28

$ABCD$ is a cyclic quadrilateral in which AB and CD when produced meet in E and $EA = ED$.
Prove that:

(i) $AD \parallel BC$
(ii) $EB = EC$.

Solution



Given $ABCD$ is a cyclic quadrilateral in which $EA = ED$

To prove (i) $AD \parallel BC$ (ii) $EB = EC$

Proof (i) Since $EA = ED$

Then, $\angle EAD = \angle EDA$ ---(1) [Oppo. angles to equal sides]

Since, $ABCD$ is a cyclic quadrilateral

Then, $\angle ABC + \angle ADC = 180°$

But $\angle ABC + \angle EBC = 180°$ [Lilear pair of angles]

then, $\angle ADC = EBC$ ---(2)

Compare equations (1) and (2)

$\angle EAD = \angle EBC$ ---(3)

Since, corresponding angles are equal

Then, $BC \parallel AD$

(ii) From equation (3)

$\angle EAD = \angle EBC$ ---(3)

Similarly $\angle EDA = \angle ECB$ ---(4)

Compare equations (1)(3) and (4)

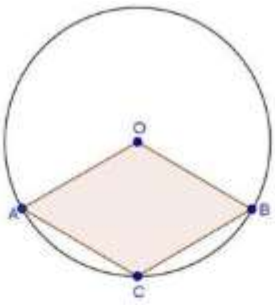
$\angle EBC = \angle ECB$

$\Rightarrow$ $EB = EC$ [Opposite angles to equal sides]

Q29

Prove that the angle in a segment shorter than a semicircle is greater than a right angle.

Solution



Given:- $\angle ACB$ is an angle in minor segment.

To prove:- $\angle ACB > 90°$

Proof:- By degree measure theorem

Reflex $\angle AOB = 2\angle ACB$

And reflex $\angle AOB > 180°$

Then, $2\angle ACB > 180°$

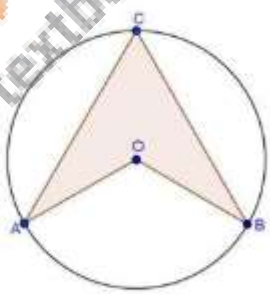
$\Rightarrow \quad \angle ACB > \frac{180°}{2}$

$\Rightarrow \quad \angle ACB > 90°$

Q30

Prove that the angle in a segment greater than a semi-circle is less than a right angle.

Solution



Given:- $\angle ACB$ is an angle in major segment.

To prove:- $\angle ACB < 90°$

Proof:- By degree measure theorem

$\angle AOB = 2\angle ACB$

And $\angle AOB < 180°$

Then, $2\angle ACB < 180°$

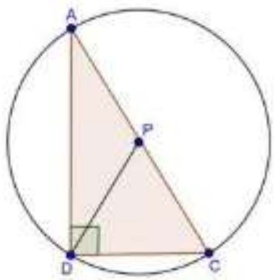
$\Rightarrow \quad \angle ACB < \frac{180°}{2}$

$\Rightarrow \quad \angle ACB < 90°$

Q31

Prove that the line segment joining the mid-point of the hypotenuse of a right triangle to its opposite vertex is half of the hypotenuse.

Solution



Let ΔABC be a right triangle right angled at B. Let P be the mid-point of hypotenuse AC.
Draw a circle with centre at P and AC as a diameter.

Since, $\angle ABC = 90^\circ$. Therefore, the circle passes through B.

$\therefore \quad BP = \text{Radius}$

Also, $AP = CP = \text{Radius}$

$\therefore \quad AP = BP = CP$

Hence, $BP = \frac{1}{2} AC$