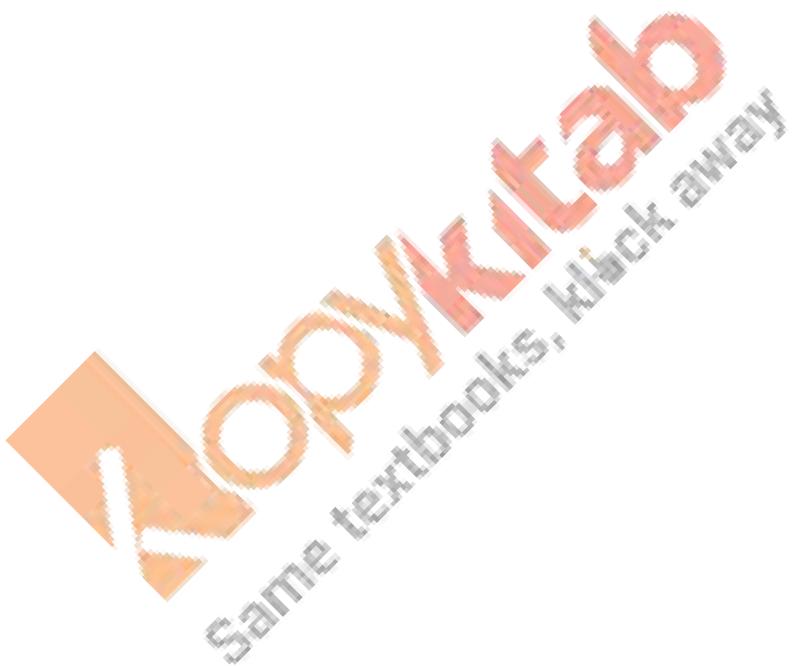


Exercise 3.1**Q1**

Akhila went to a fair in her village. She wanted to enjoy rides on the Giant Wheel and play Hoopla (a game in which you throw a ring on the items kept in the stall, and if the ring covers any object completely you get it). The number of times she played Hoopla is half the number of rides she had on the Giant Wheel. Each ride costs Rs 3, and a game of Hoopla costs Rs 4. If she spent Rs 20 in the fair, represent this situation algebraically and graphically.

Solution

The pair of equations formed is:

$$y = \frac{1}{2}x$$

$$\text{i.e., } x - 2y = 0 \quad (1)$$

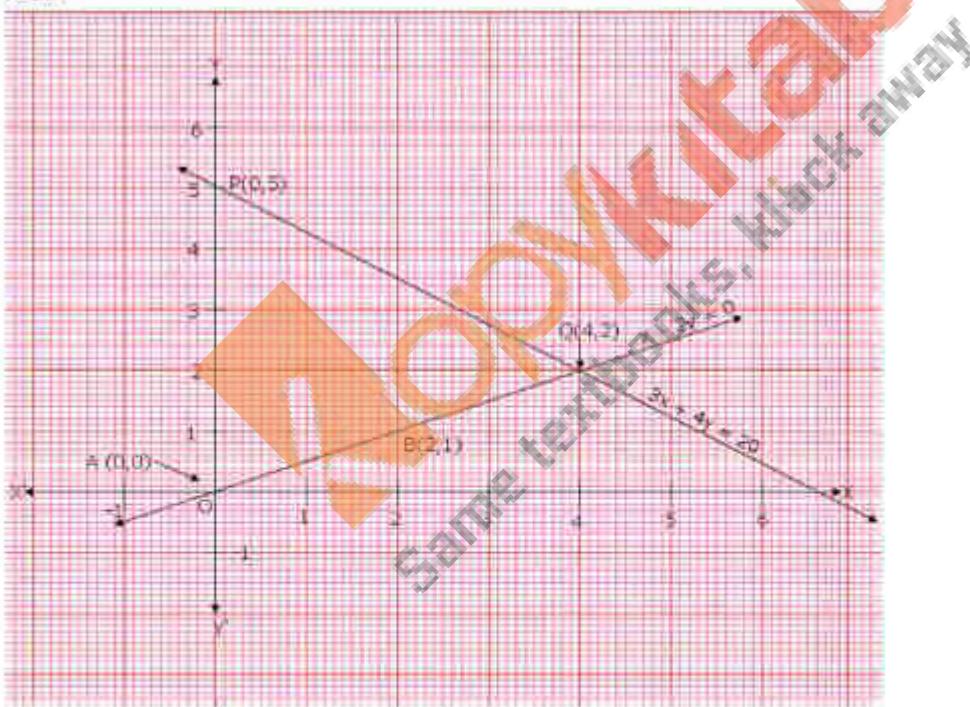
$$3x + 4y = 20 \quad (2)$$

Let us represent these equations graphically. For this, we need at least two solutions for each equation. We give these solutions in Table:

x	0	2
$y = \frac{1}{2}x$	0	1

x	0	$\frac{20}{3}$	4
$y = \frac{20 - 3x}{4}$	5	0	2

Recall from Class IX that there are infinitely many solutions of each linear equation. So, each of you choose any two values, which may not be the ones we have chosen. Can you guess why we have chosen $x = 0$ in the first equation and in the second equation? When one of the variables is zero, the equation reduces to a linear equation in one variable, which can be solved easily. For instance, putting $x = 0$ in Equation (2), we get $4y = 20$ i.e., $y = 5$. Similarly, putting $y = 0$ in Equation (2), we get $3x = 20$ i.e., $x = \frac{20}{3}$. But $\frac{20}{3}$ is not an integer; it will not be easy to plot exactly on the graph paper. So, we choose $y = 2$ which gives $x = 4$, an integral value.



Plot the points A(0,0), B(2,1) and P(0,5), C(4,2); now draw the lines AB and PQ, representing the equations $x - 2y = 0$ and $3x + 4y = 20$, as shown in fig.

In fig., observe that the two lines representing the two equations are intersecting at the point (4,2).

Q2

Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Solution

Let the present age of Aftab and his daughter be x and y respectively.

Seven years ago,
Age of Aftab = $x - 7$
Age of his daughter = $y - 7$

According to the given condition,

$$(x - 7) = 7(y - 7) \\ \Leftrightarrow x - 7 = 7y - 49 \\ \Leftrightarrow x - 7y = -42$$

Three years hence,
Age of Aftab = $x + 3$
Age of his daughter = $y + 3$

According to the given condition,

$$(x + 3) = 3(y + 3) \\ \Leftrightarrow x + 3 = 3y + 9 \\ \Leftrightarrow x - 3y = 6$$

Thus, the given conditions can be algebraically represented as
 $x - 7y = -42$
 $x - 3y = 6$

$$x - 7y = -42 \Rightarrow x = -42 + 7y$$

Three solutions of this equation can be written in a table as follows:

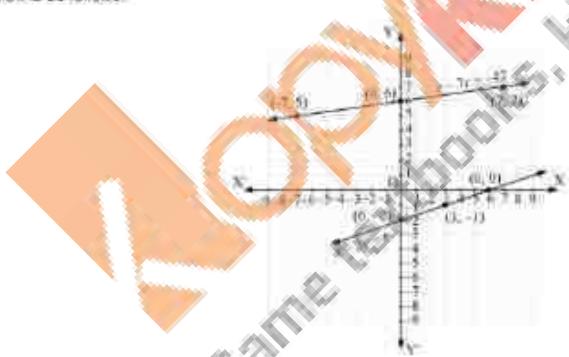
x	-7	0	7
y	5	5	7

$$x - 3y = 6 \Rightarrow x = 6 + 3y$$

Three solutions of this equation can be written in a table as follows:

x	3	0	-3	0
y	0	-1	-2	0

The graphical representation is as follows:



Concept insight: In order to represent a given situation mathematically, first see what we need to find out in the problem. Here, Aftab and his daughter's present age needs to be found so, so the ages will be represented by variables x and y . The problem talks about their ages seven years ago and three years from now. Here, the words, 'seven years ago' means we have to subtract 7 from their present ages, and 'three years from now' or 'three years hence' means we have to add 3 to their present ages. Remember in order to represent the algebraic equations graphically the solution set of equations must be taken as whole numbers only for the accuracy. Graph of the two linear equations will be represented by a straight line.

Q3

The path of a train A is given by the equation $3x + 4y - 12 = 0$ and the path of another train B is given by the equation $6x + 8y - 48 = 0$. Represent this situation graphically.

Solution

The paths of two trains are given by the following pair of linear equations.

$$3x + 4y - 12 = 0 \quad \text{--- (i)}$$

$$6x + 8y - 48 = 0 \quad \text{--- (ii)}$$

In order to represent the above pair of linear equations graphically, we need two points on the line representing each equation. That is, we find two solutions of each equations as given below:

We have,

$$3x + 4y - 12 = 0$$

Putting $y = 0$, we get

$$3x + 4 \times 0 - 12 = 0$$

$$\Rightarrow 3x = 12$$

$$\Rightarrow x = \frac{12}{3} = 4$$

Putting $x = 0$, we get

$$3 \times 0 + 4y - 12 = 0$$

$$\Rightarrow 4y = 12$$

$$\Rightarrow y = \frac{12}{4} = 3$$

Thus, two solutions of equation $3x + 4y - 12 = 0$ are:

(0,3) and (4,0)

We have,

$$6x + 8y - 48 = 0$$

Putting $x = 0$, we get

$$6 \times 0 + 8y - 48 = 0$$

$$\Rightarrow 8y = 48$$

$$\Rightarrow y = \frac{48}{8}$$

$$\Rightarrow y = 6$$

Putting $y = 0$, we get

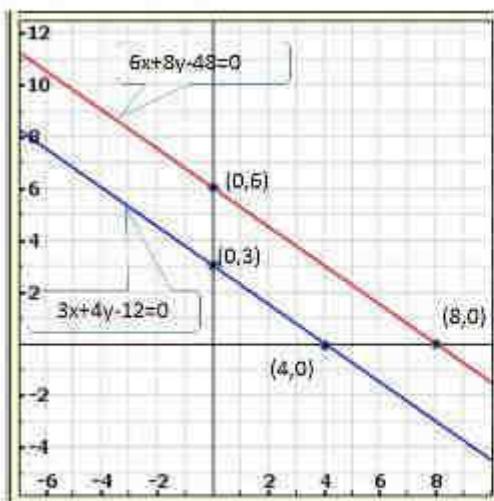
$$6x + 8 \times 0 - 48 = 0$$

$$\Rightarrow 6x = 48$$

$$\Rightarrow x = \frac{48}{6} = 8$$

Thus, two solutions of equation $6x + 8y - 48 = 0$ are:

(0,6) and (8,0)



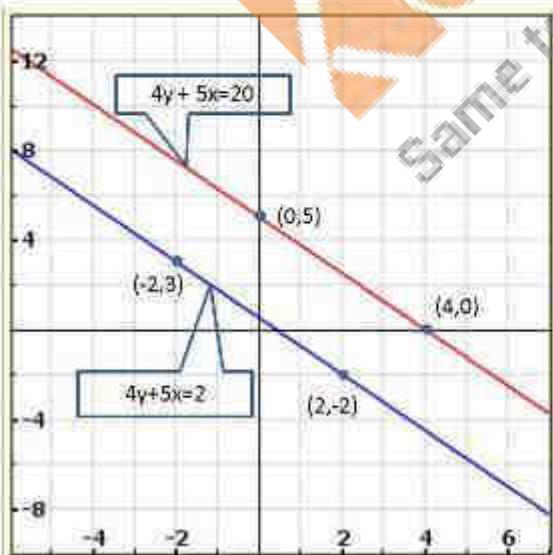
We observe that the lines are parallel and they do not intersect anywhere.

Q4

Gloria is walking along the path joining $(-2, 3)$ and $(2, -2)$, while Suresh is walking along the path joining $(0, 5)$ and $(4, 0)$. Represent this situation graphically.

Solution

It is given that Gloria is walking along the path joining $(-2, 3)$ and $(2, -2)$, while Suresh is walking along the path joining $(0, 5)$ and $(4, 0)$.



We observe that the lines are parallel and they do not intersect anywhere.

Q5

On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, and without drawing them, find out whether

the lines representing the following pairs of linear equations intersect at a point, are parallel or coincide:

$$5x - 4y + 8 = 0$$

$$7x + 6y - 9 = 0$$

Solution

We have,

$$5x - 4y + 8 = 0$$

$$7x + 6y - 9 = 0$$

Here, $a_1 = 5$, $b_1 = -4$, $c_1 = 8$,

$a_2 = 7$, $b_2 = 6$, $c_2 = -9$

We have,

$$\frac{a_1}{a_2} = \frac{5}{7}, \quad \frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3} \quad \text{and} \quad \frac{c_1}{c_2} = \frac{8}{-9} = \frac{-8}{9}$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

\therefore Two lines are intersecting with each other at a point.

Q6

On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, and without drawing them, find out whether

the lines representing the following pairs of linear equations intersect at a point, are parallel or coincide:

$$9x + 3y + 12 = 0$$

$$18x + 6y + 24 = 0$$

Solution

We have,

$$9x + 3y + 12 = 0$$

$$18x + 6y + 24 = 0$$

Here, $a_1 = 9$, $b_1 = 3$, $c_1 = 12$,
 $a_2 = 18$, $b_2 = 6$, $c_2 = 24$

Now,

$$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2},$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2},$$

$$\text{and } \frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

\therefore Both the lines coincide.

Q7

On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, and without drawing them, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincide:

$$6x - 3y + 10 = 0$$

$$2x - y + 9 = 0$$

Solution

We have,

$$6x - 3y + 10 = 0$$

$$2x - y + 9 = 0$$

Here, $a_1 = 6, b_1 = -3, c_1 = 10,$
 $a_2 = 2, b_2 = -1, c_2 = 9$

Now,

$$\frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1},$$

$$\frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1},$$

and $\frac{c_1}{c_2} = \frac{10}{9}$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

\therefore The lines are parallel.

Q8

Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is intersecting lines.

Solution

We have,

$$2x + 3y - 8 = 0$$

Let another equation of line is:

$$4x + 9y - 4 = 0$$

Here,

$$a_1 = 2, b_1 = 3, c_1 = -8,$$

$$a_2 = 4, b_2 = 9, c_2 = -4$$

Now,

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},$$

$$\frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3},$$

$$\text{and } \frac{c_1}{c_2} = \frac{-8}{-4} = \frac{2}{1}$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\therefore 2x + 3y - 8 = 0$ and $4x + 9y - 4 = 0$ intersect each other at one point.

Hence, required equation of line is $4x + 9y - 4 = 0$.

Q9

Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is parallel lines.

Solution

We have,

$$2x + 3y - 8 = 0$$

Let another equation of line is:

$$4x + 6y - 4 = 0$$

Here,

$$a_1 = 2, b_1 = 3, c_1 = -8,$$

$$a_2 = 4, b_2 = 6, c_2 = -4$$

Now,

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2},$$

$$\text{and } \frac{c_1}{c_2} = \frac{-8}{-4} = \frac{2}{1}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

\therefore Lines are parallel to each other.

Hence, required equation of line is $4x + 6y - 4 = 0$.

Q10

Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is coincident lines.

Solution

We have,

$$2x + 3y - 8 = 0$$

Let another equation of line is:

$$4x + 6y - 16 = 0$$

Here,

$$a_1 = 2, b_1 = 3, c_1 = -8,$$

$$a_2 = 4, b_2 = 6, c_2 = -16$$

Now,

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2},$$

$$\text{and } \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

\therefore Lines are coincident.

Hence, required equation of line is $4x + 6y - 16 = 0$.

Q11

The cost of 2 kg of apples and 1 kg of grapes on a day was found to be Rs 160. On another day, the cost of 4 kg of apples and 2 kg of grapes is Rs 300. Represent the situation algebraically and geometrically.

Solution

Let the cost of 1 kg of apples and 1 kg grapes be Rs x and Rs y
The given conditions can be algebraically represented as:

$$2x + y = 160$$

$$4x + 2y = 300$$

$$2x + y = 160 \Rightarrow y = 160 - 2x$$

Three solutions of this equation can be written in a table as follows:

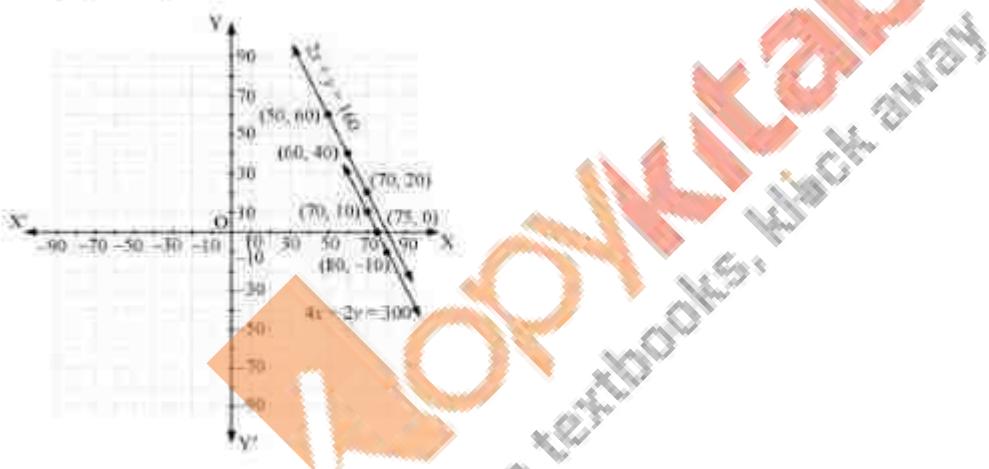
x	50	60	70
y	60	40	20

$$4x + 2y = 300 \Rightarrow y = \frac{300 - 4x}{2}$$

Three solutions of this equation can be written in a table as follows:

x	70	80	75
y	10	-10	0

The graphical representation is as follows:



Concept insight: cost of apples and grapes needs to be found so the cost of 1 kg apples and 1 kg grapes will be taken as the variables. From the given conditions of collective cost of apples and grapes, a pair of linear equations in two variables will be obtained. Then, in order to represent the obtained equations graphically, take the values of variables as whole numbers only. Since these values are large so take the suitable scale.

Exercise 3.2**Q1**

Solve the following systems of equations graphically:

$$x + y = 3$$

$$2x + 5y = 12$$

Solution

We have,

$$\begin{aligned}x + y &= 3 \\2x + 5y &= 12\end{aligned}$$

Now,

$$x + y = 3$$

When $y = 0$, we have

$$x = 3$$

When $x = 0$, we have

$$y = 3$$

Thus, we have the following table giving points on the line $x + y = 3$

x	0	3
y	3	0

Now,

$$2x + 5y = 12$$

$$\Rightarrow y = \frac{12 - 2x}{5}$$

When $x = 1$, we have

$$y = \frac{12 - 2(1)}{5} = 2$$

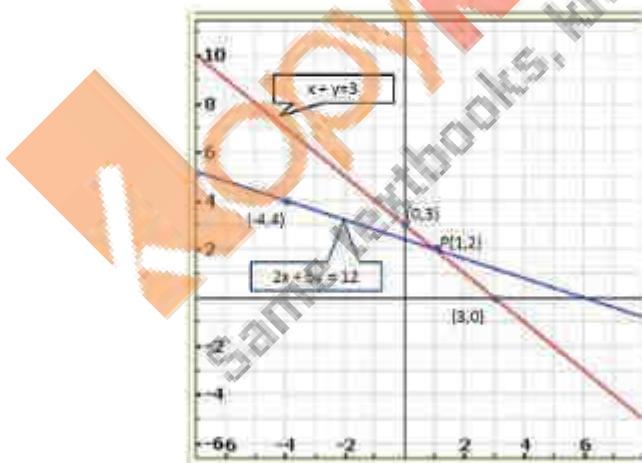
When $x = -4$, we have

$$y = \frac{12 - 2(-4)}{5} = 4$$

Thus, we have the following table giving points on the line $2x + 5y = 12$

x	1	-4
y	2	4

Graph of the equations $x + y = 3$ and $2x + 5y = 12$:



Clearly, two lines intersect at $P(1, 2)$.

Hence, $x = 1, y = 2$ is the solution of the given system of equations.

Q2

Solve the following systems of equations graphically:

$$\begin{aligned}x - 2y &= 5 \\2x + 3y &= 10\end{aligned}$$

Solution

We have,

$$x - 2y = 5$$

$$2x + 3y = 10$$

Now,

$$x - 2y = 5$$

$$\Rightarrow x = 5 + 2y$$

When $y = 0$, we have

$$x = 5 + 2 \times 0 = 5$$

When $y = -2$, we have

$$x = 5 + 2 \times (-2) = 1$$

Thus, we have the following table giving points on the line $x - 2y = 5$:

x	5	1
y	0	-2

Now,

$$2x + 3y = 10$$

$$\Rightarrow 2x = 10 - 3y$$

$$\Rightarrow x = \frac{10 - 3y}{2}$$

When $y = 0$, we have

$$x = \frac{10}{2} = 5$$

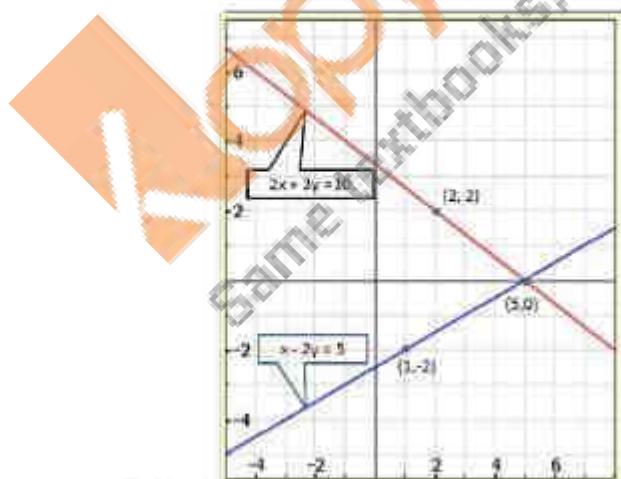
When $y = 2$, we have

$$x = \frac{10 - 3 \times 2}{2} = 2$$

Thus, we have the following table giving points on the line $2x + 3y = 10$:

x	5	2
y	0	2

Graph of the equations $x - 2y = 5$ and $2x + 3y = 10$:



Clearly, two lines intersect at $(5, 0)$.

Hence, $x = 5, y = 0$ is the solution of the given system of equations.

Q3

Solve the following systems of equations graphically:

$$3x + y + 1 = 0$$

$$2x - 3y + 8 = 0$$

Solution

We have,

$$\begin{aligned}3x + y + 1 &= 0 \\2x - 3y + 8 &= 0\end{aligned}$$

Now,

$$3x + y + 1 = 0$$

$$\Rightarrow y = -1 - 3x$$

When $x = 0$, we have

$$y = -1$$

When $x = -1$, we have

$$y = -1 - 3 \times (-1) = 2$$

Thus, we have the following table giving points on the line $3x + y + 1 = 0$

x	-1	0
y	2	-1

Now,

$$2x - 3y + 8 = 0$$

$$\Rightarrow 2x = 3y - 8$$

$$\Rightarrow x = \frac{3y - 8}{2}$$

When $y = 0$, we have

$$x = \frac{3 \times 0 - 8}{2} = -4$$

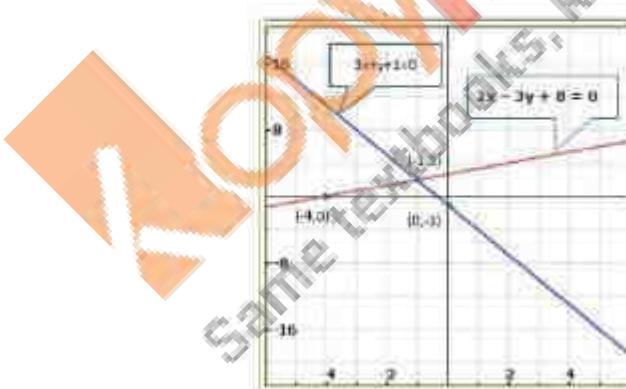
When $y = 2$, we have

$$x = \frac{3 \times 2 - 8}{2} = -1$$

Thus, we have the following table giving points on the line $2x - 3y + 8 = 0$

x	-4	-1
y	0	-2

Graph of the equations are:



Clearly, two lines intersect at $(-1, 2)$.

Hence, $x = -1, y = 2$ is the solution of the given system of equations.

Q4

Solve the following systems of equations graphically:

$$2x + y - 3 = 0$$

$$2x - 3y - 7 = 0$$

Solution

We have,

$$2x + y - 3 = 0$$

$$2x - 3y - 7 = 0$$

Now,

$$2x + y - 3 = 0$$

$$\Rightarrow y = 3 - 2x$$

When $x = 0$, we have

$$y = 3$$

When $x = 1$, we have

$$y = 1$$

Thus, we have the following table giving points on the line $2x + y - 3 = 0$

x	0	1
y	3	1

Now,

$$2x - 3y - 7 = 0$$

$$\Rightarrow 3y = 2x - 7$$

$$\Rightarrow y = \frac{2x - 7}{3}$$

When $x = 5$, we have

$$y = \frac{2 \times 5 - 7}{3} = 1$$

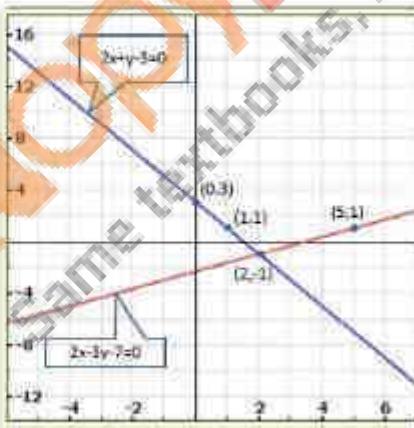
When $x = 2$, we have

$$y = \frac{2 \times 2 - 7}{3} = -1$$

Thus, we have the following table giving points on the line $2x - 3y - 7 = 0$

x	2	5
y	-1	1

Graph of the given equations are:



Clearly, two lines intersect at $(2, -1)$.

Hence, $x = 2, y = -1$ is the solution of the given system of equations.

Q5

Solve the following systems of equations graphically:

$$x + y = 5$$

$$x - y = 2$$

Solution

We have,

$$x + y = 6$$

$$x - y = 2$$

Now,

$$x + y = 6$$

$$\Rightarrow y = 6 - x$$

When $x = 2$, we have

$$y = 4$$

When $x = 3$, we have

$$y = 3$$

Thus, we have the following table giving points on the line $x + y = 6$

x	2	3
y	4	3

Now,

$$x - y = 2$$

$$\Rightarrow y = x - 2$$

When $x = 0$, we have

$$y = -2$$

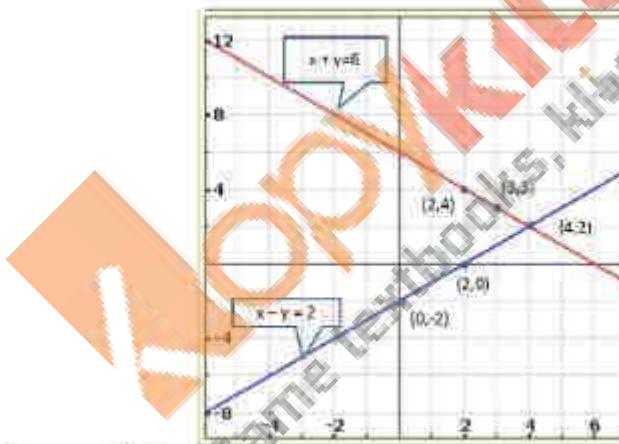
When $x = 2$, we have

$$y = 0$$

Thus, we have the following table giving points on the line $x - y = 2$

x	0	2
y	-2	0

Graph of the given equations are:



Clearly, two lines intersect at $(4, 2)$.

Hence, $x = 4, y = 2$ is the solution of the given system of equations.

Q6

Solve the following systems of equations graphically:

$$x - 2y = 6$$

$$3x - 6y = 8$$

Solution

We have,

$$x - 2y = 6$$

$$3x - 6y = 0$$

Now,

$$x - 2y = 6$$

$$\Rightarrow x = 6 + 2y$$

When $y = -2$, we have

$$x = 6 + 2 \times -2 = 2$$

When $y = -3$, we have

$$x = 6 + 2 \times -3 = 0$$

Thus, we have the following table giving points on the line $x - 2y = 6$

x	2	0
y	-2	-3

Now,

$$3x - 6y = 0$$

$$\Rightarrow 3y = 6y$$

$$\Rightarrow x = 2y$$

When $y = 0$, we have

$$x = 0$$

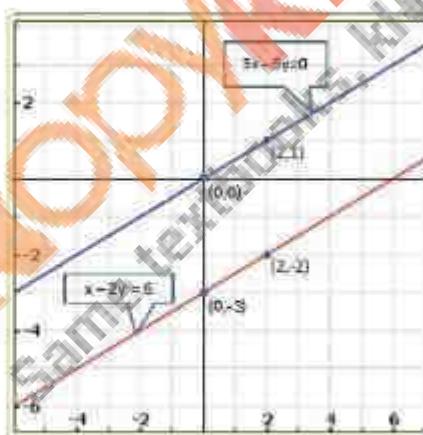
When $y = 1$, we have

$$x = 2$$

Thus, we have the following table giving points on the line $3x - 6y = 0$

x	0	2
y	0	1

Graph of the given equations are:



Clearly, two lines are parallel to each other. So, the two lines have no common point.

Hence, the given system of equations has no solution.

Q7

Solve the following systems of equations graphically:

$$x + y = 4$$

$$2x - 3y = 3$$

Solution

We have,

$$x + y = 4$$

$$2x - 3y = 3$$

Now,

$$x + y = 4$$

$$\Rightarrow x = 4 - y$$

When $y = 0$, we have

$$x = 4$$

When $y = 2$, we have

$$x = 2$$

Thus, we have the following table giving points on the line $x + y = 4$

x	4	2
y	0	2

Now,

$$2x - 3y = 3$$

$$\Rightarrow 2x - 3y + 3 = 0$$

$$\Rightarrow x = \frac{3y + 3}{2}$$

When $y = 1$, we have

$$x = 3$$

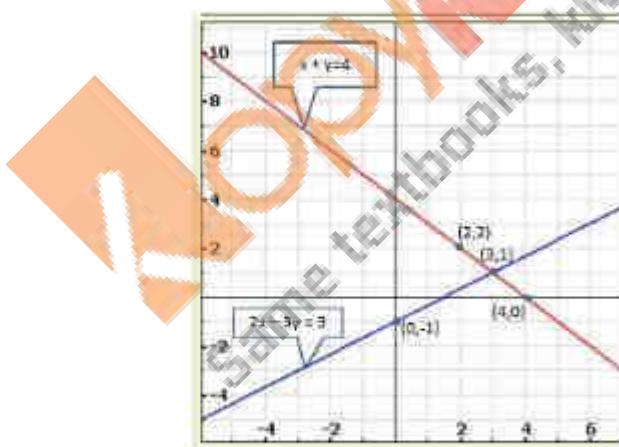
When $y = -1$, we have

$$x = 0$$

Thus, we have the following table giving points on the line $2x - 3y = 3$

x	3	0
y	1	-1

Graph of the given equations are:



Clearly, two lines intersect at $(3, 1)$.

Hence, $x = 3, y = 1$ is the solution of the given system of equations.

Q8

Solve the following systems of equations graphically:

$$2x + 3y = 4$$

$$x - y + 3 = 0$$

Solution

We have,

$$2x + 3y = 4$$

$$x - y + 3 = 0$$

Now,

$$2x + 3y = 4$$

$$\Rightarrow 2x = 4 - 3y$$

$$\Rightarrow x = \frac{4 - 3y}{2}$$

When $y = 0$, we have

$$x = \frac{4 - 3(0)}{2} = 2$$

When $y = 2$, we have

$$x = \frac{4 - 3 \times 2}{2} = -1$$

Thus, we have the following table giving points on the line $2x + 3y = 4$

x	-1	2
y	2	0

Now,

$$x - y + 3 = 0$$

$$\Rightarrow x = y - 3$$

When $y = 3$, we have

$$x = 0$$

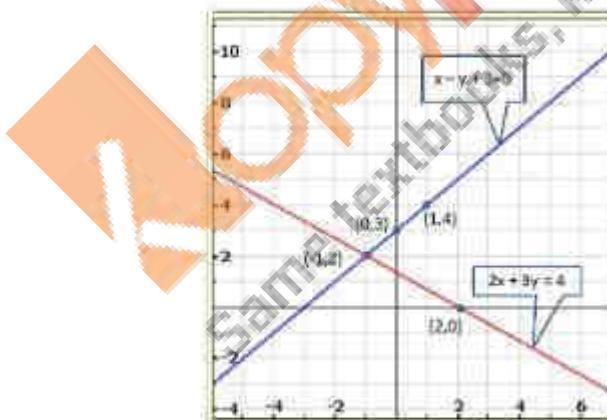
When $y = 4$, we have

$$x = 1$$

Thus, we have the following table giving points on the line $x - y + 3 = 0$

x	0	1
y	3	4

Graph of the given equations are:



Clearly, two lines intersect at $(-1, 2)$.

Hence, $x = -1$, $y = 2$ is the solution of the given system of equations.

Q9

Solve the following systems of equations graphically:

$$2x - 3y + 13 = 0$$

$$3x - 2y + 12 = 0$$

Solution

We have,

$$2x - 3y + 13 = 0$$

$$3x - 2y + 12 = 0$$

Now,

$$2x - 3y + 13 = 0$$

$$\Rightarrow 2x = 3y - 13$$

$$\Rightarrow x = \frac{3y - 13}{2}$$

When $y = 1$, we have

$$x = \frac{3 \times 1 - 13}{2} = -5$$

When $y = 3$, we have

$$x = \frac{3 \times 3 - 13}{2} = -2$$

Thus, we have the following table giving points on the line $2x - 3y + 13 = 0$

X	-5	-2
Y	1	3

Now,

$$3x - 2y + 12 = 0$$

$$\Rightarrow 3x = 2y - 12$$

$$\Rightarrow x = \frac{2y - 12}{3}$$

When $y = 0$, we have

$$x = \frac{2 \times 0 - 12}{3} = -4$$

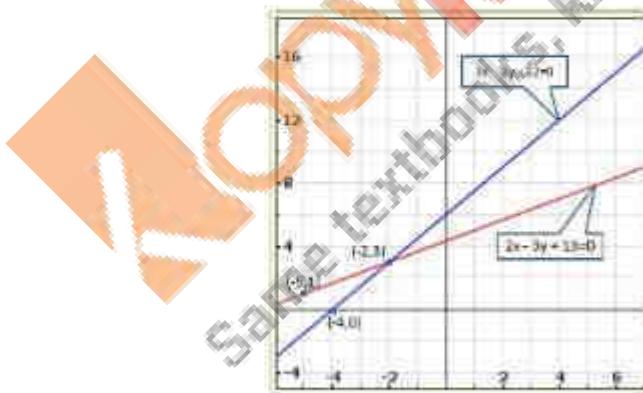
When $y = 3$, we have

$$x = \frac{2 \times 3 - 12}{3} = -2$$

Thus, we have the following table giving points on the line $3x - 2y + 12 = 0$

X	-4	-2
Y	0	3

Graph of the given equations are:



Clearly, two lines intersect at $(-2, 3)$.

Hence, $x = -2, y = 3$ is the solution of the given system of equations.

Q10

Solve the following systems of equations graphically:

$$2x + 3y + 5 = 0$$

$$3x - 2y - 12 = 0$$

Solution

We have,

$$2x + 3y + 5 = 0$$

$$3x - 2y - 12 = 0$$

Now,

$$2x + 3y + 5 = 0$$

$$\Rightarrow 2x = -3y - 5$$

$$\Rightarrow x = \frac{-3y - 5}{2}$$

When $y = 1$, we have

$$x = \frac{-3 \times 1 - 5}{2} = -4$$

When $y = -1$, we have

$$x = \frac{-3 \times (-1) - 5}{2} = -1$$

Thus, we have the following table giving points on the line $2x + 3y + 5 = 0$

x	-4	-1
y	2	-1

Now,

$$3x - 2y - 12 = 0$$

$$\Rightarrow 3x - 2y + 12$$

$$\Rightarrow x = \frac{2y + 12}{3}$$

When $y = 0$, we have:

$$x = \frac{2 \times 0 + 12}{3} = 4$$

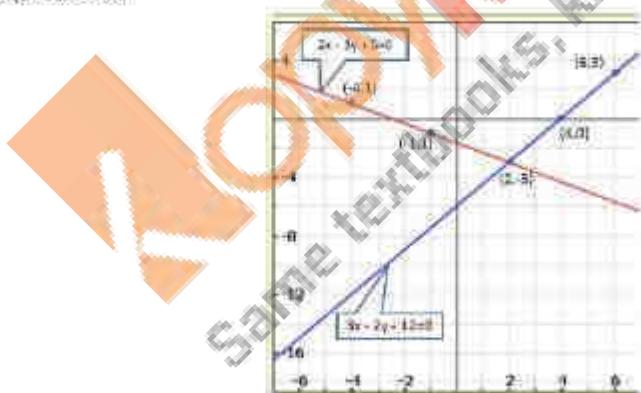
When $y = 3$, we have:

$$x = \frac{2 \times 3 + 12}{3} = 6$$

Thus, we have the following table giving points on the line $3x - 2y - 12 = 0$

x	4	6
y	0	3

Graph of the given equations are:



Clearly, two lines intersect at $(2, -3)$.

Hence, $x = 2$, $y = -3$ is the solution of the given system of equations.

Q11

Show graphically that each one of the following systems of equations has infinitely many solutions:

$$2x + 3y = 6$$

$$4x + 6y = 12$$

Solution

We have:

$$\begin{aligned} 2x + 3y &= 6 \\ 4x + 6y &= 12 \end{aligned}$$

Now,

$$\begin{aligned} 2x + 3y &= 6 \\ \Rightarrow 2x &= 6 - 3y \\ \Rightarrow x &= \frac{6 - 3y}{2} \end{aligned}$$

When $y = 0$, we have:

$$x = 3$$

When $y = 2$, we have

$$x = \frac{6 - 3 \times 2}{2} = 0$$

Thus, we have the following table giving points on the line $2x + 3y = 6$:

x	0	3
y	2	0

Now,

$$\begin{aligned} 4x + 6y &= 12 \\ \Rightarrow 4x &= 12 - 6y \\ \Rightarrow x &= \frac{12 - 6y}{4} \end{aligned}$$

When $y = 0$, we have:

$$x = 3$$

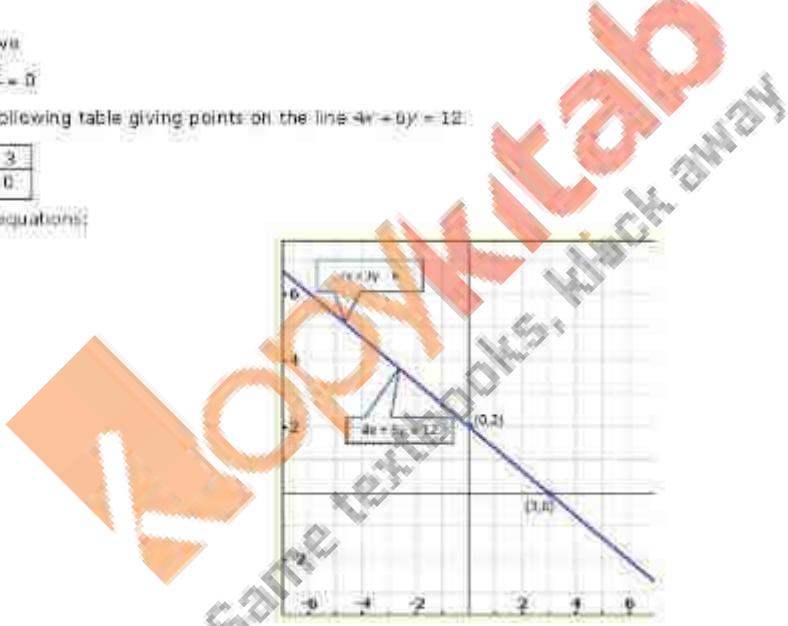
When $y = 2$, we have

$$x = \frac{12 - 6 \times 2}{4} = 0$$

Thus, we have the following table giving points on the line $4x + 6y = 12$:

x	0	3
y	2	0

Graph of the given equations:



Thus, the graphs of the two equations are coincident.

Hence, the system of equations has infinitely many solutions.

Q12

Show graphically that each one of the following systems of equations has infinitely many solutions.

$$\begin{aligned} x - 2y &= 5 \\ 2x - 6y &= 15 \end{aligned}$$

Solution

We have,

$$\begin{aligned}x - 2y &= 5 \\3x - 6y &= 15\end{aligned}$$

Now,

$$x - 2y = 5$$

$$\Rightarrow x = 2y + 5$$

When $y = -1$, we have

$$x = 2(-1) + 5 = 3$$

When $y = 0$, we have

$$x = 2 \times 0 + 5 = 5$$

Thus, we have the following table giving points on the line $x - 2y = 5$

x	3	5
y	1	0

Now,

$$3x - 6y = 15$$

$$\Rightarrow 3x = 15 + 6y$$

$$\Rightarrow x = \frac{15 + 6y}{3}$$

When $y = -2$, we have

$$x = \frac{15 + 6(-2)}{3} = 1$$

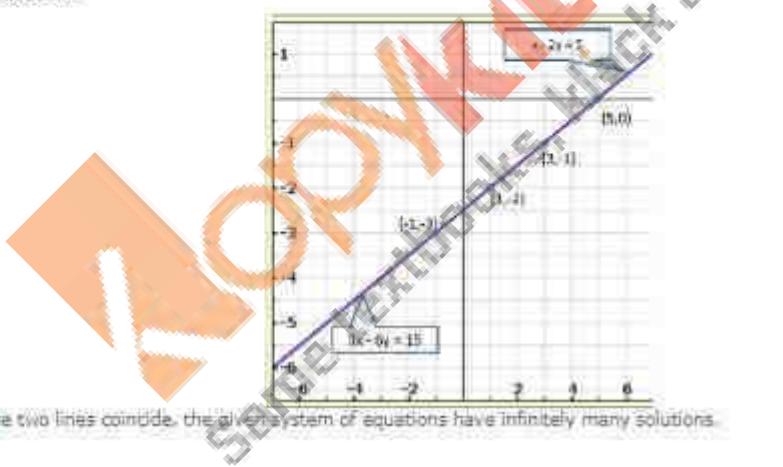
When $y = -3$, we have

$$x = \frac{15 + 6(-3)}{3} = -1$$

Thus, we have the following table giving points on the line $3x - 6y = 15$

x	1	-1
y	-2	-3

Graph of the given equations:



Since the graph of the two lines coincide, the given system of equations have infinitely many solutions.

