Exercise 4.1

1.

Sol:

(i) All circles are similar

(ii) All squares are similar

(iii) All equilateral triangles are similar

(iv) Two triangles are similar, if their corresponding angles are equal

(v) Two triangles are similar, if their corresponding sides are proportional

(vi) Two polygons of the same number of sides are similar, if (a) their corresponding angles are equal and (b) their corresponding sides are proportional.

2.

Sol:

(i) False

(ii) True

(iii) False

(iv) False

(v) True

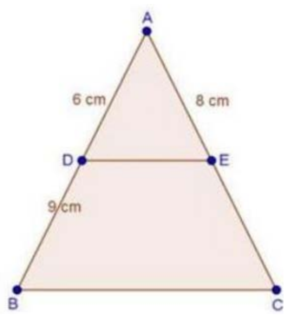
(vi) True

Exercise 4.2

1.

Sol:

(i)



We have,

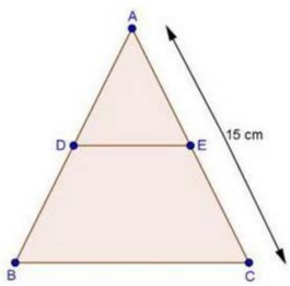
DE || BC

Therefore, by basic proportionally theorem,

We have $\frac{AD}{DB} = \frac{AE}{EC}$

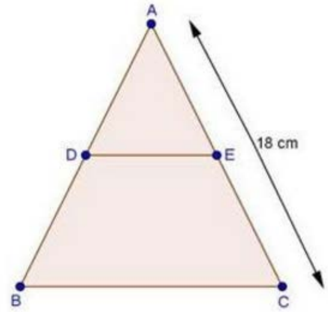
$\Rightarrow \frac{6}{9} = \frac{8}{EC}$
$\Rightarrow \frac{2}{3} = \frac{8}{EC}$
$\Rightarrow$ EC = $\frac{8\times3}{2}$
$\Rightarrow$ EC = 12 cm
$\Rightarrow$ Now, AC = AE + EC = 8 + 12 = 20 cm
∴ AC = 20 cm

(ii)



We have,
$\frac{AD}{DB} = \frac{3}{4}$ *and DE* || *BC*
Therefore, by basic proportionality theorem, we have
$\frac{AD}{DB} = \frac{AE}{EC}$
Adding 1 on both sides, we get
$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$
$\frac{3}{4} + 1 = \frac{AE+EC}{EC}$
$\Rightarrow \frac{3+4}{4} = \frac{AC}{EC}$ [∵ AE + EC = AC]
$\Rightarrow \frac{7}{4} = \frac{15}{EC}$
$\Rightarrow$ EC = $\frac{15\times4}{7}$
$\Rightarrow$ EC = $\frac{60}{7}$
Now, AE + EC = AC
$\Rightarrow$ AE + $\frac{60}{7}$ = 15
$\Rightarrow$ AE = 15 − $\frac{60}{7}$
= $\frac{105-60}{7}$
= $\frac{45}{7}$
= 6.43 cm
∴ AE = 6.43 cm

(iii)



We have,

$\frac{AD}{DB} = \frac{2}{3}$ $and\ DE \,||\, BC$

$Therefore, by\ basic\ proportionality\ theorem, we\ have,$

$\frac{AD}{DB} = \frac{EC}{AE}$

$\Rightarrow \frac{3}{2} = \frac{EC}{AE}$

Adding 1 on both sides, we get

$\frac{3}{2} + 1 = \frac{EC}{AE} + 1$

$\Rightarrow \frac{3+2}{2} = \frac{EC+AE}{AE}$

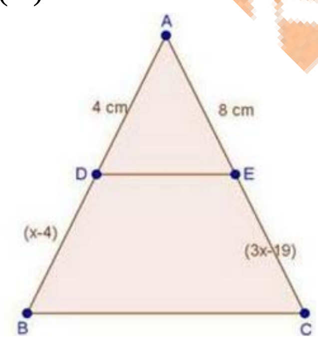
$\Rightarrow \frac{5}{2} = \frac{AC}{AE}$ [∵ AE + EC = AC]

$\Rightarrow \frac{5}{2} = \frac{18}{AE}$ [∵ AC = 18]

$\Rightarrow AE = \frac{18 \times 2}{5}$

$\Rightarrow AE = \frac{36}{5} = 7.2\ cm$

(iv)



We have,

DE || BC

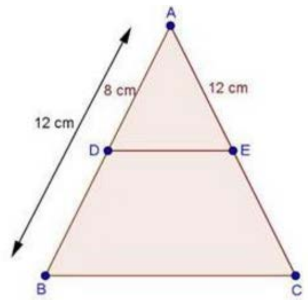
Therefore, by basic proportionality theorem, we have,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\frac{4}{x-4} = \frac{8}{3x-19}$

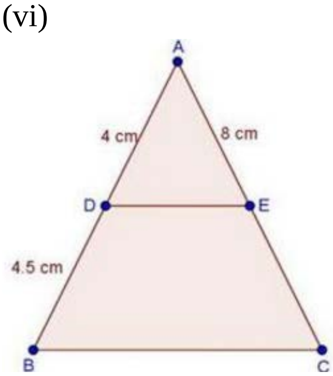
⇒ 4(3x – 19) = 8(x – 4)
⇒ 12x – 76 = 8x – 32
⇒ 12x – 8x = −32 + 76
⇒ 4x = 44
⇒ $x = \frac{44}{4} = 11cm$
∴ x = 11 cm

(v)



We have,
AD = 8cm, AB = 12 cm
∴ BD = AB – AD
= 12 – 8
⇒ BD = 4 cm
And, DE || BC
Therefore, by basic proportionality theorem, we have,
$\frac{AD}{BD} = \frac{AE}{CE}$
⇒ $\frac{8}{4} = \frac{12}{CE}$
⇒ CE = $\frac{12 \times 4}{8} = \frac{12}{2}$
⇒ CE = 6cm
∴ CE = 6cm

(vi)



We have,

DE || BC

Therefore, by basic proportionality theorem, we have,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{4}{4.5} = \frac{8}{EC}$

$\Rightarrow EC = \frac{8 \times 4.5}{4}$

⇒ EC = 9cm

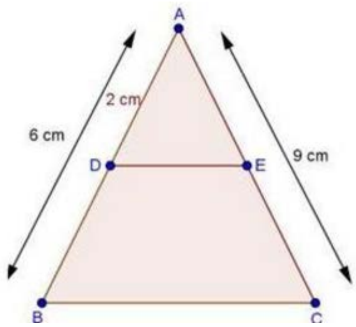
Now, AC = AE + EC

= 8 + 9

= 17 cm

∴ AC = 17 cm

(vii)



We have,

AD = 2 cm, AB = 6 cm

∴ DB = AB – AD

= 6 – 2

⇒ DB = 4 cm

And, DE || BC

Therefore, by basic proportionality theorem, we have,

$\frac{AD}{DB} = \frac{AE}{EC}$

Taking reciprocal on both sides, we get,

$\frac{DB}{AD} = \frac{EC}{AE}$

$\frac{4}{2} = \frac{EC}{AE}$

Adding 1 on both sides, we get

$\frac{4}{2} + 1 = \frac{EC}{AE} + 1$

$\Rightarrow \frac{4+2}{2} = \frac{EC+AE}{AE}$

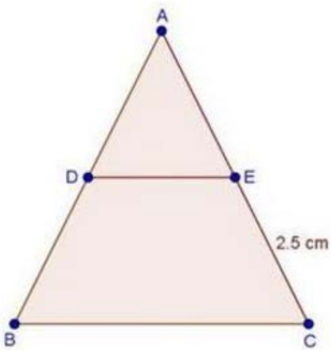
$\Rightarrow \frac{6}{2} = \frac{AC}{AE}$ [∵ EC + AE = AC]

$\Rightarrow \frac{6}{2} = \frac{9}{AE}$ [∵ AC = 9cm]

$AE = \frac{9 \times 2}{6}$

$\Rightarrow AE = 3cm$

(viii)



We have, DE || BC

Therefore, by basic proportionality theorem,

We have,

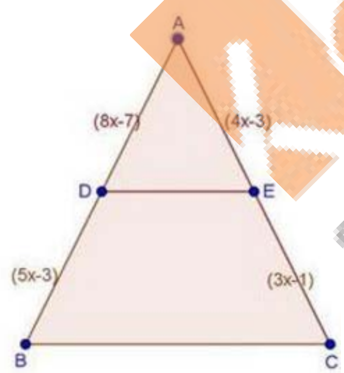
$\frac{AD}{BD} = \frac{AE}{EC}$

$\Rightarrow \frac{4}{5} = \frac{AE}{2.5}$

$\Rightarrow$ AE = $\frac{4 \times 2.5}{5}$

$\Rightarrow$ AE = 2cm

(ix)



We have,

DE || BC

Therefore, by basic proportionality theorem,

We have,

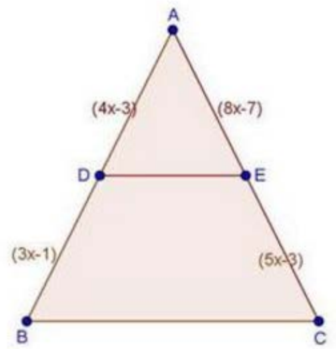
$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

$\Rightarrow x(x-1) = (x+2)(x-2)$

$\Rightarrow x^2 - x = x^2 - (2)^2$ [∵ (a – b) (a + b) = $a^2 - b^2$]
$\Rightarrow -x = -4$
⇒ x = 4 cm
∴ x = 4 cm

(x)



We have,
DE || BC
Therefore, by basic proportionality theorem, we have,
$\frac{AD}{DB} = \frac{AE}{EC}$
$\Rightarrow \frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$
$\Rightarrow (8x-7)(3x-1) = (4x-3)(5x-3)$
$\Rightarrow 24x^2 - 8x - 21x + 7 = 20x^2 - 12x - 15x + 9$
$\Rightarrow 24x^2 - 20x^2 - 29x + 27x + 7 - 9 = 0$
$\Rightarrow 4x^2 - 2x - 2 = 0$
$\Rightarrow 2[2x^2 - x - 1] = 0$
$\Rightarrow 2x^2 - x - 1 = 0$
$\Rightarrow 2x^2 - 2x + 1x - 1 = 0$
$\Rightarrow 2x(x-1) + 1(x-1) = 0$
⇒ (2x + 1) (x – 1) = 0
⇒ 2x + 1 = 0 or x – 1 = 0
⇒ x = $-\frac{1}{2}$ $or\ x = 1$
$x = -\frac{1}{2}$ $is\ not\ possible$
∴ x = 1

(xi)
We have, DE || BC
Therefore, by basic proportionality theorem,
We have,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$

$\Rightarrow (4x-3)(5x-3) = (8x-7)(3x-1)$

$\Rightarrow 4x(5x-3) - 3(5x-3) = 8x(3x-1) - 7(3x-1)$

$\Rightarrow 20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$

$\Rightarrow 4x^2 - 2x - 2 = 0$

$\Rightarrow 2(2x^2 - x - 1) = 0$

$\Rightarrow 2x^2 - x - 1 = 0$

$\Rightarrow 2x^2 - 2x + 1x - 1 = 0$

$\Rightarrow 2x(x-1) + 1(x-1) = 0$

$\Rightarrow (2x + 1)(x - 1) = 0$

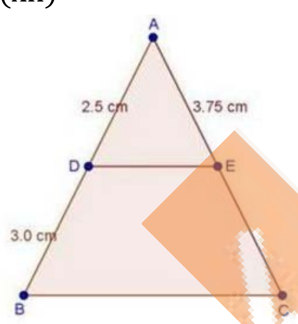
$\Rightarrow 2x + 1 = 0$ or $x - 1 = 0$

$\Rightarrow x = -\frac{1}{2}$ or $x = 1$

$x = -\frac{1}{2}$ is not possible

$\therefore x = 1$

(xii)



We have, DE || BC

Therefore, by basic proportionality theorem, we have,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{2.5}{3.0} = \frac{3.75}{EC}$

$\Rightarrow EC = \frac{3.75 \times 3}{2.5} = \frac{375 \times 3}{250}$

$\Rightarrow EC = \frac{15 \times 3}{10}$

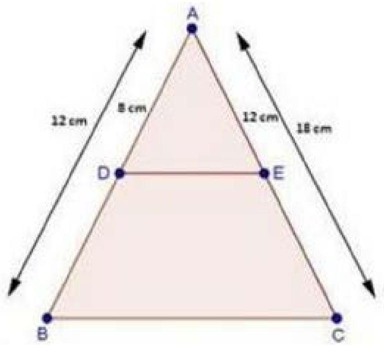
$= \frac{45}{10} = 4.5\ cm$

Now, AC = AE + EC = 3.75 + 4.5 = 8.25

$\therefore$ AC = 8.25 cm

2.

Sol:



AB = 12 cm, AD = 12 cm and AC = 18 cm.

∴ DB = AB – AD

= 12 – 8

⇒ DB = 4 cm

And, EC = AC – AE

= 18 – 12

⇒ EC = 6 cm

Now, $\frac{AD}{DB} = \frac{8}{4} = \frac{2}{1}$ [∵ DB = 4 cm]

And, $\frac{AE}{EC} = \frac{12}{6} = \frac{2}{1}$ [∵ EC = 6 cm]

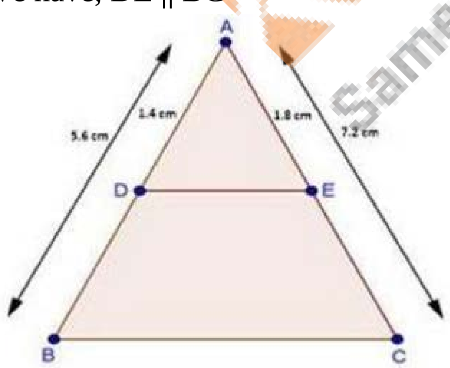
⇒ $\frac{AD}{DB} = \frac{AE}{EC}$

Thus, DE divides sides AB and AC of ΔABC in the same ratio.

Therefore, by the converse of basic proportionality theorem,

(ii)

We have, DE || BC



We have,

AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm

∴ DB = AB – AD

= 5.6 – 1.4

⇒ DB = 4.2 cm

And, EC = AC – AE

= 7.2 – 1.8

⇒ EC = 5.4 cm

Now, $\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3}$ [∵ DB = 4.2 cm]

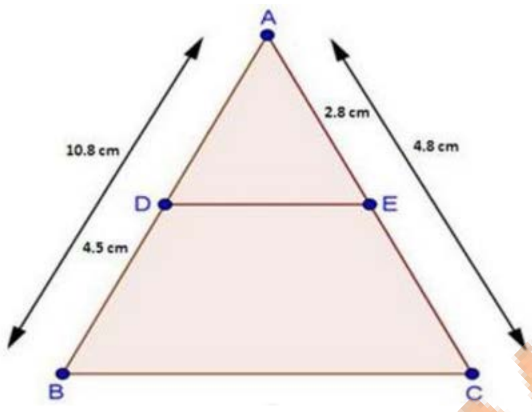
And, $\frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$ [∵ EC = 5.4 cm]

Thus, DE divides sides AB and AC of ΔABC in the same ratio.

