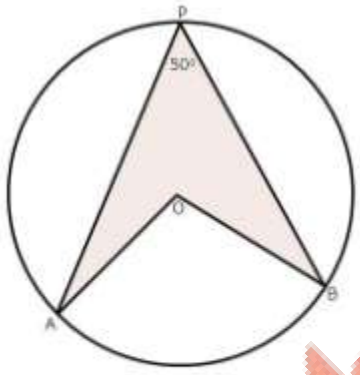
Exercise 15.4

Q1

In fig., O is the centre of the circle. If ∠APB = 50°, find ∠AOB and ∠OAB.



Solution

$\angle APB = 50^\circ$

by degree measure theorem

$\angle AOB = 2\angle APB$

$\Rightarrow \angle APB = 2 \times 50^\circ = 100^\circ$

Since, OA = OB [Radii of circle]

Then $\angle OAB = \angle OBA$ [Angles opposite to equal sides]

Let $\angle OAB = x$

In ΔOAB, by angle sum property

$\angle OAB + \angle OBA + \angle AOB = 180^\circ$

$\Rightarrow x + x + 100 = 180^\circ$

$\Rightarrow 2x = 180^\circ - 100^\circ$

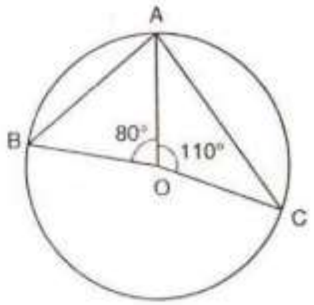
$\Rightarrow 2x = 80^\circ$

$\Rightarrow x = \frac{80}{2} = 40^\circ$

$\therefore \angle OAB = \angle OBA = 40^\circ$

Q2

In fig., O is the centre of the circle. Find ∠BAC.



Solution

We have $\angle AOB = 80^\circ$

And $\angle AOC = 110^\circ$

$\therefore \quad \angle AOB + \angle AOC + \angle BOC = 360^\circ$ [Complete angle]

$\Rightarrow \quad 80^\circ + 110^\circ + \angle BOC = 360^\circ$

$\Rightarrow \quad \angle BOC = 360^\circ - 80^\circ - 110^\circ$

$\Rightarrow \quad \angle BOC = 170^\circ$

By degree measure theorem

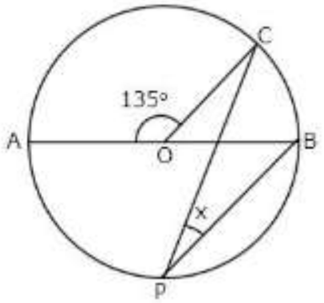
$\angle BOC = 2\angle BAC$

$\Rightarrow \quad 170^\circ = 2\angle BAC$

$\Rightarrow \quad \angle BAC = \frac{170^\circ}{2} = 85^\circ$

Q3

If O is the centre of the circle. Find the value of x in the following figure:



Solution

$\angle AOC = 135^\circ$

$\therefore \angle AOC + \angle BOC = 180^\circ$ [Linear pair of angles]

$\Rightarrow 135^\circ + \angle BOC = 180^\circ$

$\Rightarrow \angle BOC = 180^\circ - 135^\circ = 45^\circ$

By degree measure theorem

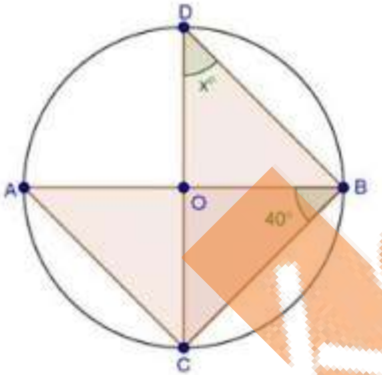
$\angle BOC = 2\angle CDB$

$\Rightarrow 45^\circ = 2x$

$\Rightarrow x = \frac{45^\circ}{2} = 22\frac{1}{2}^\circ$

Q4

If O is the centre of the circle. Find the value of x in the following figure:



Solution

We have

$\angle ABC = 40^\circ$
$\angle ACB = 90^\circ$ [Angle in semicircle]

In $\triangle ABC$, by angle sum property

$\angle CAB + \angle ACB + \angle ABC = 180^\circ$

$\Rightarrow \quad \angle CAB + 90^\circ + 40^\circ = 180^\circ$
$\Rightarrow \quad \angle CAB = 180^\circ - 90^\circ - 40^\circ$

$\Rightarrow \quad \angle CAB = 50^\circ$

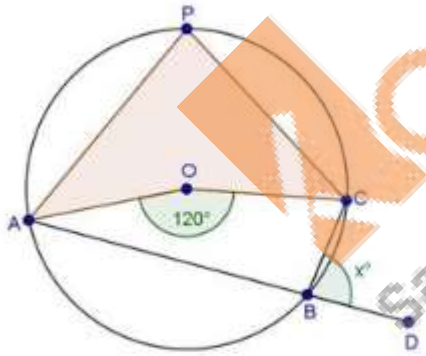
Now,

$\angle CDB = \angle CAB$ [Angle in same segment]

$\Rightarrow \quad x^\circ = 50^\circ$

Q5

If O is the centre of the circle. Find the value of x in the following figure:



Solution

We have

$\angle AOC = 120^\circ$

By degree measure theorem

$\angle AOC = 2\angle APC$

$\Rightarrow \quad 120^\circ = 2\angle APC$

$\Rightarrow \quad \angle APC = \dfrac{120^\circ}{2} = 60^\circ$

$\therefore \quad \angle APC + \angle ABC = 180^\circ$ [Opposite angles of cyclic quadrilateral]

$\Rightarrow \quad 60^\circ + \angle ABC = 180^\circ$

$\Rightarrow \quad \angle ABC = 180^\circ - 60^\circ = 120^\circ$

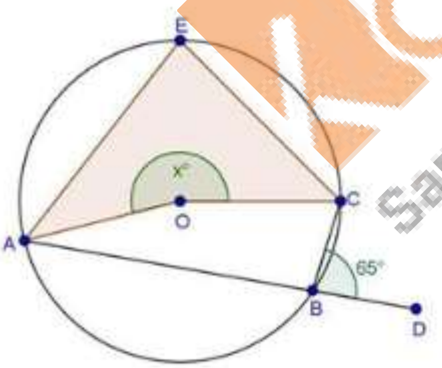
$\therefore \quad \angle ABC + \angle DBC = 180^\circ$ [Linear pair of angles]

$\Rightarrow \quad 120^\circ + x = 180^\circ$

$\Rightarrow \quad x = 180^\circ - 120^\circ = 60^\circ$

Q6

If O is the centre of the circle. Find the value of x in the following figure:



Solution

We have

$\angle CBD = 65^\circ$

$\therefore \quad \angle ABC + \angle CBD = 180^\circ$ [Linear pair of angles]

$\Rightarrow \quad \angle ABC + 65^\circ = 180^\circ$

$\Rightarrow \quad \angle ABC = 180^\circ - 65^\circ = 115^\circ$

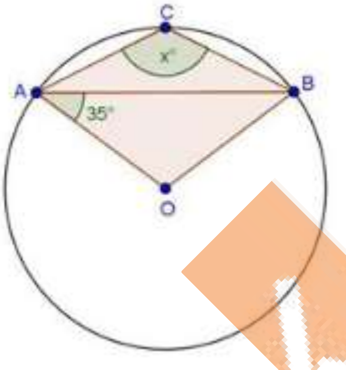
$\therefore \quad \text{Reflex } \angle AOC = 2\angle ABC$ [By degree measure theorem]