Q13

Show graphically that each one of the following systems of equations has infinitely many solutions.

$$3x + y = 8$$

$$6x + 2y = 16$$

Solution

We have,

$$\begin{aligned} 3x + y &= 8 \\ 6x + 2y &= 16 \end{aligned}$$

Now,

$$\begin{aligned} 3x + y &= 8 \\ \Rightarrow y &= 8 - 3x \end{aligned}$$

When $x = 2$, we have

$$y = 8 - 3 \times 2 = 2$$

When $x = 3$, we have

$$y = 8 - 3 \times 3 = -1$$

Thus, we have the following table giving points on the line $3x + y = 8$:

x	2	3
y	2	-1

Now,

$$\begin{aligned} 6x + 2y &= 16 \\ \Rightarrow 2y &= 16 - 6x \\ \Rightarrow y &= \frac{16 - 6x}{2} \end{aligned}$$

When $x = 1$, we have

$$y = \frac{16 - 6 \times 1}{2} = 5$$

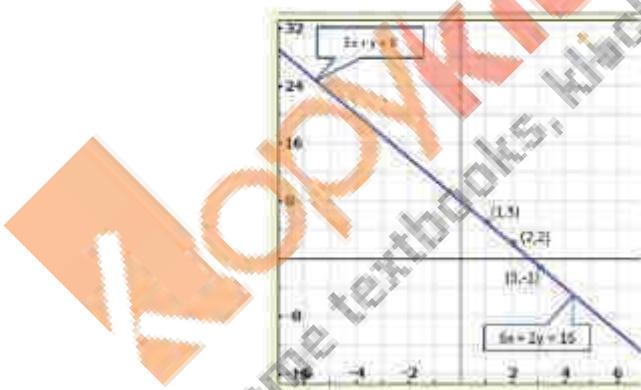
When $x = 3$, we have

$$y = \frac{16 - 6 \times 3}{2} = -1$$

Thus, we have the following table giving points on the line $6x + 2y = 16$:

x	1	3
y	5	-1

Graph of the given equations:



Thus, the graphs of the two equations are coincident.

Hence, the system of equations has infinitely many solutions.

Q14

Show graphically that each one of the following systems of equations has infinitely many solutions:

$$\begin{aligned} x - 2y + 13 &= 0 \\ 3x - 6y + 33 &= 0 \end{aligned}$$

Solution

We have,

$$\begin{aligned}x - 2y + 11 &= 0 \\3x - 6y + 33 &= 0\end{aligned}$$

Now,

$$x - 2y + 11 = 0$$

$$\Rightarrow x - 2y = -11$$

When $y = 5$, we have

$$x = 2 \times 5 - 11 = -1$$

When $y = 4$, we have

$$x = 2 \times 4 - 11 = -3$$

Thus, we have the following table giving points on the line $x - 2y + 11 = 0$

x	-1	-3
y	5	4

Now,

$$3x - 6y + 33 = 0$$

$$\Rightarrow 3x + 6y = 33$$

$$\Rightarrow x = \frac{6y - 33}{3}$$

When $y = 5$, we have

$$x = \frac{6 \times 5 - 33}{3} = 1$$

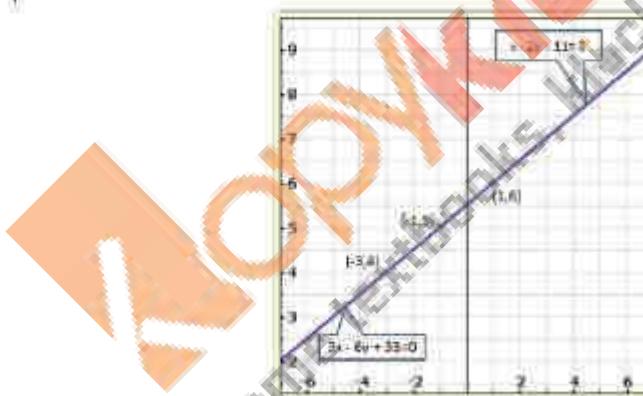
When $y = 4$, we have:

$$x = \frac{6 \times 4 - 33}{3} = -1$$

Thus, we have the following table giving points on the line $3x - 6y + 33 = 0$

x	1	-1
y	5	4

Graph of the given equations:



Thus, the graphs of the two equations are coincident.

Hence, the system of equations has infinitely many solutions.

Q15

Show graphically that each one of the following systems of equations is in-consistent (i.e. has no solution):

$$3x - 5y = 20$$

$$6x - 10y = -40$$

Solution

We have,

$$3x - 5y = 20$$

$$6x - 10y = -40$$

Now,

$$3x - 5y = 20$$

$$\Rightarrow 3x = 5y + 20$$

$$\Rightarrow x = \frac{5y + 20}{3}$$

When $y = -1$, we have

$$x = \frac{5(-1) + 20}{3} = 5$$

When $y = -4$, we have

$$x = \frac{5(-4) + 20}{3} = 0$$

Thus, we have the following table giving points on the line $3x - 5y = 20$.

x	5	0
y	-1	-4

Now,

$$6x - 10y = -40$$

$$\Rightarrow 6x = -40 + 10y$$

$$\Rightarrow x = \frac{-40 + 10y}{6}$$

When $y = 4$, we have

$$x = \frac{-40 + 10 \times 4}{6} = 0$$

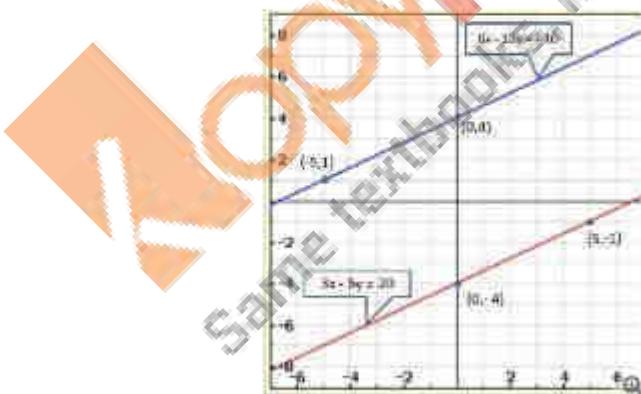
When $y = 1$, we have

$$x = \frac{-40 + 10 \times 1}{6} = -5$$

Thus, we have the following table giving points on the line $6x - 10y = -40$.

x	0	-5
y	-4	1

Graph of the given equations:



Clearly, there is no common point between these two lines.

Hence, given system of equations is inconsistent.

Q16

Show graphically that each one of the following systems of equations is inconsistent (i.e., has no solution):

$$x - 2y = 0$$

$$3x - 6y = 0$$

Solution

We have,

$$\begin{aligned}x - 2y &= 6 \\3x - 6y &= 0\end{aligned}$$

Now,

$$\begin{aligned}x - 2y &= 6 \\&\Rightarrow x = 6 + 2y\end{aligned}$$

When $y = 0$, we have

$$x = 6 + 2 \times 0 = 6$$

When $y = -2$, we have

$$x = 6 + 2 \times (-2) = 2$$

Thus, we have the following table giving points on the line $x - 2y = 6$

x	6	2
y	0	-2

Now,

$$\begin{aligned}3x - 6y &= 0 \\&\Rightarrow 3x = 6y \\&\Rightarrow x = \frac{6y}{3} \\&\Rightarrow x = 2y\end{aligned}$$

When $y = 0$, we have

$$x = 2 \times 0 = 0$$

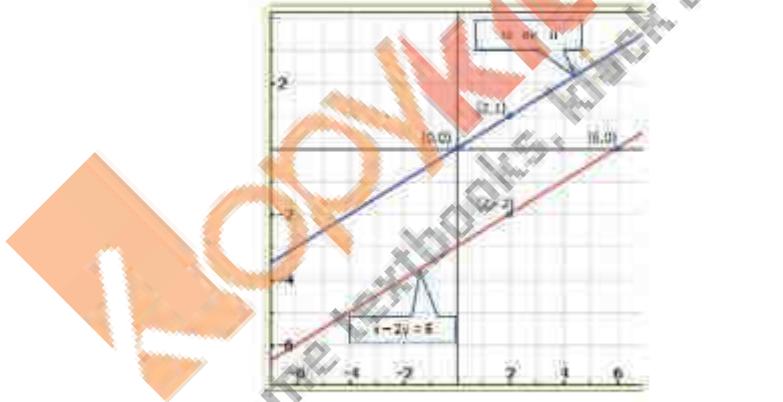
When $y = 1$, we have

$$x = 2 \times 1 = 2$$

Thus, we have the following table giving points on the line $3x - 6y = 0$

x	0	2
y	0	1

Graph of the given equations:



We find the lines represented by equations $x - 2y = 6$ and $3x - 6y = 0$ are parallel. So, the two lines have no common point.

Hence, the given system of equations is inconsistent.

Q17

Show graphically that each one of the following systems of equations is inconsistent (i.e., has no solution):

$$\begin{aligned}2y - x &= 9 \\6y - 3x &= 21\end{aligned}$$

Solution

We have,

$$\begin{aligned} 2y - x &= 9 \\ 6y - 3x &= 21 \end{aligned}$$

Now,

$$2y - x = 9$$

$$\Rightarrow 2y - 9 = x$$

$$\Rightarrow x = 2y - 9$$

When $y = 3$, we have

$$x = 2 \times 3 - 9 = -3$$

When $y = 4$, we have

$$x = 2 \times 4 - 9 = -1$$

Thus, we have the following table giving points on the line $2x - y = 9$

x	-3	-1
y	3	4

Now,

$$6y - 3x = 21$$

$$\Rightarrow 6y - 21 = 3x$$

$$\Rightarrow 3y - 7 = 2x$$

$$\Rightarrow x = \frac{3(2y - 7)}{3}$$

$$\Rightarrow x = 2y - 7$$

When $y = 2$, we have

$$x = 2 \times 2 - 7 = -3$$

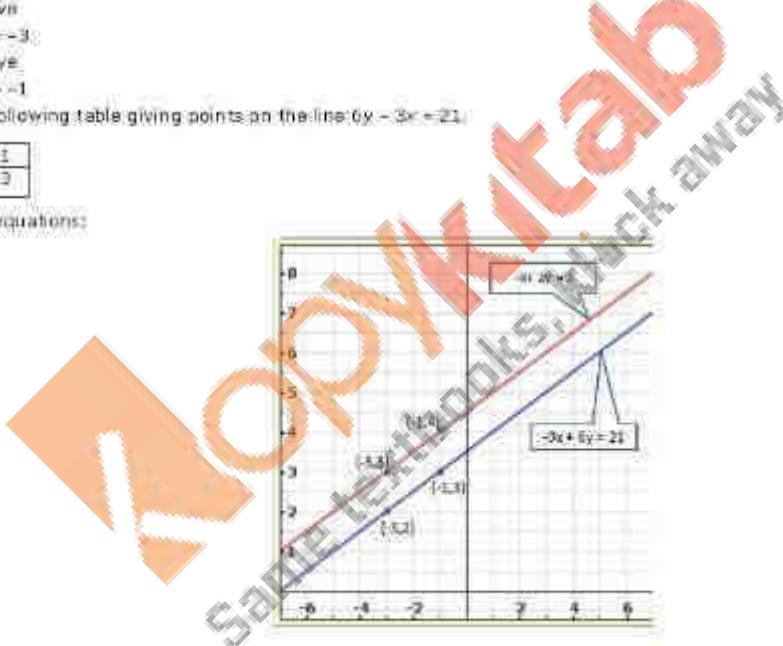
When $y = 3$, we have

$$x = 2 \times 3 - 7 = -1$$

Thus, we have the following table giving points on the line $6y - 3x = 21$.

x	-3	-1
y	2	3

Graph of the given equations:



We find the lines represented by equations $2y - x = 9$ and $6y - 3x = 21$ are parallel. So, the two lines have no common point.

Hence, the given system of equations is inconsistent.

Q18

Show graphically that each one of the following systems of equations is inconsistent (i.e. has no solution).

$$3x - 4y - 1 = 0$$

$$2x - \frac{8}{3}y + 5 = 0$$

Solution

We have,

$$3x - 4y - 1 = 0$$

$$2x - \frac{8}{3}y + 5 = 0$$

Now,

$$3x - 4y - 1 = 0$$

$$\Rightarrow 3x = 1 + 4y$$

$$\Rightarrow x = \frac{1+4y}{3}$$

When $y = 2$, we have,

$$x = \frac{1+4 \times 2}{3} = 3$$

When $y = -1$, we have,

$$x = \frac{1+4 \times (-1)}{3} = -1$$

Thus, we have the following table giving points on the line $3x - 4y - 1 = 0$.

x	-1	3
y	-1	3

Now,

$$2x - \frac{8}{3}y + 5 = 0$$

$$\Rightarrow 6x - 8y + 15 = 0$$

$$\Rightarrow 6x - 8y + 15 = 0$$

$$\Rightarrow 6x = 8y - 15$$

$$\Rightarrow x = \frac{8y - 15}{6}$$

When $y = 0$, we have,

$$x = \frac{0 \times 0 - 15}{6} = -2.5$$

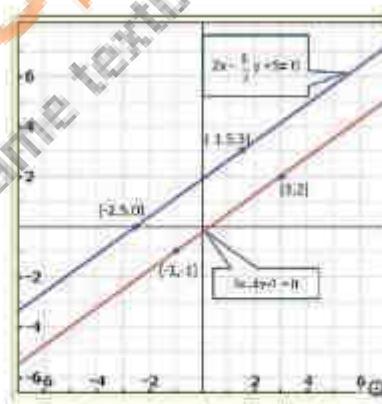
When $y = 3$, we have,

$$x = \frac{8 \times 3 - 15}{6} = 1.5$$

Thus, we have the following table giving points on the line $2x - \frac{8}{3}y + 5 = 0$.

x	-2.5	1.5
y	0	3

Graph of the given equations:



We find the lines represented by equations $3x - 4y - 1 = 0$ and $2x - \frac{8}{3}y + 5 = 0$ are parallel. So, the two lines have no common point.

Hence, the given system of equations is inconsistent.

Determine graphically the vertices of the triangle, the equations of whose sides are given below:

$$2y - x = 8, 5y - x = 14 \text{ and } y - 2x = 1$$

Solution

We have,

$$\begin{aligned} 2y - x &= 8 \\ 5y - x &= 14 \\ y - 2x &= 1 \end{aligned}$$

Now,

$$2y - x = 8$$

$$\Rightarrow 2y - 8 = x$$

$$\Rightarrow x = 2y - 8$$

When $y = 2$, we have

$$x = 2 \times 2 - 8 = -4$$

When $y = 4$, we have

$$x = 2 \times 4 - 8 = 0$$

Thus, we have the following table giving points on the line $2y - x = 8$:

x	-4	0
y	2	4

We have,

$$5y - x = 14$$

$$\Rightarrow 5y - 14 = x$$

$$\Rightarrow x = 5y - 14$$

When $y = 2$, we have

$$x = 5 \times 2 - 14 = -4$$

When $y = 3$, we have

$$x = 5 \times 3 - 14 = 1$$

Thus, we have the following table giving points on the line $5y - x = 14$:

x	-4	1
y	2	3

We have,

$$y - 2x = 1$$

$$\Rightarrow y - 1 = 2x$$

$$\Rightarrow x = \frac{y-1}{2}$$

When $y = 3$, we have

$$x = \frac{3-1}{2} = 1$$

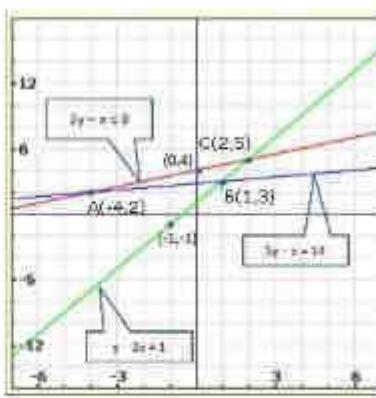
When $y = -1$, we have

$$x = \frac{-1-1}{2} = -1$$

Thus, we have the following table giving points on the line $y - 2x = 1$:

x	-1	1
y	1	3

Graph of the given equations:



From the graph of the lines represented by the given equations, we observe that the lines taken in pairs intersect each other at points $A\{-4, 2\}$, $B\{1, 3\}$ and $C\{2, 5\}$.

Hence, the vertices of the triangle are $A\{-4, 2\}$, $B\{1, 3\}$ and $C\{2, 5\}$.

Q20

Determine graphically the vertices of the triangle, the equations of whose sides are given below:

$$y = x, y = 0 \text{ and } 3x + 3y = 10$$

Solution

The given system of equations is

$$y = x$$

$$y = 0$$

$$3x + 3y = 10$$

We have,

$$y = x$$

When $x = 1$, we have

$$y = 1$$

When $x = -2$, we have

$$y = -2$$

Thus, we have the following table giving points on the line $y = x$

x	1	-2
y	1	-2

We have,

$$3x + 3y = 10$$

$$\Rightarrow 3y = 10 - 3x$$

$$\Rightarrow y = \frac{10 - 3x}{3}$$

When $x = 1$, we have

$$y = \frac{10 - 3 \times 1}{3} = \frac{7}{3}$$

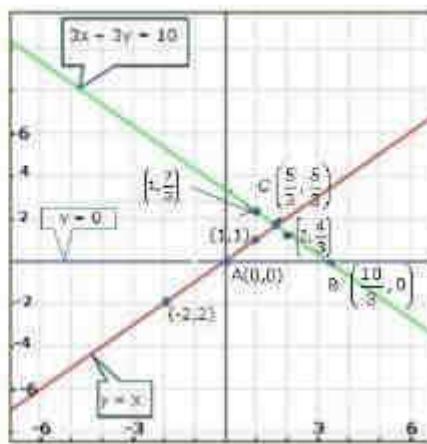
When $x = 2$, we have

$$y = \frac{10 - 3 \times 2}{3} = \frac{4}{3}$$

Thus, we have the following table giving points on the line $3x + 3y = 10$

x	1	2
y	$7/3$	$4/3$

Graph of the given equations:



From the graph of the lines represented by the given equations, we observe that:

the lines taken in pairs intersect each other at points $A(0, 0)$, $B\left(\frac{10}{3}, 0\right)$ and $C\left(\frac{5}{3}, \frac{5}{3}\right)$

Hence, the required vertices of the triangle are $A(0, 0)$, $B\left(\frac{10}{3}, 0\right)$ and $C\left(\frac{5}{3}, \frac{5}{3}\right)$.

Q21

Determine, graphically whether the system of equations $x - 2y = 2$, $4x - 2y = 5$ is consistent or inconsistent.

Solution

We have,

$$\begin{aligned}x - 2y &= 2 \\4x - 2y &= 5\end{aligned}$$

Now,

$$\begin{aligned}x - 2y &= 2 \\x &= 2 + 2y\end{aligned}$$

When $y = 0$, we have

$$x = 2 + 2 \times 0 = 2$$

When $y = -1$, we have

$$x = 2 + 2 \times (-1) = 0$$

Thus, we have the following table giving points on the line $x - 2y = 2$:

x	2	0
y	0	-1

Now,

$$4x - 2y = 5$$

$$\Rightarrow 4x = 5 + 2y$$

$$\Rightarrow x = \frac{5+2y}{4}$$

When $y = 0$, we have

$$x = \frac{5+2 \times 0}{4} = \frac{5}{4}$$

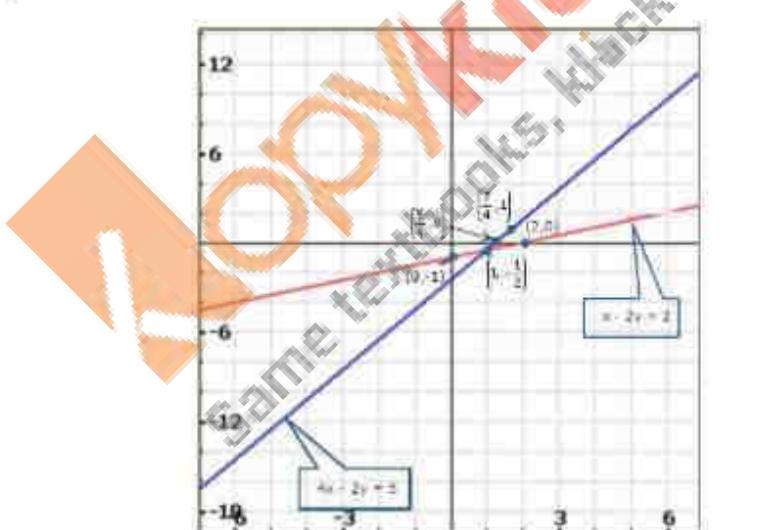
When $y = 1$, we have

$$x = \frac{5+2 \times 1}{4} = \frac{7}{4}$$

Thus, we have the following table giving points on the line $4x - 2y = 5$:

x	$\frac{5}{4}$	$\frac{7}{4}$
y	0	-1

Graph of the given equations:



Clearly, the two lines intersect at $\left(1, -\frac{1}{2}\right)$.

Hence, the system of equations is consistent.

Q22

Determine, by drawing graphs, whether the following system of linear equations has a unique solution or not.

$$2x - 3y = 6$$

$$x + y = 1$$

Solution

We have,

$$2x - 3y = 5$$

$$x + y = 1$$

Now,

$$2x - 3y = 5$$

$$\Rightarrow 2x = 5 + 3y$$

$$\Rightarrow x = \frac{5 + 3y}{2}$$

When $y = 0$, we have

$$x = \frac{5 + 3 \times 0}{2} = 3$$

When $y = -2$, we have

$$x = \frac{5 + 3 \times (-2)}{2} = 0$$

Thus, we have the following table giving points on the line $2x - 3y = 5$.

x	3	0
y	0	-2

Now,

$$x + y = 1$$

$$\Rightarrow x = 1 - y$$

When $y = 1$, we have

$$x = 1 - 1 = 0$$

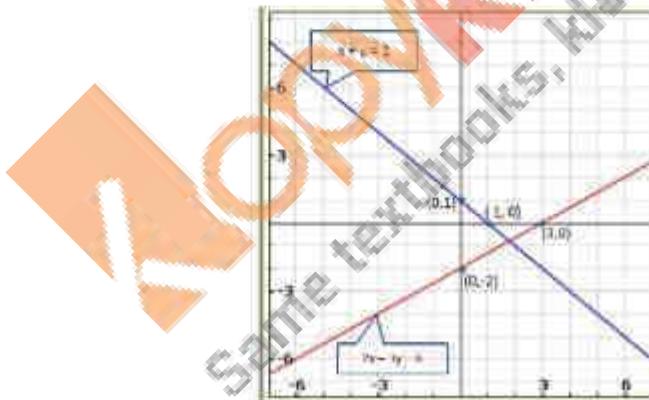
When $y = 0$, we have

$$x = 1 - 0 = 1$$

Thus, we have the following table giving points on the line $x + y = 1$.

x	0	1
y	1	0

Graph of the given equations:



Clearly, the two lines intersect at one point.

- The system of linear equations has a unique solution.

Q23

Determine, by drawing graphs, whether the following system of linear equations has a unique solution or not.

$$2y = 4x - 6$$

$$2x = y + 3$$

Solution

We have,

$$\begin{aligned} 2y &= 4x - 6 \\ 2y + 6 &= 4x \end{aligned}$$

Now,

$$\begin{aligned} 2y &= 4x - 6 \\ \Rightarrow 2y + 6 &= 4x \\ \Rightarrow 4x &= 2y + 6 \\ \Rightarrow x &= \frac{2y + 6}{4} \end{aligned}$$

When $y = -1$, we have

$$x = \frac{2 \times (-1) + 6}{4} = 1$$

When $y = 5$, we have

$$x = \frac{2 \times 5 + 6}{4} = 4$$

Thus, we have the following table giving points on the line $2y = 4x - 6$:

x	y
-1	5

Now,

$$\begin{aligned} 2x &= y + 3 \\ \Rightarrow x &= \frac{y + 3}{2} \end{aligned}$$

When $y = 1$, we have

$$x = \frac{1 + 3}{2} = 2$$

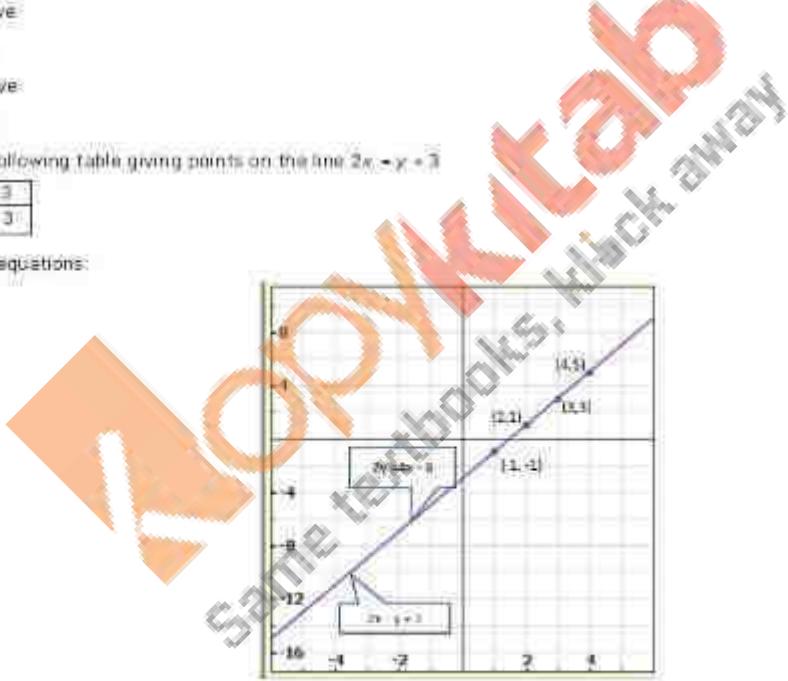
When $y = 3$, we have

$$x = \frac{3 + 3}{2} = 3$$

Thus, we have the following table giving points on the line $2x = y + 3$:

x	y
2	1

Graph of the given equations:



We find the graphs of the two equations are coincident.

Hence, the system of equations has infinitely many solutions.

Q24

Solve graphically each of the following systems of linear equations. Also find the coordinates of the points where the lines meet axis of y.

$$2x - 5y + 4 = 0$$

$$2x + y - 8 = 0$$

Solution

We have,

$$\begin{aligned} 2x - 3y + 4 &= 0 \\ 2x + y - 8 &= 0 \end{aligned}$$

Now,

$$2x - 3y + 4 = 0$$

\Rightarrow

$$2x + 3y = 4$$

\Rightarrow

$$x = \frac{3y - 4}{2}$$

When $y = 2$, we have

$$x = \frac{6 - 4}{2} = 1$$

When $y = 4$, we have

$$x = \frac{12 - 4}{2} = 4$$

Thus, we have the following table giving points on the line $2x - 3y + 4 = 0$:

x	1	4
y	2	4

Now,

$$2x + y - 8 = 0$$

\Rightarrow

$$2x = 8 - y$$

\Rightarrow

$$x = \frac{8 - y}{2}$$

When $y = 2$, we have

$$x = \frac{6 - 2}{2} = 2$$

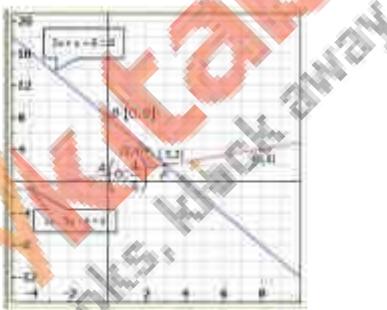
When $y = 4$, we have

$$x = \frac{4 - 2}{2} = 1$$

Thus, we have the following table giving points on the line $2x + y - 8 = 0$:

x	1	2	4
y	4	2	0

Graph of the given equations:



Clearly, two intersect at $(3, 2)$.

Hence, $x = 3, y = 2$ is the solution of the given system of equations.

We also observe that the lines represented by $2x - 3y + 4 = 0$ and $2x + y - 8 = 0$ meet

y-axis at $4\left(0, \frac{4}{3}\right)$ and $8\left(0, 8\right)$ respectively.

Q25

Solve graphically each of the following systems of linear equations. Also find the coordinates of the points where the lines meet axis of y .

$$3x + 2y = 12$$

$$5x - 2y = 4$$

Solution

We have:

$$3x + 2y = 12$$

$$5x - 2y = 4$$

Now,

$$3x + 2y = 12$$

$$3x + 12 = 2y$$

$$\therefore x = \frac{12 - 2y}{3}$$

When $y = 3$, we have

$$x = \frac{12 - 2 \times 3}{3} = 2$$

When $y = -3$, we have

$$x = \frac{12 - 2 \times (-3)}{3} = 6$$

Thus, we have the following table giving points on the line $3x + 2y = 12$:

x	2	6
y	3	-3

Now,

$$5x - 2y = 4$$

$$5x = 4 + 2y$$

$$\therefore x = \frac{4 + 2y}{5}$$

When $y = 3$, we have

$$x = \frac{4 + 2 \times 3}{5} = 2$$

When $y = -3$, we have

$$x = \frac{4 + 2 \times (-3)}{5} = -2$$

Thus, we have the following table giving points on the line $5x - 2y = 4$:

x	-2	2
y	3	-3

Graph of the given equations:



Clearly, they intersect at $P(3, 2)$.

Hence, $x = 3$, $y = 2$ is the solution of the given system of equations.

We also observe that the lines represented by $3x + 2y = 12$ and $5x - 2y = 4$ meet the y-axis at $\{0, 6\}$ and $\{0, -2\}$ respectively.

Q26

Solve graphically each of the following systems of linear equations. Also find the coordinates of the points where the lines meet axis of y.

$$2x + y - 11 = 0$$

$$x - y - 1 = 0$$

Solution

We have,

$$2x+y-11=0$$

$$x-y-1=0$$

Now,

$$2x+y-11=0$$

$$\Rightarrow y = 11 - 2x$$

When $x = 4$, we have

$$y = 11 - 2 \times 4 = 3$$

When $x = 5$, we have

$$y = 11 - 2 \times 5 = 1$$

Thus, we have the following table giving points on the line $2x+y-11=0$

3	4	5
y	3	1

Now,

$$x-y-1=0$$

$$\Rightarrow x-1=y$$

$$\Rightarrow y=x-1$$

When $x = 2$, we have

$$y = 2 - 1 = 1$$

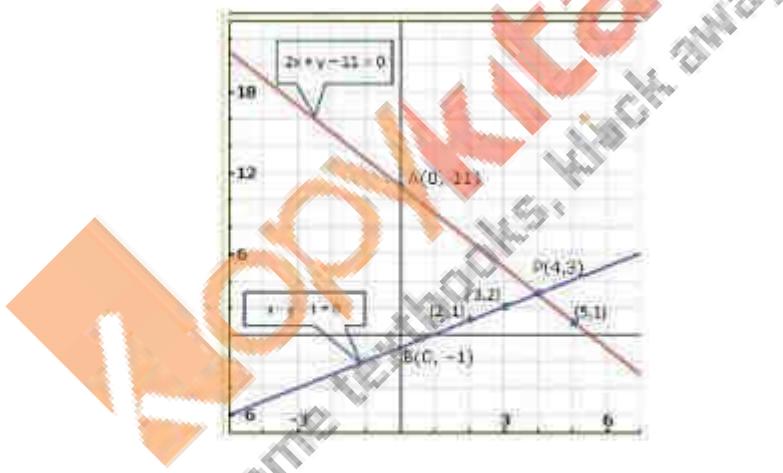
When $x = 3$, we have

$$y = 3 - 1 = 2$$

Thus, we have the following table giving points on the line $x-y-1=0$

3	2	1
y	1	2

Graph of the given equations:



Clearly, two intersect at $P(4, 3)$

Hence, $x = 4, y = 3$ is the solution of the given system of equations.

We also observe that the lines represented by $2x+y-11=0$ and $x-y-1=0$ meet y-axis at A(0, 11) and B(0, -1) respectively.

Q27

Solve graphically each of the following systems of linear equations. Also find the coordinates of the points where the lines meet axis of y.

$$x+2y-7=0$$

$$2x-y-4=0$$

Solution

We have, $x+2y-7=0$

$$2x-y-4=0$$

Now, $x+2y-7=0$

$$x=7-2y$$

When $y=1, x=5$

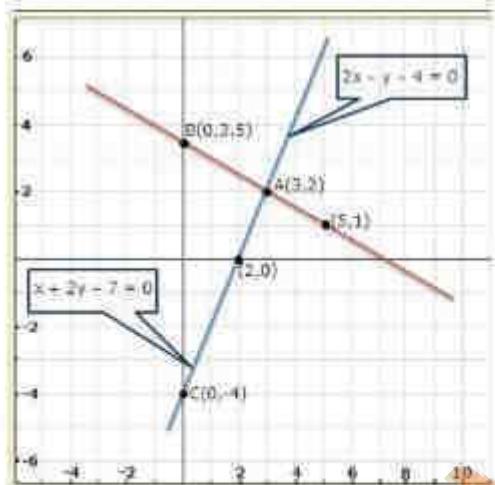
$$y=2, x=3$$

x	5	3
y	1	2

Also, $2x-y-4=0$

$$y=2x-4$$

x	2	0
y	0	-4



From the graph, the solution is $A(3, 2)$.

Also, the coordinates of the points where the lines meet the y-axis are B(0, 5) and C(0, -4).

Q28

Solve graphically each of the following systems of linear equations. Also find the coordinates of the points where the lines meet the axis of y.

$$3x+y-5=0$$

$$2x-y-5=0$$

Solution

We have,

$$3x + y - 5 = 0$$

$$2x - y - 5 = 0$$

Now,

$$3x + y - 5 = 0$$

$$\Rightarrow y = 5 - 3x$$

When $x = 1$, we have

$$y = 5 - 3 \times 1 = 2$$

When $x = 2$, we have

$$y = 5 - 3 \times 2 = -1$$

Thus, we have the following table giving points on the line $3x + y - 5 = 0$

x	1	2
y	2	-1

Now,

$$2x - y - 5 = 0$$

$$\Rightarrow 2x - 5 = y$$

$$\Rightarrow y = 2x - 5$$

When $x = 0$, we have

$$y = -5$$

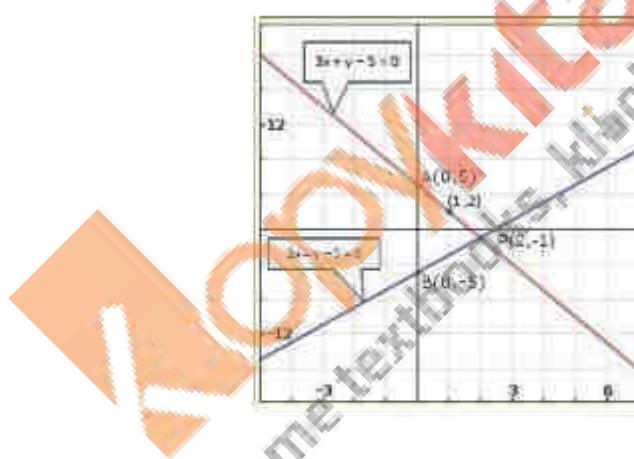
When $x = 2$, we have

$$y = 2 \times 2 - 5 = -1$$

Thus, we have the following table giving points on the line $2x - y - 5 = 0$

x	0	2
y	-5	-1

Graph of the given equations:



Clearly, two intersect at $E(2, -1)$.

Hence, $x = 2, y = -1$ is the solution of the given system of equations.

We also observe that the lines represented by $3x + y - 5 = 0$ and $2x - y - 5 = 0$ meet y-axis at $A(0, 5)$ and $C(0, -5)$ respectively.

Q29

Solve graphically each of the following systems of linear equations. Also find the coordinates of the points where the lines meet axes of y .

$$2x - y - 5 = 0$$

$$x + y - 3 = 0$$

Solution

We have,

$$2x - y - 5 = 0$$

$$x - y - 3 = 0$$

Now,

$$2x - y - 5 = 0$$

$$\Rightarrow 2x - 5 = y$$

$$\Rightarrow y = 2x - 5$$

When $x = 1$, we have

$$y = 2 \times 1 - 5 = -3$$

When $x = 2$, we have

$$y = 2 \times 2 - 5 = -1$$

Thus, we have the following table giving points on the line $2x - y - 5 = 0$.

x	1	2
y	-3	-1

Now,

$$x - y - 3 = 0$$

$$\Rightarrow x - 3 = y$$

$$\Rightarrow y = x - 3$$

When $x = 3$, we have

$$y = 3 - 3 = 0$$

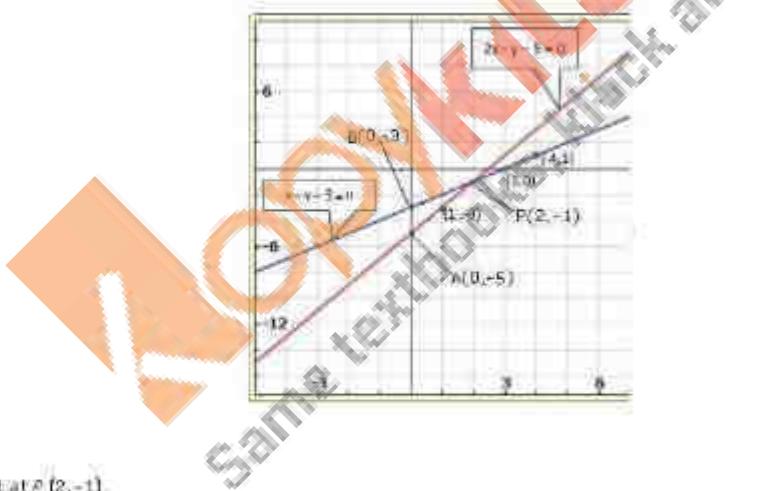
When $x = 4$, we have

$$y = 4 - 3 = 1$$

Thus, we have the following table giving points on the line $x - y - 3 = 0$.

x	3	4
y	0	1

Graph of the given equations:



Clearly, two intersect at $P(2, -1)$.

Hence, $x = 2, y = -1$ is the solution of the given system of equations.

We also observe that the lines represented by $2x - y - 5 = 0$ and $x - y - 3 = 0$ meet y-axis at $A(0, -5)$ and $B(0, -3)$ respectively.

Q30

Solve the following system of linear equations graphically and shade the region between the two lines and x-axis:

$$2x + 3y = 12$$

$$x - y = 1$$

Solution

The system of the given equations is,

$$2x + 3y = 12$$

$$x - y = 1$$

Now,

$$2x + 3y = 12$$

$$\Rightarrow 2x = 12 - 3y$$

$$\therefore x = \frac{12 - 3y}{2}$$

When $y = 2$, we have

$$x = \frac{12 - 3 \times 2}{2} = 3$$

When $y = 4$, we have

$$x = \frac{12 - 3 \times 4}{2} = 0$$

Thus, we have the following table:

x	0	3
y	4	2

We have,

$$x - y = 1$$

$$\Rightarrow x = 1 + y$$

When $y = 0$, we have

$$x = 1$$

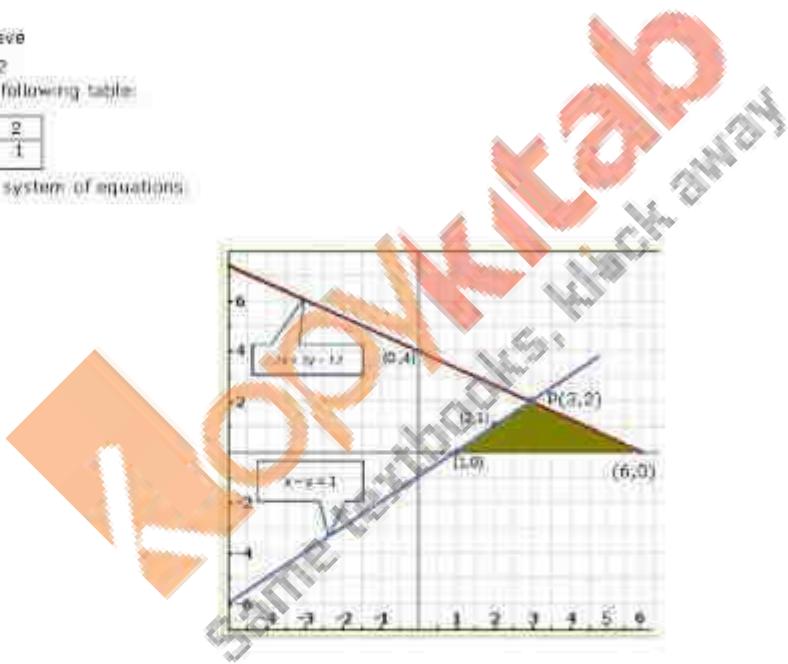
When $y = 1$, we have

$$x = 1 + 1 = 2$$

Thus, we have the following table:

x	1	2
y	0	1

Graph of the given system of equations



Clearly, the two lines intersect at $P(3, 2)$.

Hence, $x = 3, y = 2$ is the solution of the given system of equations.

Q31

Solve the following system of linear equations graphically and shade the region between the two lines and x-axis.

$$3x + 2y - 4 = 0$$

$$2x - 3y - 7 = 0$$

Solution

The system of the given equations is,

$$3x + 2y - 4 = 0$$

$$2x - 3y - 7 = 0$$

Now,

$$3x + 2y - 4 = 0$$

$$\Rightarrow 3x = 4 - 2y$$

$$\Rightarrow x = \frac{4 - 2y}{3}$$

When $y = 5$, we have:

$$x = \frac{4 - 2 \times 5}{3} = -2$$

When $y = 8$, we have:

$$x = \frac{4 - 2 \times 8}{3} = -4$$

Thus, we have the following table:

x	-2	-4
y	5	0

We have,

$$2y - 3x - 7 = 0$$

$$\Rightarrow 2y = 3x + 7$$

$$\Rightarrow y = \frac{3x + 7}{2}$$

When $y = 1$, we have:

$$x = \frac{3 \times 1 + 7}{2} = 5$$

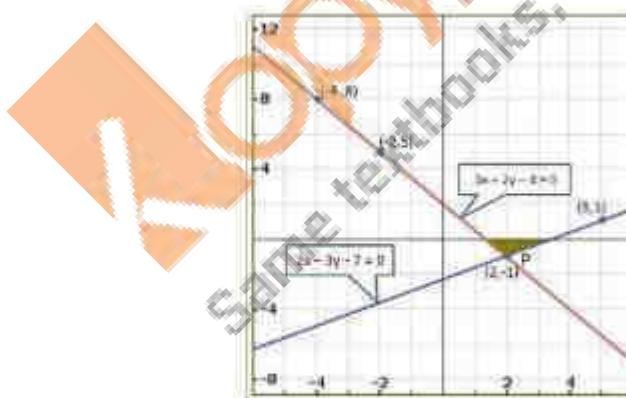
When $y = -1$, we have:

$$x = \frac{3 \times (-1) + 7}{2} = 2$$

Thus, we have the following table:

x	5	2
y	1	-1

Graph of the given system of equations:



Clearly, the two lines intersect at $P(2, -1)$.

Hence, $x = 2, y = -1$ is the solution of the given system of equations.