Therefore, by the converse of basic proportionality theorem,

(iii)

We have,



We have,

AB = 10.8cm, BD = 4.5cm, AC = 4.8 cm and AE = 2.8cm

∴ AD = AB – DB = 10.8 – 4.5

⇒ AD = 6.3 cm

And, EC = AC – AE

= 4.8 – 2.8

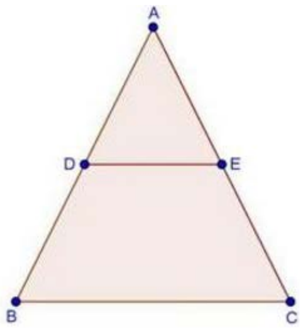
⇒ EC = 2 cm

Now, $\frac{AD}{DB} = \frac{6.3}{4.5} = \frac{7}{5}$ [∵ AD = 6.3 cm]

And, $\frac{AE}{EC} = \frac{2.8}{2} = \frac{28}{20} = \frac{75}{5}$ [∵ EC = 2 cm]

Thus, DE divides sides AB and AC of ΔABC in the same ratio. Therefore, by the converse of basic proportionality theorem.

(iv)



We have,

DE || BC

We have, AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm and EC = 5.5 cm

Now $\frac{AD}{BD} = \frac{5.7}{9.5} = \frac{57}{95}$

$\Rightarrow \frac{AD}{BD} = \frac{3}{5}$

And, $\frac{AE}{EC} = \frac{3.3}{5.5} = \frac{33}{55}$

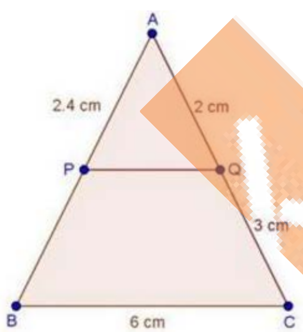
$\Rightarrow \frac{AE}{EC} = \frac{3}{5}$

Thus DE divides sides AB and AC of ΔABC in the same ratio.

Therefore, by the converse of basic proportionality theorem. We have DE || BC

3.

Sol:



We have || BC

Therefore, by BPT

We have,

$\frac{AP}{PB} = \frac{AQ}{QC}$

$\frac{2.4}{PB} = \frac{2}{3}$

$\Rightarrow$ PB = $\frac{3\times2.4}{2} = \frac{3\times24}{2} = \frac{3\times6}{5} = \frac{18}{5}$

$\Rightarrow$ PB = 3.6 cm

Now, AB = AP + PB

= 2.4 + 3.6 = 6cm

Now, In ΔAPQ and ΔABC

∠A = ∠A [common]

∠APQ = ∠ABC [∵ PQ ∥ BC ⇒ Corresponding angles are equal]

⇒ ΔAPQ ~ ΔABC [By AA criteria]

⇒ $\frac{AB}{AP} = \frac{BC}{PQ}$ [corresponding sides of similar triangles are proportional]

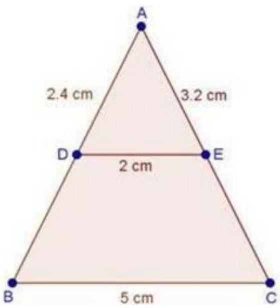
⇒ PQ = $\frac{6\times2.4}{6}$

⇒ PQ = 2.4 cm

Hence, AB = 6 cm and PO = 2.4 cm

4.

Sol:



We have,

DE ∥ BC

Now, In ΔADE and ΔABC

∠A = ∠A [common]

∠ADE = ∠ABC [∵ DE ∥ BC ⇒ Corresponding angles are equal]

⇒ ΔADE ~ ΔABC [By AA criteria]

⇒ $\frac{AB}{BC} = \frac{AD}{DE}$ [corresponding sides of similar triangles are proportional]

⇒ AB = $\frac{2.4\times5}{2}$

⇒ AB = 1.2 × 5 = 6.0 cm

⇒ AB = 6 cm

∴ BD = 6 cm

BD = AB – AD

= 6 – 2.4 = 3.6 cm

⇒ DB = 3.6 cm

Now,

$\frac{AC}{BC} = \frac{AE}{DE}$ [∵ Corresponding sides of similar triangles are equal]

⇒ $\frac{AC}{5} = \frac{3.2}{2}$

⇒ $AC = \frac{3.2\times5}{2} = 1.6 \times 5 = 8.0\ cm$

⇒ AC = 8 cm

∴ CE = AC – AE

= 8 – 3.2 = 4.8 cm

Hence, BD = 3.6 cm and CE = 4.8 cm

5.

Sol:

We have,

DP = 3.9 cm, PE = 3cm, DQ = 3.6 cm and QF = 2.4 cm

Now, $\frac{DP}{PE} = \frac{3.9}{3} = \frac{1.3}{1} = \frac{13}{10}$

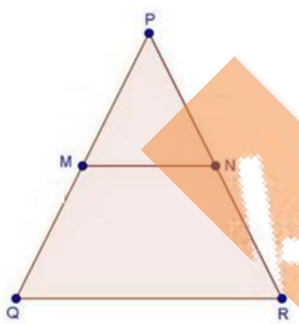
And, $\frac{DQ}{QF} = \frac{3.6}{2.4} = \frac{36}{24} = \frac{3}{2}$

$\Rightarrow \frac{DP}{PE} \neq \frac{DQ}{QF}$

So, PQ is not parallel to EF

6.

Sol:



(i) We have, PM = 4cm, QM = 4.5 cm, PN = 4 cm and NR = 4.5 cm

Hence, $\frac{PM}{QM} = \frac{4}{4.5} = \frac{8}{9}$

Also, $\frac{PN}{NR} = \frac{4}{4.5} = \frac{8}{9}$

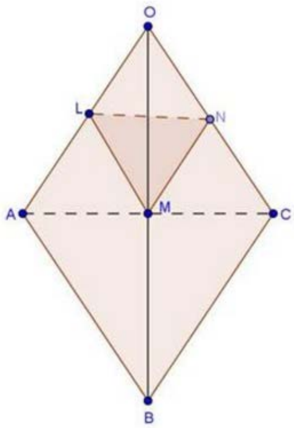
Hence, $\frac{PM}{QM} = \frac{PN}{NR}$

By converse of proportionality theorem

MN || QR

7.

Sol:



We have,

LM || AB and MN || BC

Therefore, by basic proportionality theorem,

We have,

$\frac{QL}{AL} = \frac{OM}{MB}$ ...(i)

$and, \frac{ON}{NC} = \frac{OM}{MB}$ ...(ii)

$Comparing\ equation\ (i) and\ equation\ (ii), we\ get,$

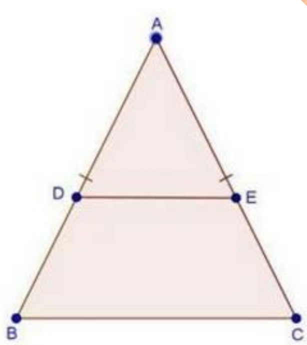
$\frac{ON}{AL} = \frac{ON}{NC}$

Thus, LN divides sides OA and OC of ΔOAC in the same ratio. Therefore, by the converse of basic proportionality theorem,

we have, LN || AC

8.

Sol:



We have, DE || BC

Therefore, by BPT, we have,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{AD}{DB} = \frac{AE}{DB}$ [∵BD = CE]

⇒ AD = AE

Adding DB on both sides

⇒ AD + DB = AE + DB

⇒ AD + DB = AE + EC [∴ BD = CE]

⇒ AB = AC

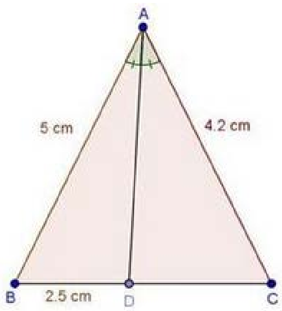
⇒ Δ ABC is isosceles

Exercise 4.3

1.

Sol:

(i)



We have,

∠BAD = ∠CAD

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

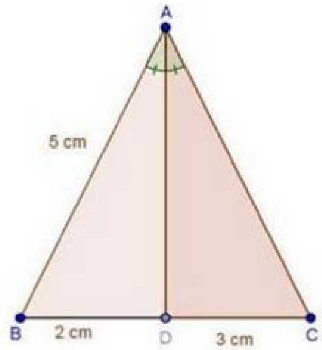
$\Rightarrow \frac{2.5}{DC} = \frac{5}{4.2}$

$\Rightarrow DC = \frac{2.5 \times 4.2}{5}$

$= \frac{25 \times 42}{5 \times 100} = \frac{5 \times 42}{100} = \frac{210}{100} = 2.1\ cm$

∴ DC = 2.1 cm

(ii)



We have,

AD is the bisector of ∠A

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

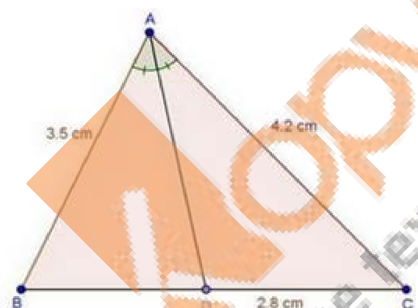
$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{2}{3} = \frac{5}{AC}$

$\Rightarrow$ AC $= \frac{5\times3}{2} = \frac{15}{2}$

$\Rightarrow$ AC = 7.5 cm

(iii)



In ΔABC, AD is the bisector of ∠A.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

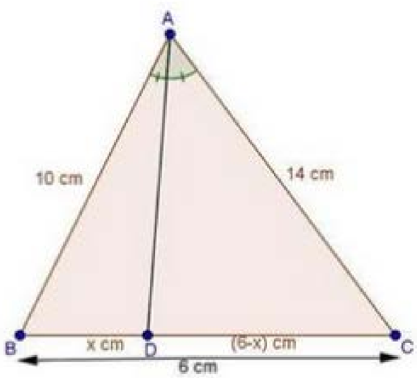
$\Rightarrow \frac{BD}{2.8} = \frac{3.5}{4.2}$

$= \frac{3.5\times2}{3}$

$= \frac{7}{3} = 2.33\ cm$

$\therefore BD = 2.3\ cm$

(iv)



In ΔABC, AD is the bisector of ∠A

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{x}{6-x} = \frac{10}{14}$

⇒ 14x = 10(6 – x)

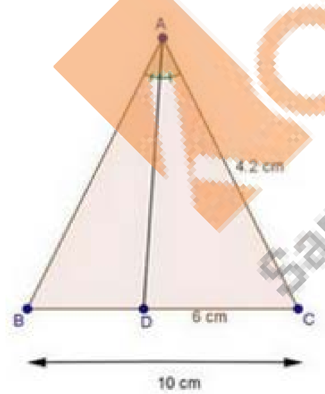
⇒ 24x = 60

$\Rightarrow x = \frac{60}{24} = \frac{5}{2} = 2.5cm$

Since, DC = 6 – x = 6 – 2.5 = 3.5 cm

Hence, BD = 2.5cm, and DC = 3.5 cm

(v)



We have,

BC = 10 cm, DC = 6 cm and AC = 4.2 cm

∴ BD = BC – DC = 10 – 6 = 4 cm

⇒ BD = 4 cm

In ΔABC, AD is the bisector of ∠A.

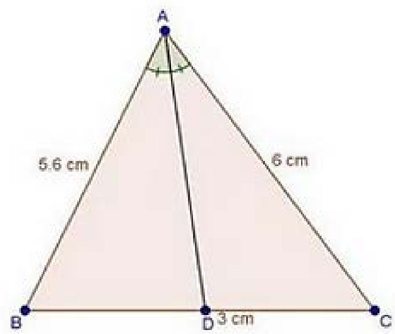
We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{4}{6} = \frac{AB}{4.2}$ [∵ BD = 4 cm]

$\Rightarrow$ AB = 2.8 cm

(vi)



We have, In ΔABC, AD is the bisector of ∠A.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{BD}{3} = \frac{5.6}{6}$

$\Rightarrow$ BD = $\frac{5.6\times3}{6} = \frac{5.6}{2} = 2.8cm$

$\Rightarrow$ BD = 2.8 cm

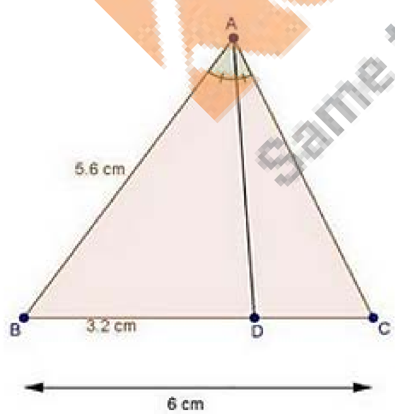
Since, BC = BD + DC

= 2.8 + 3

= 5.8 cm

∴ BC = 5.8 cm

(vii)



We have,

In ΔABC, AD is the bisector of ∠A.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the containing the angle.

$\therefore \frac{AB}{AC} = \frac{BD}{DC}$

$\frac{5.6}{AC} = \frac{3.2}{6-3.2}$ [∵ DC = BC – BD]

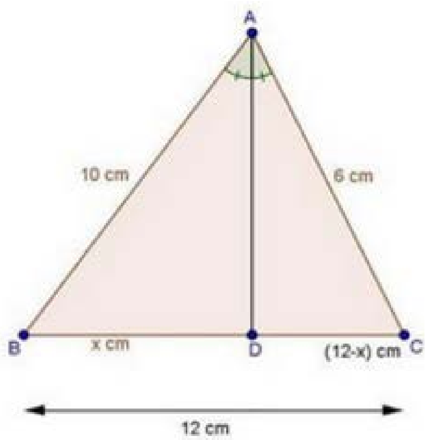
$\Rightarrow \frac{5.6}{AC} = \frac{3.2}{2.8}$

$\Rightarrow AC = \frac{5.6 \times 2.8}{3.2}$

$= \frac{5.6 \times 7}{8} = 0.7 \times 7$

$= 4.9\ cm$

(viii)



In ΔABC, AD is the bisector of ∠A.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{x}{12-z} = \frac{10}{6}$

⇒ 6x = 10(12 – x)

⇒ 6x = 120

$\Rightarrow x = \frac{120}{16} = 7.5\ cm$

∴ BD = 7.5 cm and DC = 12 – x = 12 – 7.5 = 4.5 cm

Hence, BD = 7.5 cm and DC = 4.5 cm

2.

Sol:

In ΔABC, AD is the bisector of ∠A.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC} \Rightarrow \frac{x}{12-x} = \frac{10}{6}$

⇒ 6(12 + x) = 10x

⇒ 72 + 6x = 10x

⇒ 4x – 72

⇒ x = $\frac{72}{4} = 18\ cm$

∴ CE = 18 cm

3.

Sol:

We have, if a line through one vertex of a triangle divides the opposite side in the ratio of the other two sides, then the line bisects the angle at the vertex.