$\Rightarrow \quad x = 2 \times 115^\circ$

$\Rightarrow \quad x = 230^\circ$

Q7

If O is the centre of the circle. Find the value of x in the following figure:



Solution

We have

$\angle OAB = 35^\circ$

Then, $\angle OBA = \angle OAB = 35^\circ$ [Angles Opposite to equal radii]

In $\triangle AOB$, by angle sum property

$\angle AOB + \angle OAB + \angle OBA = 180^\circ$

$\Rightarrow \quad \angle AOB + 35^\circ + 35^\circ = 180^\circ$

$\Rightarrow \quad \angle AOB = 180^\circ - 35^\circ - 35^\circ = 110^\circ$

$\therefore \quad \angle AOB + \text{reflex } \angle AOB = 360^\circ$ [Complete angle]

$\Rightarrow \quad 110^\circ + \text{reflex } \angle AOB = 360^\circ$

$\Rightarrow \quad \text{Reflex } \angle AOB = 360^\circ - 110^\circ = 250^\circ$

By degree measure theorem

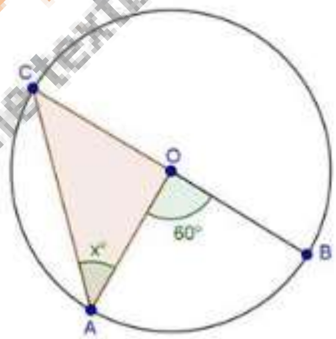
$\text{Reflex } \angle AOB = 2\angle ACB$

$\Rightarrow \quad 250^\circ = 2x$

$\Rightarrow \quad x = \frac{250^\circ}{2} = 125^\circ$

Q8

If O is the centre of the circle. Find the value of x in the following figure:



Solution

We have

$\angle AOB = 60°$

By degree measure theorem

$\angle AOB = 2\angle ACB$

$\Rightarrow \quad 60° = 2\angle ACB$

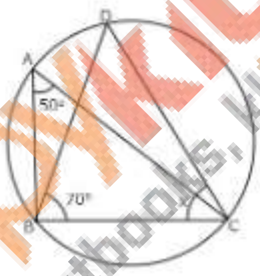
$\Rightarrow \quad \angle ACB = \frac{60°}{2} = 30°$

$\therefore \quad \angle OAC = \angle OCA$ [Angles Opposite to equal radii]

$\Rightarrow \quad x = 30°$

Q9

If O is the centre of the circle. Find the value of x in the following figure:



Solution

We have

$\angle BAC = 50°$
and $\angle DBC = 70°$

$\therefore \quad \angle BDC = \angle BAC = 50°$ [Angle in same segment]

In $\triangle BDC$, by angle sum property

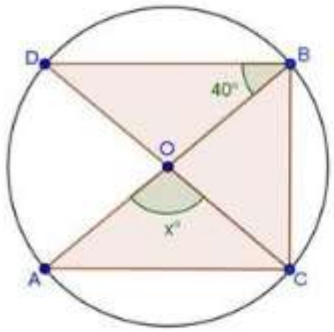
$\angle BDC + \angle BCD + \angle DBC = 180°$

$\Rightarrow \quad 50° + x + 70° = 180°$

$\Rightarrow \quad x = 180° - 50° - 70° = 60°$

Q10

If O is the centre of the circle. Find the value of x in the following figure:



Solution

We have

$\angle DBO = 40^\circ$

$\angle DBC = 90^\circ$ [Angle in semicircle]

$\Rightarrow \quad \angle DBO + \angle OBC = 90^\circ$

$\Rightarrow \quad 40^\circ + \angle OBC = 90^\circ$

$\Rightarrow \quad \angle OBC = 90^\circ - 40^\circ = 50^\circ$

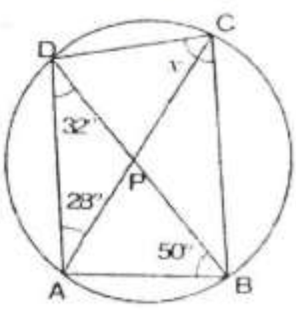
By degree measure theorem

$\angle AOC = 2\angle OBC$

$\Rightarrow \quad x = 2 \times 50^\circ = 100^\circ$

Q11

If O is the centre of the circle. Find the value of x in the following figure:



Solution

In $\triangle DAB$, by angle sum property

$\angle ADB + \angle DAB + \angle ABD = 180^\circ$

$\Rightarrow \quad 32^\circ + \angle DAB + 50^\circ = 180^\circ$

$\Rightarrow \quad \angle DAB = 180^\circ - 32^\circ - 50^\circ$

$\Rightarrow \quad \angle DAB = 98^\circ$

Now,

$\angle DAB + \angle DCB = 180^\circ$ [Opposite angles of cyclic quadrilateral]

$\Rightarrow \quad 98^\circ + x = 180^\circ$

$\Rightarrow \quad x = 180^\circ - 98^\circ = 82^\circ$

Q12

If O is the centre of the circle. Find the value of x in the following figure:

Solution

We have

$\angle BAC = 35^\circ$

$\therefore \quad \angle BDC = \angle BAC = 35^\circ$ [Angle in same segment]

In $\triangle BCD$, by angle sum property

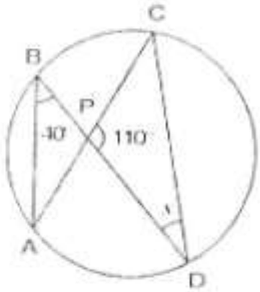
$\angle BDC + \angle BCD + \angle DBC = 180^\circ$

$\Rightarrow \quad 35^\circ + x + 65^\circ = 180^\circ$

$\Rightarrow \quad x = 180^\circ - 35^\circ - 65^\circ = 80^\circ$

Q13

If O is the centre of the circle. Find the value of x in the following figure:



Solution

We have

$\angle ABD = 40°$

$\therefore \quad \angle ACD = \angle ABD = 40°$ [Angle in same segment]

In $\triangle PCD$, by angle sum property

$\angle PCD + \angle CPD + \angle PDC = 180°$

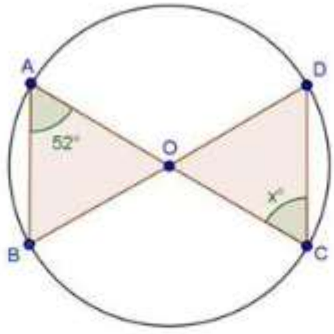
$\Rightarrow \quad 40° + 110° + x° = 180°$

$\Rightarrow \quad x = 180° - 40° - 110°$

$\Rightarrow \quad x = 30°$

Q14

If O is the centre of the circle. Find the value of x in the following figure:



Solution

We have

$\angle BAC = 52^\circ$

Then, $\angle BDC = \angle BAC = 52^\circ$ [Angle in same segment]

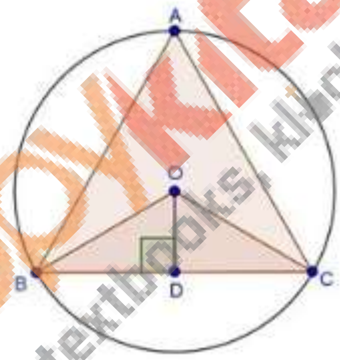
Since, $OD = OC$ [Radii of circle]
Then, $\angle ODC = \angle OCD$ [Opposite angles to equal radii]

$\Rightarrow \quad 52^\circ = x$

Q15

O is the circumcentre of the triangle ABC and OD is perpendicular on BC. Prove That $\angle BOD = \angle A$.