Q32

Solve the following system of linear equations graphically and shade the region between the two lines and x -axis.

$$3x + 2y - 11 = 0$$

$$2x - 3y + 10 = 0$$

Solution

The system of the given equations is,

$$2x + 2y - 11 = 0$$

$$2x - 3y + 10 = 0$$

Now,

$$2x + 2y - 11 = 0$$

$$\Rightarrow 2x = 11 - 2y$$

$$\Rightarrow x = \frac{11 - 2y}{2}$$

When $y = 1$, we have

$$x = \frac{11 - 2 \times 1}{2} = 4$$

When $y = 4$, we have

$$x = \frac{11 - 2 \times 4}{2} = 1$$

Thus, we have the following table:

x	3	1
y	1	4

We have,

$$2x - 3y + 10 = 0$$

$$\Rightarrow 2x = 3y - 10$$

$$\Rightarrow x = \frac{3y - 10}{2}$$

When $y = 0$, we have

$$x = \frac{3 \times 0 - 10}{2} = -5$$

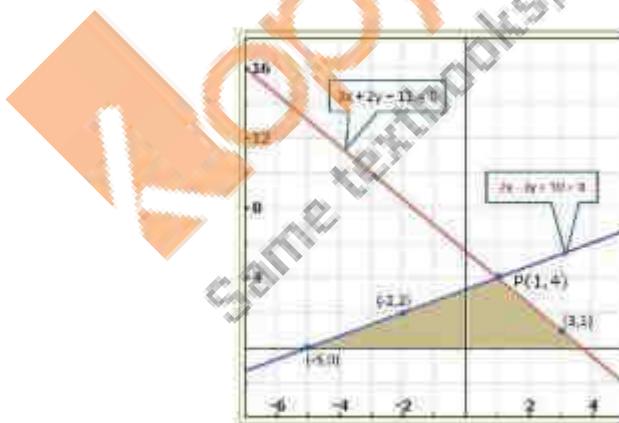
When $y = 2$, we have

$$x = \frac{3 \times 2 - 10}{2} = -2$$

Thus, we have the following table:

x	-5	-2
y	0	2

Graph of the given system of equations:



Clearly, the two lines intersect at $P(1, 4)$

Hence, $x = 1, y = 4$ is the solution of the given system of equations.

Q33

Draw the graphs of the following equations on the same graph paper:

$$2x + 3y = 12$$

$$x - y = 1$$

Find the coordinates of the vertices of the triangle formed by the two straight lines and the y-axis.

Solution

The system of the given equations is,

$$2x + 3y = 12$$

$$x - y = 1$$

Now,

$$2x + 3y = 12$$

$$\Rightarrow 2x = 12 - 3y$$

$$\Rightarrow x = \frac{12 - 3y}{2}$$

When $y = 0$, we have

$$x = \frac{12 - 3 \times 0}{2} = 6$$

When $y = 2$, we have

$$x = \frac{12 - 3 \times 2}{2} = 3$$

Thus, we have the following table:

x	6	3
y	0	2

We have,

$$x - y = 1$$

$$\Rightarrow x = 1 + y$$

When $y = 0$, we have

$$x = 1$$

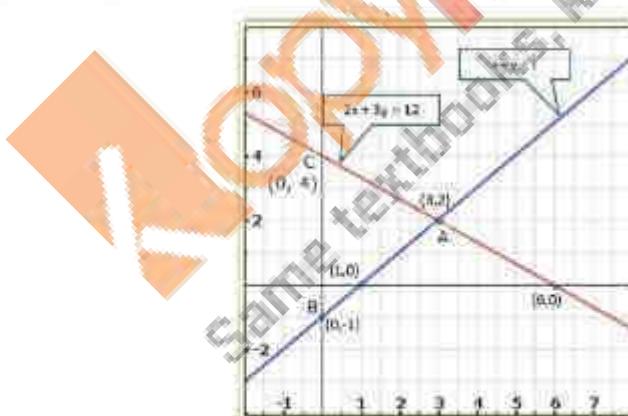
When $y = -1$, we have

$$x = 1 - 1 = 0$$

Thus, we have the following table:

x	1	0
y	0	-1

Graph of the given system of equations:



Clearly, the two lines intersect at A(3, 2).

We also observe that the lines represented by the equations $2x + 3y = 12$ and $x - y = 1$ meet y-axis at B(0, -1) and C(0, 4).

Hence, the vertices of the required triangle are A(3, 2), B(0, -1) and C(0, 4).

Q34

Draw the graphs of $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and x-axis and shade the triangular area. Calculate the area bounded by these lines and x-axis.

Solution

The given system of equations is:

$$x - y + 1 = 0$$

$$3x + 2y - 12 = 0$$

Now,

$$x - y + 1 = 0$$

$$\Rightarrow x = y - 1$$

When $y = 3$, we have:

$$x = 3 - 1 = 2$$

When $y = -1$, we have:

$$x = -1 - 1 = -2$$

Thus, we have the following table:

x	2	-2
y	3	-1

We have,

$$3x + 2y - 12 = 0$$

$$\Rightarrow 3x = 12 - 2y$$

$$\Rightarrow x = \frac{12 - 2y}{3}$$

When $y = 6$, we have:

$$x = \frac{12 - 2 \times 6}{3} = 0$$

When $y = 3$, we have:

$$x = \frac{12 - 2 \times 3}{3} = 2$$

Thus, we have the following table:

x	0	2
y	6	3

Graph of the given system of equations:



Clearly, the two lines intersect at A(2, 3).

We also observe that the lines represented by the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$ meet x-axis at B(-1, 0) and C(4, 0) respectively.

Thus, $x = 2$, $y = 3$ is the solution of the given system of equations.

Draw AD perpendicular from A on x-axis.

Clearly, we have

$$\begin{aligned}AD &= y - \text{coordinate of point } A(2, 3) \\&\Rightarrow AD = 3 \\&\text{and, } BC = y - (-1) = 4 + 1 = 5 \\&\therefore \text{Area of the shaded region} = \text{Area of } \triangle ABC \\&\Rightarrow \text{Area of the shaded region} = \frac{1}{2}(\text{Base} \times \text{Height}) \\&= \frac{1}{2} \times (BC \times AD) \\&= \frac{1}{2} \times 5 \times 3 \\&= 7.5 \text{ sq. units}\end{aligned}$$

\therefore Area of the shaded region = 7.5 sq. units.

Q35

Solve graphically the system of linear equations

$$\begin{aligned}4x - 3y + 4 &= 0 \\4x + 3y - 20 &= 0\end{aligned}$$

Find the area bounded by these lines and x-axis.

Solution

The given system of equations is:

$$\begin{aligned}4x - 3y + 4 &= 0 \\4x + 3y - 20 &= 0\end{aligned}$$

Now,

$$4x - 3y + 4 = 0$$

$$\Rightarrow 4x = 3y - 4$$

$$\Rightarrow x = \frac{3y - 4}{4}$$

When $y = 0$, we have:

$$x = \frac{3 \times 0 - 4}{4} = -1$$

When $y = 4$, we have:

$$x = \frac{3 \times 4 - 4}{4} = 2$$

Thus, we have the following table:

x	-1	2
y	0	4

We have,

$$4x + 3y - 20 = 0$$

$$\Rightarrow 4x = 20 - 3y$$

$$\Rightarrow x = \frac{20 - 3y}{4}$$

When $y = 0$, we have

$$x = \frac{20 - 3 \times 0}{4} = 5$$

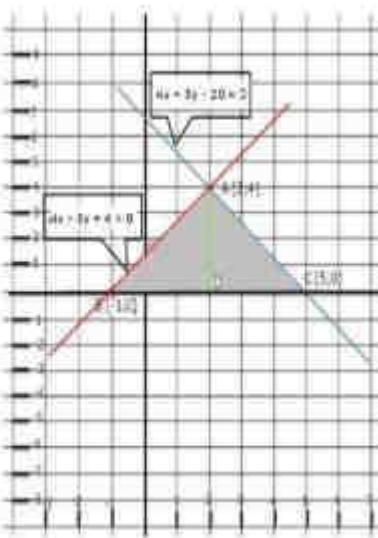
When $y = 4$, we have:

$$x = \frac{20 - 3 \times 4}{4} = 2$$

Thus, we have the following table:

x	2	5
y	4	0

Graph of the given system of equations:



Clearly, the two lines intersect at $A(2, 4)$. Hence $x = 2, y = 4$ is the solution of the given system of equations.

We also observe that the lines represented by the equations $4x + 3y + 4 = 0$ and $4x + 3y - 20 = 0$ meet x -axis at $B(-1, 0)$ and $C(5, 0)$, respectively.

Thus, $x = 2, y = 4$ is the solution of the given system of equations.

Draw AD perpendicular from A on x -axis.

Clearly, we have

$$AD = y - \text{coordinate of point } A(2, 4)$$

$$\Rightarrow AD = 4$$

$$\text{and, } BC = 5 - (-1) = 5 + 1 = 6$$

$$\text{Area of the shaded region} = \text{Area of } \triangle ABC$$

$$\Rightarrow \text{Area of the shaded region} = \frac{1}{2} (\text{base } BC \times \text{height})$$

$$= \frac{1}{2} \times (BC \times AD)$$

$$= \frac{1}{2} \times 6 \times 4$$

$$= 6 \times 2$$

$$= 12 \text{ sq. units}$$

$$\therefore \text{Area of the shaded region} = 12 \text{ sq. units}$$

Q36

Solve the following system of linear equations graphically:

$$3x + y - 11 = 0, x - y - 1 = 0$$

Shade the region bounded by these lines and y -axis. Also, find the area of the region bounded by these lines and y -axis.

Solution

The given system of equations is

$$3x + y - 11 = 0$$

$$x - y - 1 = 0$$

Now,

$$3x + y - 11 = 0$$

$$\Rightarrow y = 11 - 3x$$

When $x = 0$, we have

$$y + 11 - 3 \times 0 = 11$$

When $x = 3$, we have

$$y + 11 - 3 \times 3 = 2$$

Thus, we have the following table:

x	0	3
y	11	2

We have,

$$x + y - 11 = 0$$

$$\Rightarrow x - 1 = y$$

$$\Rightarrow y = x - 1$$

When $x = 0$, we have

$$y = 0 - 1 = -1$$

When $x = 3$, we have

$$y = 3 - 1 = 2$$

Thus, we have the following table:

x	0	3
y	-1	2

Graph of the given system of equations:



Clearly, the two lines intersect at $A(3, 2)$. Hence $(x = 3, y = 2)$ is the solution of the given system of equations.

We also observe that the lines represented by the equations $x + y - 11 = 0$ and $x - y - 1 = 0$ meet y -axis at $B(0, 11)$ and $C(0, -1)$ respectively.

Thus, $x = 3, y = 2$ is the solution of the given system of equations.
Draw AD perpendicular from A on y -axis.

Clearly, we have:

$$AD = y - \text{coordinate of point } A(3, 2)$$

$$\Rightarrow AD = 3$$

$$\text{and, } BC = 11 - (-1) = 11 + 1 = 12$$

$$\text{Area of the shaded region} = \text{Area of } ABC$$

$$\Rightarrow \text{Area of the shaded region} = \frac{1}{2} (\text{base} \times \text{height})$$

$$= \frac{1}{2} \times \{BC \times AD\}$$

$$= \frac{1}{2} \times 12 \times 3$$

$$= 6 \times 3$$

$$= 18 \text{ sq. units.}$$

$$\therefore \text{Area of the shaded region} = 18 \text{ sq. units.}$$

Solve graphically each of the following systems of linear equations. Also, find the coordinates of the points where the lines meet the axis of x in each system:

$$\begin{aligned} 2x + y &= 6 \\ x - 2y &= -2 \end{aligned}$$

Solution

The given system of equations is

$$\begin{aligned} 2x + y &= 6 \\ x - 2y &= -2 \end{aligned}$$

Now,

$$\begin{aligned} 2x + y &= 6 \\ \Rightarrow x &= \frac{6-y}{2} \end{aligned}$$

When $y = 0$, we have

$$x = \frac{6-0}{2} = 3$$

When $y = 2$, we have

$$x = \frac{6-2}{2} = 2$$

Thus, we have the following table:

x	3	2
y	0	2

We have,

$$\begin{aligned} x - 2y &= -2 \\ \Rightarrow x &= 2y - 2 \end{aligned}$$

When $y = 0$, we have

$$x = 2 \times 0 - 2 = -2$$

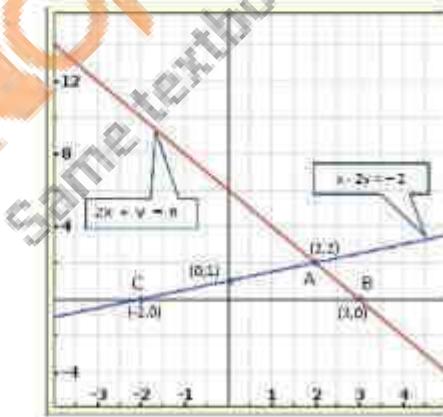
When $y = 1$, we have

$$x = 2 \times 1 - 2 = 0$$

Thus, we have the following table:

x	-2	0
y	0	1

Graph of the given system of equations:



Clearly, the two lines intersect at A(2, 2). Hence $x = 2, y = 2$ is the solution of the given system of equations.

We also observe that the lines represented by the equations $2x + y = 6$ and $x - 2y = -2$ meet x -axis at B(3, 0) and C(-2, 0) respectively.

Solve graphically each of the following systems of linear equations. Also, find the coordinates of the points where the lines meet the axis of x in each system:

$$2x - y = 2$$

$$4x - y = 8$$

Solution

The system of the given equations is

$$2x - y = 2$$

$$4x - y = 8$$

Now,

$$2x - y = 2$$

$$\Rightarrow 2x = y + 2$$

$$\Rightarrow x = \frac{y+2}{2}$$

When $y = 0$, we have

$$x = \frac{0+2}{2} = 1$$

When $y = 2$, we have

$$x = \frac{2+2}{2} = 2$$

Thus, we have the following table:

x	1	2
y	0	2

We have,

$$4x - y = 8$$

$$\Rightarrow 4x - y = 8$$

$$\Rightarrow x = \frac{y+8}{4}$$

When $y = 0$, we have

$$x = \frac{0+8}{4} = 2$$

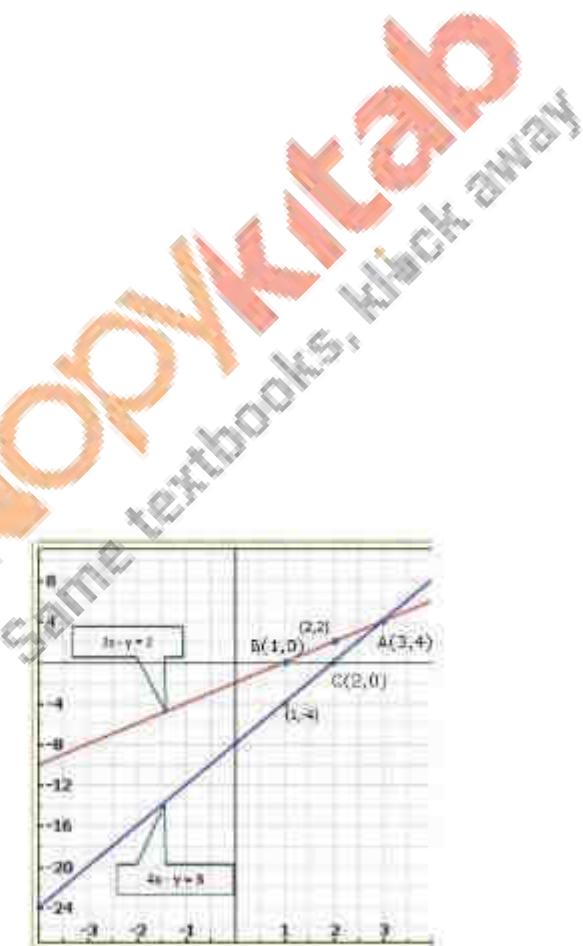
When $y = -4$, we have

$$x = \frac{-4+8}{4} = 1$$

Thus, we have the following table:

x	2	1
y	0	-4

Graph of the given system of equations:



Clearly, the two lines intersect at $A(3, 4)$. Hence $x = 3, y = 4$ is the solution of the given system of equations.

We also observe that the lines represented by the equations $2x - y = 2$ and $4x - y = 8$ meet x -axis at $B(1, 0)$ and $C(2, 0)$, respectively.

Q39

Solve graphically each of the following systems of linear equations. Also, find the coordinates of the points where the lines meet the axis of x in each system:

$$\begin{aligned}x + 2y &= 5 \\2x - 3y &= -4\end{aligned}$$

Solution

The system of the given equations is:

$$\begin{aligned}x + 2y &= 5 \\2x - 3y &= -4\end{aligned}$$

Now,

$$\begin{aligned}x + 2y &= 5 \\ \Rightarrow x &= 5 - 2y\end{aligned}$$

When $y = 2$, we have:

$$x = 5 - 2 \times 2 = 1$$

When $y = 3$, we have:

$$x = 5 - 2 \times 3 = -1$$

Thus, we have the following table:

x	1	-1
y	2	3

We have,

$$\begin{aligned}2x - 3y &= -4 \\ \Rightarrow 2x &= 3y - 4 \\ \Rightarrow x &= \frac{3y - 4}{2}\end{aligned}$$

When $y = 0$, we have:

$$x = \frac{3 \times 0 - 4}{2} = -2$$

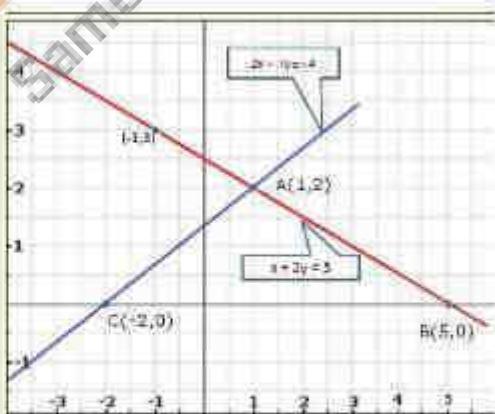
When $y = 2$, we have:

$$x = \frac{3 \times 2 - 4}{2} = 1$$

thus, we have the following table:

x	-2	1
y	0	2

Graph of the given system of equations:



Clearly, the two lines intersect at $A(1, 2)$. Hence $x = 1, y = 2$ is the solution of the given system of equations.

We also observe that the lines represented by the equations $x + 2y = 5$ and $2x - 3y = -4$ meet x -axis at $B(5, 0)$ and $C(-2, 0)$ respectively.

Q40

Solve graphically each of the following systems of linear equations. Also, find the coordinates of the points where the lines meet the axis of x in each system:

$$\begin{aligned} 2x + 3y &= 8 \\ x - 2y &= -3 \end{aligned}$$

Solution

The given system of equations is -

$$\begin{aligned} 2x + 3y &= 8 \\ x - 2y &= -3 \end{aligned}$$

Now,

$$\begin{aligned} 2x + 3y &= 8 \\ \Rightarrow 2x &= 8 - 3y \\ \Rightarrow x &= \frac{8 - 3y}{2} \end{aligned}$$

When $y = 2$, we have

$$x = \frac{8 - 3 \times 2}{2} = 1$$

When $y = 4$, we have

$$x = \frac{8 - 3 \times 4}{2} = -2$$

Thus, we have the following table:

x	1	-2
y	2	4

We have,

$$x - 2y = -3$$

$$\Rightarrow x = 2y - 3$$

When $y = 0$, we have

$$x = 2 \times 0 - 3 = -3$$

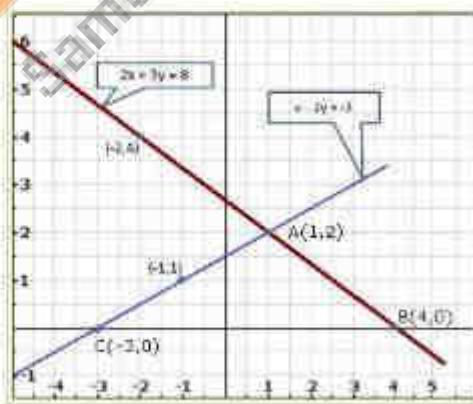
When $y = 1$, we have

$$x = 2 \times 1 - 3 = -1$$

Thus, we have the following table:

x	-3	-1
y	0	1

Graph of the given system of equations:



Clearly, the two lines intersect at $A(1, 2)$. Hence $x = 1, y = 2$ is the solution of the given system of equations.

We also observe that the lines represented by the equations $2x + 3y = 8$ and $x - 2y = -3$ meet x -axis at $B(4, 0)$ and $C(-3, 0)$ respectively.

Q41

Draw the graphs of the following equations:

$$2x - 3y + 5 = 0$$

$$2x + 3y - 18 = 0$$

$$y - 2 = 0$$

Find the vertices of the triangle so obtained. Also, find the area of the triangle.

Solution

The given system of equations is:

$$2x - 3y + 5 = 0$$

$$2x + 3y - 18 = 0$$

$$y - 2 = 0$$

Now,

$$2x - 3y + 5 = 0$$

$$\Rightarrow 2x = 3y - 5$$

$$\Rightarrow x = \frac{3y - 5}{2}$$

When $y = 0$, we have

$$x = \frac{3 \times 0 - 5}{2} = -\frac{5}{2}$$

When $y = 2$, we have

$$x = \frac{3 \times 2 - 5}{2} = \frac{1}{2}$$

Thus, we have the following table:

x	-5/2	1/2
y	0	2

We have,

$$2x + 3y - 18 = 0$$

$$\Rightarrow 2x = 18 - 3y$$

$$\Rightarrow x = \frac{18 - 3y}{2}$$

When $y = 0$, we have

$$x = \frac{18 - 3 \times 0}{2} = 9$$

When $y = 6$, we have

$$x = \frac{18 - 3 \times 6}{2} = 0$$

Thus, we have the following table:

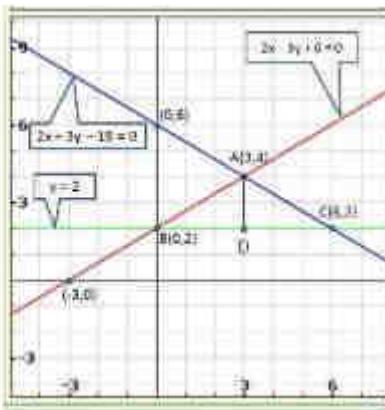
x	9	0
y	0	6

We have,

$$y - 2 = 0$$

$$\Rightarrow y = 2$$

Graph of the given system of equations:



From the graph of the three equations, we find that the three lines taken in pairs intersect each other at points $A(3, 4)$, $B(0, 2)$ and $C(6, 2)$.

Hence, the vertices of the required triangle are $(3, 4)$, $(0, 2)$ and $(6, 2)$.

From graph, we have

$$AD = 4 - 2 = 2$$

$$BC = 6 - 0 = 6$$

$$\text{Area of } \triangle ABC = \frac{1}{2} (\text{Base} \times \text{Height})$$

$$= \frac{1}{2} \times BC \times AD$$

$$= \frac{1}{2} \times 6 \times 2$$

$$= 6 \text{ sq. units.}$$

∴ Area of $\triangle ABC = 6$ sq. units.

Q42

Solve the following system of equations graphically.

$$2x - 3y + 6 = 0$$

$$2x + 3y - 18 = 0$$

Also, find the area of the region bounded by these two lines and y -axis.

Solution

The given system of equations is:

$$2x + 3y + 6 = 0$$

$$2x + 3y - 16 = 0$$

Now,

$$2x + 3y + 6 = 0$$

$$\Rightarrow 2x + 6 = -3y$$

$$\Rightarrow 3y = -2x - 6$$

$$\Rightarrow y = \frac{-2x - 6}{3}$$

When $x = 0$, we have

$$y = \frac{2 \times 0 + 6}{3} = 2$$

When $x = -3$, we have

$$y = \frac{2 \times (-3) + 6}{3} = 0$$

Thus, we have the following table:

x	0	-3
y	2	0

We have,

$$2x + 3y - 16 = 0$$

$$\Rightarrow 3y = 16 - 2x$$

$$\Rightarrow y = \frac{16 - 2x}{3}$$

When $x = 0$, we have

$$y = \frac{16 - 2 \times 0}{3} = 6$$

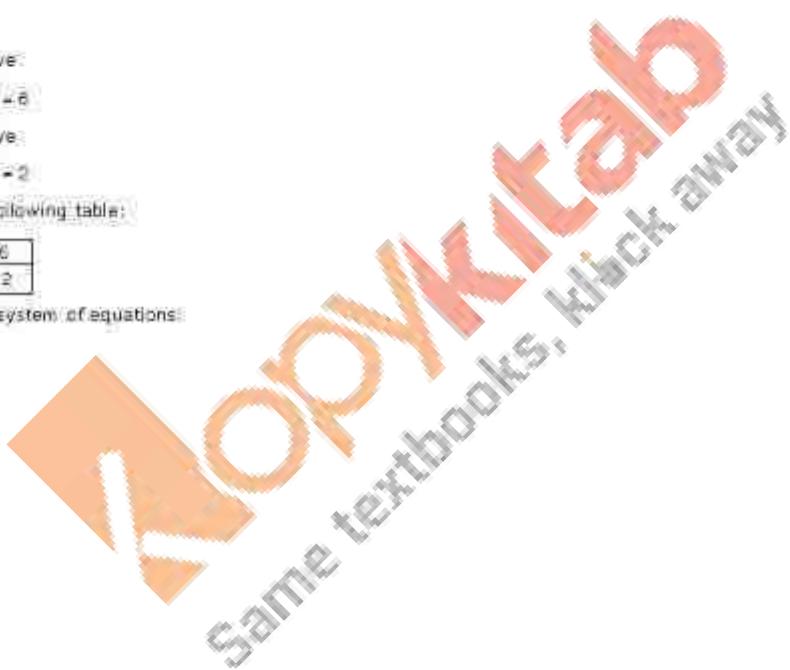
When $x = 6$, we have

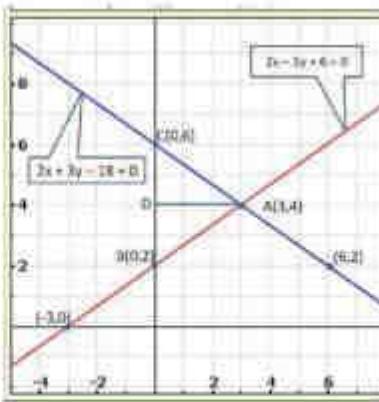
$$y = \frac{16 - 2 \times 6}{3} = 2$$

Thus, we have the following table:

x	0	6
y	6	2

Graph of the given system of equations:





Clearly, the two lines intersect at $A(3, 4)$. Hence, $x = 3$, $y = 4$ is the solution of the given system of equations.

We also observe that the lines represented by the equations $2x - 3y + 6 = 0$ and $2x + 3y - 18 = 0$ meet y -axis at $B(0, 2)$ and $C(0, 6)$ respectively.

Thus, $x = 3$, $y = 4$ is the solution of the given system of equations.

Draw AD perpendicular from A on y -axis.

Clearly, we have,

$$AD = y - \text{coordinate of point } A(3, 4)$$

$$\Rightarrow AD = 4$$

$$\text{and, } BC = 6 - 2 = 4.$$

$$\therefore \text{Area of the shaded region} = \text{Area of } \triangle ABC$$

$$\therefore \text{Area of the shaded region} = \frac{1}{2}(\text{Base} \times \text{Height})$$

$$= \frac{1}{2}(BC \times AD)$$

$$= \frac{1}{2} \times 4 \times 3$$

$$= 2 \times 3$$

$$= 6 \text{ sq. units}$$

∴ Area of the region bounded by these two lines and y -axis is 6 sq. units.

Q43

Solve the following system of linear equations graphically:

$$4x - 5y - 20 = 0$$

$$3x + 5y - 15 = 0$$

Determine the vertices of the triangle formed by the lines representing the above equation and the y -axis.

Solution

The given system of equations is:

$$4x - 5y - 20 = 0$$

$$3x + 5y - 15 = 0$$

Now,

$$4x - 5y - 20 = 0$$

$$\Rightarrow 4x = 5y + 20$$

$$\Rightarrow x = \frac{5y + 20}{4}$$

When $y = 0$, we have:

$$x = \frac{5 \times 0 + 20}{4} = 5$$

When $y = -4$, we have:

$$x = \frac{5 \times (-4) + 20}{4} = 0$$

Thus, we have the following table:

x	5	0
y	0	-4

We have,

$$3x + 5y - 15 = 0$$

$$\Rightarrow 3x = 15 - 5y$$

$$\Rightarrow x = \frac{15 - 5y}{3}$$

When $y = 0$, we have:

$$x = \frac{15 - 5 \times 0}{3} = 5$$

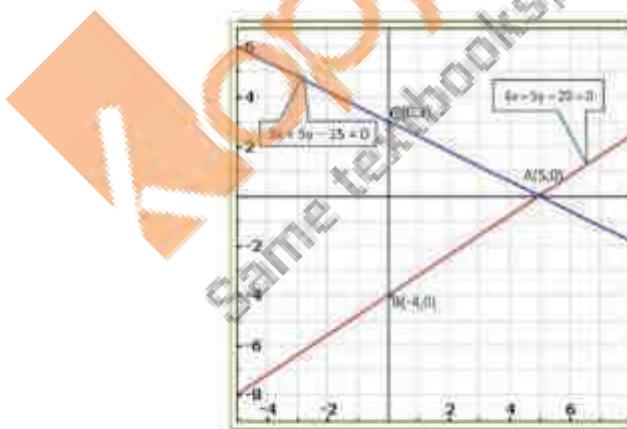
When $y = 3$, we have:

$$x = \frac{15 - 5 \times 3}{3} = 0$$

Thus, we have the following table:

x	5	0
y	0	3

Graph of the given system of equations:



Clearly, the two lines intersect at $A(5, 0)$. Hence, $x = 5, y = 0$ is the solution of the given system of equations.

We also find that the two lines represented by the equations $4x - 5y - 20 = 0$ and $3x + 5y - 15 = 0$ meet y-axis at $B(0, -4)$ and $C(0, 3)$ respectively.

The vertices of the required triangle are $(5, 0)$, $(0, -4)$ and $(0, 3)$.

Q44

Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the y-axis. Calculate the area of the triangle so formed.

Solution

$$5x - y = 5 \Rightarrow y = 5x - 5$$

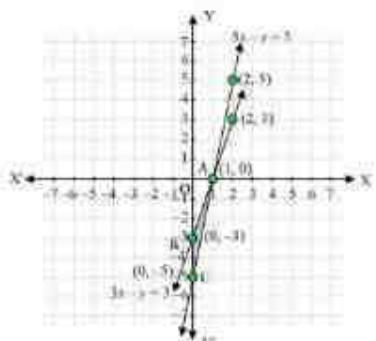
Three solutions of this equation can be written in a table as follows:

x	0	1	2
y	-5	0	5

$$3x - y = 3 \Rightarrow y = 3x - 3$$

x	0	1	2
y	-3	0	3

The graphical representation of the two lines will be as follows:



It can be observed that the required triangle is $\triangle ABC$.

The coordinates of its vertices are $A(1, 0)$, $B(0, -3)$, $C(0, -5)$.

Area space of space Triangle space increment ABC space equals space 1 half cross times BC cross times AC equals 1 half cross times 2 cross times 1 equals 1 space sq. space unit.

Concept insight: In order to find the coordinates of the vertices of the triangle so formed, find the points where the two lines intersects the y -axis and also where the two lines intersect each other. Here, note that the coordinates of the intersection of lines with y -axis is taken and not with x -axis, this is because the question says to find the triangle formed by the two lines and the y -axis.

Q45

Form the pair of linear equations in the following problems, and find their solutions graphically.

- 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.
- 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and that of one pen.
- Champa went to a 'Sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, "The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased". Help her friends to find how many pants and skirts Champa bought.

Solution

(i) Let the number of girls and boys in the class be x and y respectively.

According to the given conditions, we have

$$x + y = 10$$

$$x - y = 4$$

$$x + y = 10 \Rightarrow x = 10 - y$$

Three solutions of this equation can be written in a table as follows:

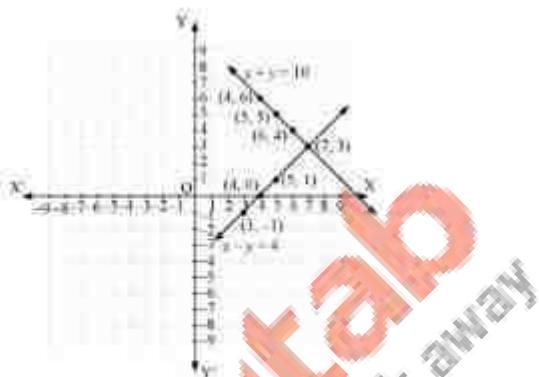
x	4	5	6
y	6	5	4

$$x - y = 4 \Rightarrow x = 4 + y$$

Three solutions of this equation can be written in a table as follows:

x	5	4	3
y	1	0	-1

The graphical representation is as follows:



From the graph, it can be observed that the two lines intersect each other at the point (7, 3).
So, $x = 7$ and $y = 3$.

Thus, the number of girls and boys in the class are 7 and 3 respectively.

KopyKitab
Same textbooks, klick away

(ii) Let the cost of one pencil and one pen be Rs x and Rs y , respectively.

According to the given conditions, we have:

$$\begin{aligned} 5x + 7y &= 50 \\ 7x + 5y &= 40 \\ 5x + 7y = 50 &\quad | - 7y \\ \hline 5x &= 50 - 7y \end{aligned}$$

Three solutions of this equation can be written in a table as follows:

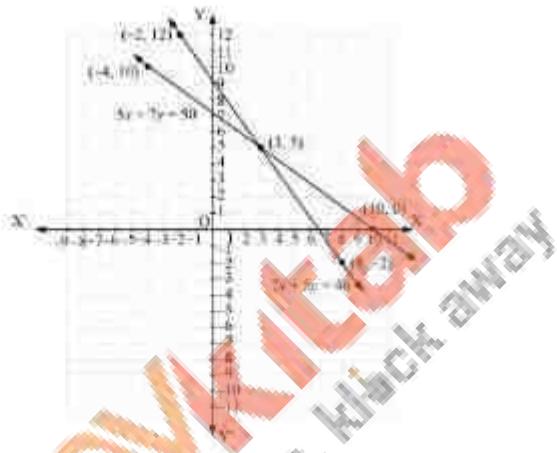
x	3	10	-4
y	5	0	10

$$7x + 5y = 40 \Rightarrow x = \frac{40 - 5y}{7}$$

Three solutions of this equation can be written in a table as follows:

x	8	3	-2
y	-2	5	12

The graphical representation is as follows:



From the graph, it can be observed that the two lines intersect each other at the point (3, 5). So, $x = 3$ and $y = 5$.

Therefore, the cost of one pencil and one pen are Rs 3 and Rs 5 respectively.

(ii)

Let us denote the number of pants by x and the number of skirts by y . Then the equations formed are:

$$y = 2x - 2 \quad \dots(1)$$

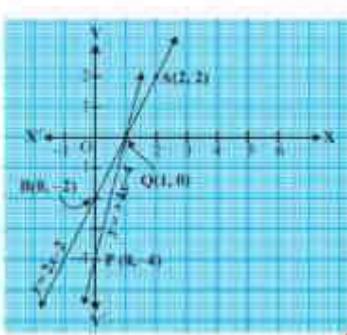
$$\text{and } y = 4x - 4 \quad \dots(2)$$

Let us draw the graphs of Equations (1) and (2) by finding two solutions for each of the equations.

They are given in Table

x	0	1
$y = 2x - 2$	2	0

x	0	1
$y = 4x - 4$	-4	0



Plot the points and draw the lines passing through them to represent the equations, as shown in fig.

The two lines intersect at the point (1, 0). So, $x = 1, y = 0$ is the required solution of the pair of linear equations, i.e., the number of pants she purchased is 1 and she did not buy any skirt.

Concept insight: Read the question carefully and examine what are the unknowns. Represent the given conditions with the help of equations by taking the unknown quantities as variables. Also carefully state the variables as whole solution is based on it. On the graph paper, mark the points accurately and neatly using a sharp pencil. Also, take at least three points satisfying the two equations in order to obtain the correct straight line of the equation. Since joining any two points gives a straight line and if one of the points is computed incorrect will give a wrong line and taking third point will give a correct line. The point where the two straight lines will intersect will give the values of the two variables, i.e., the solution of the two linear equations. State the solution point.

Q46

Solve the following system of equations graphically.
Shade the region between the lines and the y -axis.

$$3x - 4y = 7$$

$$5x + 2y = 3$$

Solution

The given system of equations is:

$$3x - 4y = 7$$

$$5x + 2y = 3$$

Now,

$$3x - 4y = 7$$

$$\Rightarrow 3x = 7 + 4y$$

$$\Rightarrow 4y = 3x - 7$$

$$\Rightarrow y = \frac{3x - 7}{4}$$

When $x = 1$, we have:

$$y = \frac{3 \times 1 - 7}{4} = -1$$

When $x = -3$, we have:

$$y = \frac{3 \times (-3) - 7}{4} = -4$$

Thus, we have the following table:

x	1	-3
y	-1	-4

We have,

$$5x + 2y = 3$$

$$\Rightarrow 2y = 3 - 5x$$

$$\Rightarrow y = \frac{3 - 5x}{2}$$

When $x = 1$, we have:

$$y = \frac{3 - 5 \times 1}{2} = -1$$

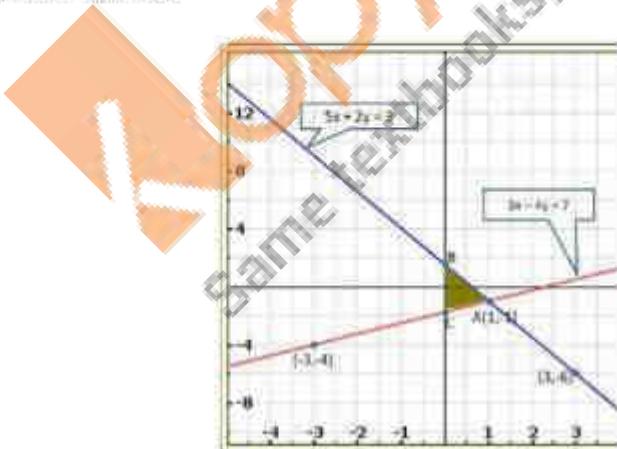
When $x = 3$, we have:

$$y = \frac{3 - 5 \times 3}{2} = -6$$

Thus, we have the following table:

x	-3	3
y	-1	-6

Graph of the given system of equations:



Clearly, the two lines intersect at $A(1, -1)$. Hence, $x = 1, y = -1$ is the solution of the given system of equations.

We also observe that the required shaded region is $\triangle ABC$.

Q47

Solve the following system of equations graphically:

Shade the region between the lines and the y -axis.

$$4x - y = 4$$

$$3x + 2y = 14$$

Solution

The given system of equations is:

$$4x - y = 4$$

$$3x + 2y = 14$$

Now,

$$4x - y = 4$$

$$\therefore 4x - 4 = y$$

$$\therefore y = 4x - 4$$

When $x = 0$, we have

$$y = 4 \times 0 - 4 = -4$$

When $x = 1$, we have

$$y = 4 \times (-1) - 4 = -8$$

Thus, we have the following table:

x	0	-1
y	-4	-8

We have,

$$3x + 2y = 14$$

$$\therefore 2y = 14 - 3x$$

$$\therefore y = \frac{14 - 3x}{2}$$

When $x = 0$, we have

$$y = \frac{14 - 3 \times 0}{2} = 7$$

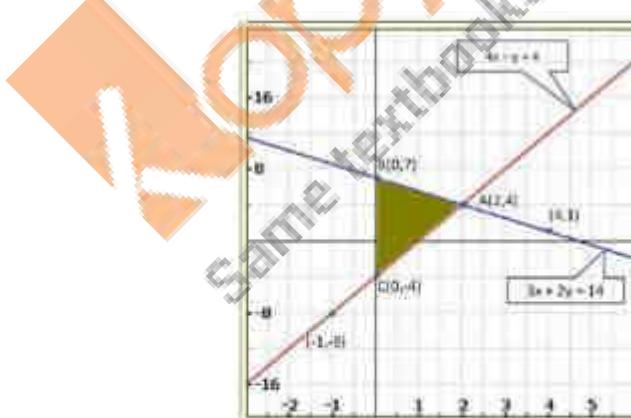
When $x = 4$, we have

$$y = \frac{14 - 3 \times 4}{2} = 1$$

Thus, we have the following table:

x	0	4
y	7	1

Graph of the given system of equations:



Clearly, the two lines intersect at $A(2, 4)$. Hence, $x = 2, y = 4$ is the solution of the given system of equations.

We also observe $\triangle ABC$ is the required shaded region.

Q48

Represent the following pair of equations graphically and write the coordinates of points where the lines intersects y -axis.

$$x + 3y = 5$$

$$2x - 3y = 12$$

Solution

The given system of equations is:

$$x + 3y = 6$$

$$2x - 3y = 12$$

Now,

$$x + 3y = 6$$

$$\Rightarrow 3y = 6 - x$$

$$\Rightarrow y = \frac{6-x}{3}$$

When $x = 0$, we have:

$$y = \frac{6-0}{3} = 2$$

When $x = 3$, we have:

$$y = \frac{6-3}{3} = 1$$

Thus, we have the following table:

x	0	3
y	2	1

We have,

$$2x - 3y = 12$$

$$\Rightarrow 2x - 12 = 3y$$

$$\Rightarrow 3y = 2x - 12$$

$$\Rightarrow y = \frac{2x-12}{3}$$

When $x = 0$, we have:

$$y = \frac{2 \times 0 - 12}{3} = -4$$

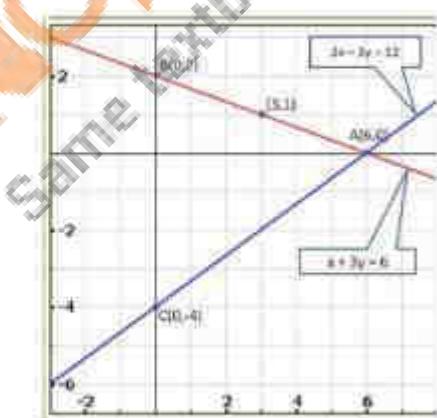
When $x = 6$, we have:

$$y = \frac{2 \times 6 - 12}{3} = 0$$

Thus, we have the following table:

x	0	6
y	-4	0

Graph of the given system of equations:



We observe that the lines represented by the equations $x + 3y = 6$ and $2x - 3y = 12$ meet y-axis at $E(0, 2)$ and $C(0, -4)$ respectively.

Hence, the required co-ordinates are $(0, 2)$ and $(0, -4)$.

Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

- (i) Intersecting lines
- (ii) parallel lines
- (iii) Coincident lines

Solution

(i) For the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ to be intersecting, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, the other linear equation can be $5x + 6y - 16 = 0$

$$\text{as } \frac{a_1}{a_2} = \frac{2}{5}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

(ii) For the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ to be parallel, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the other linear equation can be $5x + 0y + 24 = 0$

$$\text{as } \frac{a_1}{a_2} = \frac{2}{5} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{3}{0} = \frac{1}{0}, \frac{c_1}{c_2} = \frac{-8}{24} = \frac{-1}{3}$$

(iii) For the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ to be coincident, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, the other linear equation can be $8x + 12y - 32 = 0$

$$\text{as } \frac{a_1}{a_2} = \frac{2}{3} = \frac{1}{4}, \frac{b_1}{b_2} = \frac{3}{12} = \frac{1}{4}, \frac{c_1}{c_2} = \frac{-8}{-32} = \frac{1}{4}$$

Concept Insight: In order to answer such type of problems, just remember the conditions for two lines to be intersecting, parallel, and coincident. This problem will have multiple answers as there can be many equations satisfying the required conditions.

Q50

Determine graphically the coordinates of the vertices of triangle, the equations of whose sides are:

$$y = x, y = 2x \text{ and } y + x = 5$$

Solution

The system of the given equations is:

$$y = x$$

$$y = 2x$$

$$x + y = 6$$

Now,

$$y = x$$

When $x = 0$, we have

$$y = 0$$

When $x = -1$, we have

$$y = -1$$

Thus, we have the following table:

x	0	-1
y	0	-1

We have,

$$y = 2x$$

When $x = 0$, we have

$$y = 2 \times 0 = 0$$

When $x = -1$, we have

$$y = 2 \times \{-1\} = -2$$

Thus, we have the following table:

x	0	-1
y	0	-2

We have,

$$y + x = 6$$

$$\Rightarrow y = 6 - x$$

When $x = 2$, we have

$$y = 6 - 2 = 4$$

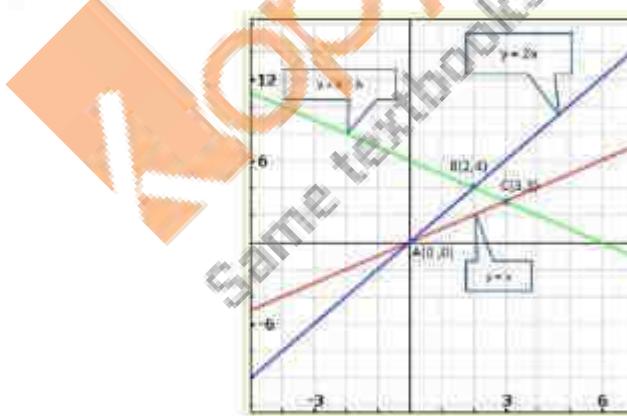
When $x = 4$, we have

$$y = 6 - 4 = 2$$

Thus, we have the following table:

x	2	4
y	4	2

Graph of the given system of equations:-



From the graph of the three equations, we find that the three lines taken in pairs intersect each other at points A(0,0), B(2,4) and C(3,3).

Hence, the vertices of the required triangle are (0,0), (2,4) and (3,3).

Q51

Determine graphically the coordinates of the vertices of triangle, the equations of whose sides are:

$$y = x; 2y = x \text{ and } x + y = 8$$

Solution

The system of the given equations is,

$$\begin{aligned}y &= x \\2y &= x \\x + y &= 8\end{aligned}$$

Now,

$$y = x$$

$$\Rightarrow x = y$$

When $y = 0$, we have

$$x = 0$$

When $y = -3$, we have

$$x = -3$$

Thus, we have the following table:

x	0	-3
y	0	-3

We have,

$$2y = x$$

$$\Rightarrow x = 2y$$

When $y = 0$, we have

$$x = 2 \times 0 = 0$$

When $y = -1$, we have

$$x = 2 \times (-1) = -2$$

Thus, we have the following table:

x	0	-2
y	0	-1

We have,

$$x + y = 8$$

$$\Rightarrow x = 8 - y$$

When $y = 4$, we have

$$x = 8 - 4 = 4$$

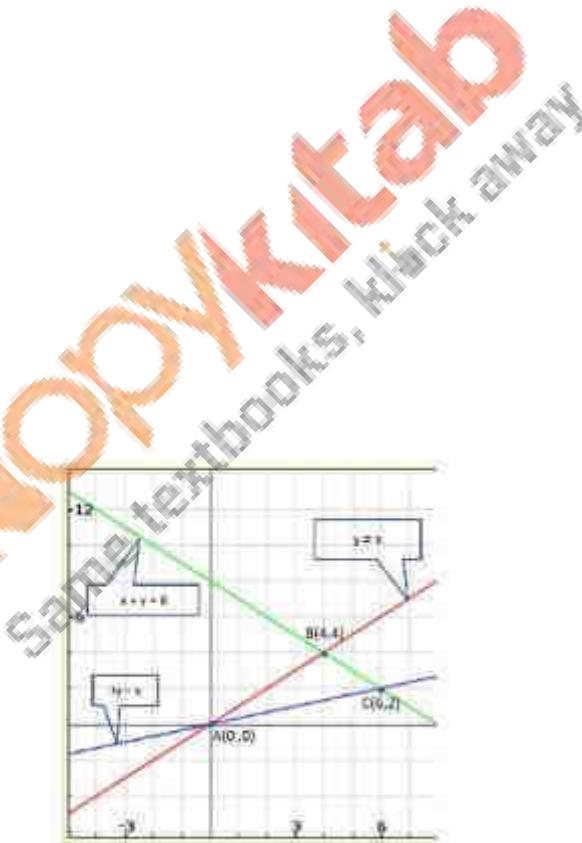
When $y = 5$, we have

$$x = 8 - 5 = 3$$

Thus, we have the following table:

x	4	3
y	4	3

Graph of the given system of equations.



From the graph of the three equations, we find that the three lines taken in pairs intersect each other at points A(0,0), B(4,4), and C(6,2).

Hence, the vertices of the required triangle are (0,0), (4,4), and (6,2).

Graphically, solve the following pair of equations:

$$\begin{aligned} 2x + y &= 6 \\ \Rightarrow y &= 6 - 2x \end{aligned}$$

Find the ratio of the areas of the two triangles formed by the lines representing these equations with the x-axis and the lines with the y-axis.

Solution

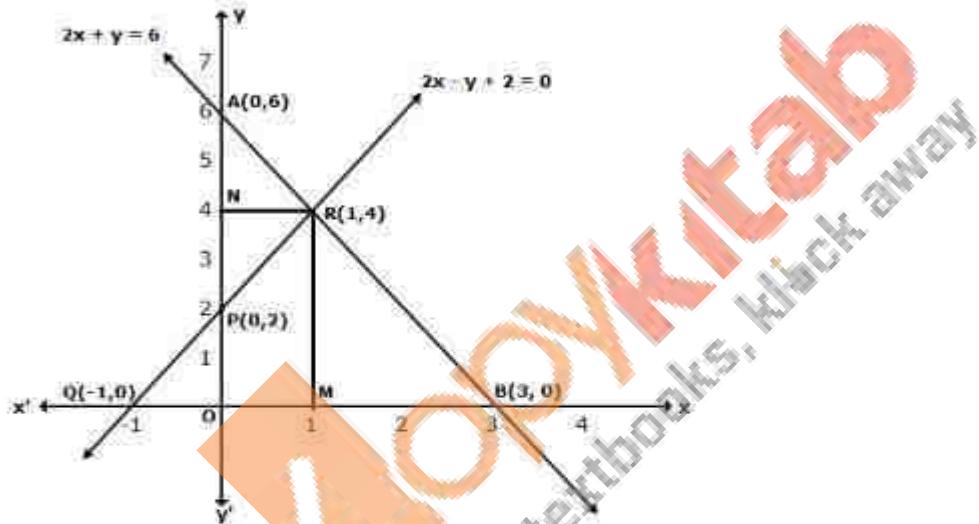
$$\begin{aligned} 2x + y &= 6 \\ \Rightarrow y &= 6 - 2x \end{aligned}$$

x	0	3
y	6	0

$$\begin{aligned} 2x - y + 2 &= 0 \\ \Rightarrow y &= 2x + 2 \end{aligned}$$

x	0	-1
y	2	0

Graph of the given system of equation is as follows:



The lines AB and CD intersect at point R(1, 4). Hence, the solution of the given pair of linear equations is $x = 1, y = 4$.

From R, draw RM \perp X-axis and RN \perp Y-axis.

Then, from graph, we have

$RM = 4$ units, $RN = 1$ unit, $AP = 4$ units, $BD = 4$ units

Area of triangle formed by the lines and X-axis

$$= A(\Delta RQB)$$

$$= \frac{1}{2} \times BQ \times RM$$

$$= \frac{1}{2} \times 4 \times 4$$

$$= 8 \text{ sq. units}$$

Area of triangle formed by the lines and Y-axis

$$= A(\Delta ARP)$$

$$= \frac{1}{2} \times AP \times RN$$

$$= \frac{1}{2} \times 4 \times 1$$

$$= 2 \text{ sq. units}$$

$$\frac{A(\Delta RQB)}{A(\Delta ARP)} = \frac{8}{2} = \frac{4}{1} = 4:1$$

Determine, graphically, the vertices of the triangle formed by the lines $y = x$, $3y = x$, $x + y = 8$.

Solution

$$y = x$$

x	0	4
y	0	4

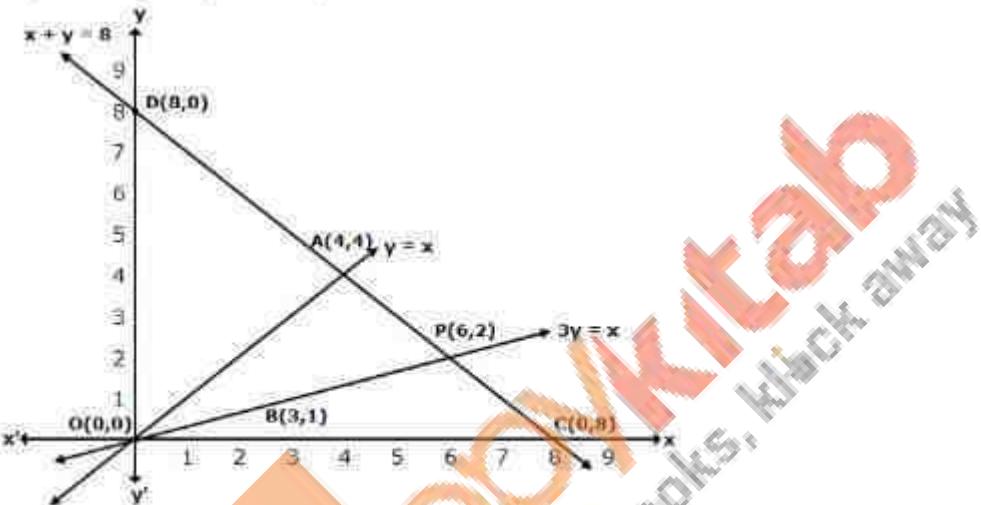
$$3y = x$$

x	0	3
y	0	1

$$x + y = 8 \Rightarrow x = 8 - y$$

x	0	8
y	8	0

Graph of the given system of equation is as follows:



From the graph, the vertices of the triangle AOP formed by the given lines are A(4, 4), O(0, 0) and P(6, 2).

Q54

Draw the graph of the equations $x = 3$, $x = 5$ and $2x - y - 4 = 0$. Also, find the area of the quadrilateral formed by the lines and the x-axis.

Solution

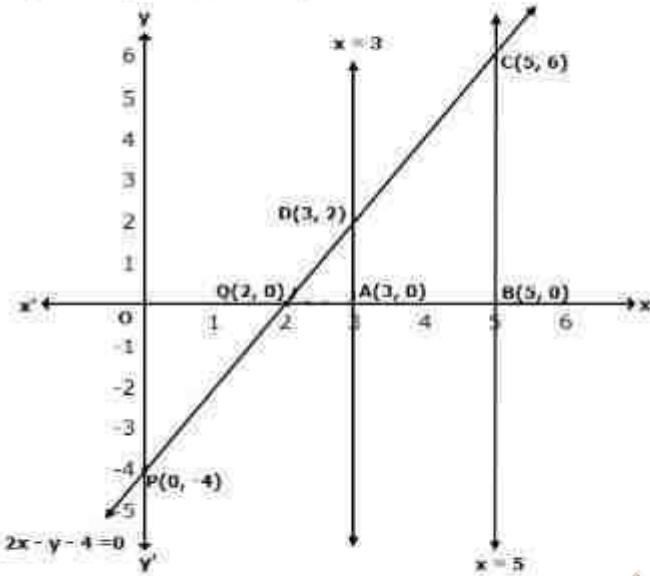
The graph of $x = 3$ is a straight line parallel to Y-axis at a distance of 3 units to the right of Y-axis.

The graph of $x = 5$ is a straight line parallel to Y-axis at a distance of 5 units to the right of Y-axis.

For the graph of $2x - y - 4 = 0$ i.e. $y = 2x - 4$, we have

x	0	2
y	-4	0

Graph of the given system of equation is as follows:



From the graph,

Area of trapezum ABCD

$$= \frac{1}{2} \times (AD + BC) \times AB$$

$$= \frac{1}{2} \times (2+6) \times 2$$

$$= 8 \text{ sq. units}$$

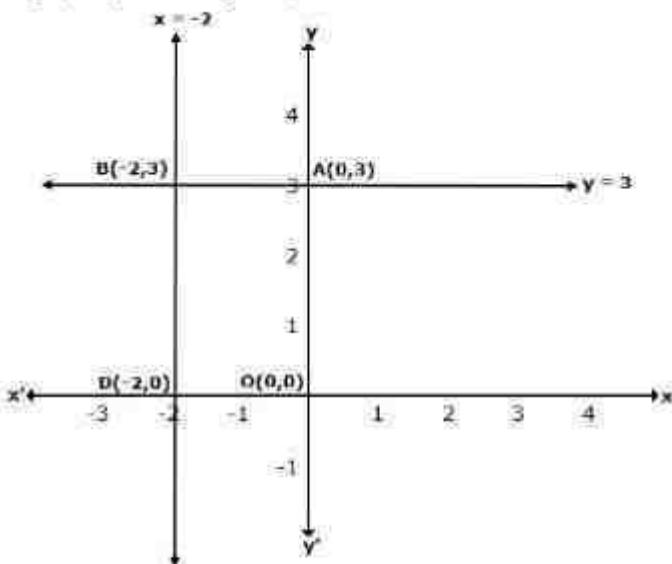
Q55

Draw the graphs of the lines $x = -2$, and $y = 3$. Write the vertices of the figure formed by these lines, the x-axis and the y-axis. Also, find the area of the figure.

Solution

The graph of $x = -2$ is a straight line parallel to Y-axis at a distance of 2 units to the left of Y-axis.

The graph of $y = 3$ is a straight line parallel to X-axis at a distance of 3 units above X-axis.



From the graph, the vertices of the figure formed by given lines, X-axis and Y-axis are

$O(0, 0)$, $A(0, 3)$, $B(-2, 3)$ and $D(-2, 0)$.

Now, $OD = AB = 2$ units and $BD = OA = 3$ units

$\Rightarrow OABD$ is a rectangle.

Hence, area of the figure formed

- Area of rectangle $OABD$

- $OAx AB$

- 3×2

- 6 sq. units

Q56

Draw the graphs of the pair of linear equations $x - y + 2 = 0$ and $4x - y - 4 = 0$. Calculate the area of the triangle formed by the lines so drawn and the x-axis.

Solution

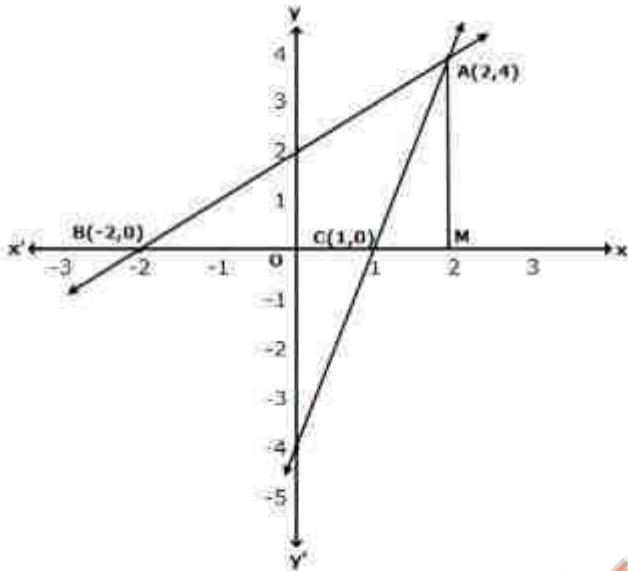
$$x - y + 2 = 0 \Rightarrow y = x + 2$$

x	0	-2
y	2	0

$$4x - y - 4 = 0 \Rightarrow y = 4x - 4$$

x	0	1
y	-4	0

Graph of the given system of equation is as follows:



From the graph,

Area of $\triangle ABC$

$$= \frac{1}{2} \times BC \times AM$$

$$= \frac{1}{2} \times 3 \times 4$$

$$= 6 \text{ sq units}$$

KopyKitab
Same textbooks, klick away

Exercise 3.3**Q1**

Solve the following systems of equations:

$$11x + 15y + 23 = 0 \quad \text{---(i)}$$

$$7x - 2y - 20 = 0 \quad \text{---(ii)}$$

Solution

The given system of equations is:

$$\begin{aligned} 11x + 15y + 23 &= 0 & \text{---(i)} \\ 7x - 2y - 20 &= 0 & \text{---(ii)} \end{aligned}$$

From (ii), we get

$$\begin{aligned} 2y &= 7x - 20 \\ \Rightarrow y &= \frac{7x - 20}{2} \end{aligned}$$

Substituting $y = \frac{7x - 20}{2}$ in (i), we get

$$\begin{aligned} 11x + 15\left(\frac{7x - 20}{2}\right) + 23 &= 0 \\ 11x + \frac{105x - 300}{2} + 23 &= 0 \\ 22x + 105x - 300 + 46 &= 0 \\ 127x - 254 &= 0 \\ 127x &= 254 \\ \Rightarrow x &= \frac{254}{127} = 2 \end{aligned}$$

Putting $x = 2$ in $y = \frac{7x - 20}{2}$, we get

$$\begin{aligned} \Rightarrow y &= \frac{7 \times 2 - 20}{2} \\ &= \frac{14 - 20}{2} \\ &= \frac{-6}{2} \\ &= -3 \end{aligned}$$

Hence, the solution of the given system of equations is $x = 2, y = -3$.**Q2**

Solve the following systems of equations:

$$3x - 7y + 10 = 0$$

$$y + 2x - 3 = 0$$

Solution

The given system of equations is:

$$\begin{aligned} 3x - 2y + 10 &= 0 \quad \text{---(i)} \\ y - 2x - 3 &= 0 \quad \text{---(ii)} \end{aligned}$$

From (ii), we get

$$y = 2x + 3$$

Substituting $y = 2x + 3$ in (i), we get

$$\begin{aligned} 3x - 2(2x + 3) + 10 &= 0 \\ \Rightarrow 3x - 4x - 6 + 10 &= 0 \\ \Rightarrow -x - 11 &= 0 \\ \Rightarrow -x &= 11 \\ \Rightarrow x &= \frac{11}{-1} = -1 \end{aligned}$$

Putting $x = -1$ in $y = 2x + 3$, we get

$$\begin{aligned} y &= 2 \times (-1) + 3 \\ &= -2 + 3 \\ &= 1 \\ \Rightarrow y &= 1 \end{aligned}$$

Hence, the solution of the given system of equations is $x = -1$, $y = 1$.

Q3

Solve the following systems of equations:

$$\begin{aligned} 0.4x + 0.3y &= 1.7 \\ 0.7x - 0.2y &= 0.8 \end{aligned}$$

Solution

The given system of equations is:

$$\begin{aligned} 0.4x + 0.3y &= 1.7 \quad \text{---(i)} \\ 0.7x - 0.2y &= 0.8 \quad \text{---(ii)} \end{aligned}$$

Multiplying both sides of (i) and (ii) by 10, we get

$$\begin{aligned} 4x + 3y &= 17 \quad \text{---(iii)} \\ 7x - 2y &= 8 \quad \text{---(iv)} \end{aligned}$$

From (iv), we get

$$7x = 8 + 2y$$

$$\Rightarrow x = \frac{8+2y}{7}$$

Substituting $x = \frac{8+2y}{7}$ in (iii), we get

$$4\left(\frac{8+2y}{7}\right) + 3y = 17$$

$$\Rightarrow \frac{32+8y}{7} + 3y = 17$$

$$\Rightarrow \frac{32+8y+21y}{7} = 17$$

$$\Rightarrow 32+29y = 17 \times 7$$

$$\Rightarrow 29y = 119 - 32$$

$$\Rightarrow 29y = 87$$

$$\Rightarrow y = \frac{87}{29} = 3$$

Putting $y = 3$ in $x = \frac{8+2y}{7}$, we get

$$\begin{aligned} x &= \frac{8+2 \times 3}{7} \\ &= \frac{8+6}{7} \\ &= \frac{14}{7} \\ &= 2 \end{aligned}$$

Hence, the solution of the given system of equations is $x = 2$, $y = 3$.

Q4

Solve the following systems of equations:

$$\frac{x}{2} + y = 0.8$$

$$\frac{7}{x+y} = 10$$

Solution

$$\frac{x}{2} + y = 0.8 \text{ and } \frac{7}{x+y} = 10$$

$$\frac{x}{2} + y = 0.8 \text{ and } \frac{7}{2x+y} = 10$$

$$x+2y = 1.6 \text{ and } \frac{7 \times 2}{2x+y} = 10$$

$$x+2y = 1.6 \text{ and } \frac{7}{2x+y} = 3$$

$$x+2y = 1.6 \text{ and } 7 = 10x+3y$$

Multiply first equation by 10

$$10x+20y = 16 \text{ and } 10x+3y = 7$$

Subtracting the two equations.

$$15y = 9$$

$$y = \frac{9}{15} = \frac{3}{5}$$

$$x = 1.6 - 2\left(\frac{3}{5}\right) = 1.6 - \frac{6}{5} = \frac{2}{5}$$

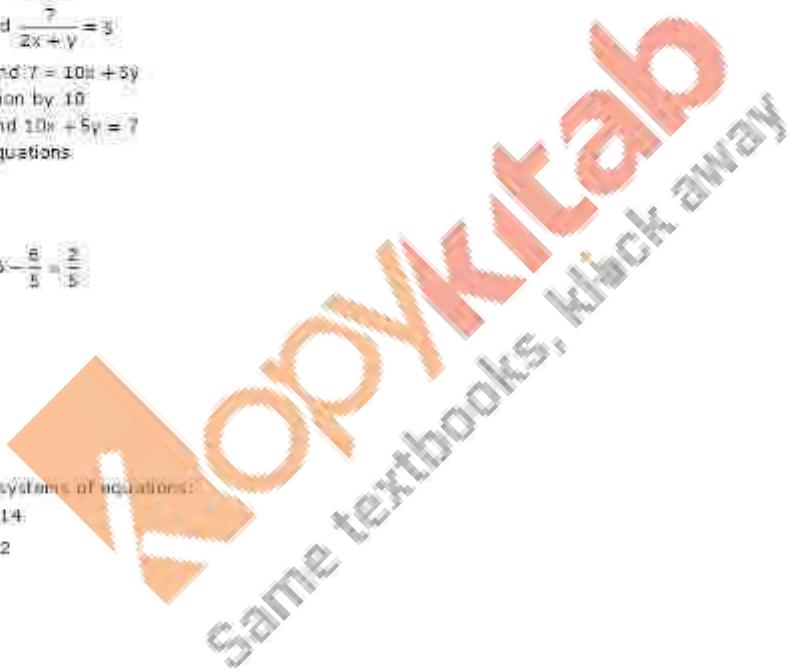
$$\text{Solution is } \left(\frac{2}{5}, \frac{3}{5}\right)$$

Q5

Solve the following systems of equations:

$$7(x+3) - 3(x+2) = 14$$

$$4(y-2) + 3(y-3) = 2$$

Solution

The given system of equations is

$$\begin{aligned} 7(y+2) - 2(x+2) &= 14 & \text{---(i)} \\ 4(y-2) + 3(x-3) &= 2 & \text{---(ii)} \end{aligned}$$

From (i), we get

$$\begin{aligned} 7y + 14 - 2x - 4 &= 14 \\ \Rightarrow 7y + 14 + 4 - 21 + 2x &= 0 \\ \Rightarrow y + \frac{2x - 3}{7} &= 0 \end{aligned}$$

From (ii), we get

$$\begin{aligned} 4y - 8 + 3x - 9 &= 2 \\ \Rightarrow 4y + 3x - 17 - 2 &= 0 \\ \Rightarrow 4y + 3x - 19 &= 0 & \text{---(iii)} \end{aligned}$$

Substituting $y = \frac{2x - 3}{7}$ in (iii), we get

$$\begin{aligned} 4\left(\frac{2x - 3}{7}\right) + 3x - 19 &= 0 \\ \Rightarrow \frac{8x - 12}{7} + 3x - 19 &= 0 \\ \Rightarrow 8x - 12 + 21x - 133 &= 0 \\ \Rightarrow 29x - 145 &= 0 \\ \Rightarrow 29x &= 145 \\ \Rightarrow x &= \frac{145}{29} = 5 \end{aligned}$$

Putting $x = 5$ in $y = \frac{2x - 3}{7}$, we get

$$\begin{aligned} y &= \frac{2 \times 5 - 3}{7} \\ &= \frac{10 - 3}{7} \\ &= \frac{7}{7} \\ &= 1 \\ \Rightarrow y &= 1 \end{aligned}$$

Hence, the solution of the given system of equations is $x = 5, y = 1$.

Q6

Solve the following systems of equations:

$$\begin{aligned} \frac{x}{7} + \frac{y}{3} &= 5 \\ \frac{x}{2} - \frac{y}{9} &= 5 \end{aligned}$$

Solution

The given system of equations is:

$$\frac{x}{7} + \frac{y}{3} = 5 \quad \text{---(i)}$$

$$\frac{x}{2} - \frac{y}{9} = 6 \quad \text{---(ii)}$$

From (i), we get

$$\frac{3x + 7y}{21} = 5$$

$$\Rightarrow 3x + 7y = 105$$

$$\Rightarrow 3x = 105 - 7y$$

$$\therefore x = \frac{105 - 7y}{3}$$

From (ii), we get

$$\frac{9x - 2y}{18} = 6$$

$$\Rightarrow 9x - 2y = 108 \quad \text{---(iii)}$$

Substituting $x = \frac{105 - 7y}{3}$ in (iii), we get

$$9\left(\frac{105 - 7y}{3}\right) - 2y = 108$$

$$\Rightarrow \frac{945 - 63y}{3} - 2y = 108$$

$$\Rightarrow 945 - 63y - 6y = 108 \times 3$$

$$\Rightarrow 945 - 69y = 324$$

$$\Rightarrow 945 - 324 = 69y$$

$$\Rightarrow 69y = 621$$

$$\therefore y = \frac{621}{69} = 9$$

Putting $y = 9$ in $x = \frac{105 - 7y}{3}$, we get:

$$x = \frac{105 - 7 \times 9}{3} = \frac{105 - 63}{3}$$

$$\therefore x = \frac{42}{3} = 14$$

Hence, the solution of the given system of equations is $x = 14, y = 9$.

Q7

Solve the following systems of equations.

$$\frac{x}{3} + \frac{y}{4} = 11$$

$$\frac{5x}{6} - \frac{y}{3} = -7$$

Solution

The given system of equations is:

$$\frac{x}{3} + \frac{y}{4} = 11 \quad \text{---(i)}$$

$$\frac{5x}{6} - \frac{y}{2} = -7 \quad \text{---(ii)}$$

From (i), we get

$$\begin{aligned} 4x + 3y &= 11 \\ \frac{12x + 9y}{12} &= 11 \\ 4x + 3y &= 132 \quad \text{---(iii)} \end{aligned}$$

From (ii), we get

$$\begin{aligned} 5x - 2y &= -7 \\ \frac{30x - 12y}{6} &= -7 \\ 5x - 2y &= -42 \quad \text{---(iv)} \end{aligned}$$

Let us eliminate y from the given equations. The coefficients of y in the equations (iii) and (iv) are 3 and 2 respectively. The LCM of 3 and 2 is 6. So, we make the coefficient of y equal to 6 in the two equations.

Multiplying (iii) by 2 and (iv) by 3, we get

$$\begin{aligned} 8x + 6y &= 264 \quad \text{---(v)} \\ 15x - 6y &= -126 \quad \text{---(vi)} \end{aligned}$$

Adding (v) and (vi), we get

$$\begin{aligned} 8x + 15x &= 264 - 126 \\ 23x &= 138 \\ x &= \frac{138}{23} = 6 \end{aligned}$$

Substituting $x = 6$ in (iii), we get

$$\begin{aligned} 4x + 3y &= 132 \\ 4 \times 6 + 3y &= 132 \\ 24 + 3y &= 132 \\ 3y &= 108 \\ y &= \frac{108}{3} = 36 \end{aligned}$$

Hence, the solution of the given system of equations is $x = 6, y = 36$.

Q8

Solve the following systems of equations:

$$\frac{4}{x} + \frac{3}{y} = 8$$

$$\frac{6}{x} - \frac{4}{y} = -5$$

Solution

Taking $\frac{1}{x} = u$, then given equations become

$$4u + 3v = 8 \quad \text{---(i)}$$

$$6u - 4v = -5 \quad \text{---(ii)}$$

From (i), we get:

$$\begin{aligned} 4u &= 8 - 3v \\ \Rightarrow u &= \frac{8 - 3v}{4} \end{aligned}$$

Substituting $u = \frac{8 - 3v}{4}$ in (ii), we get:

From (ii), we get:

$$\begin{aligned} 6\left(\frac{8 - 3v}{4}\right) - 4v &= -5 \\ \Rightarrow \frac{3(8 - 3v)}{2} - 4v &= -5 \\ \Rightarrow \frac{24 - 9v}{2} - 4v &= -5 \\ \Rightarrow \frac{24 - 9v - 8v}{2} &= -5 \\ \Rightarrow 24 - 17v &= -10 \\ \Rightarrow -17v &= -10 - 24 \\ \Rightarrow -17v &= -34 \\ \Rightarrow v &= \frac{-34}{-17} = 2 \end{aligned}$$

Putting $v = 2$, in $u = \frac{8 - 3v}{4}$, we get:

$$u = \frac{8 - 3 \times 2}{4} = \frac{8 - 6}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\text{Hence, } x = \frac{1}{u} = 2$$

So, the solution of the given system of equations is $x = 2, y = 2$.

Q9

Solve the following systems of equations.

$$x + \frac{y}{2} = 4$$

$$\frac{x}{3} + 2y = 5$$

Solution

The given system of equations is

$$x + \frac{y}{2} = 4 \quad \text{---(i)}$$

$$\frac{x}{3} + 2y = 5 \quad \text{---(ii)}$$

From (i), we get

$$\frac{2x + y}{2} = 4$$

$$2x + y = 8$$

$$y = 8 - 2x$$

From (ii), we get

$$x + 6y = 15 \quad \text{---(iii)}$$

Substituting $y = 8 - 2x$ in (iii), we get:

$$x + 6(8 - 2x) = 15$$

$$\Rightarrow x + 48 - 12x = 15$$

$$\Rightarrow -11x = 15 - 48$$

$$\Rightarrow -11x = -33$$

$$\Rightarrow x = \frac{-33}{-11} = 3$$

Putting $x = 3$, in $y = 8 - 2x$, we get:

$$y = 8 - 2 \times 3$$

$$= 8 - 6$$

$$= 2$$

$$\therefore y = 2$$

Hence, Solution of the given system of equation is $x = 3$, $y = 2$.

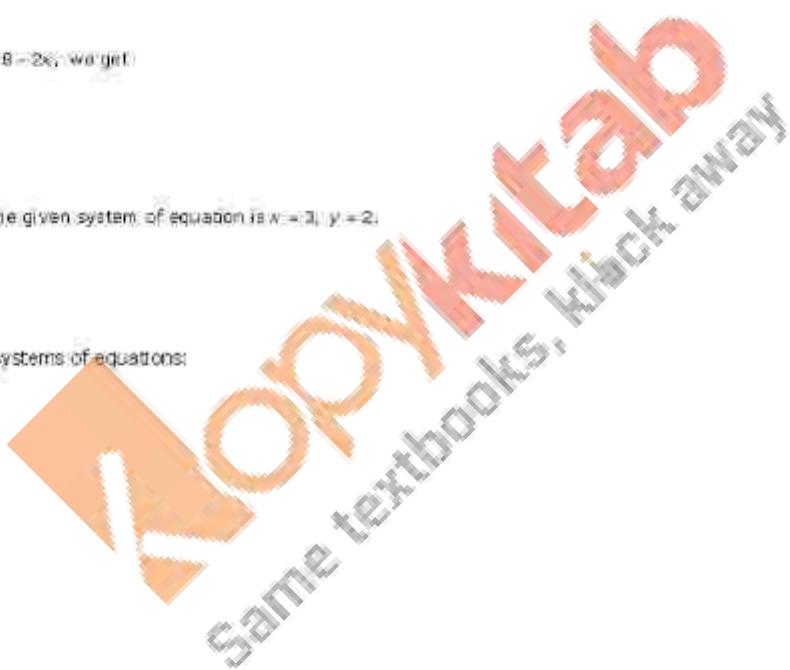
Q10

Solve the following systems of equations:

$$x + 2y = \frac{3}{2}$$

$$2x + y = \frac{3}{2}$$

Solution



The given system of equations is:

$$\begin{aligned}x+2y = \frac{3}{2} &\quad \text{---(i)} \\2x+y = \frac{3}{2} &\quad \text{---(ii)}\end{aligned}$$

Let us eliminate y from the given equations. The coefficients of y in the given equations are 2 and 1 respectively. The L.C.M. of 2 and 1 is 2. So, we make the coefficient of y equal to 2 in the two equations.

Multiplying (i) by 1 and (ii) by 2, we get:

$$\begin{aligned}x+2y = \frac{3}{2} &\quad \text{---(iii)} \\4x+2y = 3 &\quad \text{---(iv)}\end{aligned}$$

Subtracting (iii) from (iv), we get:

$$\begin{aligned}4x-x+2y-2y &= 3-\frac{3}{2} \\3x &= \frac{3}{2} \\3x &= \frac{3}{2} \\x &= \frac{3}{2 \times 3} \\x &= \frac{1}{2}\end{aligned}$$

Putting $x = \frac{1}{2}$, in equation (iv), we get:

$$\begin{aligned}4 \times \frac{1}{2} + 2y &= 3 \\2+2y &= 3 \\2y &= 3-2 \\2y &= 1 \\y &= \frac{1}{2}\end{aligned}$$

Hence, Solution of the given system of equations is $(\frac{1}{2}, \frac{1}{2})$.

Q11

Solve the following system of equations:

$$\sqrt{2}x - \sqrt{3}y = 0$$

$$\sqrt{3}x - \sqrt{8}y = 0$$

Solution

$$\sqrt{2}x - \sqrt{3}y = 0 \quad \dots (i)$$

$$\sqrt{3}x - \sqrt{8}y = 0 \quad \dots (ii)$$

From equation (i), we obtain:

$$x = \frac{\sqrt{3}y}{\sqrt{2}} \quad \dots (iii)$$

Substituting this value in equation (ii), we obtain:

$$\sqrt{3}\left(\frac{\sqrt{3}y}{\sqrt{2}}\right) - \sqrt{8}y = 0$$

$$\frac{3y}{\sqrt{2}} - 2\sqrt{2}y = 0$$

$$y\left(\frac{3}{\sqrt{2}} - 2\sqrt{2}\right) = 0$$

Substituting the value of y in equation (iii), we obtain:

$$x = 0$$

$$\therefore x = 0, y = 0$$

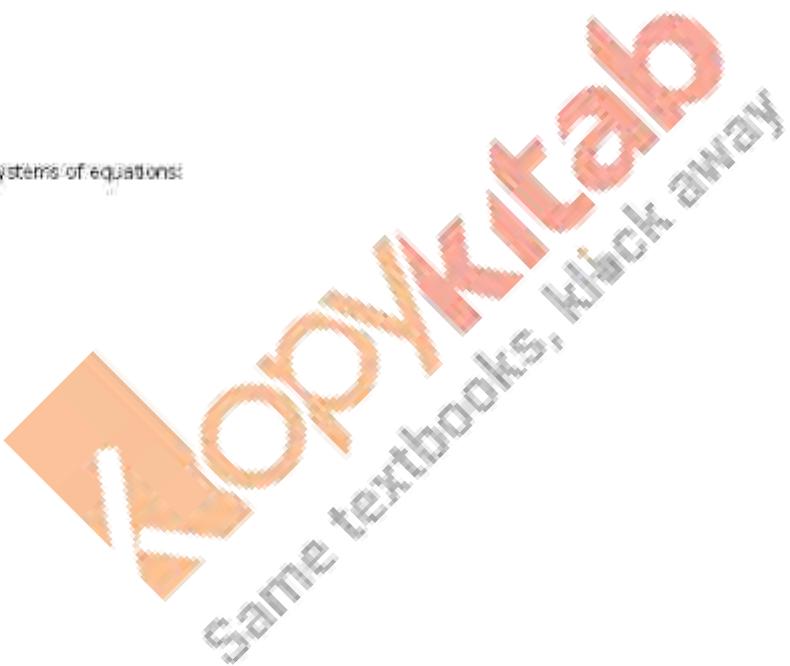
Q12

Solve the following systems of equations:

$$3x - \frac{y+7}{3} + 2 = 10$$

$$2y + \frac{x+11}{7} - 10 = 0$$

Solution



The given system of equations is:

$$\begin{aligned} 3x - \frac{y+7}{11} + 2 &= 10 \\ 2y + \frac{x+11}{7} - 10 &= 0 \end{aligned} \quad \text{---(i)}$$

From (i), we get:

$$\begin{aligned} \frac{33x - y - 7 + 22}{11} &= 10 \\ \Rightarrow 33x - y + 15 &= 10 \times 11 \\ \Rightarrow 33x + 15 - 110 &= y \\ \Rightarrow y &= 33x - 95 \end{aligned}$$

From (ii), we get:

$$\begin{aligned} \frac{14y + x + 11}{7} &= 0 \\ \Rightarrow 14y + x + 11 &= 0 \times 7 \\ \Rightarrow 14y + x + 11 &= 0 \\ \Rightarrow 14y + x &= 0 - 11 \\ \Rightarrow 14y + x &= -11 \quad \text{---(ii)} \end{aligned}$$

Substituting $y = 33x - 95$ in (ii), we get:

$$\begin{aligned} 14(33x - 95) + x &= -11 \\ \Rightarrow 462x - 1330 + x &= -11 \\ \Rightarrow 463x - 1330 &= -11 \\ \Rightarrow 463x &= 1330 - 11 \\ \Rightarrow x &= \frac{1319}{463} = 3 \end{aligned}$$

Putting $x = 3$, in $y = 33x - 95$, we get:

$$\begin{aligned} y &= 33 \times 3 - 95 \\ \Rightarrow y &= 99 - 95 \\ &\quad + 4 \\ \Rightarrow y &= 4 \end{aligned}$$

Hence, Solution of the given system of equation is $x = 3$, $y = 4$.

Q13

Solve the following system of equations:

$$\begin{aligned} 2x - \frac{3}{y} &= 0 \\ 3y + \frac{7}{x} &= 2, \quad y \neq 0 \end{aligned}$$

Solution

The given systems of equations is

$$2y - \frac{3}{y} = 9 \quad \text{---(i)}$$

$$3x + \frac{2}{y} = 2, y \neq 0 \quad \text{---(ii)}$$

Taking $\frac{1}{y} = u$, the given equations becomes:

$$2y - 3u = 9 \quad \text{---(iii)}$$

$$3x + 2u = 2 \quad \text{---(iv)}$$

From (iii), we get

$$2y = 9 + 3u$$

$$\Rightarrow y = \frac{9+3u}{2}$$

Substituting $y = \frac{9+3u}{2}$ in (iv), we get

$$3\left(\frac{9+3u}{2}\right) + 2u = 2$$

$$\Rightarrow 27 + 9u + 4u = 2$$

$$\Rightarrow 27 + 13u = 2$$

$$\Rightarrow 13u = 2 - 27$$

$$\Rightarrow u = \frac{-25}{13} = -1$$

$$\text{Hence, } y = \frac{1}{u} = \frac{1}{-1} = -1$$

Putting $u = -1$ in $x = \frac{9+3u}{2}$, we get

$$x = \frac{9+3(-1)}{2} = \frac{9-3}{2} = \frac{6}{2} = 3$$

$$\Rightarrow x = 3$$

Hence, solution of the given system of equations is $x = 3, y = -1$.

Q14

Solve the following systems of equations:

$$0.5x + 0.7y = 0.74$$

$$0.3x + 0.5y = 0.5$$

Solution

The given system of equations is

$$0.5x + 0.7y = 0.74 \quad \text{---(i)}$$

$$0.3x + 0.5y = 0.5 \quad \text{---(ii)}$$

Multiplying (i) and (ii) by 100, we get

$$50x + 70y = 74 \quad \text{---(iii)}$$

$$30x + 50y = 50 \quad \text{---(iv)}$$

From (iii), we get:

$$50x = 74 - 70y$$

$$\Rightarrow x = \frac{74 - 70y}{50}$$

Substituting $x = \frac{74 - 70y}{50}$ in equation (iv), we get

$$30\left(\frac{74 - 70y}{50}\right) + 50y = 50$$

$$\Rightarrow \frac{3(74 - 70y)}{5} + 50y = 50$$

$$\Rightarrow \frac{222 - 210y}{5} + 50y = 50$$

$$\Rightarrow 222 - 210y + 250y = 250$$

$$\Rightarrow 40y = 250 - 222$$

$$\Rightarrow 40y = 28$$

$$\Rightarrow y = \frac{28}{40} = \frac{14}{20} = \frac{7}{10} = 0.7$$

Putting $y = 0.7$ in $x = \frac{74 - 70y}{50}$, we get:

$$x = \frac{74 - 70 \times 0.7}{50}$$

$$= \frac{74 - 49}{50}$$

$$= \frac{25}{50}$$

$$= \frac{1}{2}$$

$$= 0.5$$

Hence, Solution of the given system of equation is $x = 0.5, y = 0.7$

Q15

Solve the following systems of equations:

$$\frac{1}{7x} + \frac{1}{6y} = 3$$

$$\frac{1}{2x} - \frac{1}{3y} = \frac{5}{6}$$

Solution

$$\frac{1}{7x} + \frac{1}{6y} = 3 \quad \dots (1)$$

$$\frac{1}{4x} - \frac{1}{3y} = 5 \quad \dots (2)$$

Multiplying (2) by $\frac{1}{3}$, we get:

$$\frac{1}{4x} - \frac{1}{6y} = \frac{5}{3} \quad \dots (3)$$

Solving (1) and (3), we get:

$$\begin{aligned} \frac{1}{7x} + \frac{1}{6y} &= 3 \\ \frac{1}{4x} - \frac{1}{6y} &= \frac{5}{3} \\ \frac{1}{7x} + \frac{1}{4x} &= 3 + \frac{5}{2} \quad (\text{Adding the equations}) \\ \Rightarrow \frac{4+7}{28x} &= \frac{6+5}{2} \\ \Rightarrow \frac{11}{28x} &= \frac{11}{2} \\ \Rightarrow x &= \frac{11 \times 2}{28 \times 11} = \frac{1}{14} \end{aligned}$$

When $x = \frac{1}{14}$, we get:

$$\begin{aligned} \frac{1}{7\left(\frac{1}{14}\right)} + \frac{1}{6y} &= 3 \quad (\text{Using (1)}) \\ \Rightarrow 2 + \frac{1}{6y} &= 3 \\ \Rightarrow \frac{1}{6y} &= 3 - 2 = 1 \\ \Rightarrow y &= \frac{1}{6} \end{aligned}$$

Thus, the solution of given equations is $x = \frac{1}{14}$ and $y = \frac{1}{6}$.