∴ ∠1 = ∠2

In ΔABC

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 70° + 50° = 180°$ [$\because \angle B = 70°$ and $\angle C = 50°$]

$\Rightarrow \angle A = 180° - 120° = 60°$

$\Rightarrow \angle 1 + \angle 2 = 60°$

$\Rightarrow \angle 1 + \angle 1 = 60°$ [$\because \angle 1 = \angle 2$]

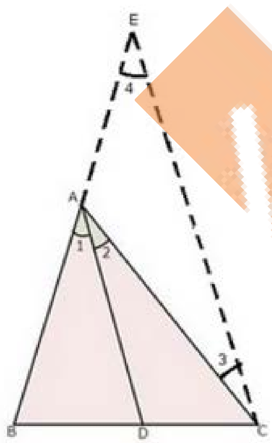
$\Rightarrow 2\angle 1 = 60°$

$\Rightarrow \angle 1 = 30°$

$\therefore \angle BAD = 30°$

4.

Sol:



Given: A ΔABC in which ∠1 = ∠2

To prove: $\frac{AB}{AC} = \frac{BD}{DC}$

Construction: Draw CE || DA to meet BA produced in E.

Proof: since, CE || DA and AC cuts them.

$\therefore \angle 2 - \angle 3$ .... (i) [Alternate angles]

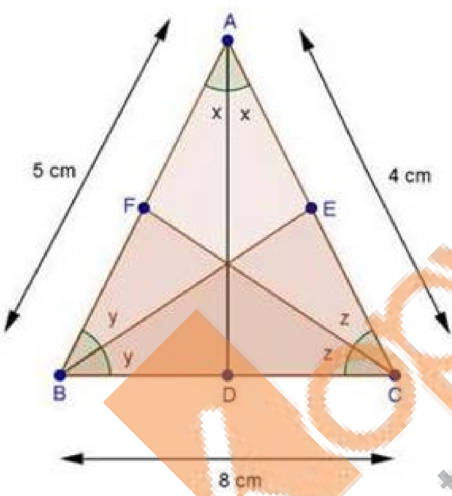
And, $\angle 1 - \angle 4$ ....(ii) [Corresponding angles]

But, $\angle 1 - \angle 2$ [Given]

From (i) and (ii), we get
$\angle 3 - \angle 4$
Thus, in ΔACE, we have
$\angle 3 = \angle 4$
$\Rightarrow$ AE = AC ... (iii) [Sides opposite to equal angles are equal]
Now, In ΔBCE, we have
DA || CE
$\Rightarrow \frac{BD}{\boldsymbol{DC}} = \frac{BA}{\boldsymbol{AE}}$ [Using basic proportionality theorem]
$\Rightarrow \frac{BD}{DC} = \frac{AB}{AC}$ [∵ BA – AB and AE – AC from (iii)]
Hence, $\frac{AB}{AC} = \frac{BD}{DC}$

5.

Sol:



In ΔABC, CF bisects ∠C.
We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.
$\therefore \frac{AF}{FB} = \frac{AC}{BC}$
$\Rightarrow \frac{AF}{5-AF} = \frac{4}{8}$ [∵ FB = AB – AF = 5 – AF]
$\Rightarrow \frac{AF}{5-AF} = \frac{1}{2}$
$\Rightarrow$ 2AF = 5 – AF
$\Rightarrow$ 2AF + AF = 5
$\Rightarrow$ 3AF = 5
$\Rightarrow$ AF = $\frac{5}{3}$ cm
Again, In ΔABC, BE bisects ∠B.
$\therefore \frac{AE}{EC} = \frac{AB}{BC}$

$\Rightarrow \frac{4-CE}{CE} = \frac{5}{8}$ [∵ AE = AC – CE = 4 – CE]

$\Rightarrow 8(4 - CE) = 5 \times CE$

⇒ 32 – 8CE = 5CE

⇒ 32 = 13CE

⇒ CE = $\frac{32}{13}$ cm

Similarly,

$\frac{BD}{DC} = \frac{AD}{AC}$

$\Rightarrow \frac{BD}{8-BD} = \frac{5}{4}$ [∵ DC = BC – BD = 8 – BD]

⇒ 4BD = 40 – 5BD

⇒ 9BD = 40

⇒ BD = $\frac{40}{9}$ cm

Hence, AF = $\frac{5}{3}\ cm$, CE = $\frac{32}{13}$ cm and BD = $\frac{40}{9}$ cm.

6.

Sol:

Now,

$\frac{BD}{CD} = \frac{1.5}{3.5} = \frac{3}{7}$

And, $\frac{AB}{AC} = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow \frac{BD}{CD} \neq \frac{AB}{AC}$

⇒ AD is not the bisector of ∠A.

Now,

$\frac{AB}{AC} = \frac{4}{6} = \frac{2}{3}$

And, $\frac{BD}{CD} = \frac{1.6}{2.4} = \frac{2}{3}$

$\Rightarrow \frac{AB}{AC} = \frac{BD}{CD}$

⇒ AD is the bisector of ∠A.

Now, $\frac{AB}{AC} = \frac{8}{24} = \frac{1}{3}$

And, $\frac{BD}{CD} = \frac{BD}{BC-BD}$ [∵ CD = BC – BD]

$= \frac{BD}{24-6}$

$= \frac{6}{18}$

$= \frac{1}{3}$

$\therefore \frac{AB}{AC} = \frac{BD}{CD}$

$\therefore AD\ is\ the\ bisector\ of\ \angle A\ of\ \Delta ABC.$

$\frac{AB}{AC} = \frac{6}{8} = \frac{3}{4}$

And, $\frac{BD}{CD} = \frac{2.5}{BC-BD}$ [∵ CD = BC – BD]

$= \frac{2.5}{9-2.5}$

$= \frac{2.5}{6.5}$

$= \frac{1}{3}$

$\therefore \frac{AB}{AC} \neq \frac{BD}{CD}$

∴ AD is not the bisector of ∠A of ΔABC.

Exercise 4.4

1.

Sol:

Since diagonals of a trapezium divide each other proportionally.

$\therefore \frac{AO}{OC} = \frac{BO}{OD}$

$\Rightarrow \frac{4}{4x-2} = \frac{x+1}{2x+4}$

$\Rightarrow 4(2x+4) = (x+1)(4x-2)$

⇒ 8x + 16 = x(4x – 2) +1(4x – 2)

⇒ $8x + 16 = 4x^2 + 2x - 2$

$\Rightarrow 4x^2 + 2x - 8x - 2 - 16 = 0$

$\Rightarrow 4x^2 - 6x - 18 = 0$

$\Rightarrow 2[2x^2 - 3x - 9] = 0$

$\Rightarrow 2x^2 - 3x - 9 = 0$

$\Rightarrow 2x(x-3) + 3(x-3) = 0$

$\Rightarrow (x-3)(2x+3) = 0$

$\Rightarrow x - 3 = 0$ or $2x + 3 = 0$

$\Rightarrow x = 3$ or $x = -\frac{3}{2}$

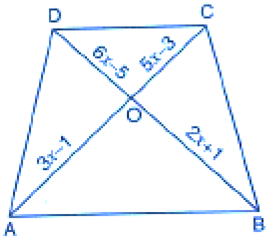
⇒ x = 3 or $x = -\frac{3}{2}$

$x = -\frac{3}{2}$ is not possible, because OB = x + 1 = $-\frac{3}{2} + 1 = -\frac{1}{2}$

Length cannot be negative

$\therefore \frac{AO}{OC} = \frac{BO}{OD}$

(ii) In the below fig., If AB || CD, find the value of x.



$\Rightarrow \frac{3x-1}{5x-3} = \frac{2x+1}{6x-5}$

$\Rightarrow$ (3x – 1) (6x – 5) = (2x + 1) (5x – 3)

$\Rightarrow$ 3x (6x – 5) – 1(6x – 5) = 2x (5x – 3) + 1 (5x – 3)

$\Rightarrow 18x^2 - 15x - 6x + 5 = 10x^2 - 6x + 5x - 3$

$\Rightarrow 8x^2 - 20x + 8 = 0$

$\Rightarrow 4(2x^2 - 5x + 2) = 0$

$\Rightarrow 2x^2 - 4x - 1x + 2 = 0$

$\Rightarrow 2x(x-2) - 1(x-2) = 0$

$\Rightarrow (2x-1)(x-2) = 0$

$\Rightarrow$ 2x – 1 = 0 or x – 2 = 0

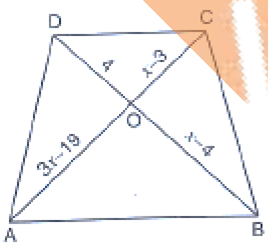
$\Rightarrow x = \frac{1}{2}\ or\ x = 2$

$x = \frac{1}{2}$ is not possible, because, OC = 5x – 3

$= 5\left(\frac{1}{2}\right) - 3$

$= \frac{5-6}{2} = -\frac{1}{2}$

(iii) In below fig., AB || CD. If OA = 3x – 19, OB = x – 4, OC = x – 3 and OD = 4, find x.



Since diagonals of a trapezium divide each other proportionally.

$\therefore \frac{AO}{OC} = \frac{BO}{OD}$

$\Rightarrow \frac{3x-19}{x-3} = \frac{x-4}{4}$

$\Rightarrow 4(3x-19) = (x-4)(x-3)$

$\Rightarrow$ 12x – 76 = x (x – 3) −4(x – 3)

$\Rightarrow 12x - 76 = x^2 - 3x - 4x + 12$

$\Rightarrow x^2 - 7x - 12x + 12 + 76 = 0$

$\Rightarrow x^2 - 19x + 88 = 0$

$\Rightarrow x^2 - 11z - 8z + 88 = 0$

$\Rightarrow x(x-11)-8(x-11)=0$
$\Rightarrow (x-11)(x-8)=0$
$\Rightarrow x-11=0$ or x – 8 = 0
⇒ x = 11 or x = 8

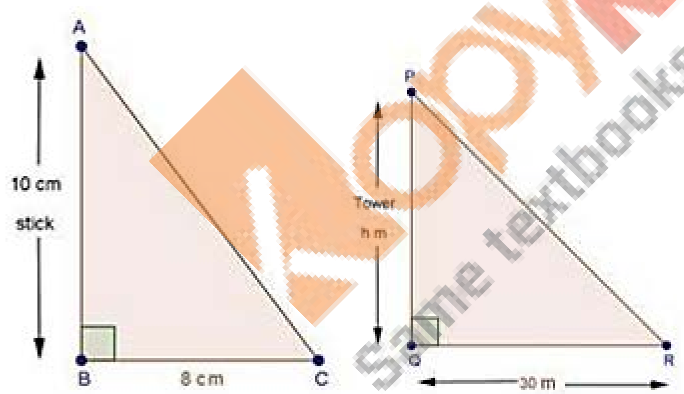
Exercise 4.5

1.

Sol:
Given ΔACB ~ ΔAPQ
Then, $\frac{AC}{AP}=\frac{BC}{PQ}=\frac{AB}{AQ}$ [corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{AC}{2.8}=\frac{8}{4}=\frac{6.5}{AQ}$
$\Rightarrow \frac{AC}{2.8}=\frac{8}{4}\, and \,\frac{8}{4}-\frac{6.5}{AQ}$
⇒ AC = $\frac{8}{4}\times 2.8\ and\ AQ=6.5\times\frac{4}{8}$
⇒ AC = 5.6 cm and AQ = 3.25 cm

2.

Sol:



Length of stick = 10 cm
Length of shadow of stick = 8 cm
Length of shadow of tower = h cm
In ΔABC and ΔPQR
∠B = ∠Q = 90°
And, ∠C = ∠R [Angular elevation of sun]
Then, ΔABC ~ ΔPQR [By AA similarity]
$\therefore \frac{AB}{PQ}=\frac{BC}{QR}$
$\Rightarrow \frac{10cm}{8\ cm}=\frac{h\ cm}{3000}$

$\Rightarrow h = \frac{10}{8} \times 3000 = 3750\ cm = 37.5\ m$

3.

Sol:
We have, ΔPAB and ΔPQR

$\angle P = \angle P$	[common]
$\angle PAB = \angle PQR$	[corresponding angles]
Then, ΔPAB ~ ΔPQR	[By AA similarity]
$\therefore \frac{PB}{PR} = \frac{AB}{QR}$	[Corresponding parts of similar Δare proportional]

$\Rightarrow \frac{PB}{6} = \frac{3}{9}$

$\Rightarrow PB = \frac{3}{9} \times 6 = 2\ cm$

4.

Sol:
We have, XY || BC
In ΔAXY and ΔABC

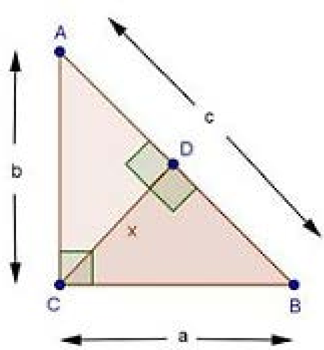
$\angle A = \angle A$	[common]
$\angle AXY = \angle ABC$	[corresponding angles]
Then, ΔAXY ~ ΔABC	[By AA similarity]
$\therefore \frac{AX}{AB} = \frac{XY}{BC}$	[Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{1}{4} = \frac{XY}{6}$

$\Rightarrow XY = \frac{6}{4} = 1.5cm$

5.

Sol:
We have: $\angle C = 90°$ and $CD \perp AB$



In ΔACB and ΔCDB

∠B = ∠B [common]
∠ACB = ∠CDB [Each 90°]
Then, ΔACB ~ ΔCDB [By AA similarity]
$\therefore \frac{AC}{CD} = \frac{AB}{CB}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{b}{x} = \frac{c}{a}$
⇒ ab = cx

6.

Sol:
We have, ∠ABC = 90° and BD ⊥ AC
Now, ∠ABD + ∠DBC − 90° ...(i) [∵ ∠ABC − 90°]
And, ∠C + ∠DBC − 90° ...(ii) [By angle sum prop. in ΔBCD]
Compare equations (i) & (ii)
∠ABD = ∠C ...(iii)
In ΔABD and ΔBCD
∠ABD = ∠C [From (iii)]
∠ADB = ∠BDC [Each 90°]
Then, ΔABD ~ ΔBCD [By AA similarity]
$\therefore \frac{BD}{CD} = \frac{AD}{BD}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{8}{CD} = \frac{4}{8}$
$\Rightarrow$ CD $= \frac{8\times8}{4} = 16\ cm$

7.

Sol:
We have, ∠ABC = 90° and BD ⊥ AC
In ΔABC and ΔBDC
∠ABC = ∠BDC [Each 90°]
∠C = ∠C [Common]
Then, ΔABC ~ ΔBDC [By AA similarity]
$\therefore \frac{AB}{BD} = \frac{BC}{DC}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{5.7}{3.8} = \frac{BC}{5.4}$
$\Rightarrow$ BC $= \frac{5.7}{3.8} \times 8.1\ cm$

8.

Sol:

We have, DE || BC, AB = 6 cm and AE = $\frac{1}{4}AC$

In ΔADE and ΔABC

∠A = ∠A [Common]

∠ADE = ∠ABC [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{AD}{6} = \frac{\frac{1}{4}AC}{AC}$ [∵ AE = $\frac{1}{4}AC\ given$]

$\Rightarrow \frac{AD}{6} = \frac{1}{4}$

⇒ AD = $\frac{6}{4}$ = 1.5 cm

9.