Solution



Given, O is the circumcentre of $\triangle ABC$ and $OD \perp BC$
To prove $\angle BOD = 2\angle A$

Proof
In $\triangle OBD$ and $\triangle OCD$
$\angle ODB = \angle ODC$ [Each 90°]
$OB = OC$ [Radii of circle]
$OD = OD$ [Common]

Then, $\triangle OBD \cong \triangle OCD$ [By RHS condition]
$\therefore \quad \angle BOD = \angle COD$ ---(1) [c.p.c.t]

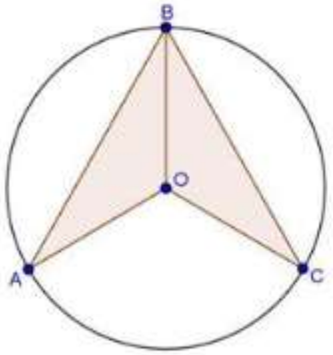
By degree measure theorem

$\angle BOC = 2\angle BAC$

$\Rightarrow \quad 2\angle BOD = 2\angle BAC$ [By using (1)]
$\Rightarrow \quad \angle BOD = \angle BAC$

Q16

In fig., O is the centre of the circle, BO is the bisector of ∠ABC. Show that AB = AC.



Solution

Given, BO is the bisector of $\angle ABC$
To prove $AB = BC$

Proof

Since, BO is the bisector of $\angle ABC$
Then, $\angle ABO = \angle CBO$ ---(1)

Since, $OB = OA$ [Radii of circle]
Then, $\angle ABO = \angle OAB$ ---(2) [Opposite angles to equal sides]

Since, $OB = OC$ [Radii of circle]
Then, $\angle CBO = \angle OCB$ ---(3) [Opposite angles to equal sides]

Compare equations (1)(2) & (3)

$\angle OAB = \angle OCB$ ---(4)

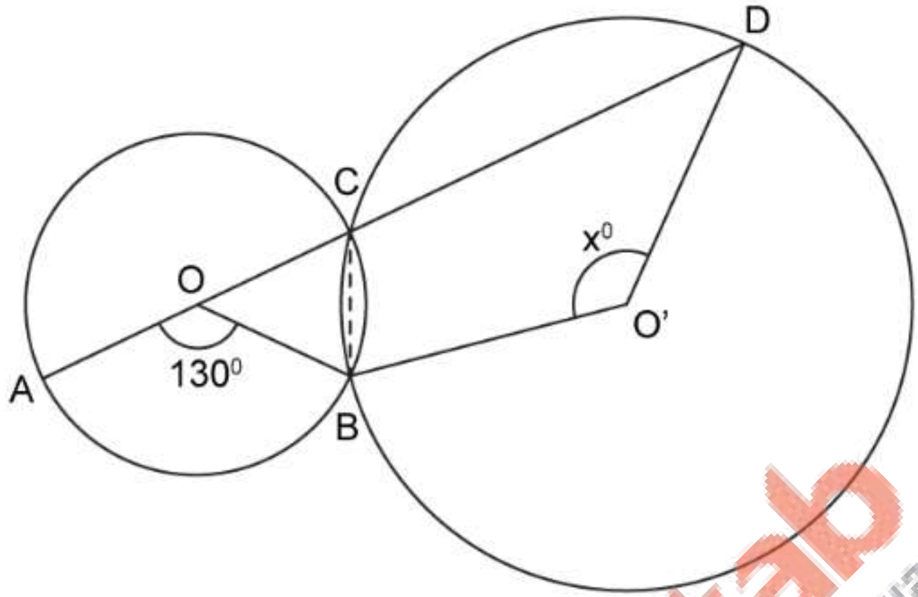
In $\triangle OAB$ and $\triangle OCB$

$\angle OAB = \angle OCB$ [From (4)]
$\angle OBA = \angle OBC$ [Given]
$OB = OB$ [Common]
Then, $\triangle OAB \cong \triangle OCB$ [by AAS condition]

$\therefore$ $AB = BC$ [c.p.c.t]

Q17

In fig., O and O' are centres of two circles intersecting at B and C. ABD is straight line, find x.