Q16

Solve the following systems of equations:

$$\frac{1}{2x} + \frac{1}{3y} = 2$$

$$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$$

Solution

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the given equations become:

$$\begin{aligned} \frac{u}{2} + \frac{v}{3} &= 2 \\ \Rightarrow \frac{3u+2v}{6} &= 2 \\ \Rightarrow 3u+2v &= 12 \quad \text{---(i)} \end{aligned}$$

$$\begin{aligned} \text{And, } \frac{u}{3} + \frac{v}{2} &= \frac{13}{5} \\ \Rightarrow \frac{2u+3v}{6} &= \frac{13}{5} \\ \Rightarrow 2u+3v &= 13 \quad \text{---(ii)} \end{aligned}$$

Let us eliminate 'v' from equations (i) and (ii).

Multiplying equation (i) by 3 and (ii) by 2, we get:

$$\begin{aligned} 9u+6v &= 36 \quad \text{---(iii)} \\ 4u+6v &= 26 \quad \text{---(iv)} \end{aligned}$$

Subtracting equation (iv) from equation (iii), we get:

$$\begin{aligned} 9u-4u+6v-6v &= 36-26 \\ \Rightarrow 5u &= 10 \\ \Rightarrow u &= \frac{10}{5} = 2 \end{aligned}$$

Putting $u = 2$ in equation (i), we get:

$$\begin{aligned} 3 \times 2 + 2v &= 12 \\ \Rightarrow 6+2v &= 12 \\ \Rightarrow 2v &= 12-6 \\ \Rightarrow v &= \frac{6}{2} = 3 \end{aligned}$$

Hence, $x = \frac{1}{u} = \frac{1}{2}$ and $y = \frac{1}{v} = \frac{1}{3}$

So, the solution of the given system of equation is $x = \frac{1}{2}$, $y = \frac{1}{3}$.

Q17

Solve the following systems of equations:

$$\begin{aligned} \frac{1}{u} + \frac{2}{v} &= 17 \\ \frac{1}{u} - \frac{1}{v} &= \frac{36}{5} \end{aligned}$$

Solution

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$. Then, the given system of equations becomes

$$\begin{aligned} 15u + 2v &= 17 \quad \text{---(i)} \\ u+v &= \frac{36}{5} \quad \text{---(ii)} \end{aligned}$$

From (i), we get

$$\begin{aligned} 2v &= 17 - 15u \\ \Rightarrow v &= \frac{17 - 15u}{2} \end{aligned}$$

Substituting $v = \frac{17 - 15u}{2}$ in equation (ii), we get

$$\begin{aligned} u + \frac{17 - 15u}{2} &= \frac{36}{5} \\ \frac{2u + 17 - 15u}{2} &= \frac{36}{5} \\ \frac{-13u + 17}{2} &= \frac{36}{5} \\ 5(-13u + 17) &= 36 \times 2 \\ -65u + 85 &= 72 \\ -65u &= 72 - 85 \\ -65u &= -13 \\ \Rightarrow u &= \frac{-13}{-65} = \frac{1}{5} \end{aligned}$$

Putting $u = \frac{1}{5}$ in equation (ii), we get:

$$\begin{aligned} \frac{1}{5} + v &= \frac{36}{5} \\ \Rightarrow v &= \frac{36}{5} - \frac{1}{5} \\ &= \frac{36 - 1}{5} \\ &= \frac{35}{5} = 7 \end{aligned}$$

Hence, $u = \frac{1}{5}$ and $v = \frac{1}{7}$.

So, the solution of the given system of equation is $u = 5$, $v = 7$.

Q18

Solve the following systems of equations:

$$\begin{aligned} \frac{3}{x} - \frac{1}{y} &= -9 \\ \frac{2}{x} + \frac{3}{y} &= 5 \end{aligned}$$

Solution

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$. Then, the given system of equations becomes:

$$\begin{aligned} 3u + v &= -9 \\ 2u + 3v &= 5 \end{aligned} \quad \begin{array}{l} \text{---(i)} \\ \text{---(ii)} \end{array}$$

Multiplying equation (i) by 3 and equation (ii) by 1, we get

$$\begin{aligned} 9u + 3v &= -27 \\ 2u + 3v &= 5 \end{aligned} \quad \begin{array}{l} \text{---(iii)} \\ \text{---(iv)} \end{array}$$

Adding equation (iii) and equation (iv), we get

$$\begin{aligned} 9u + 2u - 3v + 3v &= -27 + 5 \\ \Rightarrow 11u &= -22 \end{aligned}$$

$$\Rightarrow u = \frac{-22}{11} = -2$$

Putting $u = -2$ in equation (iv), we get

$$\begin{aligned} 2 \times (-2) + 3v &= 5 \\ \Rightarrow -4 + 3v &= 5 \\ \Rightarrow 3v &= 5 + 4 \\ \Rightarrow v &= \frac{9}{3} = 3 \end{aligned}$$

Hence, $x = \frac{1}{u} = \frac{1}{-2} = -\frac{1}{2}$ and $y = \frac{1}{v} = \frac{1}{3}$.

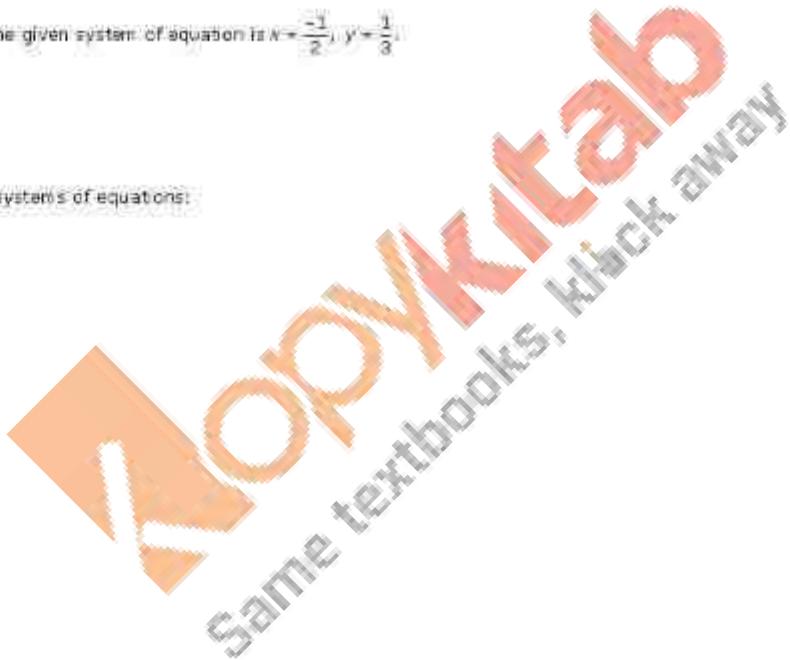
So, the solution of the given system of equation is $x = -\frac{1}{2}$, $y = \frac{1}{3}$.

Q19

Solve the following systems of equations:

$$\begin{aligned} \frac{2}{x} + \frac{5}{y} &= 1 \\ \frac{60}{x} + \frac{40}{y} &= 19 \end{aligned}$$

Solution



Taking $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the given becomes:

$$\begin{aligned} 3u + 5v &= 1 & \text{---(i)} \\ 60u + 40v &= 19 & \text{---(ii)} \end{aligned}$$

Let us eliminate 'u' from equations (i) and (ii). Multiplying equation (i) by 60 and equation (ii) by 2, we get:

$$\begin{aligned} 120u + 300v &= 60 & \text{---(iii)} \\ 120u + 80v &= 38 & \text{---(iv)} \end{aligned}$$

Subtracting (iv) from (iii), we get:

$$300v - 80v = 60 - 38$$

$$\Rightarrow 220v = 22$$

$$\Rightarrow v = \frac{22}{220} = \frac{1}{10}$$

Putting $v = \frac{1}{10}$ in equation (i), we get:

$$3u + 5 \times \frac{1}{10} = 1$$

$$\Rightarrow 3u + \frac{5}{2} = 1$$

$$\Rightarrow 3u = 1 - \frac{5}{2}$$

$$\Rightarrow 3u = \frac{2-5}{2} = -\frac{3}{2}$$

$$\Rightarrow u = -\frac{1}{2}$$

$$\Rightarrow u = -\frac{1}{4}$$

Hence, $x = -\frac{1}{u} = 4$ and $y = \frac{1}{v} = 10$.

So, the solution of the given system of equation is $x = -4$, $y = 10$.

Q20

Solve the following systems of equations:

$$\begin{aligned} \frac{1}{5x} + \frac{1}{6y} &= 10 \\ \frac{1}{3x} - \frac{3}{7y} &= 8 \end{aligned}$$

Solution

Taking $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the given equations become

$$\begin{aligned} \text{I: } & \frac{u}{5} + \frac{v}{6} = 12 \\ & 6u + 5v = 12 \times 30 \\ \Rightarrow & 6u + 5v = 360 \quad \text{---(i)} \end{aligned}$$

$$\begin{aligned} \text{and, } & \frac{u}{3} - \frac{3v}{7} = 8 \\ \text{II: } & \frac{7u - 9v}{21} = 8 \\ \Rightarrow & 7u - 9v = 168 \quad \text{---(ii)} \end{aligned}$$

Let us eliminate ' v ' from equations (i) and (ii). Multiplying equation (i) by 9 and equation (ii) by 5, we get

$$\begin{aligned} 54u + 45v &= 3240 \quad \text{---(iii)} \\ 35u - 45v &= 840 \quad \text{---(iv)} \end{aligned}$$

Adding equation (iii) and equation (iv), we get

$$\begin{aligned} 89u &= 3240 + 840 \\ \Rightarrow & 89u = 4080 \\ \Rightarrow & u = \frac{4080}{89} \end{aligned}$$

Putting $u = \frac{4080}{89}$ in equation (i), we get

$$\begin{aligned} 5 \times \frac{4080}{89} + 5v &= 360 \\ \frac{20400}{89} + 5v &= 360 \\ \Rightarrow & 5v = 360 - \frac{20400}{89} \\ \Rightarrow & 5v = \frac{32400 - 20400}{89} \\ \Rightarrow & 5v = \frac{7560}{89} \\ \Rightarrow & v = \frac{7560}{5 \times 89} \\ \Rightarrow & v = \frac{1512}{89} \end{aligned}$$

Hence, $x = \frac{1}{u} = \frac{89}{4080}$ and $y = \frac{1}{v} = \frac{89}{1512}$

So, the solution of the given system of equations is $x = \frac{89}{4080}, y = \frac{89}{1512}$.

Q21

Solve the following system of equations:

$$\begin{aligned} \frac{4}{x} + 3y &= 14 \\ \frac{3}{x} - 4y &= 23 \end{aligned}$$

Solution

$$\begin{aligned}\frac{4}{x} + 3y &= 14 \\ \frac{3}{x} - 4y &= 23 \\ \text{Let } \frac{1}{x} &= p\end{aligned}$$

The given equations reduce to:
 $4p + 3y = 14 \quad \dots (1)$

$$\Rightarrow 4p + 3y - 14 = 0 \quad \dots (1)$$

$$3p - 4y = 23$$

$$\Rightarrow 3p - 4y - 23 = 0 \quad \dots (2)$$

Using cross-multiplication method, we obtain:

$$\begin{array}{rcl} p & y & z \\ \hline -69 - 56 & -42 - (-92) & -16 - 9 \\ -125 & 50 & 25 \\ \hline p & -1 & -1 \\ -125 & 25 & 50 & 25 \\ p = 5, y = -2 & & & \end{array}$$

$$p = \frac{1}{x} = 5$$

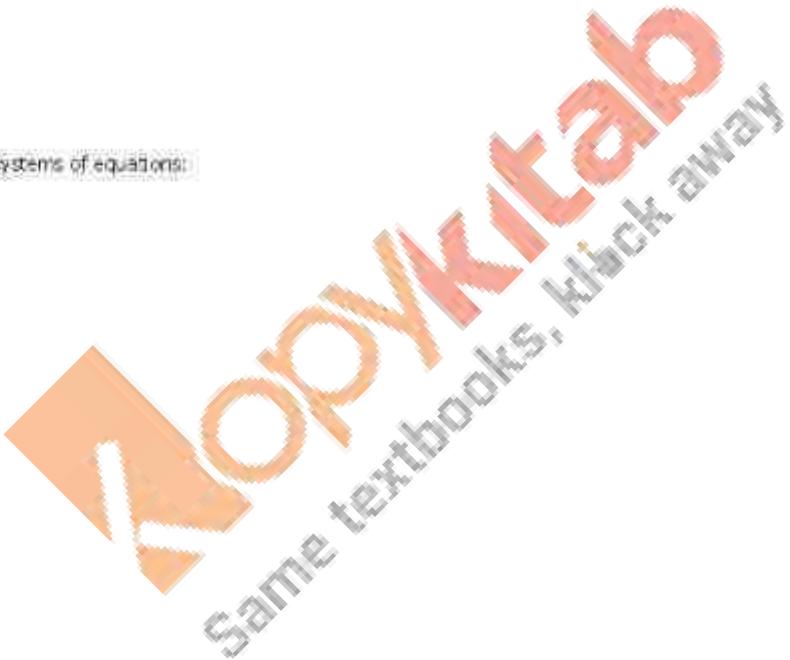
$$\begin{aligned}x &= \frac{1}{5}, y = -2 \\ x &= 5\end{aligned}$$

Q22

Solve the following systems of equations:

$$\begin{aligned}\frac{4}{x} + 5y &= 7 \\ \frac{3}{x} + 4y &= 5\end{aligned}$$

Solution



The given system of equations is

$$\begin{aligned} \frac{4}{x} + 5y &= 7 & \text{---(i)} \\ \frac{3}{x} + 4y &= 5 & \text{---(ii)} \end{aligned}$$

Multiplying equation (i) by 3 and equation (ii) by 4, we get:

$$\begin{aligned} \frac{12}{x} + 15y &= 21 & \text{---(iii)} \\ \frac{12}{x} + 16y &= 20 & \text{---(iv)} \end{aligned}$$

Subtracting equation (iv) from equation (iii), we get:

$$\begin{aligned} \frac{12}{x} - \frac{12}{x} + 15y - 16y &= 21 - 20 \\ \Leftrightarrow y &= -1 \end{aligned}$$

Putting $y = -1$ in equation (i), we get:

$$\begin{aligned} \frac{4}{x} + 5 \times (-1) &= 7 \\ \frac{4}{x} - 5 &= 7 \\ \frac{4}{x} &= 7 + 5 \\ \frac{4}{x} &= 12 \\ \Rightarrow 4 &= 12x \\ \Rightarrow \frac{4}{12} &= x \\ \Rightarrow x &= \frac{4}{12} \\ \Rightarrow x &= \frac{1}{3} \end{aligned}$$

Hence, solution of the given system of equations is $x = \frac{1}{3}, y = -1$.

Q23

Solve the pair of equations:

$$\begin{aligned} \frac{2}{x} + \frac{3}{y} &= 13 \\ \frac{5}{x} - \frac{4}{y} &= -2 \end{aligned}$$

Solution

Let us write the given pair of equations as:

$$\begin{aligned} 2\left(\frac{1}{x}\right) + 3\left(\frac{1}{y}\right) &= 13 & \text{(1)} \\ 5\left(\frac{1}{x}\right) - 4\left(\frac{1}{y}\right) &= -2 & \text{(2)} \end{aligned}$$

These equations are not in the form $ax + by + c = 0$. However, if we substitute

$\frac{1}{x} = p$ and $\frac{1}{y} = q$ in Equations (1) and (2), we get:

$$2p + 3q = 13$$

$$5p - 4q = -2$$

So, we have expressed the equations as a pair of linear equations. Now, you can use any method to solve these equations, and get $p = 3, q = 2$.

You know that $p = \frac{1}{x}$ and $q = \frac{1}{y}$.

Substitute the values of p and q to get:

$$\frac{1}{x} = 3 \text{ i.e., } x = \frac{1}{3} \text{ and } \frac{1}{y} = 2 \text{ i.e., } y = \frac{1}{2}$$

Q24

Solve the following system of equations:

$$\begin{aligned}\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} &= 2 \\ \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} &= -1\end{aligned}$$

Solution

$$\begin{aligned}\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} &= 2 \\ \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} &= -1\end{aligned}$$

$$\text{Let } \frac{1}{\sqrt{x}} = p \text{ and } \frac{1}{\sqrt{y}} = q$$

The given equations reduce to:

$$\begin{aligned}2p + 3q &= 2 & \dots (1) \\ 4p - 9q &= -1 & \dots (2)\end{aligned}$$

Multiplying equation (1) by 3, we obtain:

$$6p + 9q = 6 \quad \dots (3)$$

Adding equation (2) and (3), we obtain:

$$\begin{aligned}10p &= 5 \\ p &= \frac{1}{2}\end{aligned}$$

Putting the value of p in equation (1), we obtain:

$$\begin{aligned}2 \times \frac{1}{2} + 3q &= 2 \\ 3q &= 1 \\ q &= \frac{1}{3} \\ p &= \frac{1}{\sqrt{x}} = \frac{1}{2} \\ \sqrt{x} &= 2 \\ x &= 4 \\ q &= \frac{1}{\sqrt{y}} = \frac{1}{3} \\ \sqrt{y} &= 3 \\ y &= 9 \\ \therefore x &= 4, y = 9\end{aligned}$$

Q25

Solve the following systems of equation:

$$\begin{aligned}x + y &= 2, \\ xy &\\ \frac{x - y}{xy} &= 6\end{aligned}$$

Solution

The given system of equations is:

$$\begin{aligned}x + y &= 2 \\ \frac{x}{xy} &= \frac{2}{xy} \\ \Rightarrow x + y &= 2xy \quad \dots (i)\end{aligned}$$

$$\text{And, } \frac{x-y}{xy} = 6$$

$$\Rightarrow x - y = 6xy \quad \dots (ii)$$

Adding equations (i) and (ii), we get

$$2x = 8xy$$

$$\Rightarrow 1 = 4y$$

$$\Rightarrow y = \frac{1}{4}$$

Substituting $y = \frac{1}{4}$ in (i), we get:

$$x + \frac{1}{4} = 2x \times \frac{1}{4}$$

$$\Rightarrow \frac{4x+1}{4} = \frac{x}{2}$$

$$\Rightarrow 8x + 2 = 4x$$

$$\Rightarrow 4x = -2$$

$$\Rightarrow x = -\frac{1}{2}$$

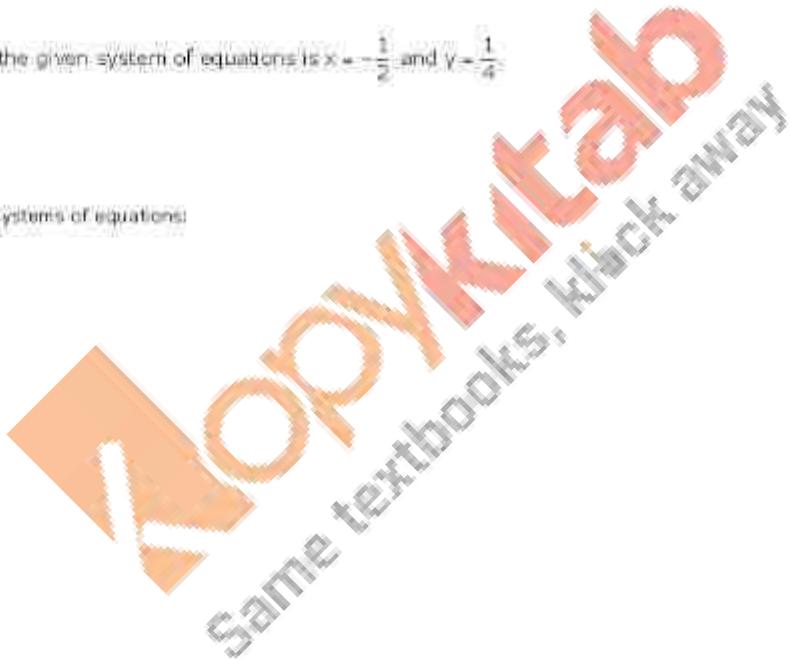
Hence, solution of the given system of equations is $x = -\frac{1}{2}$ and $y = \frac{1}{4}$.

Q26

Solve the following systems of equations:

$$\begin{aligned}\frac{2}{x} + \frac{3}{y} &= \frac{9}{14} \\ \frac{4}{x} + \frac{9}{y} &= \frac{21}{14}\end{aligned}$$

Solution



The system of given equations is

$$\begin{aligned} \frac{2}{x} + \frac{3}{y} &= \frac{9}{xy} \\ \frac{4}{x} + \frac{3}{y} &= \frac{21}{xy} \end{aligned} \quad \begin{array}{l} \text{---(i)} \\ \text{---(ii)} \end{array}$$

Multiplying equation (i) and (ii) by xy , we get

$$\begin{aligned} 2y + 3x &= 9 & \text{---(iii)} \\ 4y + 3x &= 21 & \text{---(iv)} \end{aligned}$$

From (iii), we get

$$\begin{aligned} 3x &= 9 - 2y \\ \Rightarrow x &= \frac{9-2y}{3} \end{aligned}$$

Substituting $x = \frac{9-2y}{3}$ in equation (iv), we get

$$\begin{aligned} 4y + 3\left(\frac{9-2y}{3}\right) &= 21 \\ \Rightarrow 4y + 3(9-2y) &= 21 \\ \Rightarrow 4y + 27 - 6y &= 21 \\ \Rightarrow -2y &= 21 - 27 \\ \Rightarrow -2y &= -6 \\ \Rightarrow y &= \frac{-6}{-2} = 3 \end{aligned}$$

Putting $y = 3$ in $x = \frac{9-2y}{3}$, we get

$$\begin{aligned} x &= \frac{9-2 \times 3}{3} \\ &= \frac{9-6}{3} \\ &= \frac{3}{3} \\ &= 1 \end{aligned}$$

Hence, solution of the system of equations is $x = 1, y = 3$.

Q27

Solve the following systems of equations:

$$\begin{aligned} \frac{6}{x+y} - \frac{7}{x-y} &= 3 \\ \frac{1}{2(x+y)} - \frac{1}{3(x-y)} &= 1 \end{aligned}$$

where $x+y \neq 0$ and $x-y \neq 0$

Solution

Let $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$. Then, the given system of equations becomes:

$$\Rightarrow 6u - 7v = 3 \quad \text{---(i)}$$

$$\text{and, } \frac{v}{2} = \frac{1}{3}$$

$$\Rightarrow 3v = 2y \quad \text{---(ii)}$$

$$\Rightarrow 3u - 2v = 0 \quad \text{---(iii)}$$

Multiplying equation (ii) by 2, and equation (i) by 1, we get

$$6u - 7v = 3 \quad \text{---(iv)}$$

$$6u - 4v = 0 \quad \text{---(v)}$$

Subtracting equation (v) from equation (iv), we get

$$-7v + 4v = 3$$

$$\Rightarrow -3v = 3$$

$$\Rightarrow v = -1$$

Putting $v = -1$ in equation (ii), we get

$$2u + 2 \times (-1) = 0$$

$$\Rightarrow 2u + 2 = 0$$

$$\Rightarrow 2u = -2$$

$$\Rightarrow u = \frac{-2}{3}$$

Now,

$$u = \frac{-2}{3}$$

$$\Rightarrow \frac{1}{x+y} = \frac{-2}{3}$$

$$\Rightarrow x+y = \frac{-3}{2} \quad \text{---(vi)}$$

$$\text{and, } v = -1$$

$$\Rightarrow \frac{1}{x-y} = -1$$

$$\Rightarrow x-y = -1 \quad \text{---(vii)}$$

Adding equation (vi) and equation (vii), we get

$$2x = \frac{-3}{2} - 1$$

$$\Rightarrow 2x = \frac{-5}{2}$$

$$\Rightarrow 2x = \frac{-5}{2}$$

$$\Rightarrow x = \frac{-5}{4}$$

Putting $x = \frac{-5}{4}$ in equation (vi), we get

$$\frac{-5}{4} - y = -1$$

$$\Rightarrow \frac{-5}{4} + 1 = y$$

$$\Rightarrow \frac{-5+4}{4} = y$$

$$\Rightarrow \frac{-1}{4} = y$$

$$\Rightarrow y = \frac{-1}{4}$$

Hence, solution of the system of equations is $x = \frac{-5}{4}$, $y = \frac{-1}{4}$.

Q28

Solve the following systems of equations:

$$\frac{xy}{x+y} = \frac{6}{5}$$

$$\frac{xy}{y-x} = \frac{5}{3}$$

$$\frac{xy}{y-x} = 5$$

where $x+y \neq 0$, and $y-x \neq 0$.

Solution

The given system of equation is

$$\begin{aligned} \frac{xy}{x+y} &= 6 \\ \Rightarrow xy &= 6(x+y) \\ \Rightarrow xy &= 6x+6y \quad \text{---(i)} \end{aligned}$$

$$\text{And, } \frac{xy}{y-x} = 6$$

$$\begin{aligned} \Rightarrow xy &= 6(y-x) \\ \Rightarrow xy &= 6y - 6x \quad \text{---(ii)} \end{aligned}$$

Adding equation (i) and equation (ii), we get

$$\begin{aligned} 6xy &= 6y + 6x \\ \Rightarrow 6xy &= 12y \\ \Rightarrow x &= \frac{12y}{6y} = 2 \end{aligned}$$

Putting $x = 2$ in equation (i), we get:

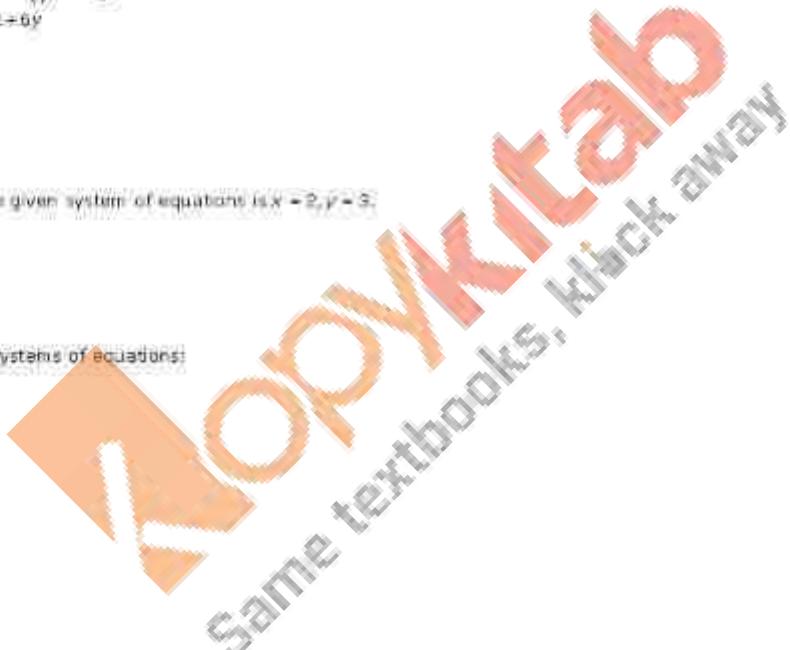
$$\begin{aligned} 5 \times 2 \times y &= 6 \times 2 + 6y \\ \Rightarrow 10y &= 12 + 6y \\ \Rightarrow 10y - 6y &= 12 \\ \Rightarrow 4y &= 12 \\ \Rightarrow y &= \frac{12}{4} = 3 \end{aligned}$$

Hence, solution of the given system of equations is $x = 2, y = 3$.

Q29

Solve the following systems of equations:

$$\begin{aligned} \frac{2x}{x+y} + \frac{15}{x-y} &= 5 \\ \frac{5x}{x+y} + \frac{45}{x-y} &= 14 \end{aligned}$$

Solution

Let $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$. Then, the given system of equations becomes

$$\begin{aligned} 22u + 15v &= 5 & \text{---(i)} \\ 55u + 15v &= 14 & \text{---(ii)} \end{aligned}$$

Multiplying equation (i) by 3, and equation (ii) by 1, we get

$$\begin{aligned} 66u + 45v &= 15 & \text{---(iii)} \\ 55u + 15v &= 14 & \text{---(iv)} \end{aligned}$$

Subtracting equation (iv) from equation (iii), we get

$$\begin{aligned} 66u - 55u &= 15 - 14 \\ \Rightarrow 11u &= 1 \\ \Rightarrow u &= \frac{1}{11} \end{aligned}$$

Putting $u = \frac{1}{11}$ in equation (i), we get

$$\begin{aligned} 22 \times \frac{1}{11} + 15v &= 5 \\ \Rightarrow 2 + 15v &= 5 \\ \Rightarrow 15v &= 5 - 2 \\ \Rightarrow 15v &= 3 \\ \Rightarrow v &= \frac{3}{15} = \frac{1}{5} \end{aligned}$$

$$\begin{aligned} \text{Now, } u &= \frac{1}{x+y} \\ \Rightarrow \frac{1}{x+y} &= \frac{1}{11} \\ \Rightarrow x+y &= 11 & \text{---(v)} \end{aligned}$$

$$\begin{aligned} \text{And, } v &= \frac{1}{x-y} \\ \Rightarrow \frac{1}{x-y} &= \frac{1}{5} \\ \Rightarrow x-y &= 5 & \text{---(vi)} \end{aligned}$$

Adding equation (v) and equation (vi), we get

$$\begin{aligned} 2x &= 11+5 \\ \Rightarrow 2x &= 16 \\ \Rightarrow x &= \frac{16}{2} = 8 \end{aligned}$$

Putting $x = 8$ in equation (v), we get

$$\begin{aligned} 8+y &= 11 \\ \Rightarrow y &= 11-8 = 3 \end{aligned}$$

Hence, solution of the given system of equations is $x = 8, y = 3$.

Q30

Solve the following systems of equations:

$$\begin{aligned} \frac{5}{x+y} - \frac{2}{x-y} &= -1 \\ \frac{15}{x+y} + \frac{7}{x-y} &= 10 \end{aligned}$$

Solution

Let $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$. Then, the given system of equations becomes

$$\begin{aligned} 5u - 2v &= -1 \quad \text{---(i)} \\ 15u + 7v &= 10 \quad \text{---(ii)} \end{aligned}$$

Multiplying equation (i) by 7, and equation (ii) by 2, we get

$$\begin{aligned} 35u - 14v &= -7 \quad \text{---(iii)} \\ 30u + 14v &= 20 \quad \text{---(iv)} \end{aligned}$$

Adding equation (iii) and equation (iv), we get:

$$\begin{aligned} \therefore 35u + 30u &= -7 + 20 \\ \therefore 65u &= 13 \\ \therefore u &= \frac{13}{65} = \frac{1}{5} \end{aligned}$$

Putting $u = \frac{1}{5}$ in equation (i), we get

$$\begin{aligned} \frac{5}{x+y} - 2v &= -1 \\ 1 - 2v &= -1 \\ -2v &= -1 - 1 \\ -2v &= -2 \\ \therefore v &= \frac{-2}{-2} = 1 \end{aligned}$$

Now, $u = \frac{1}{x+y}$

$$\begin{aligned} u &= \frac{1}{x+y} = \frac{1}{5} \\ \therefore x+y &= 5 \quad \text{---(v)} \end{aligned}$$

$$\begin{aligned} \text{And, } v &= \frac{1}{x-y} = 1 \\ \therefore x-y &= 1 \quad \text{---(vi)} \end{aligned}$$

Adding equation (v) and equation (vi), we get:

$$\begin{aligned} 2x &= 5+1 \\ 2x &= 6 \\ \therefore x &= \frac{6}{2} = 3 \end{aligned}$$

Putting $x = 3$ in equation (v), we get:

$$\begin{aligned} 3+y &= 5 \\ \therefore y &= 5-3=2 \end{aligned}$$

Hence, solution of the given system of equations is $x = 3, y = 2$.

Q31

Solve the following systems of equations:

$$\begin{aligned} \frac{3}{x+y} + \frac{2}{x-y} &= 2 \\ \frac{9}{x+y} + \frac{4}{x-y} &= 1 \end{aligned}$$

Solution

Let $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$. Then, the given system of equations becomes

$$\begin{aligned} 3u + 2v &= 2 & \text{---(i)} \\ 5u - 4v &= 1 & \text{---(ii)} \end{aligned}$$

Multiplying equation (i) by 2, and equation (ii) by 1, we get:

$$\begin{aligned} 6u + 4v &= 4 & \text{---(iii)} \\ 5u - 4v &= 1 & \text{---(iv)} \end{aligned}$$

Adding equation (iii) and equation (iv), we get:

$$\begin{aligned} 6u + 9u &= 4 + 1 \\ \Rightarrow 15u &= 5 \\ \Rightarrow u &= \frac{5}{15} = \frac{1}{3} \end{aligned}$$

Putting $u = \frac{1}{3}$ in equation (i), we get:

$$\begin{aligned} 3 \times \frac{1}{3} + 2v &= 2 \\ \Rightarrow 1 + 2v &= 2 \\ \Rightarrow 2v &= 2 - 1 \\ \Rightarrow v &= \frac{1}{2} \end{aligned}$$

Now, $u = \frac{1}{x+y}$

$$\begin{aligned} \Rightarrow \frac{1}{x+y} &= \frac{1}{3} \\ \Rightarrow x+y &= 3 & \text{---(v)} \end{aligned}$$

And, $v = \frac{1}{x-y}$

$$\begin{aligned} \Rightarrow \frac{1}{x-y} &= \frac{1}{2} \\ \Rightarrow x-y &= 2 & \text{---(vi)} \end{aligned}$$

Adding equation (v) and equation (vi), we get:

$$\begin{aligned} 2x &= 3 + 2 \\ \Rightarrow x &= \frac{5}{2} \end{aligned}$$

Putting $x = \frac{5}{2}$ in equation (v), we get:

$$\begin{aligned} \frac{5}{2} + y &= 3 \\ \Rightarrow y &= 3 - \frac{5}{2} \\ \Rightarrow y &= \frac{6-5}{2} = \frac{1}{2} \end{aligned}$$

Hence, solution of the given system of equations is $x = \frac{5}{2}, y = \frac{1}{2}$.

Q32

Solve the following systems of equations:

$$\begin{aligned} \frac{1}{2(x+2y)} + \frac{5}{3(3x-2y)} &= -3 \\ \frac{5}{4(x+2y)} - \frac{3}{2(3x-2y)} &= \frac{61}{60} \end{aligned}$$

Solution

Let $\frac{1}{x+2y} = u$ and $\frac{1}{3x-2y} = v$. Then, the given system of equations becomes

$$\begin{aligned} & \frac{u}{2} + \frac{5v}{3} = -3 \\ \Rightarrow & \frac{3u+10v}{6} = -3 \\ \Rightarrow & 3u+10v = -3 \times 6 \\ \Rightarrow & 3u+10v = -18 \quad \text{---(i)} \end{aligned}$$

$$\begin{aligned} \text{And, } & \frac{5u}{4} - \frac{3v}{5} = \frac{61}{60} \\ \Rightarrow & \frac{25u-12v}{20} = \frac{61}{60} \\ \Rightarrow & 25u-12v = \frac{61}{3} \quad \text{---(ii)} \end{aligned}$$

Multiplying equation (i) by 12 and equation (ii) by 10, we get

$$\begin{aligned} & 36u+120v = -108 \quad \text{---(iii)} \\ & 250u-120v = \frac{610}{3} \quad \text{---(iv)} \end{aligned}$$

Adding equation (iii) and equation (iv), we get

$$\begin{aligned} & 36u+250u = \frac{530}{3}-108 \\ \Rightarrow & 286u = \frac{510-324}{3} \\ \Rightarrow & 286u = \frac{186}{3} \\ \Rightarrow & u = \frac{1}{3} \end{aligned}$$

Putting $u = \frac{1}{3}$ in equation (i), we get

$$\begin{aligned} & 3 \times \frac{1}{3} + 10v = -9 \\ \Rightarrow & 1+10v = -9 \\ \Rightarrow & 10v = -9-1 \\ \Rightarrow & v = \frac{-10}{10} = -1 \end{aligned}$$

$$\text{Now, } u = \frac{1}{x+2y}$$

$$\Rightarrow \frac{1}{x+2y} = \frac{1}{3} \\ \Rightarrow x+2y = 3 \quad \text{---(v)}$$

$$\text{And, } v = \frac{1}{3x-2y}$$

$$\Rightarrow \frac{1}{3x-2y} = -1 \\ \Rightarrow 3x-2y = -1 \quad \text{---(vi)}$$

Adding equation (v) and equation (vi), we get

$$\begin{aligned} & x+3x = 3-1 \\ \Rightarrow & 4x = 2 \\ \Rightarrow & x = \frac{2}{4} = \frac{1}{2} \end{aligned}$$

KopyKitab
Same textbooks, klick away!

Putting $x = \frac{1}{2}$ in equation (v), we get

$$\begin{aligned} & \frac{1}{2} + 2y = 3 \\ \Rightarrow & 2y = 3 - \frac{1}{2} \\ \Rightarrow & 2y = \frac{6-1}{2} \\ \Rightarrow & y = \frac{5}{4} \end{aligned}$$

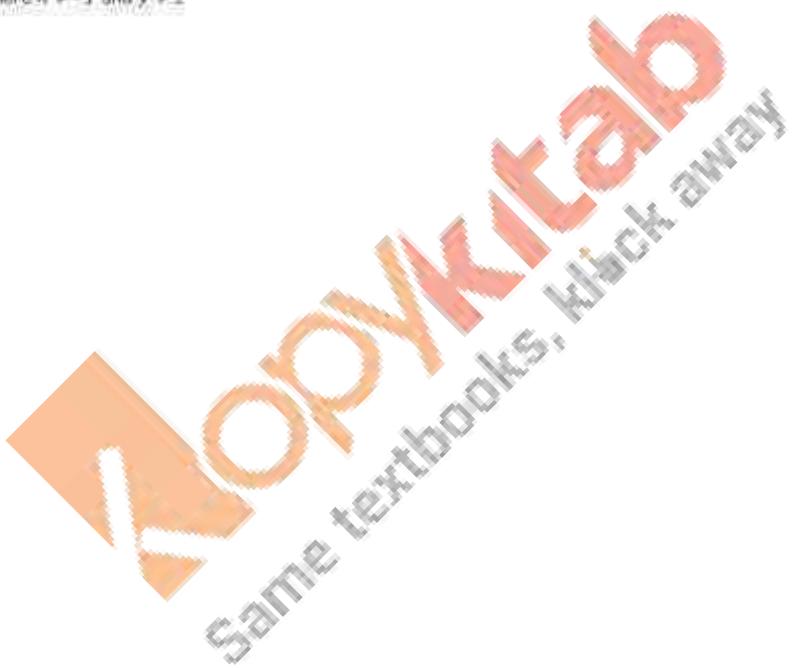
Hence, solution of the given system of equations is $x = \frac{1}{2}, y = \frac{5}{4}$.

Q33

Solve the following systems of equations:

$$\begin{aligned} & \frac{5}{x+1} - \frac{2}{y-1} = \frac{1}{2} \\ & \frac{10}{x+1} + \frac{2}{y-1} = \frac{5}{2}, \text{ where } x \neq -1 \text{ and } y \neq 1 \end{aligned}$$

Solution



Let $\frac{2}{x+1} = u$ and $\frac{1}{y-1} = v$. Then, the given system of equations becomes

$$\begin{aligned} \Rightarrow & 5u + 2v = \frac{1}{2} \quad \text{---(i)} \\ \Rightarrow & 10u + 4v = \frac{5}{2} \quad \text{---(ii)} \end{aligned}$$

Adding equation (i) and equation (ii), we get

$$\begin{aligned} 5u + 10u = \frac{1}{2} + \frac{5}{2} \\ \Rightarrow & 15u = \frac{1+5}{2} \\ \Rightarrow & 15u = \frac{6}{2} = 3 \\ \Rightarrow & u = \frac{3}{15} = \frac{1}{5} \end{aligned}$$

Putting $u = \frac{1}{5}$ in equation (i), we get

$$\begin{aligned} 5 \times \frac{1}{5} + 2v = \frac{1}{2} \\ \Rightarrow & 1 + 2v = \frac{1}{2} \\ \Rightarrow & 2v = \frac{1}{2} - 1 \\ \Rightarrow & 2v = \frac{1-2}{2} \\ \Rightarrow & 2v = \frac{-1}{2} \\ \Rightarrow & v = \frac{-1}{2 \times 2} = \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \text{Now, } u = \frac{1}{x+1} \\ \Rightarrow & \frac{1}{x+1} = \frac{1}{5} \\ \Rightarrow & x+1 = 5 \\ \Rightarrow & x = 5-1=4 \end{aligned}$$

$$\begin{aligned} \text{And, } v = \frac{1}{y-1} \\ \Rightarrow & \frac{1}{y-1} = \frac{1}{4} \\ \Rightarrow & y-1 = 4 \\ \Rightarrow & y = 4+1=5 \end{aligned}$$

Hence, solution of the given system of equations is $x=4, y=5$

Q34

Solve the following systems of equations:

$$\begin{aligned} x+y &= 5xy \\ 3x+2y &= 13xy, \quad x \neq 0, \quad y \neq 0 \end{aligned}$$

Solution

The given system of equation is

$$\begin{aligned}x + y &= 5xy \quad \text{---(i)} \\3x + 2y &= 13xy \quad \text{---(ii)}\end{aligned}$$

Multiplying equation (i) by 2 and equation (ii) by 1, we get

$$\begin{aligned}2x + 2y &= 10xy \quad \text{---(iii)} \\3x + 2y &= 13xy \quad \text{---(iv)}\end{aligned}$$

Subtracting equation (iii) from equation (iv), we get

$$\begin{aligned}3x - 2x + 13xy - 10xy &= 0 \\x &= 3xy \\ \frac{x}{3x} &= y \\ \frac{1}{3} &= y\end{aligned}$$

Putting $y = \frac{1}{3}$ in equation (i), we get

$$\begin{aligned}x + \frac{1}{3} &= 5 \times x \times \frac{1}{3} \\x + \frac{1}{3} &= \frac{5x}{3} \\x - \frac{5x}{3} &= -\frac{1}{3} \\-\frac{4x}{3} &= -\frac{1}{3} \\4x &= 1 \\x &= \frac{1}{4}\end{aligned}$$

Hence, solution of the given system of equations is $x = \frac{1}{4}, y = \frac{1}{3}$

Q35

Solve the following systems of equations:

$$\begin{aligned}x + y &= 2xy \\ \frac{x - y}{xy} &= 5, x \neq 0, y \neq 0\end{aligned}$$

Solution

The system of the given equation is:

$$x + y = 2xy \quad \text{--- (i)}$$

and, $\frac{x - y}{xy} = 6$

$$x - y = 6xy \quad \text{--- (ii)}$$

Adding equation (i) and equation (ii), we get

$$2x = 2xy + 6xy$$

$$\Rightarrow 2x = 8xy$$

$$\Rightarrow \frac{2x}{8x} = y$$

$$\Rightarrow y = \frac{1}{4}$$

Putting $y = \frac{1}{4}$ in equation (i), we get

$$x + \frac{1}{4} = 2x \times \frac{1}{4}$$

$$\Rightarrow x + \frac{1}{4} = \frac{x}{2}$$

$$\Rightarrow x - \frac{x}{2} = -\frac{1}{4}$$

$$\Rightarrow \frac{2x - x}{2} = -\frac{1}{4}$$

$$\Rightarrow x = \frac{-2}{4} = \frac{-1}{2}$$

Hence, solution of the given system of equations is $x = -\frac{1}{2}, y = \frac{1}{4}$.