Sol:

We have, PA ⊥ AC, QB ⊥ AC and RC ⊥ AC

Let, AB = a and BC = b

In ΔCQB and ΔCPA

∠QCB = ∠PCA [Common]

∠QBC = ∠PAC [Each 90°]

Then, ΔCQB ~ ΔCPA [By AA similarity]

$\therefore \frac{QB}{PA} = \frac{CB}{CA}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{y}{x} = \frac{b}{a+b}$ ….(i)

In Δ AQB and ΔARC

∠QAB = ∠RAC [common]

∠ABQ = ∠ ACR [Each 90°]

Then, ΔAQB ~ ΔARC [By AA similarity]

$\therefore \frac{QB}{RC} = \frac{AB}{AC}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{y}{z} = \frac{a}{a+b}$ ….(ii)

Adding equations (i) & (ii)

$\frac{y}{x} + \frac{y}{z} = \frac{b}{a+b} + \frac{a}{a+b}$

$\Rightarrow y\left(\frac{1}{x} + \frac{1}{z}\right) = \frac{b+a}{a+b}$

$\Rightarrow y\left(\frac{1}{x} + \frac{1}{z}\right) = 1$

$\Rightarrow \frac{1}{x} + \frac{1}{z} = \frac{1}{y}$

10.

Sol:

We have, ∠A = ∠CED

In ΔCAB and ΔCED

∠C = ∠C [Common]

∠A = ∠CED [Given]

Then, ΔCAB ~ ΔCED [By AA similarity]

$\therefore \frac{CA}{CE} = \frac{AB}{ED}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{15}{10} = \frac{9}{x}$

$\Rightarrow x = \frac{10\times9}{15} = 6\ cm$

11.

Sol:

Assume ABC and PQR to be 2 triangles

We have,

ΔABC ~ ΔPQR

Perimeter of Δ ABC = 25 cm

Perimeter of Δ PQR = 15 cm

AB = 9 cm

PQ = ?

Since, ΔABC ~ ΔPQR

Then, ratio of perimeter of triangles = ratio of corresponding sides

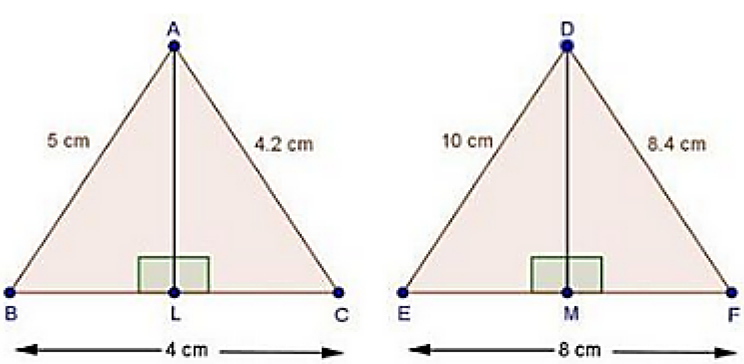
$\Rightarrow \frac{25}{12} = \frac{AB}{PQ}$

$\Rightarrow \frac{25}{15} = \frac{9}{PQ}$

$\Rightarrow$ PQ = $\frac{15\times9}{25} = 5.4\ cm$

12.

Sol:



Since, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DE} = \frac{1}{2}$

Then, ΔABC ~ ΔDEF [By SSS similarity]

Now, In ΔABL ~ ΔDEM

∠B = ∠E [Δ ABC ~ ΔDEF]

∠ALB = ∠DME [Each 90°]

Then, ΔABL ~ ΔDEM [By AA similarity]

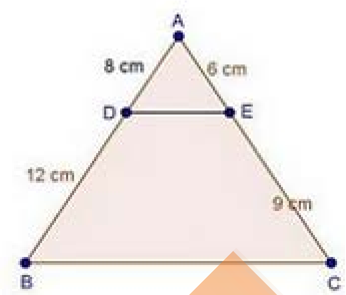
$\therefore \frac{AB}{DE} = \frac{AL}{DM}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{5}{10} = \frac{AL}{DM}$

$\Rightarrow \frac{1}{2} = \frac{AL}{DM}$

13.

Sol:



We have,

$\frac{AD}{DB} = \frac{8}{12} = \frac{2}{3}$

And, $\frac{AE}{EC} = \frac{6}{9} = \frac{2}{3}$

Since, $\frac{AD}{DB} = \frac{AE}{EC}$

Then, by converse of basic proportionality theorem

DE || BC

In ΔADE and ΔABC

∠A = ∠A [Common]

∠ADE = ∠B [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

$\therefore \frac{AD}{AB} = \frac{DE}{BC}$ [Corresponding parts of similar Δ are proportional]

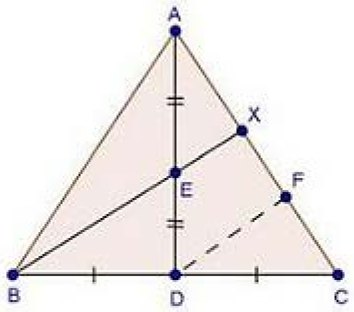
$\Rightarrow \frac{8}{20} = \frac{DE}{BC}$

$\Rightarrow \frac{2}{5} = \frac{DE}{BC}$

$\Rightarrow BC = \frac{5}{2}\ DE$

14.

Sol:

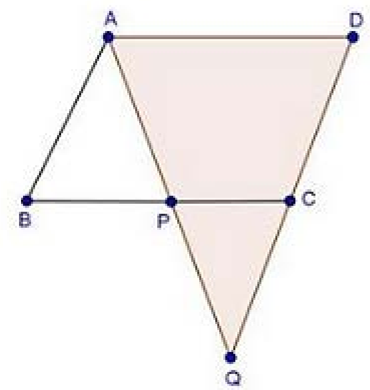


Given: In ΔABC, D is the mid-point of BC and E is the mid-point of AD.
To prove: BE : EX = 3 : 1
Const: Through D, draw DF || BX
Proof: In ΔEAX and ΔADF

∠EAX = ∠ADF	[Common]
∠AXE = ∠DAF	[Corresponding angles]
Then, ΔAEX ~ ΔADF	[By AA similarity]
$\therefore \frac{EX}{DF} = \frac{AE}{AD}$	[Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{EX}{DF} = \frac{AE}{2AE}$	[AE = ED given]
⇒ DF = 2EX	…. (i)
In ΔCDF and ΔCBX	[By AA similarity]
$\therefore \frac{CD}{CB} = \frac{DF}{BX}$	[Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{1}{2} = \frac{DF}{BE+EX}$	[BD = DC given]
⇒ BE + EX = 2DF	
⇒ BE + EX = 4EX	
⇒ BE = 4EX – EX	[By using (i)]
⇒ BE = 4EX – EX	
$\Rightarrow \frac{BE}{EX} = \frac{3}{1}$	

15.

Sol:



Given: ABCD is a parallelogram

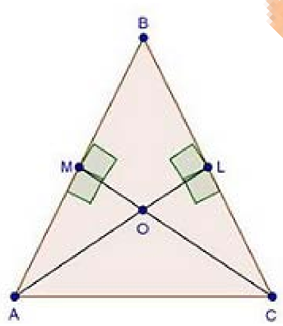
To prove: BP × DQ = AB × BC

Proof: In ΔABP and ΔQDA

∠B = ∠D	[Opposite angles of parallelogram]
∠BAP = ∠AQD	[Alternate interior angles]
Then, ΔABP ~ ΔQDA	[By AA similarity]
$\therefore \frac{AB}{QD} = \frac{BP}{DA}$	[Corresponding parts of similar Δ are proportional]
But, DA = BC	[Opposite sides of parallelogram]
Then, $\frac{AB}{QD} = \frac{BP}{BC}$	
⇒ AB × BC = QD × BP	

16.

Sol:



We have,

AL ⊥ BC and CM ⊥ AB

In Δ OMA and ΔOLC

∠MOA = ∠LOC	[Vertically opposite angles]
∠AMO = ∠CLO	[Each 90°]
Then, ΔOMA ~ ΔOLC	[By AA similarity]

$\therefore \frac{OA}{OC} = \frac{OM}{OL}$ [Corresponding parts of similar Δ are proportional]

17.

Sol:

We have AB || CD || EF. If AB = 6 cm, CD = x cm, EF = 10 cm, BD = 4 cm and DE = y cm

In ΔECD and ΔEAB

∠CED = ∠AEB [common]

∠ECD = ∠EAB [corresponding angles]

Then, ΔECD ~ ΔEAB ….(i) [By AA similarity]

$\therefore \frac{EC}{EA} = \frac{CD}{AB}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{EC}{EA} = \frac{x}{6}$ ….(ii)

In ΔACD and ΔAEF

∠CAD = ∠EAF [common]

∠ACD = ∠AEF [corresponding angles]

Then, ΔACD ~ ΔAEF [By AA similarity]

$\therefore \frac{AC}{AE} = \frac{CD}{EF}$

$\Rightarrow \frac{AC}{AE} = \frac{x}{10}$ …(iii)

Add equations (iii) & (ii)

$\therefore \frac{EC}{EA} + \frac{AC}{AE} = \frac{x}{6} + \frac{x}{10}$

$\Rightarrow \frac{AE}{AE} = \frac{5x+3x}{30}$

$\Rightarrow 1 = \frac{8x}{30}$

$\Rightarrow x = \frac{30}{8} = 3.75$ cm

From (i) $\frac{DC}{AB} = \frac{ED}{BE}$

$\Rightarrow \frac{3.75}{6} = \frac{y}{y+4}$

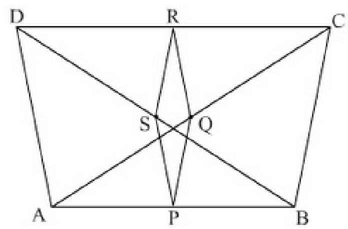
⇒ 6y = 3.75y + 15

⇒ 2.25y = 15

$\Rightarrow y = \frac{15}{2.25} = 6.67\ cm$

18.

Sol:



AD = BC and P, Q, R and S are the mid-points of sides AB, AC, CD and BD respectively, show that PQRS is a rhombus.

In ΔBAD, by mid-point theorem

PS || AD and PS = $\frac{1}{2}$ AD ...(i)

In ΔCAD, by mid-point theorem

QR || AD and QR = $\frac{1}{2}$ AD ...(ii)

Compare (i) and (ii)

PS || QR and PS = QR

Since one pair of opposite sides is equal as well as parallel then

PQRS is a parallelogram ...(iii)

Now, In ΔABC, by mid-point theorem

PQ || BC and PQ = $\frac{1}{2}$ BC ...(iv)

And, AD = BC ...(v) [given]

Compare equations (i) (iv) and (v)

PS = PQ ...(vi)

From (iii) and (vi)

Since, PQRS is a parallelogram with PS = PQ then PQRS is a rhombus

19.

Sol:

Given: AB ⊥ BC, DC ⊥ BC and DE ⊥ AC

To prove: ΔCED ~ ΔABC

Proof:

$\angle BAC + \angle BCA = 90°$ ...(i) [By angle sum property]

And, $\angle BCA + \angle ECD = 90°$ ...(ii) [DC ⊥ BC given]

Compare equation (i) and (ii)

$\angle BAC = \angle ECD$ ...(iii)

In ΔCED and ΔABC

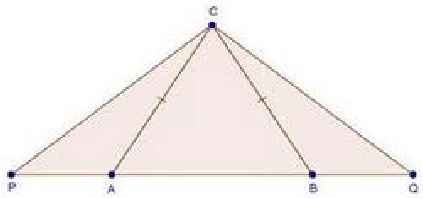
$\angle CED = \angle ABC$ [Each 90°]

$\angle ECD = \angle BAC$ [From (iii)]

Then, $\Delta CED \sim \Delta ABC$ [By AA similarity]

20.

Sol:



Given: In ΔABC, CA = CB and AP × BQ = AC2

To prove: ΔAPC ~ ΔBCQ

Proof:

$AP \times BQ = AC^2$ [Given]

$\Rightarrow AP \times BQ = AC \times AC$

$\Rightarrow AP \times BQ = AC \times BC$ [AC = BC given]

$\Rightarrow \frac{AP}{BC} = \frac{AC}{BQ}$ …(i)

Since, CA = CB [Given]

Then, ∠CAB = ∠CBA …(ii) [Opposite angles to equal sides]

Now, ∠CAB + ∠CAP = 180° …(iii) [Linear pair of angles]

And, ∠CBA + ∠CBQ = 180° …(iv) [Linear pair of angles]

Compare equation (ii) (iii) & (iv)

∠CAP = ∠CBQ …(v)

In ΔAPC and ΔBCQ

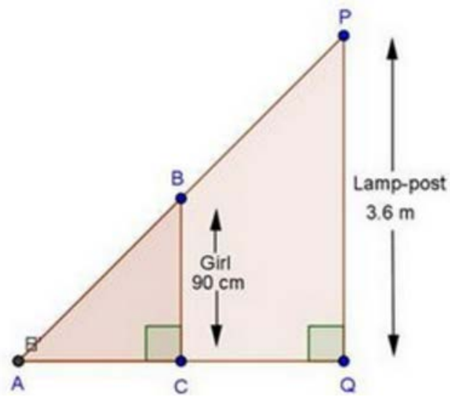
∠CAP = ∠CBQ [From (v)]

$\frac{AP}{BC} = \frac{AC}{BQ}$ [From (i)]

$Then, \Delta APC \sim \Delta BCQ$ [By SAS similarity]

21.

Sol:



We have,

Height of girl = 90 cm = 0.9 m

Height of lamp-post = 3.6 m

Speed of girl = 1.2 m/sec

∴ Distance moved by girl (CQ) = Speed × Time

= 1.2 × 4 = 4.8m

Let length of shadow (AC) = x cm

In ΔABC and ΔAPQ

∠ACB = ∠AQP [Each 90°]

∠BAC = ∠PAQ [Common]

Then, ΔAB ~ ΔAPQ [By AA similarity]

∴ $\frac{AC}{AQ} = \frac{BC}{PQ}$ [Corresponding parts of similar Δ are proportional]

⇒ $\frac{x}{x+4.8} = \frac{0.9}{3.6}$

⇒ $\frac{x}{x+4.8} = \frac{1}{4}$

⇒ 4x = x + 4.8

⇒ 4x – x = 4.8

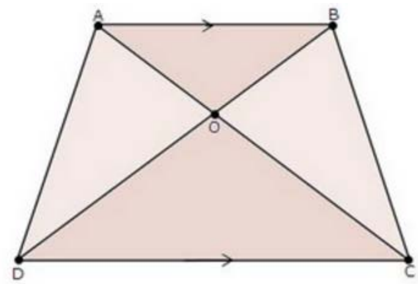
⇒ 3x = 4.8

⇒ x = $\frac{4.8}{3} = 1.6\ m$

∴ Length of shadow = 1.6m

22.

Sol:



We have,

ABCD is a trapezium with AB || DC

In ΔAOB and ΔCOD

∠AOB = ∠COD [Vertically opposite angles]

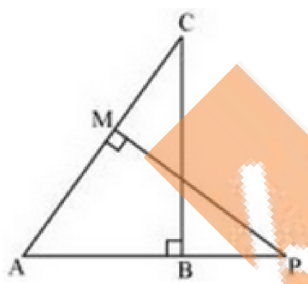
∠OAB = ∠OCD [Alternate interior angles]

Then, ΔAOB ~ ΔCOD [By AA similarity]

$\therefore \frac{OA}{OC} = \frac{OB}{OD}$ [Corresponding parts of similar Δ are proportional]

23.