Solution

By degree measure theorem

$\angle AOB = 2\angle ACB$

$\Rightarrow \quad 130^\circ = 2\angle ACB$

$\Rightarrow \quad \angle ACB = \frac{130^\circ}{2} = 65^\circ$

$\therefore \quad \angle ACB + \angle BCD = 180^\circ$ [Linear pair of angles]

$\Rightarrow \quad 65^\circ + \angle BCD = 180^\circ$

$\Rightarrow \quad \angle BCD = 180^\circ - 65^\circ = 115^\circ$

By degree measure theorem

Reflex $\angle BO'D = 2\angle BCD$

$\Rightarrow$ Reflex $\angle BO'D = 2 \times 115^\circ = 230^\circ$

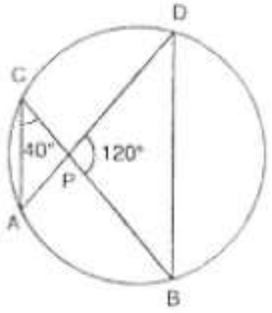
Now, reflex $\angle BO'D + \angle BO'D = 360^\circ$ [Complete angle]

$\Rightarrow \quad 230^\circ + x = 360^\circ$

$\Rightarrow \quad x = 360^\circ - 230^\circ = 130^\circ$

Q18

In fig., if ∠ACB = 40°, ∠DPB = 120°, find ∠CBD.



Solution

We have,

$\angle ACB = 40^\circ$
$\angle DPB = 120^\circ$

$\therefore \quad \angle ADB = \angle ACB = 40^\circ$ [Angle in same segment]

In $\triangle PDB$, by angle sum property

$\angle PDB + \angle PBD + \angle BPD = 180^\circ$
$\Rightarrow \quad 40^\circ + \angle PBD + 120^\circ = 180^\circ$
$\Rightarrow \quad \angle PBD = 180^\circ - 40^\circ - 120^\circ$

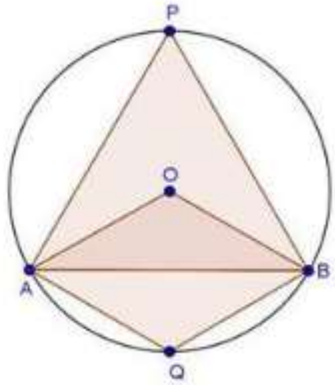
$\Rightarrow \quad \angle PBD = 20^\circ$

$\therefore \quad \angle CBD = 20^\circ$

Q19

A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Solution



We have,

Radius OA = Chord AB

$\Rightarrow \quad OA = OB = AB$

Then, $\triangle OAB$ is an equilateral triangle.

$\therefore \quad \angle AOB = 60^\circ$ [One angle of equilateral $\triangle$]

By degree measure theorem

$\angle AOB = 2\angle APB$

$\Rightarrow \quad 60^\circ = 2\angle APB$

$\Rightarrow \quad \angle APB = \frac{60^\circ}{2} = 30^\circ$

Now, $\angle APB + \angle AQB = 180^\circ$ [Opposite angles of cyclic quad.]

$\Rightarrow \quad 30^\circ + \angle AQB = 180^\circ$

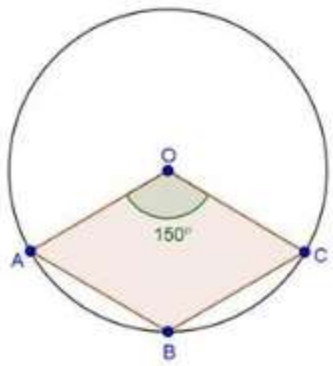
$\Rightarrow \quad \angle AQB = 180^\circ - 30^\circ = 150^\circ$

$\therefore$ Angle by chord AB at minor arc = 150°

Angle by chord AB at major arc = 30°

Q20

In fig., it is given given that O is the centre of the circle and ∠AOC = 150°. Find ∠ABC.