Q36

Solve the following systems of equations:

$$2(3u - v) = 5uv$$

$$2(u + 3v) = 5uv$$

Solution

The system of the given equation is:

$$2(3u - v) = 5uv \quad \text{--- (i)}$$

$$\Rightarrow 6u - 2v = 5uv$$

$$\text{and, } 2(u + 3v) = 5uv$$

$$\Rightarrow 2u + 6v = 5uv \quad \text{--- (ii)}$$

Multiplying equation (i) by 3 and equation (ii) by 1, we get:

$$18u - 6v = 15uv \quad \text{--- (iii)}$$

$$2u + 6v = 5uv \quad \text{--- (iv)}$$

Adding equation (iii) and equation (iv), we get:

$$18u + 2u = 15uv + 5uv$$

$$\Rightarrow 20u = 20uv$$

$$\Rightarrow \frac{20u}{20u} = v$$

$$\Rightarrow v = 1$$

Putting $v = 1$ in equation (i), we get:

$$6u - 2 \times 1 = 5u \times 1$$

$$\Rightarrow 6u - 2 = 5u$$

$$\Rightarrow 6u - 5u = 2$$

$$\Rightarrow u = 2$$

Hence, solution of the given system of equations is $u = 2, v = 1$.

Q37

Solve the following systems of equations:

$$\frac{2}{3x+2y} + \frac{3}{3x-2y} = \frac{17}{5}$$

$$\frac{5}{3x+2y} + \frac{1}{3x-2y} = 2$$

Solution

Let $\frac{2}{3x+2y} = u$ and $\frac{1}{3x-2y} = v$. Then, the given system of equations becomes

$$\begin{aligned} 2u + 3v &= \frac{17}{5} \\ 5u + v &= 2 \end{aligned} \quad \begin{array}{l} \text{---(i)} \\ \text{---(ii)} \end{array}$$

Multiplying equation (i) by 5, we get

$$10u + 15v = 17 \quad \text{---(iii)}$$

Subtracting equation (ii) from equation (iii), we get

$$\begin{aligned} 10u - 5u &= 17 - \frac{17}{5} \\ \Rightarrow 5u &= \frac{80-17}{5} \\ \Rightarrow 5u &= \frac{63}{5} \\ \Rightarrow u &= \frac{13}{5 \times 12} = \frac{1}{5} \end{aligned}$$

Putting $u = \frac{1}{5}$ in equation (ii), we get

$$\begin{aligned} 5 \times \frac{1}{5} + v &= 2 \\ 1 + v &= 2 \\ v &= 2 - 1 \\ \Rightarrow v &= 1 \end{aligned}$$

$$\begin{aligned} \text{Now, } u &= \frac{1}{3x+2y} \\ \Rightarrow \frac{1}{3x+2y} &= \frac{1}{5} \\ \Rightarrow 3x+2y &= 5 \end{aligned} \quad \text{---(iv)}$$

$$\begin{aligned} \text{And, } v &= \frac{1}{3x-2y} \\ \Rightarrow \frac{1}{3x-2y} &= 1 \\ \Rightarrow 3x-2y &= 1 \end{aligned} \quad \text{---(v)}$$

Adding equation (iv) and (v), we get

$$\begin{aligned} 6x &= 1 + 5 \\ \Rightarrow 6x &= 6 \\ \Rightarrow x &= 1 \end{aligned}$$

Putting $x = 1$ in equation (iv), we get

$$\begin{aligned} 3 \times 1 + 2y &= 5 \\ \Rightarrow 3y &= 5 - 3 \\ \Rightarrow 2y &= 2 \\ \Rightarrow y &= \frac{2}{2} = 1 \end{aligned}$$

Hence, solution of the given system of equations is $x = 1, y = 1$.

Q38

Solve the following systems of equations:

$$\begin{aligned} \frac{4x}{x+y} + \frac{30}{x-y} &= 10 \\ \frac{5x}{x+y} + \frac{40}{x-y} &= 13 \end{aligned}$$

Solution

Let $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$. Then, the system of the given equations becomes

$$\begin{aligned} 44u + 30v &= 10 & \text{---(i)} \\ 55u + 40v &= 13 & \text{---(ii)} \end{aligned}$$

Multiplying equation (i) by 5 and equation (ii) by 3, we get

$$\begin{aligned} 176u + 120v &= 40 & \text{---(iii)} \\ 165u + 120v &= 39 & \text{---(iv)} \end{aligned}$$

Subtracting equation (iv) by equation (iii), we get

$$176u - 165u = 40 - 39$$

$$\Rightarrow 11u = 1$$

$$\Rightarrow u = \frac{1}{11}$$

Putting $u = \frac{1}{11}$ in equation (i), we get

$$44 \times \frac{1}{11} + 30v = 10$$

$$4 + 30v = 10$$

$$\Rightarrow 30v = 10 - 4$$

$$\Rightarrow 30v = 6$$

$$\Rightarrow v = \frac{6}{30} = \frac{1}{5}$$

$$\text{Now, } u = \frac{1}{x+y}$$

$$\Rightarrow \frac{1}{x+y} = \frac{1}{11}$$

$$\Rightarrow x+y = 11 \quad \text{---(v)}$$

$$\text{and, } v = \frac{1}{x-y}$$

$$\Rightarrow \frac{1}{x-y} = \frac{1}{5}$$

$$\Rightarrow x-y = 5 \quad \text{---(vi)}$$

Adding equation (v) and (vi), we get

$$2x = 11 + 5$$

$$\Rightarrow 2x = 16$$

$$\Rightarrow x = \frac{16}{2} = 8$$

Putting $x = 8$ in equation (v), we get

$$8+y = 11$$

$$\Rightarrow y = 11-8 = 3$$

Hence, solution of the given system of equations is $x = 8, y = 3$.

Q39

Solve the pair of equations:

$$\begin{aligned} \frac{5}{x-1} + \frac{1}{y-2} &= 2 \\ \frac{6}{x-1} - \frac{3}{y-2} &= 1 \end{aligned}$$

Solution

Let us put $\frac{1}{x-1} = p$, and $\frac{1}{y-2} = q$. Then the given equations

$$5\left(\frac{1}{x-1}\right) + \frac{1}{y-2} = 2 \quad (1)$$

$$6\left(\frac{1}{x-1}\right) - 3\left(\frac{1}{y-2}\right) = 1 \quad (2)$$

$$\text{Can be written as: } 5p + q = 2 \quad (3)$$

$$6p - 3q = 1 \quad (4)$$

Equations (3) and (4) form a pair of linear equations in the general form. Now, you can use any method to solve these equations. We get $p = \frac{1}{3}$ and $q = \frac{1}{3}$. Now,

substituting $\frac{1}{x-1}$ for p , we have

$$\frac{1}{x-1} = \frac{1}{3},$$

i.e., $x-1 = 3$, i.e., $x = 4$.

Similarly, substituting $\frac{1}{y-2}$ for q , we get

$$\frac{1}{y-2} = \frac{1}{3}$$

i.e., $3 = y-2$, i.e., $y = 5$.

Hence, $x = 4, y = 5$ is the required solution of the given pair of equations.

Q40

Solve the following system of equations:

$$\frac{10}{x+y} + \frac{3}{x-y} = 4$$

$$\frac{15}{x+y} - \frac{5}{x-y} = -2$$

Solution**Q41**

Solve the following system of equations:

$$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$$

$$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$$

Solution

$$\begin{aligned}\frac{1}{3x+y} + \frac{1}{3x-y} &= \frac{3}{4} \\ \frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} &= \frac{-1}{8} \\ \text{Let } \frac{1}{3x+y} = p \text{ and } \frac{1}{3x-y} = q.\end{aligned}$$

The given equations reduce to:

$$\begin{aligned}p+q &= \frac{3}{4} \quad \dots (1) \\ \frac{p-q}{2} &= \frac{-1}{8} \\ \Rightarrow p-q &= \frac{-1}{4} \quad \dots (2)\end{aligned}$$

Adding (1) and (2), we obtain:

$$\begin{aligned}2p &= \frac{3}{4} - \frac{1}{4} \\ 2p &= \frac{1}{2} \\ p &= \frac{1}{4}\end{aligned}$$

Substituting the value of p in (2), we obtain:

$$\begin{aligned}\frac{1}{4} - q &= \frac{-1}{4} \\ q &= \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \\ p = \frac{1}{4} &= \frac{1}{3x+y} \quad \frac{1}{4} \\ 3x+y &= 4 \quad \dots (3) \\ q = \frac{1}{2} &= \frac{1}{3x-y} \quad \frac{1}{2} \\ 3x-y &= 2 \quad \dots (4)\end{aligned}$$

Adding equations (3) and (4), we obtain:

$$\begin{aligned}6x &= 6 \\ x &= 1\end{aligned}$$

Substituting the value of x in (3), we obtain:

$$\begin{aligned}3(1)+y &= 4 \\ y &= 1 \\ x=1, y=1. \quad &\end{aligned}$$

Q42

Solve the following system of equations:

$$\begin{array}{r} 7x-2y = 5 \\ \hline xy \\ 9x+7y = 15 \\ \hline xy \end{array}$$

Solution

$$\begin{aligned} 7x - 2y &= 5 \\ \frac{7}{x} - \frac{2}{y} &= 5 \quad \dots (1) \\ 8x + 7y &= 15 \\ \frac{8}{x} + \frac{7}{y} &= 15 \quad \dots (2) \end{aligned}$$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$

The given equations reduce to:

$$\begin{aligned} -2p + 7q &= 5 \\ \Rightarrow -2p + 7q - 5 &= 0 \quad \dots (3) \\ 7p + 8q &= 15 \\ \Rightarrow 7p + 8q - 15 &= 0 \quad \dots (4) \end{aligned}$$

Using cross-multiplication method, we obtain:-

$$\begin{aligned} \frac{p}{-105 - (-40)} &= \frac{q}{-35 - 30} = \frac{1}{-16 - 40} \\ \frac{p}{-65} &= \frac{q}{-65} = \frac{1}{-65} \\ \frac{p}{-65} &= \frac{1}{-65} = \frac{q}{-65} = \frac{1}{-65} \\ p &= 1, q = 1 \\ p = \frac{1}{x} &= 1, q = \frac{1}{y} = 1 \\ x &= 1, y = 1 \end{aligned}$$

Q43

Solution

Q44

Solve the following systems of equations.

$$\begin{aligned} 99x + 101y &= 499 \\ 101x + 99y &= 501 \end{aligned}$$

Solution



The given system of equation is:

$$\begin{aligned} 99x + 101y &= 499 \quad \text{---(i)} \\ 101x + 99y &= 501 \quad \text{---(ii)} \end{aligned}$$

Adding equation (i) and equation (ii), we get

$$\begin{aligned} 99x + 101y + 101x + 99y &= 499 + 501 \\ \Rightarrow 200x + 200y &= 1000 \\ \Rightarrow 200(x + y) &= 1000 \\ \Rightarrow x + y &= \frac{1000}{200} = 5 \\ \Rightarrow x + y &= 5 \quad \text{---(iii)} \end{aligned}$$

Subtracting equation (ii) by equation (i), we get

$$\begin{aligned} 101x - 99x + 99y - 101y &= 501 - 499 \\ \Rightarrow 2x - 2y &= 2 \\ \Rightarrow 2(x - y) &= 2 \\ \Rightarrow x - y &= \frac{2}{2} \\ \Rightarrow x - y &= 1 \quad \text{---(iv)} \end{aligned}$$

Adding equation (iii) and equation (iv), we get

$$\begin{aligned} 2x &= 5 + 1 \\ \Rightarrow x &= \frac{6}{2} = 3 \end{aligned}$$

Putting $x = 3$ in equation (ii), we get

$$\begin{aligned} 3 + y &= 5 \\ \Rightarrow y &= 5 - 3 = 2 \end{aligned}$$

Hence, solution of the given system of equations is $x = 3, y = 2$.

Q45

Solve the following systems of equations:

$$\begin{aligned} 23x - 29y &= 30 \\ 29x - 23y &= 110 \end{aligned}$$

Solution

The given system of equation is

$$23x - 29y = 98 \quad \text{---(i)}$$

$$29x - 23y = 110 \quad \text{---(ii)}$$

Adding equation (i) and equation (ii), we get

$$23x + 29x - 29y - 23y = 98 + 110$$

$$\Rightarrow 52x - 52y = 208$$

$$\Rightarrow 52(x - y) = 208$$

$$\Rightarrow x - y = \frac{208}{52} = 4$$

$$\Rightarrow x - y = 4 \quad \text{---(iii)}$$

Subtracting equation (i) by equation (ii), we get

$$29x - 23x - 23y + 29y = 110 - 98$$

$$\Rightarrow 6x + 6y = 12$$

$$\Rightarrow 6(x + y) = 12$$

$$\Rightarrow x + y = \frac{12}{6} = 2$$

$$\Rightarrow x + y = 2 \quad \text{---(iv)}$$

Adding equation (iii) and equation (iv), we get

$$2x + 2 + 4 = 5$$

$$\Rightarrow x = \frac{6}{2} = 3$$

Putting $x = 3$ in equation (iv), we get

$$3 + y = 2$$

$$\Rightarrow y = 2 - 3 = -1$$

Hence, solution of the given system of equations is $x = 3, y = -1$

Q46

Solve the following systems of equations:

$$x - y + z = 4$$

$$x - 2y - 2z = 9$$

$$2x + y + 3z = 1$$

Solution

KopyKitab
Same textbooks, klick away

We have,

$$\begin{aligned}x - y + z &= 4 && \text{---(i)} \\x - 2y - 2z &= 9 && \text{---(ii)} \\2x + y + 3z &= 1 && \text{---(iii)}\end{aligned}$$

From equation (i), we get

$$\begin{aligned}z &= 4 - x + y \\&\Rightarrow z = -x + y + 4\end{aligned}$$

Substituting the value of z in equation (ii), we get

$$\begin{aligned}x - 2y - 2[-x + y + 4] &= 9 \\x - 2y + 2x - 2y - 8 &= 9 \\3x - 4y &= 9 + 8 \\3x - 4y &= 17 && \text{---(iv)}\end{aligned}$$

Substituting the value of z in equation (iii), we get

$$\begin{aligned}2x + y + 3[-x + y + 4] &= 1 \\2x + y - 2x + 3y + 12 &= 1 \\-x + 4y &= 1 - 12 \\-x + 4y &= -11 && \text{---(v)}\end{aligned}$$

Adding equations (iv) and (v), we get

$$\begin{aligned}3x - x - 4y + 4y &= 17 - 11 \\2x &= 6 \\x &= \frac{6}{2} = 3\end{aligned}$$

Putting $x = 3$ in equation (iv), we get:

$$\begin{aligned}3 \times 3 - 4y &= 17 \\9 - 4y &= 17 \\-4y &= 17 - 9 \\-4y &= 8 \\y &= \frac{8}{-4} = -2\end{aligned}$$

Putting $x = 3$ and $y = -2$ in $z = -x + y + 4$, we get

$$\begin{aligned}z &= -3 + 2 + 4 \\z &= -5 + 4 \\z &= -1\end{aligned}$$

Hence, solution of the giving system of equation is $x = 3, y = -2, z = -1$.

Q47

Solve the following systems of equations:

$$\begin{aligned}x - y + z &= 4 \\x + y + z &= 2 \\2x + y - 3z &= 0\end{aligned}$$

Solution

We have,

$$\begin{aligned}x + y + z &= 4 && \text{---(i)} \\x + y + z &= 2 && \text{---(ii)} \\2x + y - 3z &= 0 && \text{---(iii)}\end{aligned}$$

From equation (i), we get

$$\begin{aligned}x + 4 - x + y \\z = -x + y + 4\end{aligned}$$

Substituting $z = -x + y + 4$ in equation (ii), we get

$$\begin{aligned}x + y + (-x + y + 4) &= 2 \\x + y - x + y + 4 &= 2 \\2y + 4 &= 2 \\2y - 2 &= 4 - 2 \\2y &= 2 \\y &= \frac{2}{2} = 1\end{aligned}$$

Substituting the value of y in equation (iii), we get

$$\begin{aligned}2x + y - 3(-x + y + 4) &= 0 \\2x + y + 3x - 3y - 12 &= 0 \\5x - 2y - 12 &= 0 \\5x - 2(-1) &= 12 && \text{---(iv)}$$

Putting $y = -1$ in equation (iv), we get

$$\begin{aligned}5x - 2(-1) &= 12 \\5x + 2 &= 12 \\5x = 12 - 2 &= 10 \\x &= \frac{10}{5} = 2\end{aligned}$$

Putting $x = 2$ and $y = -1$ in $z = -x + y + 4$, we get

$$\begin{aligned}z = -2 + (-1) + 4 \\= -2 - 1 + 4 \\= -3 + 4 \\= 1\end{aligned}$$

Hence, solution of the giving system of equation is $x = 2, y = -1, z = 1$.

Q48

Solve the following systems of equation:

$$\begin{aligned}21x + 47y &= 110 \\47x + 21y &= 162\end{aligned}$$

Solution

The given system of equations is

$$21x + 47y = 110 \quad \dots (i)$$

$$47x + 21y = 162 \quad \dots (ii)$$

Adding equations (i) and (ii), we get:

$$68x + 68y = 272$$

$$\Rightarrow 68(x + y) = 272$$

$$\Rightarrow x + y = 4 \quad \dots (iii)$$

Subtracting equation (ii) from (i), we get

$$26x - 26y = 52$$

$$\Rightarrow 26(x - y) = 52$$

$$\Rightarrow x - y = 2 \quad \dots (iv)$$

Adding equations (iii) and (iv), we get

$$2x = 6$$

$$\Rightarrow x = 3$$

Substituting $x = 3$ in equation (iii), we get

$$3 + y = 4$$

$$\Rightarrow y = 1$$

Hence, solution of the given system of equations is $x = 3$ and $y = 1$.

Q49

If $x + 1$ is a factor of $2x^3 + ax^2 + 2bx + 1$, then find the values of a and b given that $2a - 3b = 4$.

Solution

Since $(x + 1)$ is a factor of $2x^3 + ax^2 + 2bx + 1$

$\Rightarrow x = -1$ is a zero of $2x^3 + ax^2 + 2bx + 1$

$$\Rightarrow 2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0$$

$$\Rightarrow -2 + a - 2b + 1 = 0$$

$$\Rightarrow a - 2b - 1 = 0 \quad \dots (i)$$

$$\Rightarrow a - 2b = 1$$

Given that $2a - 3b = 4 \quad \dots (ii)$

Multiplying equation (i) by 2, we get

$$2a - 4b = 2 \quad \dots (iii)$$

Subtracting equation (iii) from (ii), we get

$$b = 2$$

Substituting $b = 2$ in equation (i), we have

$$a - 2(2) = 1$$

$$\Rightarrow a - 4 = 1$$

$$\Rightarrow a = 5$$

Hence, $a = 5$ and $b = 2$.

Q50

Find the solution of the pair of equations $\frac{x}{10} + \frac{y}{5} - 1 = 0$ and $\frac{x}{8} + \frac{y}{6} - 15 = 0$. Hence, find λ , if $y = \lambda x + b$.

Solution

The given system of equations is:

$$\begin{aligned} \frac{x+y}{10} + \frac{y}{5} - 1 &= 0 \\ \Rightarrow \frac{x+2y-10}{10} &= 0 \\ \Rightarrow x+2y-10 &= 0 \quad \dots(i) \end{aligned}$$

$$\text{And, } \frac{x}{5} + \frac{y}{6} = 15$$

$$\begin{aligned} \Rightarrow \frac{3x+4y}{24} &= 15 \\ \Rightarrow 3x+4y &= 360 \quad \dots(ii) \end{aligned}$$

Multiplying equation (i) by 3, we get

$$3x+6y=30 \quad \dots(iii)$$

Subtracting equation (iii) from (ii), we get

$$-2y = 330$$

$$\Rightarrow y = -165$$

Substituting $y = -165$ in (i), we have

$$x+2(-165) = 10$$

$$\Rightarrow x-330 = 10$$

$$\Rightarrow x = 340$$

$$\text{Now, } y = \lambda x + 5$$

$$\Rightarrow -165 = \lambda \times 340 + 5$$

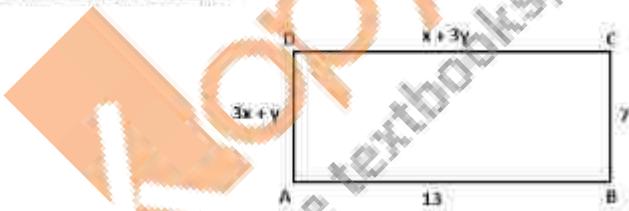
$$\Rightarrow 340\lambda = -170$$

$$\Rightarrow \lambda = \frac{-170}{340} = -\frac{1}{2}$$

Hence, $x = 340$, $y = -165$ and $\lambda = -\frac{1}{2}$

Q51

Find the values of x and y in the following rectangle.



Solution



Since ABCD is a rectangle,

$$CD = AB$$

$$\Rightarrow x + 3y = 13 \quad \dots (i)$$

Also, AD = BC

$$\Rightarrow 3x + y = 7 \quad \dots (ii)$$

Multiplying equation (ii) by 3, we get

$$9x + 3y = 21 \quad \dots (iii)$$

Subtracting equation (i) from (iii), we get

$$8x = 8$$

$$\Rightarrow x = 1$$

Substituting $x = 1$ in equation (i), we have

$$1 + 3y = 13$$

$$\Rightarrow 3y = 12$$

$$\Rightarrow y = 4$$

Hence, $x = 1$ and $y = 4$.

Q52

Write an equation of a line passing through the point representing solution of the pair of linear equations $x + y = 2$ and $2x - y = 1$. How many such lines can we find?

Solution

The given system of equations is

$$x + y = 2 \quad \dots (i)$$

$$2x - y = 1 \quad \dots (ii)$$

Adding equations (i) and (ii), we get

$$3x = 3$$

$$\Rightarrow x = 1$$

Substituting $x = 1$ in (i), we have

$$1 + y = 2$$

$$\Rightarrow y = 1$$

Hence, $x = 1$ and $y = 1$ is a solution of the given system of equations.

There are infinite number of lines which can pass through $(1, 1)$.

The general form of linear equation in two variables is $ax^2 + by + c = 0$

Hence, the equation of line is:

$3x + 2y = 5$ which satisfies $x = 1, y = 1$.

Q53

Write a pair of linear equations which has the unique solution $x = -1, y = 3$. How many such pairs can you write?

Solution

Given, $x = -1$ and $y = 3$

The general form of linear equation in two variables is $ax + by + c = 0$

We can form a pair of linear equations as follows:

$$12x + 3y = -3$$

$$2x + 2y = 4$$

Thus, there can be infinite number of lines which can pass through $(-1, 3)$.



Exercise 3.4**Q1**

Solve each of the following systems of equations by the method of cross-multiplication:

$$x + 2y + 1 = 0$$

$$2x - 3y - 12 = 0$$

Solution

The given system of equation is:

$$x + 2y + 1 = 0$$

$$2x - 3y - 12 = 0$$

Here,

$$\begin{aligned} a_1 &= 1, b_1 = 2, c_1 = 1 \\ a_2 &= 2, b_2 = -3, \text{ and } c_2 = -12 \end{aligned}$$

By cross-multiplication, we get

$$\begin{aligned} \Rightarrow \frac{x}{2 \times (-12) - 1 \times (-3)} &= \frac{-y}{1 \times (-12) - 1 \times 2} = \frac{1}{1 \times (-3) - 2 \times 2} \\ \Rightarrow \frac{x}{-24 + 3} &= \frac{-y}{-12 - 2} = \frac{1}{-3 - 4} \\ \Rightarrow \frac{x}{-21} &= \frac{-y}{-14} = \frac{1}{-7} \end{aligned}$$

Now,

$$\frac{x}{-21} = \frac{1}{-7}$$

$$\Rightarrow x = \frac{-21}{-7} = 3$$

and,

$$\frac{-y}{-14} = \frac{1}{-7}$$

$$\Rightarrow \frac{y}{14} = \frac{-1}{7}$$

$$\Rightarrow y = \frac{-14}{7} = -2$$

Hence, the solution of the given system of equations is $x = 3, y = -2$.

Q2

Solve each of the following systems of equations by the method of cross-multiplication:

$$3x + 2y + 25 = 0$$

$$2x + y + 10 = 0$$

Solution

The given system of equation is:

$$3x + 2y + 25 = 0$$

$$2x + y + 10 = 0$$

Here,

$$a_1 = 3, b_1 = 2, c_1 = 25$$

$$a_2 = 2, b_2 = 1, \text{ and } c_2 = 10$$

By cross-multiplication, we have

$$\Rightarrow \frac{x}{3 \times 10 - 25 \times 2} = \frac{-y}{3 \times 10 - 25 \times 2} = \frac{1}{3 \times 1 - 2 \times 2}$$

$$\Rightarrow \frac{x}{20 - 25} = \frac{-y}{30 - 50} = \frac{1}{3 - 4}$$

$$\Rightarrow \frac{x}{-5} = \frac{-y}{-20} = \frac{1}{-1}$$

Now,

$$\frac{x}{-5} = \frac{1}{-1}$$

$$\Rightarrow x = \frac{-5}{-1} = 5$$

and,

$$\frac{-y}{-20} = \frac{1}{-1}$$

$$\Rightarrow \frac{y}{20} = -1$$

$$\Rightarrow y = -20$$

Hence, $x = 5, y = -20$ is the solution of the given system of equations.

Q3

Solve each of the following systems of equations by the method of cross-multiplication:

$$2x + y = 35$$

$$3x + 4y = 65$$

Solution

The given system of equations may be written as

$$2x + y - 35 = 0$$

$$3x + 4y - 65 = 0$$

Here,

$$a_1 = 2, b_1 = 1, c_1 = -35$$

$$a_2 = 3, b_2 = 4, \text{ and } c_2 = -65$$

By cross-multiplication, we have

$$\Rightarrow \frac{x}{1 \times (-65) - (-35) \times 4} = \frac{-y}{2 \times (-65) - (-35) \times 3} = \frac{1}{2 \times 4 - 1 \times 3}$$

$$\Rightarrow \frac{x}{-65 + 140} = \frac{-y}{-130 + 105} = \frac{1}{8 - 3}$$

$$\Rightarrow \frac{x}{75} = \frac{-y}{-25} = \frac{1}{5}$$

$$\Rightarrow \frac{x}{75} = \frac{y}{25} = \frac{1}{5}$$

Now,

$$\frac{x}{75} = \frac{1}{5}$$

$$\Rightarrow x = \frac{75}{5} = 15$$

and,

$$\frac{y}{25} = \frac{1}{5}$$

$$\Rightarrow y = \frac{25}{5} = 5$$

Hence, $x = 15, y = 5$ is the solution of the given system of equations.

Q4

Solve each of the following systems of equations by the method of cross-multiplication:

$$\begin{aligned} 2x - y &= 6 \\ x - y &= 2 \end{aligned}$$

Solution

The given system of equations may be written as

$$2x - y - 6 = 0$$

$$x - y - 2 = 0$$

Here,

$$\begin{aligned} a_1 &= 2, b_1 = -1, c_1 = -6 \\ a_2 &= 1, b_2 = -1, \text{ and } c_2 = -2 \end{aligned}$$

By cross-multiplication, we get

$$\begin{aligned} \Rightarrow \frac{x}{(-1) \times (-2) - (-6) \times (-1)} &= \frac{-y}{2 \times (-2) - (-6) \times 1} = \frac{1}{2 \times (-1) - (-1) \times 1} \\ \Rightarrow \frac{x}{-2 + 6} &= \frac{-y}{-4 + 6} = \frac{1}{-2 + 1} \\ \Rightarrow \frac{x}{4} &= \frac{-y}{2} = 1 \\ \Rightarrow \frac{x}{4} &= \frac{-y}{2} = 1 \end{aligned}$$

Now,

$$\begin{aligned} \frac{x}{4} &= -1 \\ \Rightarrow x &= \{-4\} \times \{-1\} = 4 \end{aligned}$$

and,

$$\begin{aligned} \frac{-y}{2} &= -1 \\ \Rightarrow -y &= \{-1\} \times 2 \\ \Rightarrow -y &= -2 \\ \Rightarrow y &= 2 \end{aligned}$$

Hence, $x = 4, y = 2$ is the solution of the given system of the equations.

Q5

Solve each of the following systems of equations by the method of cross-multiplication:

$$\frac{x+y}{xy} = 2, \quad \frac{x-y}{xy} = 6$$

Solution

The given system of equations is:

$$\begin{aligned} \frac{x+y}{xy} &= 2 \\ \Rightarrow \frac{x}{xy} + \frac{y}{xy} &= 2 \\ \Rightarrow \frac{1}{y} + \frac{1}{x} &= 2 \\ \Rightarrow \frac{1}{x} + \frac{1}{y} &= 2 \quad \text{---(i)} \end{aligned}$$

and,

$$\begin{aligned} \frac{x-y}{xy} &= 6 \\ \Rightarrow \frac{x}{xy} - \frac{y}{xy} &= 6 \\ \Rightarrow \frac{1}{y} - \frac{1}{x} &= 6 \\ \Rightarrow \frac{1}{x} - \frac{1}{y} &= -6 \quad \text{---(ii)} \end{aligned}$$

Taking $u = \frac{1}{x}$ and $v = \frac{1}{y}$, we get:

$$\begin{aligned} u+v=2 &\Rightarrow u+v-2=0 \quad \text{---(iii)} \\ \text{and, } u-v=6 &\Rightarrow u-v+6=0 \quad \text{---(iv)} \end{aligned}$$

Here,

$$\begin{aligned} a_1 &= 1, b_1 = 1, c_1 = -2 \\ a_2 &= 1, b_2 = -1, \text{ and } c_2 = 6 \end{aligned}$$

By cross-multiplication,

$$\begin{aligned} \Rightarrow \frac{u}{1 \times 6 - (-2) \times (-1)} &= \frac{-v}{1 \times 6 - (-2) \times 1} = \frac{1}{1 \times (-1) - 1 \times 1} \\ \Rightarrow \frac{u}{6-2} &= \frac{-v}{6+2} = \frac{1}{-1-1} \\ \Rightarrow \frac{u}{4} &= \frac{-v}{8} = \frac{1}{-2} \\ \text{Now, } \frac{u}{4} &= \frac{1}{-2} \\ \Rightarrow u &= \frac{4}{-2} = -2 \\ \text{and, } \frac{-v}{8} &= \frac{1}{-2} \\ \Rightarrow -v &= \frac{8}{-2} = -4 \\ \Rightarrow v &= 4 \end{aligned}$$

$$\text{Now, } x = \frac{1}{u} = \frac{-1}{2} \text{ and } y = \frac{1}{v} = \frac{1}{4}$$

Hence, $x = \frac{-1}{2}$, $y = \frac{1}{4}$ is the solution of the given system of equations.

Q6

Solve each of the following systems of equations by the method of cross-multiplication:

$$\begin{aligned} ax+by &= a-b \\ bx-ay &= a+b \end{aligned}$$

Solution

The given system of equations is:

$$\begin{aligned} ax + by = a - b \\ bx - ay = a + b \end{aligned} \quad \begin{array}{l} \text{---(i)} \\ \text{---(ii)} \end{array}$$

Here,

$$\begin{aligned} a_1 = a, \quad b_1 = b, \quad c_1 = b - a \\ a_2 = b, \quad b_2 = -a, \quad c_2 = a + b \end{aligned}$$

By cross-multiplication, we get

$$\begin{aligned} & \frac{x}{\{-a-b\} \times \{b\} - \{b-a\} \times \{-a\}} = \frac{-y}{\{-a-b\} \times \{a\} - \{b-a\} \times \{b\}} = \frac{1}{-a \times a - b \times b} \\ & \Rightarrow \frac{x}{-ab - b^2 + ab - a^2} = \frac{-y}{-a^2 - ab - b^2 + ab} = \frac{1}{-a^2 - b^2} \\ & \Rightarrow \frac{x}{-b^2 - a^2} = \frac{-y}{-a^2 - b^2} = \frac{1}{-a^2 - b^2} \end{aligned}$$

Now,

$$\begin{aligned} & \frac{x}{-b^2 - a^2} = \frac{1}{-a^2 - b^2} \\ & \Rightarrow x = \frac{-b^2 - a^2}{-a^2 - b^2} \\ & = \frac{-\{b^2 + a^2\}}{-\{a^2 + b^2\}} \\ & = \frac{\{a^2 + b^2\}}{\{a^2 + b^2\}} \\ & \Rightarrow x = 1 \end{aligned}$$

and,

$$\begin{aligned} & \frac{-y}{-a^2 - b^2} = \frac{1}{-a^2 - b^2} \\ & \Rightarrow -y = \frac{1}{-a^2 - b^2} \\ & \Rightarrow -y = 1 \\ & \Rightarrow y = -1 \end{aligned}$$

Hence, $x = 1, y = -1$ is the solution of the given system of the equations.

Q7

Solve each of the following systems of equations by the method of cross-multiplication:

$$\begin{aligned} x + ay = b \\ ax - by = c \end{aligned}$$

Solution

The given system of equations may be written as

$$ax + by - d = 0$$

$$bx + ay - c = 0$$

Here,

$$a_1 = 1, b_1 = a, c_1 = -d$$

$$a_2 = a, b_2 = -b, \text{ and } c_2 = -c$$

By cross-multiplication, we get

$$\begin{aligned} &= \frac{x}{a \times (-c) - (-b) \times (-d)} = \frac{-y}{1 \times (-c) - (-b) \times a} = \frac{1}{1 \times (-d) - a \times a} \\ &\Rightarrow \frac{x}{-ac - b^2} = \frac{-y}{-c + ab} = \frac{1}{-d - a^2} \end{aligned}$$

Now,

$$\begin{aligned} &= \frac{x}{-ac - b^2} = \frac{1}{-d - a^2} \\ &\Rightarrow x = \frac{-ac - b^2}{-d - a^2} \\ &\Rightarrow x = \frac{-(b^2 + ac)}{-(a^2 + d)} \\ &= \frac{b^2 + ac}{a^2 + d} \end{aligned}$$

and,

$$\begin{aligned} &= \frac{-y}{-c + ab} = \frac{1}{-b - a^2} \\ &\Rightarrow -y = \frac{-ab - c}{-b - a^2} \\ &\Rightarrow y = \frac{ab + c}{a^2 + b} \end{aligned}$$

Hence, $x = \frac{ac + b^2}{a^2 + d}$, $y = \frac{ab + c}{a^2 + b}$ is the solution of the given system of the equations.

Q8

Solve each of the following systems of equations by the method of cross-multiplication:

$$ax + by = a^2$$

$$bx + ay = b^2$$

Solution

The system of the given equations may be written as

$$ax + by - a^2 = 0$$

$$bx + ay - b^2 = 0$$

Here,

$$a_1 = a, \quad b_1 = b, \quad c_1 = -a^2$$

$$a_2 = b, \quad b_2 = a, \quad \text{and } c_2 = -b^2$$

By cross-multiplication, we get

$$\frac{x}{b(-b^2) - (-a^2) \times a} = \frac{-y}{a(-b^2) - (-a^2) \times b} = \frac{1}{a \times a - b \times b}$$

$$\frac{x}{-b^3 + a^2} = \frac{-y}{-ab^2 + a^2b} = \frac{1}{a^2 - b^2}$$

Now,

$$\frac{x}{-b^3 + a^2} = \frac{1}{a^2 - b^2}$$

$$\begin{aligned} \text{L.H.S.} &= \frac{a^3 - b^3}{a^2 - b^2} \\ &= \frac{(a-b)(a^2 + ab + b^2)}{(a-b)(a+b)} \\ &= \frac{a^2 + ab + b^2}{a+b} \end{aligned}$$

and,

$$\frac{-y}{-ab^2 + a^2b} = \frac{1}{a^2 - b^2}$$

$$\text{L.H.S.} = \frac{a^2b - ab^2}{a^2 - b^2}$$

$$\begin{aligned} \text{L.H.S.} &= \frac{ab(a - b)}{a^2 - b^2} \\ &= \frac{ab(b - a)}{(a-b)(a+b)} \\ &= \frac{-ab(a - b)}{(a-b)(a+b)} \\ &= \frac{-ab}{a+b} \end{aligned}$$

Hence, $x = \frac{a^2 + ab + b^2}{a+b}$, $y = \frac{-ab}{a+b}$ is the solution of the given system of the equations.

Q9

Solve each of the following systems of equations by the method of cross-multiplication:

$$\frac{5}{x+y} - \frac{2}{x-y} = -1$$

$$\frac{15}{x+y} + \frac{7}{x-y} = 10,$$

where $x \neq 0$ and $y \neq 0$.

Solution

Let $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$. Then, the given system of equations becomes:

$$5u + 2v = -1$$

$$15u + 7v = 10$$

Here,

$$\begin{aligned} a_1 &= 5, \quad b_1 = -2, \quad c_1 = 1 \\ a_2 &= 15, \quad b_2 = 7 \text{ and } c_2 = -10 \end{aligned}$$

By cross-multiplication, we get:

$$\begin{aligned} \Rightarrow \frac{u}{(-2) \times (-10) - 1 \times 7} &= \frac{-v}{5 \times (-10) - 1 \times 15} = \frac{1}{5 \times 7 - (-2) \times 15} \\ \Rightarrow \frac{u}{20 - 7} &= \frac{-v}{-50 - 15} = \frac{1}{35 + 30} \\ \Rightarrow \frac{u}{13} &= \frac{-v}{-65} = \frac{1}{65} \\ \Rightarrow \frac{u}{13} &= \frac{v}{65} = \frac{1}{65} \end{aligned}$$

Now,

$$\begin{aligned} \frac{u}{13} &= \frac{1}{65} \\ \Rightarrow u &= \frac{13}{65} = \frac{1}{5} \end{aligned}$$

and,

$$\begin{aligned} \frac{v}{65} &= \frac{1}{65} \\ \Rightarrow v &= \frac{65}{65} = 1 \end{aligned}$$

Now,

$$\begin{aligned} u &= \frac{1}{x+y} \\ \Rightarrow \frac{1}{x+y} &= \frac{1}{5} \\ \Rightarrow x+y &= 5 \quad \text{---(i)} \\ \text{and,} \quad \frac{v}{x-y} &= 1 \\ \Rightarrow \frac{1}{x-y} &= 1 \\ \Rightarrow x-y &= 1 \quad \text{---(ii)} \end{aligned}$$

Adding equation (i) and (ii), we get:

$$\begin{aligned} 2x &= 5+1 \\ \Rightarrow 2x &= 6 \\ \Rightarrow x &= \frac{6}{2} = 3 \end{aligned}$$

Putting $x = 3$ in equation (i), we get:

$$\begin{aligned} 3+y &= 5 \\ \Rightarrow y &= 5-3=2 \end{aligned}$$

Hence, $x = 3, y = 2$ is the solution of the given system of equations.

Q10

Solve each of the following systems of equations by the method of cross-multiplication:

$$\frac{x}{2} + \frac{y}{3} = 13$$

$$\frac{x}{x} - \frac{y}{y} = -2, \text{ where } x \neq 0 \text{ and } y \neq 0$$

Solution

The given system of equations is

$$\begin{aligned} \frac{2}{x} + \frac{3}{y} &= 13 \\ \frac{5}{x} - \frac{4}{y} &= -2, \text{ where } x \neq 0 \text{ and } y \neq 0 \end{aligned}$$

Let $\frac{2}{x} = u$ and $\frac{3}{y} = v$. Then, the given system of equations becomes

$$\begin{aligned} 2u + 3v &= 13 \\ 5u - 4v &= -2 \end{aligned}$$

Here,

$$\begin{aligned} a_1 &= 2, b_1 = 3, c_1 = 13 \\ a_2 &= 5, b_2 = -4 \text{ and } c_2 = -2 \end{aligned}$$

By cross-multiplication, we have

$$\begin{aligned} u &= \frac{-v}{3 \times 2 - (-13) \times (-4)} = \frac{-v}{2 \times 2 - (-13) \times 5} = \frac{-v}{2 \times (-4) - 3 \times 5} \\ &= \frac{u}{5 - 52} = \frac{-v}{4 + 65} = \frac{1}{-47} = -\frac{1}{47} \\ &= \frac{u}{-46} = \frac{-v}{69} = \frac{1}{-23} \end{aligned}$$

Now,

$$\frac{u}{-46} = \frac{1}{-23}$$

$$\Rightarrow u = \frac{-46}{-23} = 2$$

and,

$$\frac{-v}{69} = \frac{1}{-23}$$

$$\Rightarrow v = \frac{-69}{-23} = 3$$

Now,

$$x = \frac{1}{u} = \frac{1}{2}$$

and,

$$y = \frac{1}{v} = \frac{1}{3}$$

Hence, $x = \frac{1}{2}, y = \frac{1}{3}$ is the solution of the given system of equations.