Sol:



We have,

∠B = ∠M = 90°

And, ∠BAC = ∠MAP

In ΔABC and ΔAMP

∠B = ∠M [Each 90°]

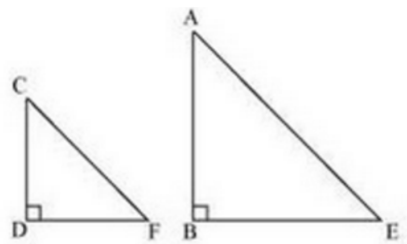
∠BAC = ∠MAP [Given]

Then, ΔABC ~ ΔAMP [By AA similarity]

$\therefore \frac{CA}{PA} = \frac{BC}{MP}$ [Corresponding parts of similar Δ are proportional]

24.

Sol:



Let AB be a tower

CD be a stick, CD = 6m

Shadow of AB is BE = 28m

Shadow of CD is DF = 4m

At same time light rays from sun will fall on tower and stick at same angle.

So, ∠DCF = ∠BAE

And ∠DFC = ∠BEA

∠CDF = ∠ABE (tower and stick are vertical to ground)

Therefore Δ ABE ~ ΔCDF (By AA similarity)

So,

$\frac{AB}{CD} = \frac{BE}{DF}$

$\frac{AB}{6} = \frac{28}{4}$

AB = $28 \times \frac{6}{4} = 42m$

So, height of tower will be 42 metres.

25.

Sol:

In ΔACB, by Pythagoras theorem

$AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = (5)^2 + (12)^2$

$\Rightarrow AB^2 = 25 + 144 = 169$

$\Rightarrow$ AB $= \sqrt{169} = 13\ cm$

In ΔAED and ΔACB

∠A = ∠A [Common]

∠AED = ∠ACB [Each 90°]

Then, ΔAED ~ ΔACB [By AA similarity]

$\therefore \frac{AE}{AC} = \frac{DE}{CB} = \frac{AD}{AB}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{AE}{5} = \frac{DE}{12} = \frac{3}{13}$

$\Rightarrow \frac{AE}{5} = \frac{3}{13}$ and $\frac{DE}{12} = \frac{3}{13}$

$\Rightarrow$ AE $= \frac{15}{13}$ cm and DE $= \frac{36}{13}$ cm

Exercise 4.6

1.

Sol:

(i)

We have,

ΔABC ~ΔDEF

Area $(\Delta ABC) = 16\ cm^2$,

Area $(\Delta DEF) = 25\ cm^2$

And BC = 2.3 cm

Since, ΔABC ~ ΔDEF

Then, $\frac{Area\ (\Delta ABC)}{Area\ (\Delta\ DEF)} = \frac{BC^2}{EF^2}$ [By area of similar triangle theorem]

$\Rightarrow \frac{16}{25} = \frac{(2.3)^2}{EF^2}$

$\Rightarrow \frac{4}{5} = \frac{2.3}{EF}$ [By taking square root]

$\Rightarrow$ EF = $\frac{11.5}{4} = 2.875\ cm$

(ii)

We have,

ΔABC ~ ΔDEF

Area(ΔABC) = 9 cm^2

Area (ΔDEF) = 64 cm^2

And DE = 5.1 cm

Since, ΔABC ~ ΔDEF

Then, $\frac{Area\ (\Delta ABC)}{Area\ (\Delta DEF)} = \frac{AB^2}{DE^2}$ [By area of similar triangle theorem]

$\Rightarrow \frac{9}{64} = \frac{AB^2}{(5.1)^2}$

$\Rightarrow \frac{3}{8} = \frac{AB}{5.1}$ [By taking square root]

$\Rightarrow$ AB $= \frac{3 \times 5.1}{8} = 1.9125\ cm$

(iii)

We have,

ΔABC ~ ΔDEF

AC = 19 cm and DF = 8 cm

By area of similar triangle theorem

$\frac{Area\ (\Delta\ ABC)}{Area\ (\Delta DEF)} = \frac{AC^2}{DF^2} = \frac{(19)^2}{8^2} = \frac{361}{64}$

We have,

ΔABC ~ ΔDEF

AC = 19 cm and DF = 8 cm

By area of similar triangle theorem

$\frac{Area\ (\Delta ABC)}{Area\ (\Delta DEF)} = \frac{AC^2}{DF^2} = \frac{(19)^2}{8^2} = \frac{361}{64}$

(iv)

We have, Area (ΔABC) = 36 cm^2

Area $(\Delta DEF) = 64\ cm^2$

DE = 6.2 cm

And, ΔABC ~ ΔDEF

By area of similar triangle theorem

$\frac{Area(\ \Delta ABC)}{Area\ (\Delta DEF)} = \frac{AB^2}{DE^2}$

$\Rightarrow \frac{36}{64} = \frac{AB^2}{(6.2)^2}$ [By taking square root]

$\Rightarrow$ AB = $\frac{6\times6.2}{8} = 4.65\ cm$

(v)

We have,

ΔABC ~ ΔDEF

AB = 1.2 cm and DF = 1.4 cm

By area of similar triangle theorem

$\frac{Area\ (\Delta\ ABC)}{Area\ (\Delta\ DEF)} = \frac{AB^2}{DE^2}$

$= \frac{(1.2)^2}{(1.4)^2}$

$= \frac{1.44}{1.96}$

$= \frac{36}{49}$

2.

Sol:

We have,

ΔACB ~ ΔAPQ

Then, $\frac{AC}{AP} = \frac{CB}{PQ} = \frac{AB}{AQ}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{AC}{2.8} = \frac{10}{5} = \frac{6.5}{AQ}$

$\Rightarrow \frac{AC}{2.8} = \frac{10}{5}\ and \frac{10}{5} = \frac{6.5}{AQ}$

$\Rightarrow$ AC = $\frac{10}{5} \times 2.8\ and\ AQ = 6.5\ \times \frac{5}{10}$

⇒ AC = 5.6 cm and AQ = 3.25 cm

By area of similar triangle theorem

$\frac{Area\ (\Delta ACB)}{Area\ (\Delta APQ)} = \frac{BC^2}{PQ^2}$

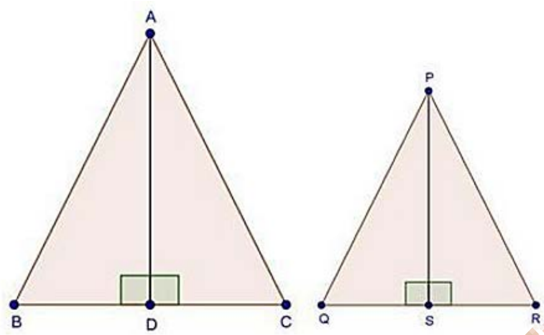
$= \frac{(10)^2}{(5)^2}$

$= \frac{100}{25}$

$= \frac{4}{1}$

3.

Sol:



We have,

ΔABC ~ ΔPQR

Area (ΔABC) = 81 cm^2,

Area (ΔPQR) = 49 cm^2

And AD and PS are the altitudes

By area of similar triangle theorem

$\frac{Area\ (\Delta ABC)}{Area\ (\Delta PQR)} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{81}{49} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{9}{7} = \frac{AB}{PQ}$ ….(i) [Taking square root]

$In\ \Delta ABD\ and\ \Delta PQS$

∠B = ∠Q [ΔABC ~ ΔPQR]

∠ ADB = ∠PSQ [Each 90°]

Then, ΔABD ~ ΔPQS [By AA similarity]

$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$ …(ii) [Corresponding parts of similar Δ are proportional]

Compare (1) and (2)

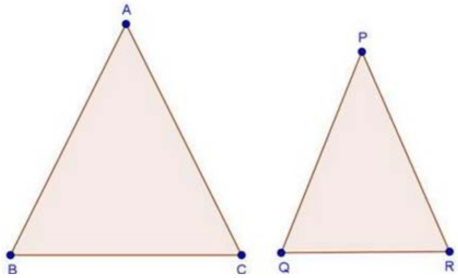
$\frac{AD}{PS} = \frac{9}{7}$

∴ Ratio of altitudes = $\frac{9}{7}$

Since, the ratio of the area of two similar triangles is equal to the ratio of the squares of the squares of their corresponding altitudes and is also equal to the squares of their corresponding medians.
Hence, ratio of altitudes = Ratio of medians = 9 : 7

4.

Sol:



We have,
$\Delta ABC \sim \Delta PQR$
Area(ΔABC) = 169 cm^2
Area(ΔPQR) = 121 cm2
And AB = 26 cm
By area of similar triangle theorem

$\frac{Area\ (\Delta ABC)}{Area\ (\Delta\ PQR)} = \frac{AB^2}{PQ^2}$

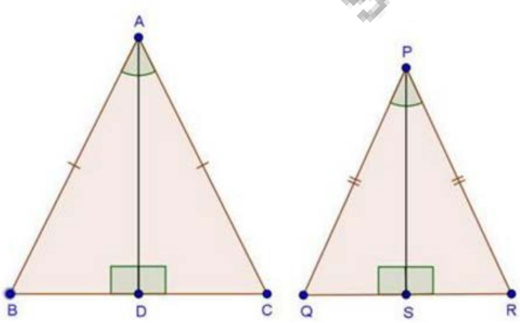
$\Rightarrow \frac{169}{121} = \frac{(26)^2}{PQ^2}$

$\Rightarrow \frac{13}{11} = \frac{26}{PQ}$ [Taking square root]

$\Rightarrow$ PQ = $\frac{11}{13} \times 26 = 22\ cm$

5.

Sol:



Given: AB = AC, PQ = PQ and $\angle$A = $\angle$P
And, AD and PS are altitudes

And, $\frac{Area\ (\Delta ABC)}{Area\ (\Delta\ PQR)} = \frac{36}{25}$ ...(i)

To find: $\frac{AD}{PS}$

Proof: Since, AB = AC and PQ = PR

Then, $\frac{AB}{AC} = 1$ and $\frac{PQ}{PR} = 1$

$\therefore \frac{AB}{AC} = \frac{PQ}{PR}$

$\Rightarrow \frac{AB}{PQ} = \frac{AC}{PR}$ …(ii)

In ΔABC and ΔPQR

∠A = ∠P [Given]

$\frac{AB}{PQ} = \frac{AC}{PR}$ [From (2)]

$Then, \Delta ABC \sim \Delta PQR$ [By SAS similarity]

$\therefore \frac{Area\ (\Delta ABC)}{Area\ (\Delta\ PQR)} = \frac{AB^2}{PQ^2}$ ….(iii) [By area of similar triangle theorem]

$Compare\ equation\ (i) and\ (iii)$

$\frac{AB^2}{PQ^2} = \frac{36}{25}$

$\Rightarrow \frac{AB}{PQ} = \frac{6}{5}$ ….(iv)

In ΔABD and ΔPQS

∠B = ∠Q [ΔABC ~ ΔPQR]

∠ADB = ∠PSQ [Each 90°]

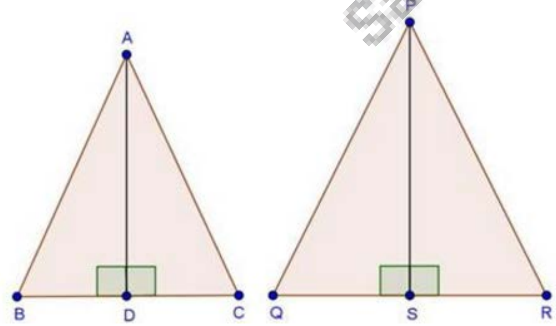
Then, ΔABD ~ ΔPQS [By AA similarity]

$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$

$\Rightarrow \frac{6}{5} = \frac{AD}{PS}$ [From (iv)]

6.

Sol:



We have,

ΔABC ~ ΔPQR

Area (ΔABC) = 25 cm^2

Area (ΔPQR) = 36 cm^2

AD = 2.4 cm
And AD and PS are the altitudes
To find: PS
Proof: Since, ΔABC ~ ΔPQR
Then, by area of similar triangle theorem

$\frac{Area\ (\Delta ABC)}{Area\ (\Delta PQR)} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{25}{36} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{5}{6} = \frac{AB}{PQ}$ ….(i)

$In\ \Delta ABD\ and\ \Delta PQS$

∠B = ∠Q [Δ ABC ~ ΔPQR]
∠ADB ~ ∠PSQ [Each 90°]
Then, ΔABD ~ ΔPQS [By AA similarity]

$\therefore \frac{AB}{PS} = \frac{AD}{PS}$ ….(ii) [Corresponding parts of similar Δ are proportional]

Compare (i) and (ii)

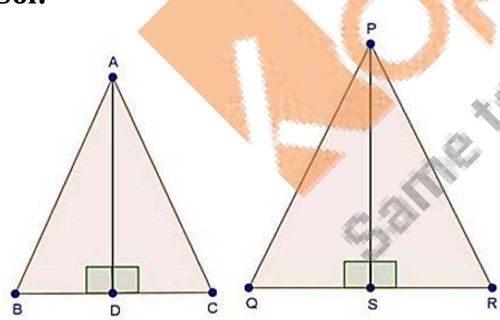
$\frac{AD}{PS} = \frac{5}{6}$

$\Rightarrow \frac{2.4}{PS} = \frac{5}{6}$

$\Rightarrow PS = \frac{2.4\times6}{5} = 2.88\ cm$

7.

Sol:



We have,
ΔABC ~ ΔPQR
AD = 6 cm
And, PS = 9 cm
By area of similar triangle theorem

$\frac{Area\ (\Delta ABC)}{Area\ (\Delta PQR)} = \frac{AB^2}{PQ^2}$ …(i)

$In\ \Delta ABD\ and\ \Delta PQS$

∠B = ∠Q [ΔABC ~ ΔPQR]

∠ADB = ∠PSQ [Each 90°]

Then, ΔABD ~ ΔPQS [By AA similarity]

$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{AB}{PQ} = \frac{6}{9}$

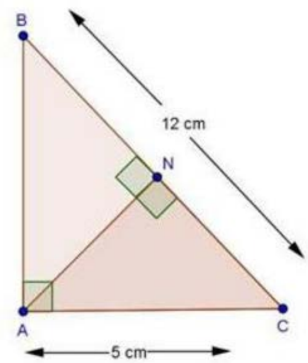
$\Rightarrow \frac{AB}{PQ} = \frac{2}{3}$ …(ii)

Compare equations (i) and (ii)

$\frac{Area(\Delta ABC)}{Area\ (\Delta PQR)} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

8.

Sol:



In ΔANC and ΔABC

∠C = ∠C [Common]

∠ANC = ∠BAC [Each 90°]

Then, ΔANC ~ ΔBAC [By AA similarity]

By area of similarity triangle theorem

$\frac{Area\ (\Delta\ ANC)}{Area\ (\Delta BAC)} = \frac{AC^2}{BC^2}$

$= \frac{5^2}{12^2}$

$= \frac{25}{144}$

9.