Solution

We have $\angle AOC = 150°$

$\therefore\ \angle AOC + \text{reflex } \angle AOC = 360°$ [Complete angle]

$\Rightarrow 150° + \text{reflex } \angle AOC = 360°$

$\Rightarrow \text{reflex } \angle AOC = 360° - 150°$

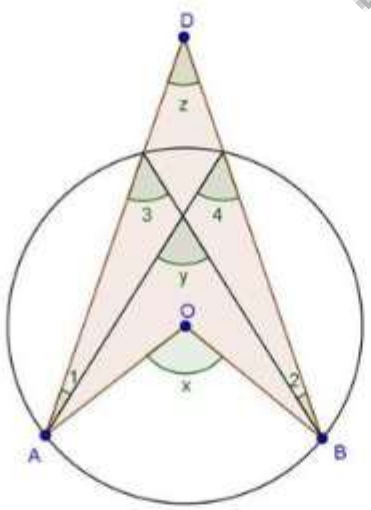
$\Rightarrow \text{reflex } \angle AOC = 210°$

$\Rightarrow 2\angle ABC = 210°$ [By degree measure theorem]

$\Rightarrow \angle ABC = \frac{210}{2} = 105°$

Q21

In fig., O is the centre of the circle, prove that ∠x = ∠y + ∠z.



Solution

We have, $\angle 3 = \angle 4$ [Angles in same segment]

$\therefore \quad \angle x = 2\angle 3$ [By degree measure theorem]

$\Rightarrow \quad \angle X = \angle 3 + \angle 3$

$\Rightarrow \quad \angle x = \angle 3 + \angle 4 \quad ---(1) \quad [\angle 3 = \angle 4]$

But $\angle y = \angle 3 + \angle 1$ [By exterior angle prop.]

$\Rightarrow \quad \angle 3 = \angle y - \angle 1 \quad ---(2)$

From (1) and (2)

$\angle x = \angle y - \angle 1 + \angle 4$

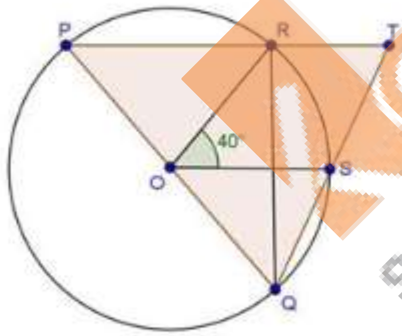
$\Rightarrow \quad \angle x = \angle y + \angle 4 - \angle 1$

$\Rightarrow \quad \angle x = \angle y + \angle z + \angle 1 - \angle 1$ [By exterior angle prop.]

$\Rightarrow \quad \angle x = \angle y + \angle z$

Q22

in fig., O is the centre of a circle and PQ is a diameter. If ∠ROS = 40°, find ∠RTS.



Solution

Since, PQ is a diameter

Then, $\angle PRQ = 90^\circ$ [Angle in semicircle]

$\therefore \quad \angle PRQ + \angle TRQ = 180^\circ$ [Linear pair of angle]

$\Rightarrow \quad \angle 90^\circ + \angle TRQ = 180^\circ$

$\Rightarrow \quad \angle TRQ = 180^\circ - 90^\circ = 90^\circ$

By degree measure theorem

$\angle ROS = 2\angle RQS$

$\Rightarrow \quad 40^\circ = 2\angle RQS$

$\Rightarrow \quad \angle RQS = \frac{40^\circ}{2} = 20^\circ$

In ΔRQT, by angle sum property

$\angle RQT + \angle QRT + \angle RTS = 180^\circ$

$\Rightarrow \quad 20^\circ + 90^\circ + \angle RTS = 180^\circ$

$\Rightarrow \quad \angle RTS = 180^\circ - 20^\circ - 90^\circ = 70^\circ$