Q11

Solve each of the following systems of equations by the method of cross-multiplication:

$$\begin{aligned} \frac{57}{x+y} + \frac{6}{x-y} &= 5 \\ \frac{38}{x+y} - \frac{21}{x-y} &= 9 \end{aligned}$$

Solution

Let $\frac{x}{x+y} = u$ and $\frac{y}{x-y} = v$. Then, the given system of equations become

$$\begin{aligned} 57u + 6v - 5 &= 0 \\ 39u + 21v - 9 &= 0 \end{aligned}$$

Here,

$$\begin{aligned} a_1 &= 57, b_1 = 6, c_1 = -5 \\ a_2 &= 39, b_2 = 21, \text{ and } c_2 = -9 \end{aligned}$$

By cross-multiplication, we have

$$\begin{aligned} \Rightarrow \frac{u}{-54+105} &= \frac{-v}{-513+180} = \frac{1}{1197-228} \\ \Rightarrow \frac{u}{51} &= \frac{-v}{-323} = \frac{1}{969} \\ \Rightarrow \frac{u}{51} &= \frac{v}{323} = \frac{1}{969} \end{aligned}$$

Now,

$$\frac{u}{51} = \frac{1}{969}$$

$$\Rightarrow u = \frac{51}{969}$$

$$\Rightarrow u = \frac{1}{19}$$

and,

$$\frac{v}{323} = \frac{1}{969}$$

$$\Rightarrow v = \frac{323}{969}$$

$$\Rightarrow v = \frac{1}{3}$$

Now,

$$u = \frac{1}{x+y}$$

$$\Rightarrow \frac{1}{x+y} = \frac{1}{19}$$

$$\Rightarrow x+y = 19$$

and,

$$v = \frac{1}{x-y}$$

$$\Rightarrow \frac{1}{x-y} = \frac{1}{3}$$

$$\Rightarrow x-y = 3$$

Adding equation (i) and equation (ii), we get

$$2x = 19 + 3$$

$$\Rightarrow 2x = 22$$

$$\Rightarrow x = \frac{22}{2} = 11$$

Putting $x = 11$ in equation (i), we get

$$11+y = 19$$

$$\Rightarrow y = 19 - 11 = 8$$

Hence, $x = 11$, $y = 8$ is the solution of the given system of the equations.

Q12

Solve each of the following systems of equations by the method of cross-multiplication:

$$\frac{x}{a} + \frac{y}{b} = 2$$

$$ax - by = a^2 - b^2$$

Solution

The system of the given equations may be written as

$$\frac{2}{a}x + \frac{2}{b}y - 2 = 0$$

$$ax - by + b^2 - a^2 = 0$$

Here,

$$a_1 = \frac{1}{a}, \quad b_1 = \frac{1}{b}, \quad c_1 = -2$$

$$a_2 = a, \quad b_2 = -b, \quad \text{and } c_2 = b^2 - a^2$$

By cross-multiplication, we get

$$\begin{aligned} &= \frac{x}{\frac{2}{b} \times (b^2 - a^2) - (-2) \times (-b)} = \frac{-y}{\frac{1}{a} \times (b^2 - a^2) - (-2) \times a} = \frac{-b \times 1 - a \times 1}{a - b} \\ &= \frac{x}{\frac{b^2 - a^2}{b} - 2b} = \frac{-y}{\frac{b^2 - a^2}{a} + 2a} = \frac{-a - b}{a - b} \\ &= \frac{x}{\frac{b^2 - a^2 - 2b^2}{b}} = \frac{-y}{\frac{b^2 - a^2 + 2a^2}{a}} = \frac{-b^2 - a^2}{a^2} \\ &= \frac{x}{\frac{-a^2 - b^2}{b}} = \frac{-y}{\frac{b^2 + a^2}{a}} = \frac{1}{ab} \end{aligned}$$

Now,

$$\frac{x}{\frac{-a^2 - b^2}{b}} = \frac{1}{\frac{-b^2 + a^2}{ab}}$$

$$\Rightarrow x = \frac{-a^2 - b^2}{b} \times \frac{ab}{-b^2 + a^2} = \frac{ab}{a^2 - b^2}$$

and,

$$\frac{-y}{\frac{b^2 + a^2}{a}} = \frac{1}{\frac{-b^2 + a^2}{ab}}$$

$$\Rightarrow -y = \frac{b^2 + a^2}{a} \times \frac{ab}{-b^2 + a^2} = \frac{ab}{a^2 - b^2}$$

$$\Rightarrow -y = \frac{(b^2 + a^2) \times 2}{-(b^2 + a^2)}$$

$$\Rightarrow y = b$$

Hence, $x = a$, $y = b$ is the solution of the given system of the equations.

Q13

Solve each of the following systems of equations by the method of cross-multiplication:

$$\frac{x}{a} + \frac{y}{b} = a + b$$

$$\frac{x}{a^2} + \frac{y}{b^2} = 2$$

Solution

The given system of equations may be written as:

$$\frac{1}{a}x + \frac{1}{b}y = (a+b) = 0$$

$$\frac{1}{a^2}x + \frac{1}{b^2}y = 2 = 0$$

Here,

$$a_1 = \frac{1}{a}, \quad b_1 = \frac{1}{b}, \quad c_1 = -(a+b)$$

$$a_2 = \frac{1}{a^2}, \quad b_2 = \frac{1}{b^2}, \quad \text{and } c_2 = -2$$

By cross-multiplication, we get

$$\Rightarrow \frac{x}{\frac{-2}{b^2}} = \frac{-y}{\frac{1}{a^2} \times -(a+b)} = \frac{1}{\frac{1}{a} \times \frac{1}{b^2} - \frac{1}{a^2} \times \frac{2}{b}}$$

$$\Rightarrow \frac{x}{\frac{-2}{b^2} + \frac{1}{b}} = \frac{-y}{\frac{2}{a} + \frac{1}{a^2} \times \frac{b}{b^2}} = \frac{1}{\frac{1}{ab^2} - \frac{2}{a^2b}}$$

$$\Rightarrow \frac{x}{\frac{a^2 - 2}{b^2}} = \frac{-y}{\frac{1}{a^2} + \frac{b}{a^2}} = \frac{1}{\frac{1}{ab^2} - \frac{2}{a^2b}}$$

$$\Rightarrow \frac{x}{\frac{a-b}{b^2}} = \frac{-y}{\frac{a-b}{a^2}} = \frac{1}{\frac{a-b}{a^2b^2}}$$

$$\Rightarrow x = \frac{a-b}{b^2} \times \frac{1}{\frac{a-b}{a^2b^2}} = a^2 \text{ and } y = \frac{a-b}{a^2} \times \frac{1}{\frac{a-b}{a^2b^2}} = b^2$$

Hence, $x = a^2, y = b^2$ is the solution of the given system of the equations.

Q14

Solve each of the following systems of equations by the method of cross-multiplication:

$$\begin{aligned} \frac{x}{a} &= \frac{y}{b} \\ ax + by &= a^2 + b^2 \end{aligned}$$

Solution

KopyKitab
Same textbooks, klick away!

$$\frac{x}{a} = \frac{y}{b}$$

$$ax + by = a^2 + b^2$$

$$\text{Here } a_1 = \frac{1}{a}, b_1 = \frac{-1}{b}, c_1 = 0$$

$$a_2 = a, b_2 = b, c_2 = -(a^2 + b^2)$$

By cross multiplication, we get

$$\frac{x}{\frac{-1}{b}(-(a^2 + b^2)) - b(0)} = \frac{-y}{\frac{1}{a}(-(a^2 + b^2)) - a(0)} = \frac{1}{\frac{1}{b}(b) - a(-\frac{-1}{b})}$$

$$\frac{x}{\frac{a^2 + b^2}{b}} = \frac{y}{\frac{a^2 + b^2}{a}} = \frac{1}{\frac{b+a}{b}}$$

$$x = \frac{b}{\frac{b+a}{b}} = \frac{b}{\frac{b^2 + a^2}{ab}} = \frac{ab}{b^2 + a^2}$$

$$y = \frac{a}{\frac{b+a}{b}} = \frac{a}{\frac{b^2 + a^2}{ab}} = \frac{ab}{b^2 + a^2}$$

Solution is (a, b)

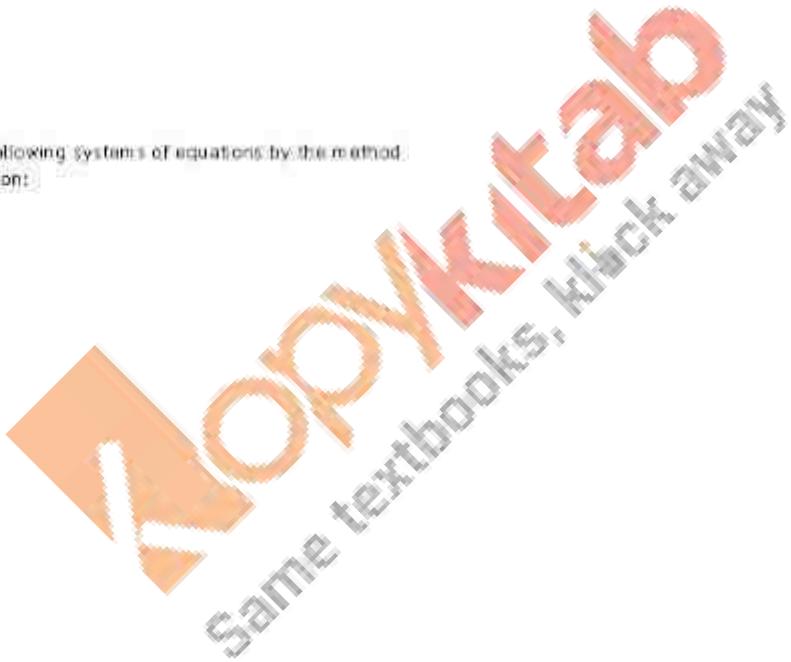
Q15

Solve each of the following systems of equations by the method of cross-multiplication:

$$2ax + 3by = a + 2b$$

$$3ax + 2by = 2a + b$$

Solution



The given system of equations is:

$$2ax + 3by = a + 2b \quad \text{---(i)}$$

$$3ax + 2by = 2a + b \quad \text{---(ii)}$$

Here,

$$a_1 = 2a, b_1 = 3b, c_1 = -(a + 2b)$$

$$a_2 = 3a, b_2 = 2b, c_2 = -(2a + b)$$

By cross-multiplication, we have

$$\Rightarrow \frac{x}{-3b \times (2a+b) - [-(a+2b)] \times 2b} = \frac{-y}{-2a \times (2a+b) - [-(a+2b)] \times 3a} = \frac{1}{2a \times 2b - 2b \times 3a}$$

$$\Rightarrow \frac{x}{-6ab - 3b^2 + 2ab + 4b^2} = \frac{-y}{-4a^2 - 2ab + 3a^2 + 6ab} = \frac{1}{4ab - 9ab}$$

$$\Rightarrow \frac{x}{-4ab + b^2} = \frac{-y}{-a^2 + 4ab} = \frac{1}{-5ab}$$

Now,

$$\frac{x}{-4ab + b^2} = \frac{1}{-5ab}$$

$$\Rightarrow x = \frac{-4ab + b^2}{-5ab} \\ = \frac{-b(4a - b)}{-5ab} \\ = \frac{4a - b}{5a}$$

and,

$$\frac{-y}{-a^2 + 4ab} = \frac{1}{-5ab}$$

$$\Rightarrow -y = \frac{-a^2 + 4ab}{-5ab} \\ = \frac{-a(a - 4b)}{-5ab} \\ = \frac{a - 4b}{5a} \\ \Rightarrow y = \frac{4b - a}{5a}$$

Hence, $x = \frac{4a - b}{5a}, y = \frac{4b - a}{5a}$ is the solution of the given system of equations.

Q16

Solve each of the following systems of equations by the method of cross-multiplication:

$$5av + 6bv = 26$$

$$3av + 4bv = 18$$

Solution

The given system of equations is

$$\begin{aligned} & 5ax + 5by = 20 \\ \Rightarrow & 5ax + 5by - 20 = 0 \quad \text{---(i)} \\ \text{and, } & 3ax + 4by = 18 \\ \Rightarrow & 3ax + 4by - 18 = 0 \quad \text{---(ii)} \end{aligned}$$

Here,

$$\begin{aligned} a_1 &= 5a, \quad b_1 = 5b, \quad c_1 = -20 \\ a_2 &= 3a, \quad b_2 = 4b \quad \text{and} \quad c_2 = -18 \end{aligned}$$

By cross-multiplication, we have:

$$\begin{aligned} & \frac{x}{6b \times (-18) - (-28) \times 4b} = \frac{-y}{5a \times (-18) - (-28) \times 3a} = \frac{1}{5a \times 4b - 5b \times 3a} \\ \Rightarrow & \frac{x}{-108b + 112b} = \frac{-y}{-90a + 84a} = \frac{1}{20ab - 15ab} \\ \Rightarrow & \frac{x}{4b} = \frac{-y}{-6a} = \frac{1}{5ab} \end{aligned}$$

Now,

$$\frac{x}{4b} = \frac{1}{5ab}$$

$$\Rightarrow x = \frac{4b}{5ab} = \frac{2}{5a}$$

and;

$$\frac{-y}{-6a} = \frac{1}{5ab}$$

$$\Rightarrow y = \frac{6a}{5ab} = \frac{3}{b}$$

Hence, $x = \frac{2}{5a}$, $y = \frac{3}{b}$ is the solution of the given system of equations.

Q17

Solve each of the following systems of equations by the method of cross-multiplication:

$$(a+2b)x + (2a-b)y = 2$$

$$(a-2b)x + (2a+b)y = 3$$

Solution

KopyKitab
Same textbooks, klick away

The given system of equations may be written as

$$(a+2b)x + (2a-b)y - 2 = 0$$

$$(a-2b)x + (2a+b)y - 3 = 0$$

Here,

$$a_1 = a+2b, \quad b_1 = 2a-b, \quad c_1 = -2$$

$$a_2 = a-2b, \quad b_2 = 2a+b, \quad c_2 = -3$$

By cross-multiplication, we have

$$\Rightarrow \frac{x}{-3(2a-b) - (-2)(2a+b)} = \frac{-y}{-3(a+2b) - (-2)(a-2b)} = \frac{1}{(a+2b)(2a+b) - (2a-b)(a-2b)}$$

$$\Rightarrow \frac{x}{-5a+5b+4ab+2b^2} = \frac{-y}{-3a-6b+2a-4b} = \frac{1}{2a^2+ab+4ab+2b^2 - [2a^2-4ab-ab+2b^2]}$$

$$\Rightarrow \frac{x}{-2a+5b} = \frac{-y}{-a-10b} = \frac{1}{2a^2+ab+4ab+2b^2 - 2a^2+4ab-ab+2b^2}$$

$$\Rightarrow \frac{x}{-2a+5b} = \frac{-y}{-a-10b} = \frac{1}{10ab}$$

$$\Rightarrow \frac{x}{-2a+5b} = \frac{y}{a+10b} = \frac{1}{10ab}$$

Now,

$$\frac{x}{-2a+5b} = \frac{1}{10ab}$$

$$\Rightarrow x = \frac{5b-2a}{10ab}$$

and,

$$\frac{y}{a+10b} = \frac{1}{10ab}$$

$$\Rightarrow y = \frac{a+10b}{10ab}$$

Hence, $x = \frac{5b-2a}{10ab}, y = \frac{a+10b}{10ab}$ is the solution of the given system of equations.

Q18

Solve each of the following systems of equations by the method of cross-multiplication:

$$x\left(\frac{a-b+\frac{ab}{a-b}}{a-b}\right) = y\left(\frac{a+b-\frac{ab}{a+b}}{a+b}\right)$$

$$x+y=2a^2$$

Solution

The given system of equation is

$$x\left(\frac{a-b+\frac{ab}{a-b}}{a-b}\right) = y\left(\frac{a+b-\frac{ab}{a+b}}{a+b}\right) \quad (i)$$

$$x+y=2a^2 \quad (ii)$$

From equation (i), we get

$$x\left(\frac{a-b+\frac{ab}{a-b}}{a-b}\right) - y\left(\frac{a+b-\frac{ab}{a+b}}{a+b}\right) = 0$$

$$\Rightarrow x\left(\frac{(a-b)^2+ab}{a-b}\right) - y\left(\frac{(a+b)^2-ab}{a+b}\right) = 0$$

$$\Rightarrow x\left(\frac{a^2+b^2-2ab+ab}{a-b}\right) - y\left(\frac{a^2+b^2+2ab-ab}{a+b}\right) = 0$$

$$= x \left(\frac{a^2 + b^2 - ab}{a+b} \right) - y \left(\frac{a^2 + b^2 + ab}{a+b} \right) = 0 \quad \text{---(i)}$$

From equation (i), we get

$$x + y - 2ab = 0 \quad \text{---(ii)}$$

Here,

$$a_1 = \frac{a^2 + b^2 - ab}{a+b}, \quad b_1 = -\left(\frac{a^2 + b^2 + ab}{a+b} \right), \quad c_1 = 0$$

$$a_2 = 1, \quad b_2 = 1 \quad \text{and} \quad c_2 = -2ab$$

By cross-multiplication, we get

$$= \frac{x}{\{-2ab\}\left(-\left(\frac{a^2 + b^2 + ab}{a+b}\right)\right)} - 0 \times 1 \cdot \frac{-y}{\{-2ab\}\left(\frac{a^2 + b^2 - ab}{a+b}\right)} - 0 \times 1 \cdot \frac{1}{\frac{a^2 + b^2 - ab}{a+b}} = \frac{1}{\left(\frac{a^2 + b^2 + ab}{a+b}\right)}$$

$$= \frac{x}{2a^2\left(\frac{a^2 + b^2 + ab}{a+b}\right)} - \frac{y}{2a^2\left(\frac{a^2 + b^2 - ab}{a+b}\right)} - \frac{1}{\frac{a^2 + b^2 - ab}{a+b} + \frac{a^2 + b^2 + ab}{a+b}}$$

$$= \frac{x}{2a^2\left(\frac{a^2 + b^2 + ab}{a+b}\right)} - \frac{y}{2a^2\left(\frac{a^2 + b^2 - ab}{a+b}\right)} - \frac{1}{(a+b)(a^2 + b^2 - ab) + (a-b)(a^2 + b^2 + ab)}$$

$$= \frac{x}{2a^2\left(\frac{a^2 + b^2 + ab}{a+b}\right)} = \frac{y}{2a^2\left(\frac{a^2 + b^2 - ab}{a+b}\right)} = \frac{1}{2a^2(a^2 + b^2 - ab)}$$

$$= \frac{x}{2a^2\left(\frac{a^2 + b^2 + ab}{a+b}\right)} = \frac{y}{2a^2\left(\frac{a^2 + b^2 - ab}{a+b}\right)} = \frac{1}{2a^2(a+b)}$$

Now,

$$\begin{aligned} & \frac{x}{2a^2\left(\frac{a^2 + b^2 + ab}{a+b}\right)} = \frac{1}{2a^2(a+b)} \\ & \Rightarrow x = \frac{2a^2(a^2 + b^2 + ab)}{a+b} \times \frac{(a-b)(a+b)}{2a^2} \\ & \quad \times \frac{(a-b)(a^2 + b^2 + ab)}{a} \\ & \Rightarrow x = \frac{a^3 - b^3}{a} \quad \left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2) \right] \end{aligned}$$

and,

$$\begin{aligned} & \frac{y}{2a^2\left(\frac{a^2 + b^2 - ab}{a+b}\right)} = \frac{1}{2a^2(a+b)} \\ & \Rightarrow y = \frac{2a^2(a^2 + b^2 - ab)}{a+b} \times \frac{(a-b)(a+b)}{2a^2} \\ & \quad \times \frac{(a+b)(a^2 + b^2 - ab)}{a} \\ & \Rightarrow y = \frac{a^3 + b^3}{a} \quad \left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2) \right] \end{aligned}$$

Hence, $x = \frac{a^3 - b^3}{a}$, $y = \frac{a^3 + b^3}{a}$ is the solution of the given system of equations.

Q19

Solve each of the following systems of equations by the method of cross-multiplication:

$$bx + cy = a + b$$

$$ay \left(\frac{1}{a+b} - \frac{1}{a+b} \right) + cy \left(\frac{1}{b-a} - \frac{1}{b+a} \right) = \frac{2a}{a+b}$$

Solution

The given system of equation is

$$bx+cy = a+b \quad \text{---(i)}$$

$$ax\left(\frac{1}{a-b} - \frac{1}{a+b}\right) + cy\left(\frac{1}{b-a} - \frac{1}{b+a}\right) = \frac{2a}{a+b} \quad \text{---(ii)}$$

From equation (i), we get

$$bx+cy-(a+b)=0 \quad \text{---(iii)}$$

From equation (ii), we get

$$ax\left[\frac{a+b-(a-b)}{(a-b)(a+b)}\right] + cy\left(\frac{b+a-(b-a)}{(b-a)(b+a)}\right) - \frac{2a}{a+b} = 0$$

$$\Rightarrow ax\left[\frac{2b}{(a-b)(a+b)}\right] + cy\left(\frac{2a}{(b-a)(b+a)}\right) - \frac{2a}{a+b} = 0$$

$$\Rightarrow ax\left[\frac{-2ab}{(a-b)(a+b)}\right] + cy\left(\frac{-2ac}{(b-a)(b+a)}\right) - \frac{2a}{a+b} = 0$$

$$\Rightarrow x\left[\frac{-2ab}{(a-b)(a+b)}\right] + y\left(\frac{-2ac}{(b-a)(b+a)}\right) - \frac{2a}{a+b} = 0$$

$$\Rightarrow x\left[\frac{-2ab}{(a-b)(a+b)}\right] + y\left(\frac{-2ac}{(b-a)(b+a)}\right) - \frac{2a}{a+b} = 0$$

$$\Rightarrow \frac{1}{a+b} \left[\frac{2abx}{a-b} - \frac{2acy}{a-b} - 2a \right] = 0$$

$$\Rightarrow \frac{2abx}{a-b} - \frac{2acy}{a-b} - 2a = 0$$

$$\Rightarrow \frac{2abx - 2acy - 2a(a-b)}{a-b} = 0$$

$$\Rightarrow 2abx - 2acy - 2a(a-b) = 0 \quad \text{---(iv)}$$

From equation (i) and equation (iv), we get

$$a_1 = b, \quad b_1 = c, \quad c_1 = -(a+b)$$

$$a_2 = 2abc, \quad b_2 = -2ac \text{ and } c_2 = 2a(a-b)$$

By cross-multiplication, we get

$$\frac{x}{-2ac(a-b) - [-(a-b)][-2ac]} = \frac{-y}{-2ab(a-b) - [-(a-b)][2ab]} = \frac{1}{-2abc - 2abc}$$

$$\Rightarrow \frac{x}{-2a^2c + 2abc - [2a^2c + 2abc]} = \frac{-y}{-2a^2b + 2ab^2 + [2a^2b + 2ab^2]} = \frac{1}{-4abc}$$

$$\Rightarrow \frac{x}{-2a^2c + 2abc - 2a^2c - 2abc} = \frac{-y}{-2a^2b + 2ab^2 + 2a^2b + 2ab^2} = \frac{1}{-4abc}$$

$$\Rightarrow \frac{x}{-4a^2c} = \frac{-y}{4ab^2} = \frac{1}{4abc}$$

Now,

$$\frac{x}{-4a^2c} = \frac{-1}{4abc}$$

$$\Rightarrow x = \frac{4a^2c}{4abc} = \frac{a}{b}$$

and,

$$\frac{-y}{4ab^2} = \frac{-1}{4abc}$$

$$\Rightarrow y = \frac{4ab^2}{4abc} = \frac{b}{c}$$

Hence, $x = \frac{a}{b}$, $y = \frac{b}{c}$ is the solution of the given system of the equations.

Q20

Solve each of the following systems of equations by the method of cross-multiplication:

$$(x - b)x + (x + b)y = 2a^2 - 2b^2$$

$$(x + b)(x + y) = 4ab$$

Solution

The given system of equation is

$$\begin{aligned} (a-b)x + (a+b)y - 2a^2 - 2b^2 &= 0 \quad \text{---(i)} \\ (a+b)(x+y) - 4ab &= 0 \quad \text{---(ii)} \end{aligned}$$

From equation (i), we get

$$\begin{aligned} (a-b)x + (a+b)y - [2a^2 + 2b^2] &= 0 \\ \Rightarrow (a-b)x + (a+b)y - 2(a^2 + b^2) &= 0 \quad \text{---(iii)} \end{aligned}$$

From equation (ii), we get

$$(a+b)x + (a+b)y - 4ab = 0 \quad \text{---(iv)}$$

Here,

$$\begin{aligned} a_1 &= a-b, \quad b_1 = a+b, \quad c_1 = -2(a^2 + b^2) \\ a_2 &= a+b, \quad b_2 = a+b, \quad \text{and } c_2 = -4ab \end{aligned}$$

By cross-multiplication, we get

$$\begin{aligned} \frac{x}{-4ab(a+b) + 2(a^2 + b^2)(a+b)} &= \frac{-y}{-4ab(a-b) + 2(a^2 + b^2)(a+b)} = \frac{1}{(a-b)(a+b) - (a+b)(a+b)} \\ \Rightarrow \frac{x}{2(a+b)[-2ab + a^2 + b^2]} &= \frac{-y}{-4ab(a-b) + 2[(a-b)(a+b)](a+b)} = \frac{1}{(a+b)[(a-b) - (a+b)]} \\ \Rightarrow \frac{x}{2(a+b)(a^2 + b^2 - 2ab)} &= \frac{-y}{2(a-b)[-2ab + (a+b)(a+b)]} = \frac{1}{(a+b)[a - b - a - b]} \\ \Rightarrow \frac{x}{2(a+b)(a^2 + b^2 - 2ab)} &= \frac{-y}{2(a-b)(-2ab + (a^2 + b^2 + 2ab))} = \frac{1}{(a+b)(-2b)} \\ \Rightarrow \frac{x}{2(a+b)(a^2 + b^2 - 2ab)} &= \frac{-y}{2(a-b)(a^2 + b^2)} = \frac{1}{-2b(a+b)} \end{aligned}$$

Now,

$$\begin{aligned} \frac{x}{2(a+b)(a^2 + b^2 - 2ab)} &= \frac{1}{-2b(a+b)} \\ \Rightarrow x &= \frac{2(a+b)(a^2 + b^2 - 2ab)}{-2b(a+b)} \\ \Rightarrow x &= \frac{a^2 + b^2 - 2ab}{-b} \\ \Rightarrow x &= \frac{-a^2 + b^2 + 2ab}{b} \\ &\quad + \frac{2ab - a^2 + b^2}{b} \end{aligned}$$

Now,

$$\begin{aligned} \frac{-y}{2(a-b)(a^2 + b^2)} &= \frac{1}{-2b(a+b)} \\ \Rightarrow -y &= \frac{2(a-b)(a^2 + b^2)}{-2b(a+b)} \\ \Rightarrow -y &= \frac{(a-b)(a^2 + b^2)}{b(a+b)} \end{aligned}$$

Hence, $x = \frac{2ab - a^2 + b^2}{b} + y = \frac{(a-b)(a^2 + b^2)}{b(a+b)}$ is the solution of the given system of equations.

Solve each of the following systems of equations by the method of cross-multiplication:

$$\begin{aligned} a^2x + b^2y &= c^2 \\ b^2x + a^2y &= d^2 \end{aligned}$$

Solution

The given system of equations may be written as

$$\begin{aligned} a^2x + b^2y - c^2 &= 0 \\ b^2x + a^2y - d^2 &= 0 \end{aligned}$$

Here,

$$\begin{aligned} a_1 &= a^2, \quad b_1 = b^2, \quad c_1 = -c^2 \\ a_2 &= b^2, \quad b_2 = a^2, \quad \text{and } c_2 = -d^2 \end{aligned}$$

By cross-multiplication, we have

$$\Rightarrow \frac{x}{-b^2c^2 + a^2c^2} = \frac{-y}{-a^2d^2 + b^2d^2} = \frac{1}{a^4 - b^4}$$

Now,

$$\frac{x}{-b^2c^2 + a^2c^2} = \frac{1}{a^4 - b^4}$$

$$\Rightarrow x = \frac{a^2c^2 - b^2c^2}{a^4 - b^4}$$

and,

$$\frac{-y}{-a^2d^2 + b^2d^2} = \frac{1}{a^4 - b^4}$$

$$\Rightarrow -y = \frac{-a^2d^2 + b^2d^2}{a^4 - b^4}$$

$$\Rightarrow y = \frac{a^2d^2 - b^2d^2}{a^4 - b^4}$$

Hence, $x = \frac{a^2c^2 - b^2c^2}{a^4 - b^4}$, $y = \frac{a^2d^2 - b^2d^2}{a^4 - b^4}$ is the solution of the given system of the equations.

Q22

Solve each of the following systems of equations by the method of cross-multiplication:

$$\begin{aligned} 2v + 3w &= \frac{s+b}{2} \\ 3v + 5w &= s \end{aligned}$$

Solution

The given system of equations is:

$$\begin{aligned} ax + by = \frac{a+b}{2} \\ 3x + 5y = 4 \end{aligned} \quad \begin{array}{l} \text{---(i)} \\ \text{---(ii)} \end{array}$$

From (i), we get:

$$\begin{aligned} 2(ax + by) = a + b \\ = 2ax + 2by = (a + b) = 0 \end{aligned} \quad \text{---(iii)}$$

From (ii), we get:

$$3x + 5y = 4 \quad \text{---(iv)}$$

Here,

$$\begin{aligned} a_1 = 2a, \quad b_1 = 2b, \quad c_1 = -(a + b) \\ a_2 = 0, \quad b_2 = 5, \quad c_2 = 4 \end{aligned}$$

By cross-multiplication, we have:

$$= \frac{x}{20 \times (-4) - [-(a + b)] \times 5} = \frac{-y}{2a \times (-4) - [-(a + b)] \times 3} = \frac{1}{2a \times 5 - 2b \times 3}$$

$$= \frac{x}{-80 - 5(a + b)} = \frac{-y}{-8a + 3(a + b)} = \frac{1}{10a - 6b}$$

$$= \frac{x}{-8b + 5a + 5b} = \frac{-y}{-8a + 3a + 3b} = \frac{1}{10a - 6b}$$

$$= \frac{x}{5a - 3b} = \frac{-y}{-5a + 3b} = \frac{1}{10a - 6b}$$

Now,

$$\frac{x}{5a - 3b} = \frac{1}{10a - 6b}$$

$$\Rightarrow x = \frac{5a - 3b}{10a - 6b} = \frac{5a - 3b}{2(5a - 3b)} = \frac{1}{2}$$

and,

$$\frac{-y}{-5a + 3b} = \frac{1}{10a - 6b}$$

$$\Rightarrow -y = \frac{-5a + 3b}{2(5a - 3b)} = \frac{-5a + 3b}{2(5a - 3b)} = \frac{1}{2}$$

$$\Rightarrow y = \frac{(-5a + 3b)}{2(5a - 3b)} = \frac{5a - 3b}{2(5a - 3b)} = \frac{1}{2}$$

$$\Rightarrow y = \frac{1}{2}$$

Hence, $x = \frac{1}{2}$, $y = \frac{1}{2}$ is the solution of the given system of equations.

Q23

Solve each of the following systems of equations by the method of cross-multiplication:

$$2\{3x - by\} + a + 4b = 0$$

$$2\{bx + ay\} + b - 4a = 0$$

Solution

The given system of equations may be written as

$$2ax - 2by + a + 4b = 0$$

$$2bx + 2ay + b - 4a = 0$$

Here,

$$a_1 = 2a, \quad A_1 = -2b, \quad C_1 = a + 4b$$

$$a_2 = 2b, \quad B_2 = 2a, \quad C_2 = b - 4a$$

By cross-multiplication, we have

$$\Rightarrow \frac{x}{(-2b)(b-4a) - (2a)(a+4b)} = \frac{-y}{(2a)(b-4a) - (2b)(a+4b)} = \frac{1}{4a^2 + 4b^2}$$

$$\Rightarrow \frac{x}{-2b^2 + 8ab - 2a^2 - 8b^2} = \frac{-y}{2ab - 8a^2 - 2ab - 8b^2} = \frac{1}{4a^2 + 4b^2}$$

$$\Rightarrow \frac{x}{-2a^2 - 2b^2} = \frac{-y}{-8a^2 - 8b^2} = \frac{1}{4a^2 + 4b^2}$$

Now,

$$\frac{x}{-2a^2 - 2b^2} = \frac{1}{4a^2 + 4b^2}$$

$$\therefore x = \frac{-2a^2 - 2b^2}{4a^2 + 4b^2}$$

$$= \frac{-2(a^2 + b^2)}{4(a^2 + b^2)}$$

$$= \frac{-1}{2}$$

and,

$$\frac{-y}{-8a^2 - 8b^2} = \frac{1}{4a^2 + 4b^2}$$

$$\therefore -y = \frac{-8a^2 - 8b^2}{4a^2 + 4b^2}$$

$$\therefore -y = \frac{-8(a^2 + b^2)}{4(a^2 + b^2)}$$

$$\therefore -y = \frac{-8}{4}$$

$$\therefore y = 2$$

Hence, $x = \frac{-1}{2}$, $y = 2$ is the solution of the given system of the equations.

Q24

Solve each of the following systems of equations by the method of cross-multiplication:

$$5(ax + by) = 3a + 2b$$

$$5(bx - ay) = 3b - 2a$$

Solution

The given system of equations is

$$6(ax + by) = 3a + 2b \quad \text{---(i)}$$

$$6(bx - ay) = 3b - 2a \quad \text{---(ii)}$$

From equation (i), we get

$$6ax + 6by - (3a + 2b) = 0 \quad \text{---(iii)}$$

From equation (ii), we get

$$6bx - 6ay - (3b - 2a) = 0 \quad \text{---(iv)}$$

Here,

$$a_1 = 6a, b_1 = 6b, c_1 = -(3a + 2b)$$

$$a_2 = 6b, b_2 = -6a, \text{ and } c_2 = -(3b - 2a)$$

By cross-multiplication, we have:

$$\frac{x}{-6b(3b - 2a) - 6a(3a + 2b)} = \frac{-y}{-6a(3b - 2a) + 6b(3a + 2b)} = \frac{1}{-36a^2 - 36b^2}$$

$$\Rightarrow \frac{x}{-18b^2 + 12ab - 12a^2 - 12ab} = \frac{-y}{-18ab + 12a^2 + 12ab + 12b^2} = \frac{1}{-36(a^2 + b^2)}$$

$$\Rightarrow \frac{x}{-18a^2 - 18b^2} = \frac{-y}{12a^2 + 12b^2} = \frac{1}{36(a^2 + b^2)}$$

Now,

$$\frac{x}{-18(a^2 + b^2)} = \frac{-1}{36(a^2 + b^2)}$$

$$\Rightarrow x = \frac{18(a^2 + b^2)}{36(a^2 + b^2)}$$

$$= \frac{1}{2}$$

and,

$$\frac{-y}{12(a^2 + b^2)} = \frac{-1}{36(a^2 + b^2)}$$

$$\Rightarrow y = \frac{12(a^2 + b^2)}{36(a^2 + b^2)}$$

$$= \frac{1}{3}$$

Hence, $x = \frac{1}{2}, y = \frac{1}{3}$ is the solution of the given system of equations.

Q25

Solve each of the following systems of equations by the method of cross-multiplication:

$$\frac{a^2}{x} - \frac{b^2}{y} = 0$$

$$\frac{a^2b}{x} + \frac{b^2a}{y} = a + b, \quad x, y \neq 0$$

Solution

Taking $\frac{1}{x} = u$ and $\frac{1}{y} = v$. Then, the given system of equations become:

$$\begin{aligned} a^2u - b^2v &= 0 \\ a^2bu + b^2av - (a+b) &= 0 \end{aligned}$$

Here,

$$\begin{aligned} a_1 &= a^2, \quad b_1 = -b^2, \quad c_1 = 0 \\ a_2 &= a^2b, \quad b_2 = b^2a, \quad \text{and } c_2 = -(a+b) \end{aligned}$$

By cross-multiplication, we have

$$= \frac{u}{b^2(a+b)-0 \times b^2a} = \frac{-v}{-a^2(a+b)-0 \times a^2b} = \frac{1}{a^2b^2+a^2b^2}$$

$$= \frac{u}{b^2(a+b)} = \frac{-v}{a^2(a+b)} = \frac{1}{a^2b^2(a+b)}$$

Now,

$$\frac{u}{b^2(a+b)} = \frac{1}{a^2b^2(a+b)}$$

$$= u = \frac{b^2(a+b)}{a^2b^2(a+b)}$$

$$\Rightarrow u = \frac{1}{a^2}$$

and,

$$\frac{v}{a^2(a+b)} = \frac{1}{a^2b^2(a+b)}$$

$$\Rightarrow v = \frac{a^2(a+b)}{a^2b^2(a+b)}$$

$$= v = \frac{1}{b^2}$$

Now,

$$x = \frac{1}{u} = a^2$$

and,

$$y = \frac{1}{v} = b^2$$

Hence, $x = a^2, y = b^2$ is the solution of the given system of equations.

Q26

Solve each of the following systems of equations by the method of cross-multiplication:

$$\begin{aligned} mx - ny &= m^2 + n^2 \\ x + y &= 2m \end{aligned}$$

Solution

The given system of equations may be written as:

$$mx + ny = \{m^2 + n^2\} = 0$$

$$x + y = 2mn = 0$$

Here,

$$a_1 = m, b_1 = -n, c_1 = -\{m^2 + n^2\}$$

$$a_2 = 1, b_2 = 1, \text{ and } c_2 = -2mn$$

By cross-multiplication, we have:

$$\frac{x}{2mn + \{m^2 + n^2\}} = \frac{-y}{-2m^2 + \{m^2 + n^2\}} = \frac{1}{m+n}$$

$$\Rightarrow \frac{x}{2mn + m^2 + n^2} = \frac{-y}{-m^2 + n^2} = \frac{1}{m+n}$$

$$\Rightarrow \frac{x}{(m+n)^2} = \frac{-y}{-m^2 + n^2} = \frac{1}{m+n}$$

Now,

$$\frac{x}{(m+n)^2} = \frac{1}{m+n}$$

$$\Rightarrow x = \frac{(m+n)^2}{m+n}$$

$$\Rightarrow x = m+n$$

and,

$$\frac{-y}{-m^2 + n^2} = \frac{1}{m+n}$$

$$\Rightarrow -y = \frac{-m^2 + n^2}{m+n}$$

$$\Rightarrow y = \frac{m^2 - n^2}{m+n}$$

$$\Rightarrow y = \frac{(m-n)(m+n)}{m+n}$$

$$\Rightarrow y = m-n$$

Hence, $x = m+n$, $y = m-n$ is the solution of the given system of equations.

Q27

Solve each of the following systems of equations by the method of cross-multiplication:

$$\begin{aligned} ax - by &= a+b \\ b &= a \end{aligned}$$

$$ax - by = 2ab$$

Solution

The given system of equations may be written as

$$\frac{a}{b}x + \frac{b}{a}y - (a+b) = 0 \\ ax + by - ab = 0$$

Here,

$$a_1 = \frac{a}{b}, \quad b_1 = -\frac{b}{a}, \quad c_1 = -(a+b) \\ a_2 = a, \quad b_2 = -b, \quad c_2 = -ab$$

By cross-multiplication, we have

$$\begin{aligned} & \frac{x}{\frac{b}{a} \times ab - b(a+b)} = \frac{-y}{\frac{a}{b} \times (-ab) + a(a+b)} = \frac{1}{-b \times \frac{a}{b} + \frac{b}{a} \times a} \\ \Rightarrow & \frac{x}{2b^2 - ab - b^2} = \frac{-y}{-2a^2 + a^2 + ab} = \frac{1}{-a+b} \\ \Rightarrow & \frac{x}{b^2 - ab} = \frac{-y}{-a^2 + ab} = \frac{1}{-a+b} \\ \Rightarrow & \frac{x}{b(b-a)} = \frac{-y}{a(-a+b)} = \frac{1}{b-a} \end{aligned}$$

Now,

$$\begin{aligned} & \frac{x}{b(b-a)} = \frac{1}{b-a} \\ \Rightarrow & x = \frac{b(b-a)}{b-a} = b \\ \text{and,} & \frac{-y}{a(-a+b)} = \frac{1}{b-a} \\ \Rightarrow & -y = \frac{a(b-a)}{b-a} \\ \Rightarrow & -y = a \\ \Rightarrow & y = -a \end{aligned}$$

Hence, $x = b$, $y = -a$ is the solution of the given system of equations.

Q28

Solve each of the following systems of equations by the method of cross-multiplication:

$$\begin{aligned} & \frac{a}{b}x + \frac{b}{a}y = a^2 + b^2 \\ & x + y = 2ab \end{aligned}$$

Solution

The given system of equations may be written as

$$\begin{aligned}\frac{b}{a}x + \frac{a}{b}y - (a^2 + b^2) &= 0 \\ x + y - 2ab &= 0\end{aligned}$$

Here,

$$\begin{aligned}a_1 &= \frac{b}{a}, \quad b_1 = \frac{a}{b}, \quad c_1 = -(a^2 + b^2) \\ a_2 &= 1, \quad b_2 = 1, \quad \text{and } c_2 = -2ab\end{aligned}$$

By cross-multiplication, we have

$$\begin{aligned}\frac{x}{-2ab \times \frac{a}{b} + a^2 + b^2} &= \frac{-y}{-2ab \times 1 + a^2 + b^2} = \frac{1}{\frac{b}{a} - \frac{a}{b}} \\ \Rightarrow \frac{x}{-2a^2 + a^2 + b^2} &= \frac{-y}{-2b^2 + a^2 + b^2} = \frac{1}{\frac{b^2 - a^2}{ab}} \\ \Rightarrow \frac{x}{b^2 - a^2} &= \frac{-y}{b^2 - a^2} = \frac{1}{b^2 - a^2}\end{aligned}$$

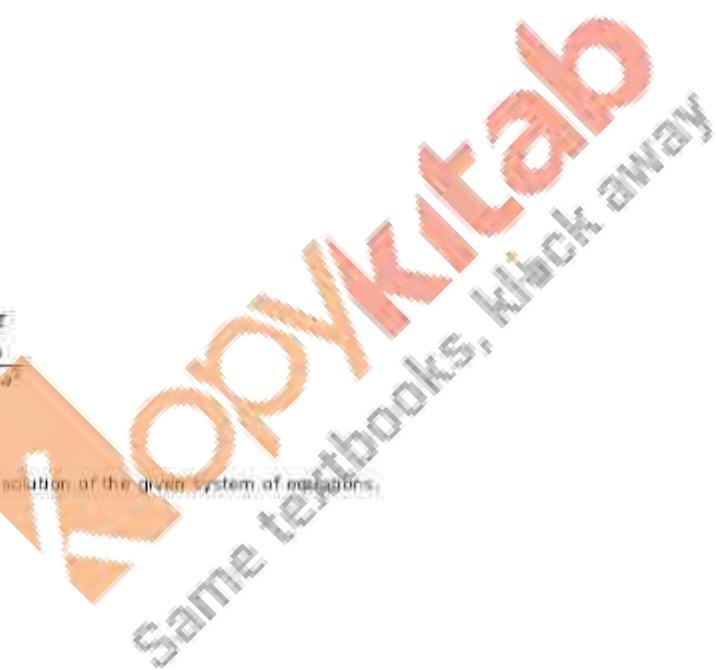
Now,

$$\begin{aligned}\frac{x}{b^2 - a^2} &= \frac{1}{b^2 - a^2} \\ \Rightarrow x - b^2 - a^2 \times \frac{ab}{b^2 - a^2} &= 0 \\ \Rightarrow x - ab &= 0\end{aligned}$$

and,

$$\begin{aligned}\frac{-y}{b^2 - a^2} &= \frac{1}{b^2 - a^2} \\ \Rightarrow -y - b^2 - a^2 \times \frac{ab}{b^2 - a^2} &= 0 \\ \Rightarrow -y - (b^2 - a^2) \times \frac{ab}{b^2 - a^2} &= 0 \\ \Rightarrow -y - ab &= 0 \\ \Rightarrow y &= ab\end{aligned}$$

Hence, $x = ab$, $y = -ab$ is the solution of the given system of equations.