Sol:

We have, DE || BC, DE = 4 cm, BC = 6 cm and area $(\Delta ADE) = 16cm^2$

In ΔADE and ΔABC

∠A = ∠A [Common]

∠ADE = ∠ABC [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

∴ By area of similar triangle theorem

$\frac{Area\ (\Delta ADE)}{Area\ (\Delta ABC)} = \frac{DE^2}{BC^2}$

$\Rightarrow \frac{16}{Area\ (\Delta ABC)} = \frac{4^2}{6^2}$

$\Rightarrow Area\ (\Delta ABC) = \frac{16\times 36}{16} = 36cm^2$

$we\ have, DE\ ||BC, DE = 4\ cm, BC = 8\ cm\ and\ area\ (\Delta ADE) = 25\ cm^2$

$In\ \Delta ADE\ and\ \Delta ABC$

∠A = ∠A [Common]

∠ADE = ∠ABC [Corresponding angles]

Then, $\Delta ADE \sim \Delta ABC$ [By AA similarity]

By area of similar triangle theorem

$\frac{Area\ (\Delta ADE)}{Area\ (\Delta\ ABC)} = \frac{DE^2}{BC^2}$

$\Rightarrow \frac{16}{Area\ (\Delta ABC)} = \frac{4^2}{6^2}$

⇒ Area (ΔABC) = $\frac{16\times 36}{16} = 36\ cm^2$

We have, DE || BC, DE = 4 cm, BC = 8 cm and area (ΔADE) = 25cm^2

In ΔADE and ΔABC

∠A = ∠A [Common]

∠ADE = ∠ABC [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

By area of similar triangle theorem

$\Rightarrow \frac{Area\ (\Delta ADE)}{Area\ (\Delta ABC)} = \frac{DE^2}{BC^2}$

$\frac{25}{Area\ (\Delta ABC)} = \frac{4^2}{8^2}$

$\Rightarrow Area\ (\Delta ABC) = \frac{25\times 64}{16} = 100\ \ cm^2$

We have, DE || BC, and $\frac{DE}{BC} = \frac{3}{5}$ ….(i)

In ΔADE and ΔABC

∠A = ∠A [Common]

∠ADE = ∠B [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

By area of similar triangle theorem

$\Rightarrow \frac{Area\ (\Delta ADE)}{Area\ (\Delta ABC)} = \frac{DE^2}{BC^2}$

$\Rightarrow \frac{ar\ (\Delta ADE)}{ar(\Delta ADE)+ar(trap.DECB)} = \frac{3^2}{5^2}$ [From (i)]

⇒ 25ar (ΔADE) = $9ar\ (\Delta ADE) + 9ar\ (trap.DECB)$

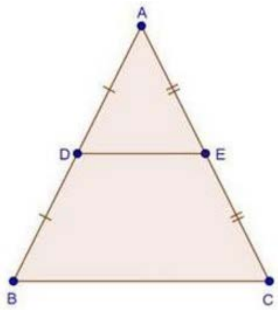
⇒ 25 ar (ΔADE – 9ar) (ΔADE) = 9ar (trap.DECB)

⇒ 16 ar(ΔADE) = 9 ar (trap. DECB)

$\Rightarrow \frac{ar\ (\Delta ADE)}{ar\ (trap.DECB)} = \frac{9}{16}$

10.

Sol:



We have, D and E as the mid-points of AB and AC

So, according to the mid-point theorem

DE ∥ BC and DE = $\frac{1}{2}BC$ …(i)

In ΔADE and ΔABC

∠A = ∠A [Common]

∠ADE = ∠B [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

By area of similar triangle theorem

$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2}$

$= \frac{\left(\frac{1}{2}BC\right)^2}{BC^2}$ [From (i)]

$= \frac{\frac{1}{4}BC^2}{BC^2}$

$= \frac{1}{4}$

11.

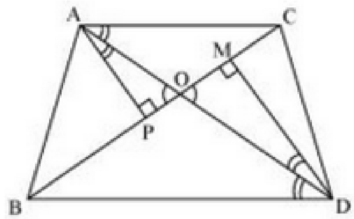
Sol:

We know that area of a triangle = $\frac{1}{2} \times Base \times height$

Since ΔABC and ΔDBC are one same base,

Therefore ratio between their areas will be as ratio of their heights.

Let us draw two perpendiculars AP and DM on line BC.



In ΔAPO and ΔDMO,

∠APO = ∠DMO (Each is 90°)

∠AOP = ∠DOM (vertically opposite angles)

∠OAP = ∠ODM (remaining angle)

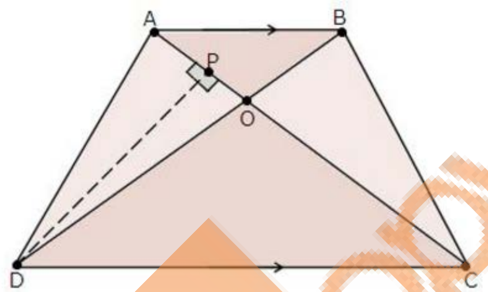
Therefore ΔAPO ~ ΔDMO (By AAA rule)

Therefore $\frac{AP}{DM} = \frac{AO}{DO}$

Therefore $\frac{area\ (\Delta ABC)}{area\ (\Delta DBC)} = \frac{AO}{DO}$

12.

Sol:



We have,

AB || DC

In ΔAOB and ΔCOD

∠AOB = ∠COD [Vertically opposite angles]

∠OAB = ∠OCD [Alternate interior angles]

Then, ΔAOB ~ ΔCOD [By AA similarity]

(a) By area of similar triangle theorem

$$\frac{ar\ (\Delta AOB)}{ar\ (\Delta COD)} = \frac{OA^2}{OC^2} = \frac{6^2}{8^2} = \frac{36}{64} = \frac{9}{16}$$

(b) Draw DP ⊥ AC

$$\therefore \frac{area\ (\Delta AOD)}{area\ (\Delta COD)} = \frac{\frac{1}{2} \times AO \times DP}{\frac{1}{2} \times CO \times DP}$$

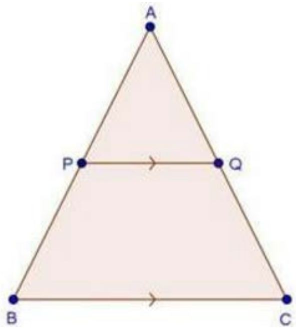
$$= \frac{AO}{CO}$$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

13.

Sol:



We have,

PQ ∥ BC

And $\frac{AP}{PB} = \frac{1}{2}$

In ΔAPQ and ΔABC

∠A = ∠A [Common]

∠APQ = ∠B [Corresponding angles]

Then, ΔAPQ ~ΔABC [By AA similarity]

By area of similar triangle theorem

$\frac{ar\ (\Delta APQ)}{ar\ (\Delta ABC)} = \frac{AP^2}{AB^2}$

$\Rightarrow \frac{ar\ (\Delta APQ)}{ar(\Delta APQ)+ar(trap.BPQC)} = \frac{1^2}{3^2}\ \left[\frac{AP}{PB} = \frac{1}{2}\right]$

$\Rightarrow$ 9ar $(APQ) = ar(\Delta APQ) + ar(trap.BPQC)$

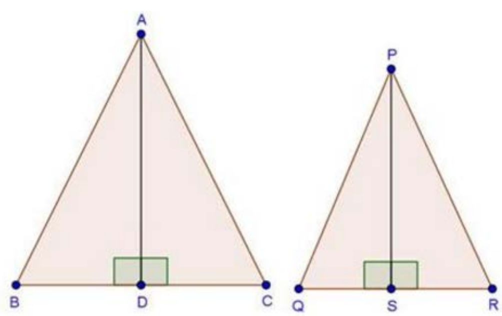
$\Rightarrow$ 9ar $(APQ) - ar(\Delta APQ) + ar(trap.BPQC)$

$\Rightarrow$ 8ar(APQ) = ar(trap. BPQC)

$\Rightarrow \frac{ar\ (\Delta APQ)}{ar(trap.BPQC)} = \frac{1}{8}$

14.

Sol:



We have, ΔABC ~ ΔPQR

Area(ΔABC) = 100 cm^2,

Area (ΔPQR) = 49 cm^2
AD = 5 cm
And AD and PS are the altitudes
By area of similar triangle theorem

$\frac{Area\ (\Delta ABC)}{Area\ (\Delta PQR)} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{100}{49} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{10}{7} = \frac{AB}{PQ}$ ...(i)

In ΔABD and ΔPQS

∠B = ∠Q [ΔABC ~ ΔPQR]
∠ADB = ∠PSQ [Each 90°]
Then, ΔABD ~ ΔPQS [By AA similarity]

$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$ ...(ii) [Corresponding parts of similar Δ are proportional]

Compare (i) and (ii)

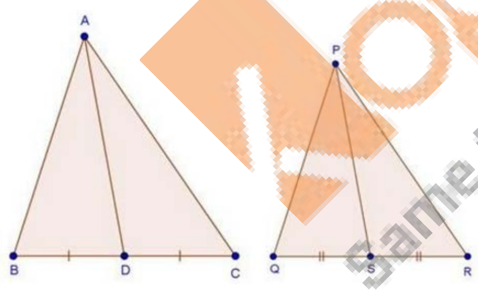
$\frac{AD}{PS} = \frac{10}{7}$

$\Rightarrow \frac{5}{PS} = \frac{10}{7}$

$\Rightarrow PS = \frac{5\times 7}{10} = 3.5\ cm$

15.

Sol:



We have,
ΔABC ~ ΔPQR
Area (ΔABC) = 121 cm^2,
Area (ΔPQR) = 64 cm^2
AD = 12.1 cm
And AD and PS are the medians
By area of similar triangle theorem

$\frac{Area(\Delta ABC)}{Area(\Delta PQR)} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{121}{64} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{11}{8} = \frac{AB}{PQ}$ …(i)

Since, ΔABC ~ ΔPQR

Then, $\frac{AB}{PQ} = \frac{BC}{QR}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QS}$ [AD and PS are medians]

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QS}$ …(ii)

In ΔABD and ΔPQS

∠B = ∠Q [ΔABC ~ ΔPQS]

$\frac{AB}{PQ} = \frac{BD}{QS}$ [From (ii)]

$Then, \Delta ABD \sim \Delta PQS$ [By SAS similarity]

$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$ …(iii) [Corresponding parts of similar Δ are proportional]

$Compare\ (i)\ and\ (iii)$

$\frac{11}{8} = \frac{AD}{PS}$

$\Rightarrow \frac{11}{8} = \frac{12.1}{PS}$

$\Rightarrow$ PS = $\frac{8 \times 12.1}{PS}$

$\Rightarrow$ PS = $\frac{8 \times 12.1}{PS} = 8.8\ cm$

16.

Sol:

We have,

ΔABC ~ ΔDEF such that AB = 5 cm,

Area (ΔABC) = 20 cm^2 and area(ΔDEF) = 45 cm^2

By area of similar triangle theorem

$\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{AB^2}{DE^2}$

$\Rightarrow \frac{20}{45} = \frac{5^2}{DE^2}$

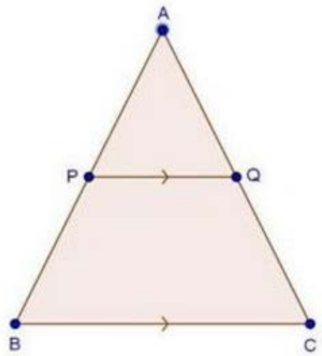
$\Rightarrow \frac{4}{9} = \frac{5^2}{DE^2}$

$\Rightarrow \frac{2}{3} = \frac{5}{DE}$ [Taking square root]

$\Rightarrow DE = \frac{3 \times 5}{2} = 7.5\ cm$

17.

Sol:



We have,

PQ || BC

And ar(ΔAPQ) = ar(trap. PQCB)

⇒ ar(ΔAPQ) = ar(ΔABC) – ar(ΔAPQ)

⇒ 2ar(ΔAPQ) = ar(ΔABC) …(i)

In ΔAPQ and ΔABC

∠A = ∠A [common]

∠APQ = ∠B [corresponding angles]

Then, ΔAPQ ~ ΔABC [By AA similarity]

∴ By area of similar triangle theorem

$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2}$

$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta APQ)} = \frac{AP^2}{AB^2}$ [By using (i)]

$\Rightarrow \frac{1}{2} = \frac{AP^2}{AB^2}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AP}{AB^2}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AP}{AB}$ [Taking square root]

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AB-BP}{AB}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AB}{AB} - \frac{BP}{AB}$

$\Rightarrow \frac{1}{\sqrt{2}} = 1 - \frac{BP}{AB}$

$= \frac{BP}{AB} = 1 - \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{BP}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$

18.

Sol:

We have,

$\Delta ABC \sim \Delta PQR$

$\frac{area\ (\Delta ABC)}{area\ (\Delta PQR)} = \frac{BC^2}{QR^2}$

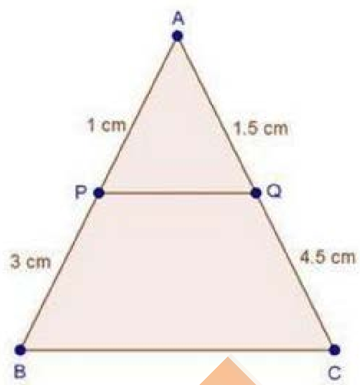
$\Rightarrow \frac{9}{16} = \frac{(4.5)^2}{QR^2}$

$\Rightarrow \frac{3}{4} = \frac{4.5}{QR}$ [Taking square root]

$\Rightarrow$ QR = $\frac{4 \times 4.5}{3} = 6cm$

19.

Sol:



We have,

AP = 1 cm, PB = 3 cm, AQ = 1.5 cm and QC = 4.5 m

In ΔAPQ and ΔABC

$\angle A = \angle A$ [Common]

$\frac{AP}{AB} = \frac{AQ}{AC}$ [Each equal to $\frac{1}{4}$]

$Then, \Delta APQ \sim \Delta ABC$ [By SAS similarity]

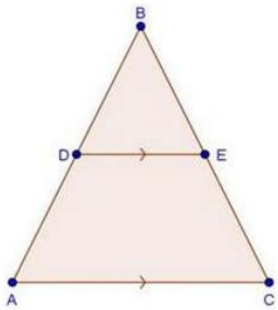
$By\ area\ of\ similar\ triangle\ theorem$

$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{1^2}{4^2}$

$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{1}{16} \times ar(\Delta ABC)$

20.

Sol:



We have,

$\frac{AD}{DB} = \frac{3}{2}$

$\Rightarrow \frac{DB}{AD} = \frac{2}{3}$

In ΔBDE and ΔBAC

∠B = ∠B [common]

∠ BDE = ∠A [corresponding angles]

Then, ΔBDE ~ ΔBAC [By AA similarity]

By area of similar triangle theorem

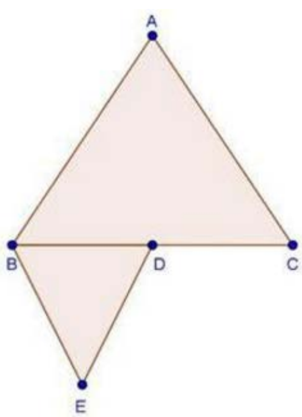
$\frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{AB^2}{BD^2}$

$= \frac{5^2}{2^2}$ $\left[\frac{AD}{DB} = \frac{3}{2}\right]$

$= \frac{25}{4}$

21.

Sol:



We have,

ΔABC and ΔBDE are equilateral triangles then both triangles are equiangular

∴ ΔABC ~ ΔBDE [By AAA similarity]

By area of similar triangle theorem

$\frac{ar(\Delta ABC)}{ar\ (\Delta BDE)} = \frac{BC^2}{BD^2}$

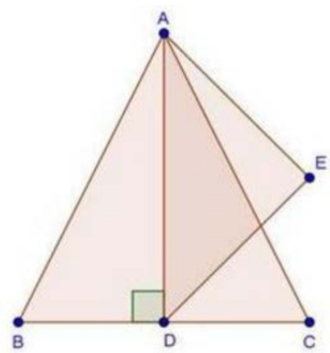
$= \frac{2(BD)^2}{BD^2}$ [D is the mid-point of BC]

$= \frac{4BD^2}{BD^2}$

$= \frac{4}{1}$

22.

Sol:



We have,

ΔABC is an equilateral triangle

Then, AB = BC = AC

Let, AB = BC = AC = 2x

Since, AD ⊥ BC then BD = DC = x

In ΔADB, by Pythagoras theorem

$AB^2 = (2x)^2 - (x)^2$

$\Rightarrow AD^2 = 4x^2 - x^2 = 3x^2$

$\Rightarrow AD = \sqrt{3}x\ cm$

$Since, \Delta ABC\ and\ \Delta ADE\ both\ are\ equilateral\ triangles\ then\ they\ are\ equiangular$

∴ ΔABC ~ ΔADE [By AA similarity]

By area of similar triangle theorem

$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{AD^2}{AB^2}$

$= \frac{(\sqrt{3}x)^2}{(2x)^2}$

$= \frac{3x^2}{4x^2}$

$= \frac{3}{4}$

Exercise 4.7

1.

Sol:

We have,

Sides of triangle

AB = 3 cm

BC = 4 cm

AC = 6 cm

$\therefore AB^2 = 3^2 = 9$

$BC^2 = 4^2 = 16$

$AC^2 = 6^2 = 36$

Since, $AB^2 + BC^2 \neq AC^2$

Then, by converse of Pythagoras theorem, triangle is not a right triangle.

2.

Sol:

We have,

a = 7 cm, b = 24 cm and c = 25 cm

$\therefore a^2 = 49, b^2 = 576 \text{ and } c^2 = 625$

Since, $a^2 + b^2 = 49 + 576$

= 625

= c^2

Then, by converse of Pythagoras theorem, given triangle is a right triangle.

We have,

a = 9 cm, b = 16 cm and c = 18 cm

$\therefore a^2 = 81, b^2 = 256 \text{ and } c^2 = 324$

Since, $a^2 + b^2 = 81 + 256 = 337$

$\neq c^2$

Then, by converse of Pythagoras theorem, given triangle is not a right triangle.

We have,

a = 1.6 cm, b = 3.8 cm and C = 4 cm

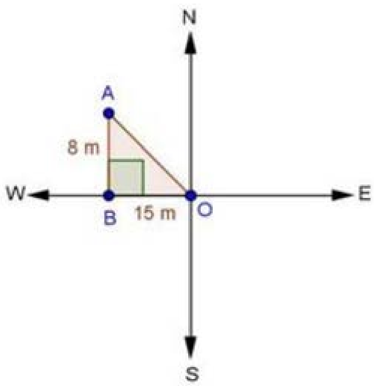
$\therefore a^2 = 64, b^2 = 100 \text{ and } c^2 = 36$

Since, $a^2 + c^2 = 64 + 36 = 100 = b^2$

Then, by converse of Pythagoras theorem, given triangle is a right triangle.

3.

Sol:



Let the starting point of the man be O and final point be A.

∴ In ΔABO, by Pythagoras theorem $AO^2 = AB^2 + BO^2$

⇒ $AO^2 = 8^2 + 15^2$

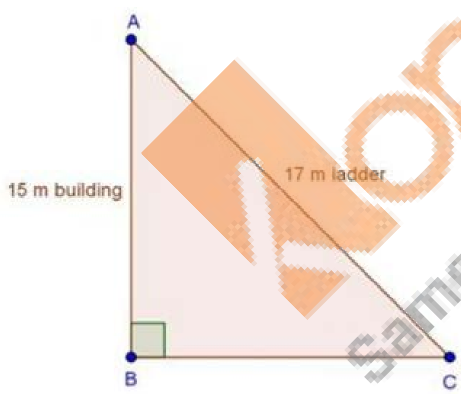
⇒ $AO^2 = 64 + 225 = 289$

⇒ AO = $\sqrt{289} = 17m$

∴ He is 17m far from the starting point.

4.

Sol:



In ΔABC, by Pythagoras theorem

$AB^2 + BC^2 = AC^2$

⇒ $15^2 + BC^2 = 17^2$

⇒ $225 + BC^2 = 17^2$

⇒ $BC^3 = 289 - 225$

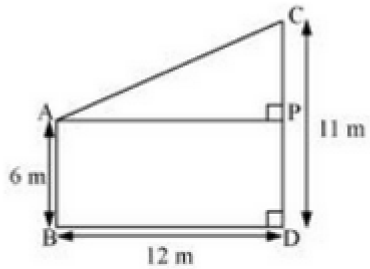
⇒ $BC^2 = 64$

⇒ $BC = 8\,m$

∴ $Distance\ of\ the\ foot\ of\ the\ ladder\ from\ building = 8\,m$

5.

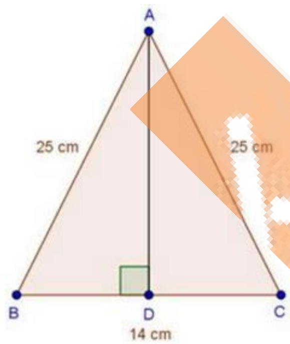
Sol:



Let CD and AB be the poles of height 11 and 6 m.
Therefore CP = 11 – 6 = 5 m
From the figure we may observe that AP = 12m
In triangle APC, by applying Pythagoras theorem
$AP^2 + PC^2 = AC^2$
$12^2 + 5^2 = AC^2$
$AC^2 = 144 + 25 = 169$
AC = 13
Therefore distance between their tops = 13m.

6.

Sol:



We have
AB = AC = 25 cm and BC = 14 cm
In ΔABD and ΔACD

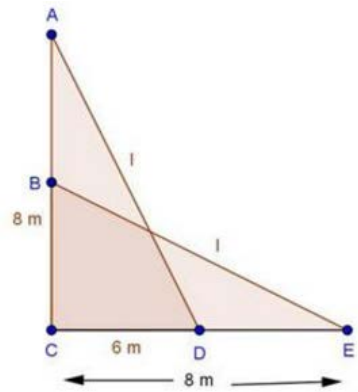
∠ADB = ∠ADC	[Each 90°]
AB = AC	[Each 25 cm]
AD = AD	[Common]
Then, ΔABD ≅ ΔACD	[By RHS condition]
∴ BD = CD = 7 cm	[By c.p.c.t]

In ΔADB, by Pythagoras theorem
$AD^2 + BD^2 = AB^2$

$\Rightarrow AD^2 + 7^2 = 25^2$
$\Rightarrow AD^2 = 625 - 49 = 576$
$\Rightarrow AD = \sqrt{576} = 24\ cm$

7.

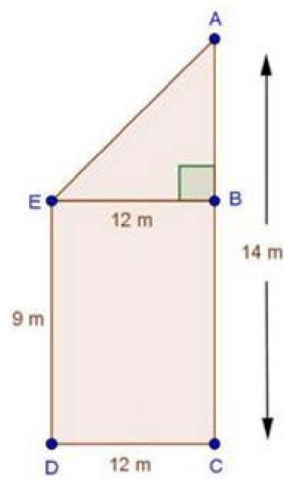
Sol:



Let, length of ladder be AD = BE = l m
In ΔACD, by Pythagoras theorem
$AD^2 = AC^2 + CD^2$
$\Rightarrow\ l^2 = 8^2 + 6^2$ ….(i)
$In\ \Delta BCE, by\ pythagoras\ theorem$
$BE^2 = BC^2 + CE^2$
$\Rightarrow\ l^2 = BC^2 + 8^2$ ….(ii)
$Compare\ (i) and\ (ii)$
$BC^2 + 8^2 = 8^2 + 6^2$
$\Rightarrow BC^2 = 6^2$
$\Rightarrow BC = 6m$

8.

Sol:



We have,

AC = 14 m, DC = 12m and ED = BC = 9m

Construction: Draw EB ⊥ AC

∴ AB = AC – BC = 14 – 9 = 5m

And, EB = DC = 12 m

In ΔABE, by Pythagoras theorem,

$AE^2 = AB^2 + BE^2$

$\Rightarrow AE^2 = 5^2 + 12^2$

$\Rightarrow AE^2 = 25 + 144 = 169$

$\Rightarrow AE = \sqrt{169} = 13\ m$

$\therefore Distance\ between\ their\ tops = 13\ m$

9.

Sol:

We have,

In ΔBAC, by Pythagoras theorem

$BC^2 = AB^2 + AC^2$

$\Rightarrow BC^2 = c^2 + b^2$

$\Rightarrow BC = \sqrt{c^2 + b^2}$ …(i)

$In\ \Delta ABD\ and\ \Delta CBA$

∠B = ∠B [Common]

∠ADB = ∠BAC [Each 90°]

Then, ΔABD ~ ΔCBA [By AA similarity]

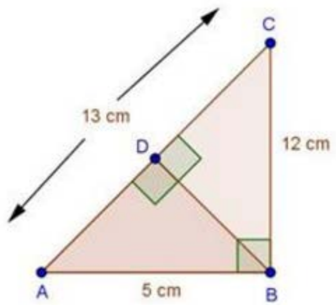
$\therefore \frac{AB}{CB} = \frac{AD}{CA}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{c}{\sqrt{c^2+b^2}} = \frac{AD}{b}$

$\Rightarrow$ AD = $\frac{bc}{\sqrt{c^2+b^2}}$

10.

Sol:



Let, AB = 5cm, BC = 12 cm and AC = 13 cm. Then, $AC^2 = AB^2 + BC^2$. This proves that ΔABC is a right triangle, right angles at B. Let BD be the length of perpendicular from B on AC.

Now, Area ΔABC = $\frac{1}{2}(BC \times BA)$

= $\frac{1}{2}(12 \times 5)$

= 30 cm^2

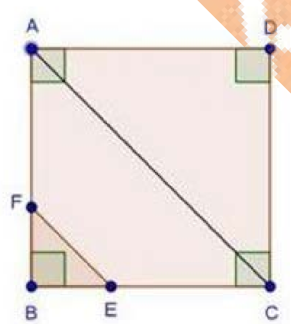
Also, Area of ΔABC = $\frac{1}{2}AC \times BD = \frac{1}{2}(13 \times BD)$

$\Rightarrow (13 \times BD) = 30 \times 2$

$\Rightarrow$ BD = $\frac{60}{13}$ cm

11.

Sol:



Since, ABCD is a square

Then, AB = BC = CD = DA = x cm

Since, F is the mid-point of AB

Then, AF = $FB = \frac{x}{2}\ cm$

Since, BE is one third of BC

Then, BE = $\frac{x}{3}$ cm

We have, area of ΔFBE = 108 cm^2

$\Rightarrow \frac{1}{2} \times BE \times FB = 108$

$\Rightarrow \frac{1}{2} \times \frac{x}{3} \times \frac{x}{2} = 108$

$\Rightarrow x^2 = 108 \times 2 \times 3 \times 2$

$\Rightarrow x^2 = 1296$

$\Rightarrow x = \sqrt{1296} = 36cm$

$In\ \Delta ABC, by\ pythagoras\ theorem\ AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = x^2 + x^2$

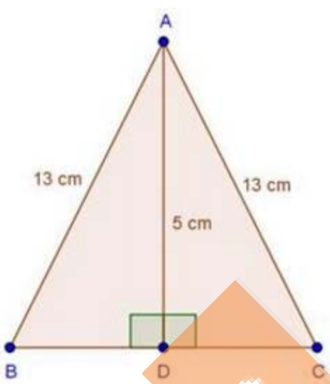
$\Rightarrow AC^2 = 2x^2$

$\Rightarrow AC^2 = 2 \times (36)^2$

$\Rightarrow AC = 36\sqrt{2} = 36 \times 1.414 = 50.904\ cm$

12.

Sol:



In ΔADB, by Pythagoras theorem

$AD^2 + BD^2 = 13^2$

$\Rightarrow 25 + BD^2 = 169$

$\Rightarrow BD^2 = 169 - 2 = 144$

$\Rightarrow BD = \sqrt{144} = 12\ cm$

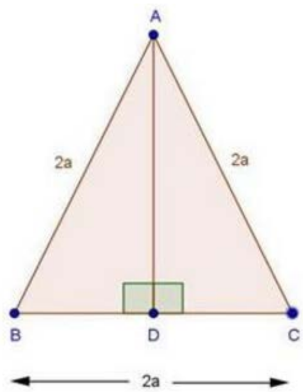
$In\ \Delta ADB\ and\ \Delta ADC$

∠ADB = ∠ADC	[Each 90°]
AB = AC	[Each 13 cm]
AD = AD	[Common]
Then, ΔADB ≅ ΔADC	[By RHS condition]
∴ BD = CD = 12 cm	[By c.p.c.t]

Hence, BC = 12 + 12 = 24 cm

13.

Sol:



(i) In ΔABD and ΔACD

∠ADB = ∠ADC [Each 90°]

AB = AC [Given]

AD = AD [Common]

Then, ΔABD ≅ ΔACD [By RHS condition]

∴ BD = CD = a [By c.p.c.t]

In ΔADB, by Pythagoras theorem

$AD^2 + BD^2 = AB^2$

$\Rightarrow AD^2 + (a)^2 = (2a)^2$

$\Rightarrow AD^2 + a^2 = 4a^2$

$\Rightarrow AD^2 = 4a^2 - a^2 = 3a^2$

$\Rightarrow AD = a\sqrt{3}$

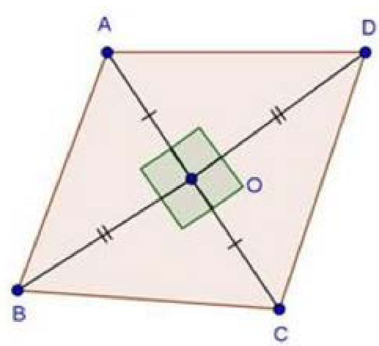
(ii) Area of ΔABC = $\frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 2a \times a\sqrt{3}$

$= \sqrt{3}a^2$

14.

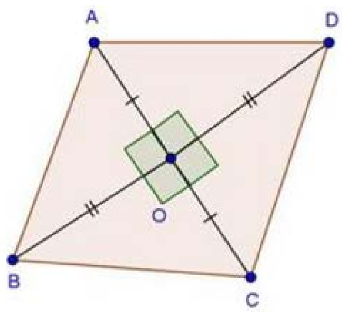
Sol:



We have,

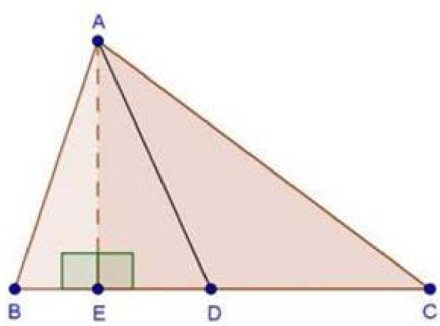
ABCD is a rhombus with diagonals AC = 10 cm and BD = 24 cm
We know that diagonal of a rhombus bisect each other at 90°
∴ AO = OC = 5 cm and BO = OD = 12 cm
In ΔAOB, by Pythagoras theorem
$AB^2 = AO^2 + BO^2$
$\Rightarrow AB^2 = 5^2 + 12^2$
$\Rightarrow AB^2 = 25 + 144 = 169$
$\Rightarrow AB = \sqrt{169} = 13\ cm$

15.
Sol:



We have,
ABCD is a rhombus with side 10 cm and diagonal BD = 16 cm
We know that diagonals of a rhombus bisect each other at 90°
∴ BO = OD = 8 cm
In ΔAOB, by pythagoras theorem
$AO^2 + BO^2 = AB^2$
$\Rightarrow AO^2 + 8^2 = 10^2$
$\Rightarrow AO^2 = 100 - 64 = 36$
$\Rightarrow AO = \sqrt{36} = 6\ cm$ [By above property]
$hence, AC = 6 + 6 = 12\ cm$

16.
Sol:



We have,

In ΔABC, AD is a median.

Draw AE ⊥ BC

In ΔAEB, by pythagoras theorem

$AB^2 = AE^2 + BE^2$

$\Rightarrow AB^2 = AD^2 - DE^2 + (BD - DE)^2$ [By Pythagoras theorem]

$\Rightarrow AB^2 = AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$

$\Rightarrow AB^2 = AD^2 + BD^2 - 2BD \times DE$

$\Rightarrow AB^2 = AD^2 + \frac{BC^2}{4} - BC \times DE$ ….(i) [BC = 2BD given]

Again, In ΔAEC, by pythagoras theorem

$AC^2 = AE^2 + EC^2$

$\Rightarrow AC^2 = AD^2 - DE^2 + (DE + CD)^2$ [By Pythagoras theorem]

$\Rightarrow AC^2 = AD^2 + CD^2 + 2CD \times DE$

$\Rightarrow AC^2 = AD^2 + \frac{BC^2}{4} + BC \times DE$ ….(ii) [BC = 2CD given]

$Add\ equations\ (i)\ and\ (ii)$

$AB^2 + AC^2 = 2AD^2 + \frac{BC^2}{2}$

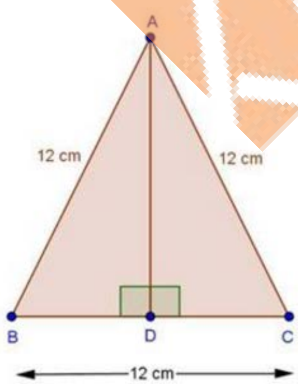
$\Rightarrow 2AB^2 + 2AC^2 = 4AD^2 + BC^2$ [Multiply by 2]

$\Rightarrow 4AD^2 = 2AB^2 + 2AC^2 - BC^2$

$\Rightarrow AD^2 = \frac{2AB^2+2AC^2-BC^2}{4}$

17.

Sol:



We have,

ΔABC is an equilateral Δ with side 12 cm.

Draw AE ⊥ BC

In ΔABD and ΔACD

∠ADB = ∠ADC [Each 90°]

AB = AC [Each 12 cm]

AD = AD [Common]

Then, ΔABD ≅ ΔACD [By RHS condition]

$\therefore AD^2 + BD^2 = AB^2$

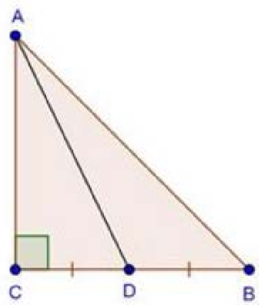
$\Rightarrow AD^2 + 6^2 = 12^2$

$\Rightarrow AD^2 = 144 - 36 = 108$

$\Rightarrow$ AD = $\sqrt{108} = 10.39$ cm

18.

Sol:



We have,

∠C = 90° and D is the mid-point of BC

In ΔACB, by Pythagoras theorem

$AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = AC^2 + (2CD)^2$ [D is the mid-point of BC]

$AB^2 = AC^2 + 4CD^2$

$\Rightarrow AB^2 = AC^2 + 4(AD^2 - AC^2)$ [In ΔACD, by Pythagoras theorem]

$\Rightarrow AB^2 = AC^2 + 4AD^2 - 4AC^2$

$\Rightarrow AB^2 = 4AD^2 - 3AC^2$

Sol:

We have, D as the mid-point of BC

(i) $AC^2 = AE^2 + EC^2$

$b^2 = AE^2 + (ED + DC)^2$ [By pythagoras theorem]

$b^2 = AD^2 + DC^2 + 2DC \times ED$

$b^2 = p^2 + \left(\frac{a}{2}\right)^2 + 2\left(\frac{a}{2}\right) \times x$ [BC = 2CD given]

$\Rightarrow b^2 = p^2 + \frac{a^2}{4} + ax$ …(i)

(ii) In ΔAEB, by pythagoras theorem

$AB^2 = AE^2 + BE^2$

$\Rightarrow c^2 = AD^2 - ED^2 + (BD - ED)^2$ [By pythagoras theorem]

$\Rightarrow c^2 = p^2 - ED^2 + BD^2 + ED^2 - 2BD \times ED$

$\Rightarrow c^2 = p^2 + \left(\frac{a}{2}\right)^2 - 2\left(\frac{a}{2}\right) \times x$ ...(ii)

(iii) Add equations (i) and (ii)

$b^2 + c^2 = 2p^2 + \frac{a^2}{2}$

19.

Sol:

In ΔADC, by pythagoras theorem

$AC^2 = AD^2 + DC^2$

$\Rightarrow b^2 = h^2 + (a - x)^2$

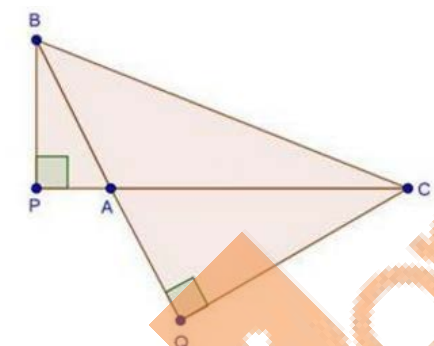
$\Rightarrow b^2 = h^2 + a^2 + x^2 - 2ax$

$\Rightarrow b^2 = a^2 + (h^2 + x^2) - 2ax$

$\Rightarrow b^2 = a^2 + c^2 - 2ax$ by Pythagoras theorem

20.

Sol:



Then, ΔAPB ~ ΔAQC [By AA similarity]

$\therefore \frac{AP}{AQ} = \frac{AB}{AC}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow AP \times AC = AQ \times AB$ ...(i)

(ii) In ΔBPC, by pythagoras theorem

$BC^2 = BP^2 + PC^2$

$\Rightarrow BC^2 = AB^2 - AP^2 + (AP + AC)^2$ [By pythagoras theorem]

$\Rightarrow BC^2 = AB^2 + AC^2 + 2AP \times AC$ ...(ii)

$In\ \Delta BQC, by\ pythagoras\ theorem,$

$BC^2 = CQ^2 + BQ^2$

$\Rightarrow BC^2 = AC^2 - AQ^2 + (AB + AQ)^2$ [By pythagoras theorem]

$\Rightarrow BC^2 = AC^2 - AQ^2 + AB^2 + AQ^2 + 2AB \times AQ$

$\Rightarrow BC^2 = AC^2 + AB^2 + 2AB \times AQ$ ...(iii)

$Add\ equations\ (ii) \&\ (iii)$

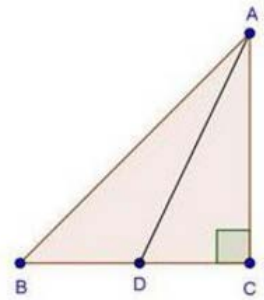
$2BC^2 = 2AC^2 + 2AB^2 + 2AP \times AC + 2AB \times AQ$

$\Rightarrow 2BC^2 = 2AC^2 + 2AB^2 + 2AP \times AC + 2AB \times AQ$

$\Rightarrow 2BC^2 = 2AC[AC + AP] + AB[AB + AQ]$
$\Rightarrow 2BC^2 = 2AC \times PC + 2AB \times BQ$
$\Rightarrow BC^2 = AC \times PC + AB \times BQ$ [Divide by 2]

21.

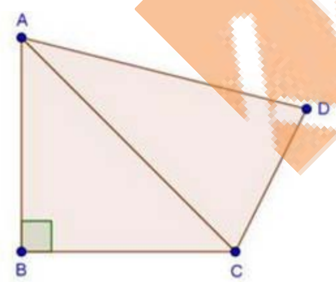
Sol:



To prove: $BC^2 = 4[AD^2 - AC^2]$
We have, ∠C = 90° and D is the mid-point of BC.
LHS = BC^2
= $(2CD)^2$ [D is the mid-point of BC]
= $4CD^2$
= $4[AD^2 - AC^2]$ [In ΔACD, by pythagoras theorem]
= RHS

22.

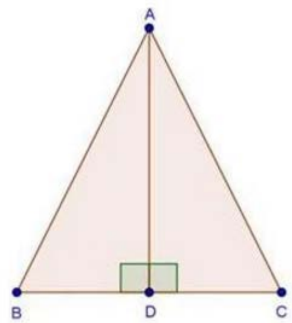
Sol:



We have, ∠B = 90° and $AD^2 = AB^2 + BC^2 + CD^2$
$\therefore AD^2 = AB^2 + BC^2 + CD^2$ [Given]
But $AB^2 + BC^2 = AC^2$ [By pythagoras theorem]
Then, $AD^2 = AC^2 + CD^2$
By converse of by pythagoras theorem
∠ACD = 90°

23.

Sol:



We have, ΔABC is an equilateral Δ and AD ⊥ BC

In ΔADB and ΔADC

∠ADB = ∠ADC [Each 90°]

AB = AC [Given]

AD = AD [Common]

Then, ΔADB ≅ ΔADC [By RHS condition]

∴ BD = CD = $\frac{BC}{2}$ …(i) [corresponding parts of similar Δ are proportional]

In, ΔABD, by Pythagoras theorem

$AB^2 = AD^2 + BD^2$

$\Rightarrow BC^2 = AD^2 + BD^2$ [AB = BC given]

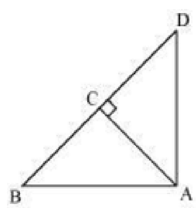
$\Rightarrow [2BD]^2 = AD^2 + BD^2$ [From (i)]

$\Rightarrow 4BD^2 - BD^2 = AD^2$

$\Rightarrow 3BD^2 = AD^2$

24.

Sol:



(i) In ΔADB and ΔCAB

∠DAB = ∠ACB = 90°

∠ABD = ∠CBA (common angle)

∠ADB = ∠CAB (remaining angle)

So, ΔADB ~ ΔCAB (by AAA similarity)

Therefore $\frac{AB}{CB} = \frac{BD}{AB}$

$\Rightarrow AB^2 = CB \times BD$

(ii) Let ∠CAB = x

In ΔCBA

∠CBA = 180° − 90° − x

∠CBA = 90° − x

Similarly in ΔCAD

∠CAD = 90° − $\angle CAD$ = 90° − x

∠CDA = 90° − ∠CAB

= 90° − x

∠CDA = 180° −90° − (90° − x)

∠CDA = x

Now in ΔCBA and ΔCAD we may observe that

∠CBA = ∠CAD

∠CAB = ∠CDA

∠ACB = ∠DCA = 90°

Therefore ΔCBA ~ ΔCAD (by AAA rule)

Therefore $\frac{AC}{DC} = \frac{BC}{AC}$

$\Rightarrow AC^2 = DC \times BC$

(iii) In ΔDCA & ΔDAB

∠DCA = ∠DAB (both are equal to 90°)

∠CDA = ∠ADB (common angle)

∠DAC = ∠DBA (remaining angle)

ΔDCA ~ ΔDAB (AAA property)

Therefore $\frac{DC}{DA} = \frac{DA}{DB}$

$\Rightarrow AD^2 = BD \times CD$

(iv) From part (i) $AB^2 = CB \times BD$

From part (ii) $AC^2 = DC \times BC$

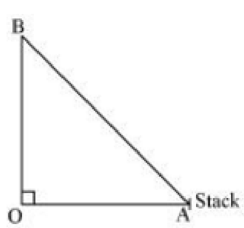
Hence $\frac{AB^2}{AC^2} = \frac{CB \times BD}{DC \times BC}$

$\frac{AB^2}{AC^2} = \frac{BD}{DC}$

Hence proved

25.

Sol:

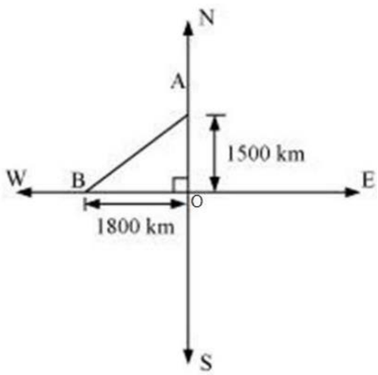


Let OB be the pole and AB be the wire. Therefore by pythagoras theorem,

$AB^2 = OB^2 + OA^2$
$24^2 = 18^2 + OA^2$
$OA^2 = 576 - 324$
OA = $\sqrt{252} = \sqrt{6 \times 6 \times 7} = 6\sqrt{7}$
Therefore distance from base = $6\sqrt{7}\ m$

26.

Sol:



Distance traveled by the plane flying towards north in $1\frac{1}{2}$ hrs
= $1000 \times 1\frac{1}{2} = 1500\ km$
Similarly, distance travelled by the plane flying towards west in $1\frac{1}{2}\ hrs$
= $1200 \times 1\frac{1}{2} = 1800\ km$
Let these distances are represented by OA and OB respectively.
Now applying Pythagoras theorem
Distance between these planes after $1\frac{1}{2}\ hrs$ AB = $\sqrt{OA^2 + OB^2}$
= $\sqrt{(1500)^2 + (1800)^2} = \sqrt{2250000 + 3240000}$
= $\sqrt{5490000} = \sqrt{9 \times 610000} = 300\sqrt{61}$
So, distance between these planes will be $300\sqrt{61}$ km, after $1\frac{1}{2}\ hrs$

27.

Sol:
Let ABC be the Δ with
AB = (a – 1) cm BC = $2\sqrt{a}$ cm, CA = (a + 1) cm
Hence, $AB^2 = (a-1)^2 = a^2 + 1 - 2a$
$BC^2 = \left(2\sqrt{a}\right)^2 = 4a$
$CA^2 = (a+1)^2 = a^2 + 1 + 2a$
Hence $AB^2 + BC^2 = AC^2$

$So\ \Delta\ ABC\ is\ right\ angled\ \Delta\ at\ B.$