Exercise 3.5**Q1**

Determine whether the following system has a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it:

$$\begin{aligned}x - 3y &= 3 \\3x - 9y &= 2\end{aligned}$$

Solution

The given system of equations may be written as

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

The given system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 1$, $b_1 = -3$, $c_1 = -3$

and, $a_2 = 3$, $b_2 = -9$, $c_2 = 2$

We have,

$$\frac{a_1}{a_2} = \frac{1}{3}$$

$$\frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$$

$$\text{and, } \frac{c_1}{c_2} = \frac{-3}{2} = \frac{3}{2}$$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system of equations has no solutions.

Q2

Determine whether the following system has a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it:

$$\begin{aligned}2x + y &= 5 \\4x + 2y &= 10\end{aligned}$$

Solution

The given system of equations may be written as

$$2x + y - 5 = 0$$

$$4x + 2y - 10 = 0$$

The given system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 2$, $b_1 = 1$, $c_1 = -5$

and, $a_2 = 4$, $b_2 = 2$, $c_2 = -10$

We have,

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{1}{2}$$

$$\text{and, } \frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$$

$$\text{Clearly, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, the given system of equations has infinitely many solutions.

Q3

Determine whether the following system has a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it:
 $3x - 5y = 20$
 $6x - 10y = 40$

Solution

$$3x - 5y = 20$$

$$6x - 10y = 40$$

Compare it with

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

we get

$$a_1 = 3, b_1 = -5, \text{ and } c_1 = -20$$

$$a_2 = 6, b_2 = -10, \text{ and } c_2 = -40$$

$$\frac{a_1}{a_2} = \frac{3}{6}, \frac{b_1}{b_2} = \frac{-5}{-10}, \text{ and } \frac{c_1}{c_2} = \frac{-20}{-40}$$

Simplifying it we get

$$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2}, \text{ and } \frac{c_1}{c_2} = \frac{1}{2}$$

Hence

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So both lines are coincident and overlap with each other

So it will have infinite or many solutions.

Q4

Determine whether the following system has a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it:
 $x - 2y = 8$
 $5x - 10y = 10$

Solution

The given system of equations may be written as:

$$x - 2y - 5 = 0$$

$$5x - 10y - 10 = 0$$

The given system of equations is of the form:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 1$, $b_1 = -2$, $c_1 = -5$

and, $a_2 = 5$, $b_2 = -10$, $c_2 = -10$

We have,

$$\frac{a_1}{a_2} = \frac{1}{5}$$

$$\frac{b_1}{b_2} = \frac{-2}{-10} = \frac{1}{5}$$

$$\text{and, } \frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$$

$$\text{Clearly, } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the given system of equations has no solution.

Q5

Find the value of k for which the following system of equations has a unique solution:

$$kx + 2y = 5$$

$$3x + y = 1$$

Solution

The given system of equations is:

$$kx + 2y - 5 = 0$$

$$3x + y - 1 = 0$$

This system of equations is of the form:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = k$, $b_1 = 2$, $c_1 = -5$

and, $a_2 = 3$, $b_2 = 1$, $c_2 = -1$

For a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{k}{3} \neq \frac{2}{1}$$

$$\therefore k \neq 6$$

So, the given system of equations will have a unique solution for all real values of k other than 6.

Q6

Find the value of k for which the following system of equations has a unique solution:

$$4x + ky + 8 = 0$$

$$2x + 2y + 2 = 0$$

Solution

Here $a_1 = 4$, $a_2 = k$, $b_1 = 2$, $b_2 = 2$.

Now for the given pair to have a unique solution: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\text{i.e., } \frac{4}{k} \neq \frac{2}{2}$$

$$\text{i.e., } k \neq 4$$

Therefore, for all values of k (except 4), the given pair of equations will have a unique solution.

Q7

Find the value of k for which the following system of equations has a unique solution:

$$4x - 5y = k$$

$$2x - 3y = 12$$

Solution

The given system of equations is

$$4x - 5y - k = 0$$

$$2x - 3y - 12 = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 4$, $b_1 = -5$, $c_1 = -k$

and, $a_2 = 2$, $b_2 = -3$, $c_2 = -12$

For a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{4}{2} \neq \frac{-5}{-3}$$

k is any real number.

So, the given system of equations will have a unique solution for all real values of k .

Q8

Find the value of k for which the following system of equations has a unique solution:

$$x + 2y = 3$$

$$5x + ky + 7 = 0$$

Solution

The given system of equations is:

$$x + 2y - 3 = 0$$

$$5x + ky + 7 = 0$$

This system of equations is of the form:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 1$, $b_1 = 2$, $c_1 = -3$

and, $a_2 = 5$, $b_2 = k$, $c_2 = 7$

For a unique solution, we must have:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{a_1}{a_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{1}{5} \neq \frac{2}{k}$$

$$\Rightarrow k \neq 10$$

So, the given system of equations will have a unique solution for all real values of k other than 10.

Q9

Find the value of k for which the following systems of equations have infinitely many solutions:

$$2x + 3y - 5 = 0$$

$$6x + ky - 15 = 0$$

Solution

The given system of equations is:

$$2x + 3y - 5 = 0$$

$$6x + ky - 15 = 0$$

This system of equations is of the form:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 2$, $b_1 = 3$, $c_1 = -5$

and, $a_2 = 6$, $b_2 = k$, $c_2 = -15$

For infinitely many solutions, we must have:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{6} = \frac{3}{k} = \frac{-5}{-15}$$

Now,

$$\frac{2}{6} = \frac{3}{k}$$

$$\Rightarrow 2k = 18$$

$$\Rightarrow k = \frac{18}{2} = 9$$

Hence, the given system of equations will have infinitely many solutions, if $k = 9$.

Q10

Find the value of k for which the following systems of equations have infinitely many solutions:

$$4x + 5y = 3$$

$$10x + 15y = 9$$

Solution

The given system of equations may be written as:

$$4x + 5y - 3 = 0$$

$$kx + 15y - 9 = 0$$

This system of equations is of the form:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 4$, $b_1 = 5$, $c_1 = -3$

and, $a_2 = k$, $b_2 = 15$, $c_2 = -9$

For infinitely many solutions, we must have:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{4}{k} = \frac{5}{15} = \frac{-3}{-9}$$

Now,

$$\frac{4}{k} = \frac{5}{15}$$

$$\frac{4}{k} = \frac{1}{3}$$

$$\Rightarrow k = 12$$

Hence, the given system of equations will have infinitely many solutions, if $k = 12$.

Q11

Find the value of k for which the following systems of equations have infinitely many solutions:

$$kx - 2y + 6 = 0$$

$$4x - 3y + 9 = 0$$

Solution

The given system of equations is:

$$kx - 2y + 6 = 0$$

$$4x - 3y + 9 = 0$$

This system of equations is of the form:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = k$, $b_1 = -2$, $c_1 = 6$

and, $a_2 = 4$, $b_2 = -3$, $c_2 = 9$

For infinitely many solution, we must have:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k}{4} = \frac{-2}{-3} = \frac{6}{9}$$

Now,

$$\frac{k}{4} = \frac{6}{9}$$

$$\frac{k}{4} = \frac{2}{3}$$

$$\Rightarrow k = \frac{2 \times 4}{3}$$

$$\Rightarrow k = \frac{8}{3}$$

Hence, the given system of equations will have infinitely many solutions, if $k = \frac{8}{3}$.

Q12

Find the value of k for which the following systems of equations have infinitely many solutions:
 $8x + 5y = 0$
 $kx + 10y = 18$

Solution

The given system of equations is:

$$8x + 5y - 0 = 0$$

$$kx + 10y - 18 = 0$$

This system of equations is of the form:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 8$, $b_1 = 5$, $c_1 = -0$

and, $a_2 = k$, $b_2 = 10$, $c_2 = -18$

For infinitely many solutions, we must have

$$\begin{aligned} & \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \\ \Rightarrow & \frac{8}{k} = \frac{5}{10} = \frac{-0}{-18} \end{aligned}$$

Now,

$$\begin{aligned} & \frac{8}{k} = \frac{5}{10} \\ \Rightarrow & 8 \times 10 = 5 \times k \\ \Rightarrow & \frac{8 \times 10}{5} = k \\ \Rightarrow & k = 8 \times 2 = 16 \end{aligned}$$

Hence, the given system of equations will have infinitely many solutions, if $k = 16$.

Q13

Find the value of k for which the following systems of equations have infinitely many solutions:
 $2x - 3y = 7$
 $(k+2)x - (2k+1)y = 3(2k-1)$

Solution

The given system of equations may be written as

$$2x - 3y - 7 = 0$$

$$(k+2)x - (2k+1)y - 3(2k-1) = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 2$, $b_1 = -3$, $c_1 = -7$

and, $a_2 = k+2$, $b_2 = -(2k+1)$, $c_2 = -3(2k-1)$

For infinitely many solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{2}{k+2} = \frac{-3}{-(2k+1)} = \frac{-7}{-3(2k-1)}$$

$$\Rightarrow \frac{2}{k+2} = \frac{-3}{-(2k+1)} \text{ and } \frac{-3}{-(2k+1)} = \frac{-7}{-3(2k-1)}$$

$$\Rightarrow 2(2k+1) = 3(k+2) \text{ and } 3 \times 3(2k-1) = 7(2k+1)$$

$$\Rightarrow 4k+2 = 3k+6 \text{ and } 18k-9 = 14k+7$$

$$\Rightarrow 4k-3k = 6-2 \text{ and } 18k-14k = 7+9$$

$$\Rightarrow k=4 \text{ and } 4k-15=k+4$$

$$\Rightarrow k=4 \text{ and } k=4$$

Hence, the given system of equations will have infinitely many solutions, if $k=4$.

Q14

Find the value of k for which the following systems of equations have infinitely many solutions:

$$2x + 3y = 2$$

$$(k+2)x + (2k+1)y = 2(k-1)$$

Solution

The given system of equations may be written as

$$\begin{aligned} 2x + 3y - 2 &= 0 \\ (k+2)x - (2k+1)y - 2(k-1) &= 0 \end{aligned}$$

This system of equations is of the form

$$\begin{aligned} a_1x + b_1y + c_1 &= 0 \\ a_2x + b_2y + c_2 &= 0 \end{aligned}$$

Where, $a_1 = 2$, $b_1 = 3$, $c_1 = -2$

and, $a_2 = k+2$, $b_2 = -(2k+1)$, $c_2 = -2(k-1)$

For infinitely many solution, we must have

$$\begin{aligned} \frac{a_1}{a_2} &= \frac{b_1}{b_2} = \frac{c_1}{c_2} \\ \Rightarrow \frac{2}{k+2} &= \frac{3}{-(2k+1)} = \frac{-2}{-2(k-1)} \\ \Rightarrow \frac{2}{k+2} &= \frac{3}{(2k+1)} \text{ and } \frac{3}{(2k+1)} = \frac{2}{2(k-1)} \\ \Rightarrow 2(2k+1) &= 3(k+2) \text{ and } 3(k-1) = (2k+1) \\ \Rightarrow 4k+2 &= 3k+6 \text{ and } 3k-3 = 2k+1 \\ \Rightarrow 4k-3k &= 6-2 \text{ and } 3k-2k = 1+3 \\ \Rightarrow k &= 4 \text{ and } k = 4 \end{aligned}$$

Hence, the given system of equations will have infinitely many solutions, if $k = 4$.

Q15

Find the value of k for which the following systems of equations have infinitely many solutions:

$$x + (k+1)y = 4$$

$$(k+1)x + 9y = (5k+2)$$

Solution

The given system of equations may be written as

$$x + (k+1)y - 4 = 0$$

$$(k+1)x + 9y - (5k+2) = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 1$, $b_1 = k+1$, $c_1 = -4$.

and, $a_2 = k+1$, $b_2 = 9$, $c_2 = -(5k+2)$

For infinitely many solution, we must have

$$\begin{aligned} & \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \\ \Rightarrow & \frac{1}{k+1} = \frac{k+1}{9} = \frac{-4}{-(5k+2)} \\ \Rightarrow & \frac{1}{k+1} = \frac{k+1}{9} \text{ and } \frac{k+1}{9} = \frac{-4}{-5k-2} \\ \Rightarrow & 9 = (k+1)^2 \text{ and } (k+1)(5k+2) = 36 \\ \Rightarrow & 9 = k^2 + 2k + 1 \text{ and } 5k^2 + 2k + 5k + 2 = 36 \\ \Rightarrow & k^2 + 2k + 1 - 9 = 0 \text{ and } 5k^2 + 7k + 2 - 36 = 0 \\ \Rightarrow & k^2 + 2k - 8 = 0 \text{ and } 5k^2 + 7k - 34 = 0 \\ \Rightarrow & k^2 + 4k - 2k - 8 = 0 \text{ and } 5k^2 + 17k - 10k - 34 = 0 \\ \Rightarrow & k(k+4) - 2(k+4) = 0 \text{ and } k(5k+17) - 2(5k+17) = 0 \\ \Rightarrow & (k+4)(k-2) = 0 \text{ and } (5k+17)(k-2) = 0 \\ \Rightarrow & (k = -4 \text{ or } k = 2) \text{ and } \left(k = \frac{-17}{5} \text{ or } k = 2\right) \\ \Rightarrow & k = 2 \text{ satisfies both the conditions.} \end{aligned}$$

Hence, the given system of equations will have infinitely many solutions, if $k = 2$.

Q16

Find the value of k for which the following systems of equations have infinitely many solutions:
 $3x + 2y = 2k + 1$
 $2(k+1)x + 9y = 7k + 1$

Solution

The given system of equations may be written as:

$$kx + 3y - \{2k + 1\} = 0$$

$$2\{k + 1\}x + 9y - \{7k + 1\} = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

$$\text{Where, } a_1 = k, b_1 = 3, c_1 = -\{2k + 1\}$$

$$\text{and, } a_2 = 2(k + 1), b_2 = 9, c_2 = -\{7k + 1\}$$

For infinitely many solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{k}{2(k+1)} = \frac{3}{9} = \frac{-\{2k+1\}}{-\{7k+1\}}$$

$$\Rightarrow \frac{k}{2(k+1)} = \frac{3}{9} \text{ and } \frac{3}{9} = \frac{2k+1}{7k+1}$$

$$\Rightarrow 9k = 3 \times 2(k+1) \text{ and } 3(7k+1) = 9(2k+1)$$

$$\Rightarrow 9k = 6(k+1) \text{ and } 21k + 3 = 18k + 9$$

$$\Rightarrow 9k = 6k + 6 \text{ and } 21k + 3 = 18k + 9$$

$$\Rightarrow 3k = 6 \text{ and } 3k = 6$$

$$\Rightarrow k = \frac{6}{3} \text{ and } k = \frac{6}{3}$$

$$\Rightarrow k = 2 \text{ and } k = 2$$

$\Rightarrow k = 2$ satisfies both the conditions.

Hence, the given system of equations will have infinitely many solutions if $k = 2$.

Q17

Find the value of k for which the following systems of equations have infinitely many solutions:

$$2x + (k - 2)y = k$$

$$6x + (2k - 1)y = 2k + 5$$

Solution

The given system of equations may be written as

$$2x + (k-2)y - k = 0$$

$$5x + (2k-1)y - (2k+5) = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 2$, $b_1 = k-2$, $c_1 = -k$

and, $a_2 = 5$, $b_2 = 2k-1$, $c_2 = -(2k+5)$

For infinitely many solution, we must have

$$\begin{aligned} \frac{a_1}{a_2} &= \frac{b_1}{b_2} = \frac{c_1}{c_2} \\ \Rightarrow \frac{2}{5} &= \frac{k-2}{2k-1} = \frac{-k}{-(2k+5)} \end{aligned}$$

$$\Rightarrow \frac{2}{5} = \frac{k-2}{2k-1} \text{ and } \frac{k-2}{2k-1} = \frac{k}{2k+5}$$

$$\Rightarrow \frac{1}{5} = \frac{k-2}{2k-1} \text{ and } (k-2)(2k+5) = k(2k-1)$$

$$\Rightarrow 2k-1 = 5(k-2) \text{ and } 2k^2+5k-4k-10 = 2k^2-k$$

$$\Rightarrow 2k-1 = 5k-10 \text{ and } k-10 = -k$$

$$\Rightarrow 2k-5k = -10+1 \text{ and } k+k = 10$$

$$\Rightarrow -3k = -9 \text{ and } 2k = 10$$

$$\Rightarrow k = \frac{-9}{-3} \text{ and } k = \frac{10}{2}$$

$$\Rightarrow k = 3 \text{ and } k = 5$$

$\Rightarrow k = 5$ satisfies both the conditions.

Hence, the given system of equations will have infinitely many solutions, if $k = 5$.

Q18

Find the value of k for which the following systems of equations have infinitely many solutions:

$$2x + 3y = 7$$

$$(k+1)x + (2k-1)y = 4k+1$$

Solution

The given system of equations may be written as

$$\begin{aligned} 2x + 3y - 7 &= 0 \\ (k+1)x + (2k-1)y - (4k+1) &= 0 \end{aligned}$$

This system of equations is of the form

$$\begin{aligned} a_1x + b_1y + c_1 &= 0 \\ a_2x + b_2y + c_2 &= 0 \end{aligned}$$

Where, $a_1 = 2$, $b_1 = 3$, $c_1 = -7$

and, $a_2 = k+1$, $b_2 = 2k-1$, $c_2 = -(4k+1)$

For infinitely many solution, we must have

$$\begin{aligned} \frac{a_1}{a_2} &= \frac{b_1}{b_2} = \frac{c_1}{c_2} \\ \Rightarrow \frac{2}{k+1} &= \frac{3}{2k-1} = \frac{-7}{-(4k+1)} \\ \Rightarrow \frac{2}{k+1} &= \frac{3}{2k-1} \text{ and } \frac{3}{2k-1} = \frac{7}{4k+1} \\ \Rightarrow 2(2k-1) &= 3(k+1) \text{ and } 3(4k+1) = 7(2k-1) \\ \Rightarrow 4k-2 &= 3k+3 \text{ and } 12k+3 = 14k-7 \\ \Rightarrow 4k-3k &= 3+2 \text{ and } 12k-14k = -7-3 \\ \Rightarrow k &= 5 \text{ and } -2k = -10 \\ \Rightarrow k &= 5 \text{ and } k = \frac{10}{2} = 5 \\ \Rightarrow k &= 5 \text{ satisfies both the conditions} \end{aligned}$$

Hence, the given system of equations will have infinitely many solutions if $k = 5$.

Q19

Find the value of k for which the following systems of equations have infinitely many solutions:

$$2x + 3y = k$$

$$(k-1)x + (k+2)y = 3k$$

Solution

The given system of equations may be written as

$$\begin{aligned} 2x + 3y - k &= 0 \\ (k-1)x + (k+2)y - 3k &= 0 \end{aligned}$$

This system of equations is of the form

$$\begin{aligned} a_1x + b_1y + c_1 &= 0 \\ a_2x + b_2y + c_2 &= 0 \end{aligned}$$

Where, $a_1 = 2$, $b_1 = 3$, $c_1 = -k$

and, $a_2 = k-1$, $b_2 = k+2$, $c_2 = -3k$

For infinitely many solution, we must have

$$\begin{aligned} \frac{a_1}{a_2} &= \frac{b_1}{b_2} = \frac{c_1}{c_2} \\ \Rightarrow \frac{2}{k-1} &= \frac{3}{k+2} = \frac{-k}{-3k} \\ \Rightarrow \frac{2}{k-1} &= \frac{3}{k+2} \text{ and } \frac{3}{k+2} = \frac{-k}{-3k} \\ \Rightarrow 2(k+2) &= 3(k-1) \text{ and } 3 \times 2 = k+2 \\ \Rightarrow 2k+4 &= 3k-3 \text{ and } 6 = k+2 \\ \Rightarrow 4+3 &= 3k-2k \text{ and } 6-2 = k \\ \Rightarrow 7 &= k \text{ and } 7 = k \\ \Rightarrow k &= 7 \text{ and } k = 7 \\ \Rightarrow k &= 7 \text{ satisfies both the conditions} \end{aligned}$$

Hence, the given system of equations will have infinitely many solutions if $k = 7$.

Q20

Find the value of k for which the following system of equations has no solution:

$$kx - 5y = 2$$

$$6x + 2y = 7$$

Solution

Given

$$kx - 5y = 2$$

$$6x + 2y = 7$$

Condition for system of equations having no solution

$$\begin{aligned} \frac{a_1}{a_2} &= \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \\ \Rightarrow \frac{k}{6} &= \frac{-5}{2} \neq \frac{2}{7} \\ \Rightarrow 2k &= -30 \\ \Rightarrow k &= -15 \end{aligned}$$

Q21

Find the value of k for which the following system of equations has no solution:

$$\begin{aligned}x + 2y &= 0 \\2x + ky &= 5\end{aligned}$$

Solution

The given system of equations may be written as:

$$x + 2y = 0$$

$$2x + ky = 5$$

This system of equations is of the form:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 1$, $b_1 = 2$, $c_1 = 0$,

and, $a_2 = 2$, $b_2 = k$, $c_2 = -5$

For no solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

We have,

$$\frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{2}{k}$$

$$\frac{c_1}{c_2} = \frac{0}{-5} = 0$$

and, $\frac{c_1}{c_2} = \frac{0}{-5} = 0$

Now, $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$$\Rightarrow \frac{1}{2} = \frac{2}{k}$$

$$\Rightarrow k = 4$$

Hence, the given system of equations has no solutions, when $k = 4$.

Q22

Find the value of k for which the following system of equations has no solution:

$$3x - 4y + 7 = 0$$

$$kx + 3y - 5 = 0$$

Solution

The given system of equations is

$$3kx - 4y + 7 = 0$$

$$kx + 3y - 5 = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 3k$, $b_1 = -4$, $c_1 = 7$

and, $a_2 = k$, $b_2 = 3$, $c_2 = -5$

For no solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

We have,

$$\frac{b_1}{b_2} = \frac{-4}{3}$$

$$\text{and, } \frac{c_1}{c_2} = \frac{-7}{5}$$

$$\text{Clearly, } \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the given system will have no solution.

$$\text{If } \frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{3k}{k} = \frac{-4}{3} \Rightarrow k = \frac{-9}{4}$$

Clearly, for this value of k , we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given system of equations has no solutions when $k = \frac{-9}{4}$.

Q23

Find the value of k for which the following system of equations has no solution:

$$2x - ky + 3 = 0$$

$$3x + 2y - 1 = 0$$

Solution

The given system of equations is

$$2x - ky + 3 = 0$$

$$3x + 2y - 1 = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 2$, $b_1 = -k$, $c_1 = 3$

and, $a_2 = 3$, $b_2 = 2$, $c_2 = -1$

For no solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

We have,

$$\frac{a_1}{a_2} = \frac{2}{3}$$

$$\text{and, } \frac{c_1}{c_2} = \frac{-3}{-1} = 3$$

$$\text{Clearly, } \frac{a_1}{a_2} \neq \frac{c_1}{c_2}$$

So, the given system of equations will have no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \text{ i.e., } \frac{2}{3} = \frac{-k}{2} \Rightarrow k = \frac{-4}{3}$$

Hence, the given system of equations will have no solution, if $k = \frac{-4}{3}$

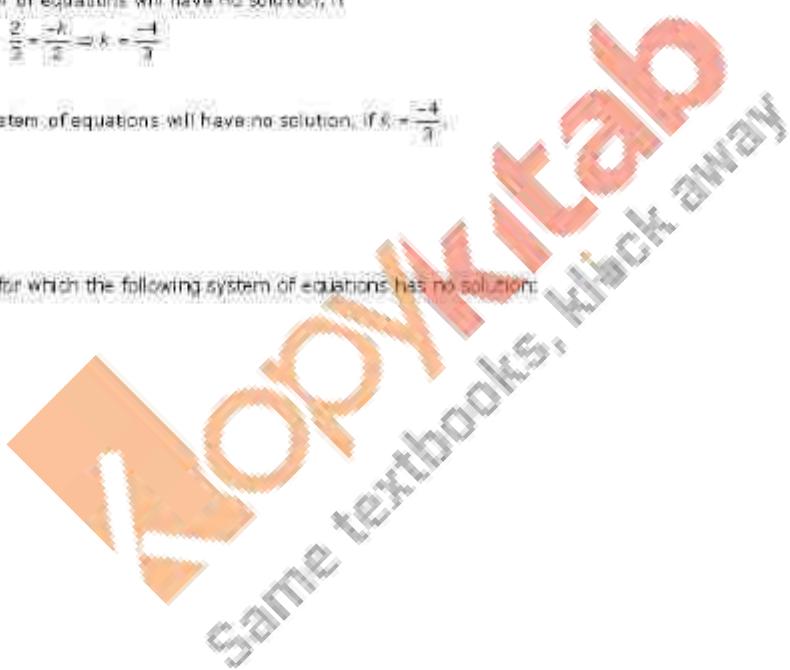
Q24

Find the value of k for which the following system of equations has no solution.

$$2x + ky = 12$$

$$5x - 3y = 5$$

Solution



The given system of equations is

$$2x + ky - 11 = 0$$

$$5x - 7y - 5 = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 2$, $b_1 = k$, $c_1 = -11$

and, $a_2 = 5$, $b_2 = -7$, $c_2 = -5$

For no solution, we must have

$$\begin{aligned} & \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \\ \Rightarrow & \frac{2}{5} = \frac{k}{-7} \neq \frac{-11}{-5} \\ \Rightarrow & \frac{2}{5} = \frac{k}{-7} \text{ and } \frac{k}{-7} \neq \frac{-11}{-5} \end{aligned}$$

Now,

$$\begin{aligned} & \frac{2}{5} = \frac{k}{-7} \\ \Rightarrow & 2 \times (-7) = 5k \\ \Rightarrow & 5k = -14 \\ \Rightarrow & k = \frac{-14}{5} \end{aligned}$$

Clearly, for $k = \frac{-14}{5}$ we have $\frac{k}{-7} \neq \frac{-11}{-5}$

Hence, the given system of equations will have no solution, if $k = \frac{-14}{5}$.

Q25

Find the value of k for which of the following system of equation has no solution:

$$\begin{aligned} kx + 3y &= k - 3 \\ 12x + ky &= 6 \end{aligned}$$

Solution

The given system of equations may be written as

$$kx + 3y - (k - 3) = 0$$

$$12x + ky - 6 = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = k$, $b_1 = 3$, $c_1 = -(k - 3)$

And, $a_2 = 12$, $b_2 = k$, $c_2 = -6$

For the system of equations to have no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{k}{12} = \frac{3}{k} \neq \frac{-(k-3)}{-6}$$

$$\Rightarrow \frac{k}{12} = \frac{3}{k} \neq \frac{k-3}{6}$$

$$\text{Now, } \frac{k}{12} = \frac{3}{k}$$

$$\Rightarrow k^2 = 36$$

$$\Rightarrow k = \pm 6$$

$$\text{And, } \frac{k}{12} = \frac{k-3}{6}$$

$$\Rightarrow 6k = 12k - 36$$

$$\Rightarrow 6k = 36$$

$$\Rightarrow k = 6$$

Hence, $k = -6$

Thus, the given system of equations will have

no solution if $k = -6$.

Q26

For what value of k , the following system of equations will be inconsistent?

$$4x + 5y = 11$$

$$2x + ky = 7$$

Solution

KopyKitab
Same textbooks, klick away

The given system of equations may be written as

$$4x + 5y - 11 = 0$$

$$2x + ky - 7 = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 4$, $b_1 = 5$, $c_1 = -11$

and, $a_2 = 2$, $b_2 = k$, $c_2 = -7$

For inconsistent, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Now,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$\Rightarrow \frac{4}{2} = \frac{5}{k}$$

$$\Rightarrow 4k = 12$$

$$\Rightarrow k = \frac{12}{4} = 3$$

Clearly, for this value of k , we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, the given system of equations is inconsistent, when $k = 3$.

Q27

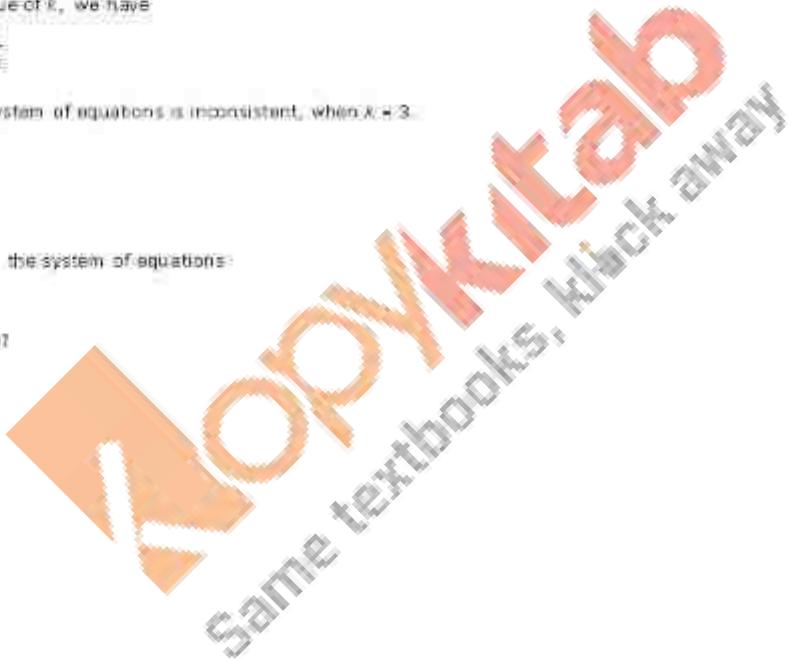
For what value of α , the system of equations:

$$\alpha x + 3y = \alpha - 3$$

$$12x + \alpha y = \alpha$$

Will have no solution?

Solution



The given system of equations may be written as

$$\alpha x + 3y - (\alpha - 3) = 0$$

$$12x + \alpha y - \alpha = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = \alpha$, $b_1 = 3$, $c_1 = -(\alpha - 3)$

and, $a_2 = 12$, $b_2 = \alpha$, $c_2 = -\alpha$

For no solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{\alpha}{12} = \frac{3}{\alpha} \neq \frac{-(\alpha - 3)}{-\alpha}$$

Now,

$$\frac{3}{\alpha} = \frac{-(\alpha - 3)}{-\alpha}$$

$$\Rightarrow \frac{3}{\alpha} = \frac{\alpha - 3}{\alpha}$$

$$\Rightarrow 3 = \alpha - 3$$

$$\Rightarrow 3 + 3 = \alpha$$

$$\Rightarrow 6 = \alpha$$

$$\Rightarrow \alpha = 6$$

and,

$$\frac{\alpha}{12} = \frac{3}{\alpha}$$

$$\Rightarrow \alpha^2 = 36$$

$$\Rightarrow \alpha = \pm 6$$

$$\Rightarrow \alpha = 6 \quad [\because \alpha \neq 6]$$

Hence, the given system of equations will have no solution, if $\alpha = -6$.

Q28

For the value of k , for which the system

$$kx + 2y = 5$$

$$3x + y = 1$$

has (i) a unique solution, and (ii) no solution.

Solution

The given system of equations may be written as

$$kx + 2y - 5 = 0$$

$$3x + y - 1 = 0$$

It is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = k$, $b_1 = 2$, $c_1 = -5$

and, $a_2 = 3$, $b_2 = 1$, $c_2 = -1$

(i) The given system will have a unique solution, if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \frac{k}{3} \neq \frac{2}{1}$$

$$\Rightarrow k \neq 6$$

So, the given system of equations will have a unique solution, if $k \neq 6$.

(ii) The given system will have no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

We have,

$$\Rightarrow \frac{b_1}{b_2} = \frac{2}{1} \text{ and } \frac{c_1}{c_2} = \frac{-5}{-1} = \frac{5}{1}$$

$$\text{Clearly, } \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the system of equations will have no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$\Rightarrow \frac{k}{3} = \frac{2}{1}$$

$$\Rightarrow k = 6$$

Hence, the given system of equations will have no solution, if $k = 6$.

Q29

Prove that there is a value of $c \neq 0$ for which the system

$$6x + 3y = c - 3$$

$$12x + 6y = c$$

Has infinitely many solutions. Find this value.

Solution

The given system of equations may be written as

$$6x + 3y - (c - 3) = 0$$

$$12x + 6y - c = 0$$

This is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 6$, $b_1 = 3$, $c_1 = -(c - 3)$

and, $a_2 = 12$, $b_2 = 6$, $c_2 = -c$.

For infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{6}{12} = \frac{3}{6} = \frac{-(c - 3)}{-c}$$

$$\Rightarrow \frac{6}{12} = \frac{3}{6} \text{ and } \frac{3}{6} = \frac{c - 3}{c}$$

$$\Rightarrow 6c = 12 \times 3 \text{ and } 3 = (c - 3)$$

$$\Rightarrow c = \frac{36}{6} \text{ and } c - 3 = 3$$

$$\Rightarrow c = 6 \text{ and } c = 6$$

Now,

$$\frac{a_1}{a_2} = \frac{6}{12} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{(6 - 3)}{-c} = \frac{1}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Clearly, for this value of c , we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given system of equations has infinitely many solutions, if $c = 6$.

Q30

Find the value of k for which the system

$$2x + ky = 1$$

$$3x - 5y = 7$$

will have (i) a unique solution, and (ii) no solution. Is there a value of k for which the system has infinitely many solutions?

Solution

The given system of equations may be written as:

$$2x + ky - 1 = 0$$

$$3x - 5y - 7 = 0$$

It is of the form:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 2$, $b_1 = k$, $c_1 = -1$

and, $a_2 = 3$, $b_2 = -5$, $c_2 = -7$

(i) The given system will have a unique solution, if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{2}{3} \neq \frac{k}{-5}$$

$$\Rightarrow -10 \neq 3k$$

$$\Rightarrow 3k \neq -10$$

$$\Rightarrow k \neq \frac{-10}{3}$$

So, the given system of equations will have a unique solution, if $k \neq \frac{-10}{3}$.

(ii) The given system will have no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

We have,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$\frac{2}{3} = \frac{k}{-5}$$

$$\Rightarrow -10 = 3k$$

$$\Rightarrow 3k = -10$$

$$\Rightarrow k = \frac{-10}{3}$$

We have,

$$\frac{a_1}{a_2} = \frac{k}{-5} = \frac{-10}{3} = \frac{2}{-3}$$

$$\text{and, } \frac{c_1}{c_2} = \frac{-1}{7} = \frac{1}{-7}$$

$$\text{Clearly, } \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the given system will have no solution, if $k = \frac{-10}{3}$.

For the given system to have infinite number of solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

We have,

$$\frac{a_1}{a_2} = \frac{2}{3}, \quad \frac{b_1}{b_2} = \frac{k}{-5}$$

$$\text{and, } \frac{c_1}{c_2} = \frac{-1}{7} = \frac{1}{-7}$$

$$\text{Clearly, } \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, whatever be the value of k , we cannot have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, there is no value of k , for which the given system of equations has infinitely many solutions.

Q31

For what value of k , the following system of equations will represent the coincident lines?

$$x + 2y + 7 = 0$$

$$2x + ky + 14 = 0$$

Solution

The given system of equations may be written as

$$x + 2y + 7 = 0$$

$$2x + ky + 14 = 0$$

The given equations are of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 1$, $b_1 = 2$, $c_1 = 7$

and, $a_2 = 2$, $b_2 = k$, $c_2 = 14$

The given equations will represent coincident lines if they have infinitely many solutions.

The condition for which is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2} = \frac{2}{k} = \frac{7}{14} \Rightarrow k = 4$$

Hence, the given system of equations will represent coincident lines, if $k = 4$.

Q32

Obtain the condition for the following system of linear equations to have a unique solution

$$ax + by = c$$

$$lx + my = n$$

Solution

The given system of equations may be written as

$$ax + by - c = 0$$

$$lx + my - n = 0$$

It is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = a$, $b_1 = b$, $c_1 = -c$

and, $a_2 = l$, $b_2 = m$, $c_2 = -n$

For unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{a}{l} \neq \frac{b}{m}$$

$$\Rightarrow am \neq bl$$

Hence, $am \neq bl$ is the required condition.

Q33

Determine the value of a and b so that the following system of linear equations have infinitely many solutions:

$$(2a-1)x + 3y - 5 = 0$$

$$2x + (b-1)y - 2 = 0$$

Solution

The given system of equations is:

$$(2a-1)x + 3y - 5 = 0$$

$$3x + (b-1)y - 2 = 0$$

It is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 2a - 1$, $b_1 = 3$, $c_1 = -5$

and, $a_2 = 3$, $b_2 = b - 1$, $c_2 = -2$

The given system of equations will have infinite number of solutions, if

$$\begin{aligned} & \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \\ \Rightarrow & \frac{2a-1}{3} = \frac{3}{b-1} = \frac{-5}{-2} \\ \Rightarrow & \frac{2a-1}{3} = \frac{-5}{-2} \text{ and } \frac{3}{b-1} = \frac{-5}{-2} \\ \Rightarrow & 2(2a-1) = 5 \times 3 \text{ and } 3 \times 2 = 5(b-1) \\ \Rightarrow & 4a-2 = 15 \text{ and } 6 = 5b-5 \\ \Rightarrow & 4a = 15+2 \text{ and } 6+5 = 5b \\ \Rightarrow & a = \frac{17}{4} \text{ and } \frac{11}{5} = b \\ \Rightarrow & a = \frac{17}{4} \text{ and } b = \frac{11}{5} \end{aligned}$$

Hence, the given system of equations will have infinitely many solutions,

$$\text{if } a = \frac{17}{4} \text{ and } b = \frac{11}{5}.$$

Q34

Find the value of a and b for which the following system of linear equations has infinite number of solutions:

$$2x - 3y = 7$$

$$(a+b)x - (a+b-3)y = 4a+b$$

Solution

KopyKitab
Same textbooks, click away

The given system of equations is:

$$\begin{aligned} 2x - 3y - 7 &= 0 \\ (a+b)x - (a+b-3)y - (4a+b) &= 0 \end{aligned}$$

It is of the form:

$$\begin{aligned} a_1x + b_1y + c_1 &= 0 \\ a_2x + b_2y + c_2 &= 0 \end{aligned}$$

Where, $a_1 = 2$, $b_1 = -3$, $c_1 = -7$

and, $a_2 = a+b$, $b_2 = -(a+b-3)$, $c_2 = -(4a+b)$

The given system of equations will have infinite number of solutions, if

$$\begin{aligned} \frac{a_1}{a_2} &= \frac{b_1}{b_2} = \frac{c_1}{c_2} \\ \Rightarrow \frac{2}{a+b} &= \frac{-3}{-(a+b-3)} = \frac{-7}{-(4a+b)} \\ \Rightarrow \frac{2}{a+b} &= \frac{3}{a+b-3} \text{ and } \frac{3}{a+b-3} = \frac{7}{4a+b} \\ \Rightarrow 2(a+b-3) &= 3(a+b) \text{ and } 3(4a+b) = 7(a+b-3) \\ \Rightarrow 2a+2b-6 &= 3a+3b \text{ and } 12a+3b = 7a+7b-21 \\ \Rightarrow -a-2a+3b-2b &= 0 \text{ and } 12a-7a+3b-7b = -21 \\ \Rightarrow -6 &= a+b \text{ and } 5a-4b = -21 \end{aligned}$$

Now,

$$\begin{aligned} a+b &= -6 \\ \Rightarrow a &= -6-b \end{aligned}$$

Substituting the value of 'a' in $5a-4b = -21$, we get

$$5(-6-b)-4b = -21$$

$$\Rightarrow -30-5b-4b = -21$$

$$\Rightarrow -9b = -21+30$$

$$\Rightarrow -9b = 9$$

$$\Rightarrow b = \frac{9}{-9} = -1$$

Putting $b = -1$ in $a = -6-b$, we get:

$$a = -(-1) - 6 = 1 - 6 = -5$$

Hence, the given system of equations will have infinitely many solutions if $a = -5$ and $b = -1$.

Q35

Find the value of p and q for which the following system of linear equations has infinite number of solutions:

$$\begin{aligned} 2x + 3y &= 9 \\ (p+q)x + (2p-q)y &= 3(p+q+2) \end{aligned}$$

Solution

The given system of equations is

$$\begin{aligned} 2x + 3y - 9 &= 0 \\ (p+q)x + \{2p-q\}y - 3(p+q+1) &= 0 \end{aligned}$$

It is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 2$, $b_1 = 3$, $c_1 = -9$

and, $a_2 = p+q$, $b_2 = 2p-q$, $c_2 = -3(p+q+1)$

The given system of equations will have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{2}{p+q} = \frac{3}{2p-q} = \frac{-9}{-3(p+q+1)}$$

$$\Rightarrow \frac{2}{p+q} = \frac{3}{2p-q} = \frac{3}{p+q+1}$$

$$\Rightarrow \frac{2}{p+q} = \frac{3}{2p-q} \text{ and } \frac{3}{2p-q} = \frac{3}{p+q+1}$$

$$\Rightarrow 2(2p-q) = 3(p+q) \text{ and } p+q+1 = 2p-q$$

$$\Rightarrow 4p - 2q = 3p + 3q \text{ and } -2p + p + q + q = -1$$

$$\Rightarrow p - 5q = 0 \text{ and } -p + 2q = -1$$

$$\Rightarrow p - 5q = p + 2q = -1 \quad [\text{On adding}]$$

$$\Rightarrow -3q = -1$$

$$\Rightarrow q = \frac{1}{3}$$

Putting $q = \frac{1}{3}$ in $p - 5q = 0$, we get

$$p - 5\left(\frac{1}{3}\right) = 0$$

$$\Rightarrow p = \frac{5}{3}$$

Hence, the given system of equations will have infinitely many solutions.

$$\text{If } p = \frac{5}{3} \text{ and } q = \frac{1}{3}$$

Q36

Find the value of a and b for which the following system of equations has infinitely many solutions:

$$(2a-1)x - 3y = 5$$

$$3x + (b+2)y = 3$$

Solution

The given system of equations is

$$(2a-1)x - 3y - 9 = 0$$

$$3x + (b-2)y - 3 = 0$$

It is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 2a - 1$, $b_1 = -3$, $c_1 = -9$

and, $a_2 = 3$, $b_2 = b - 2$, $c_2 = -3$

The given system of equations will have infinite number of solutions, if

$$\begin{aligned} & \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \\ \Rightarrow & \frac{2a-1}{3} = \frac{-3}{b-2} = \frac{-9}{-3} \\ \Rightarrow & \frac{2a-1}{3} = \frac{-3}{b-2} = \frac{3}{1} \\ \Rightarrow & \frac{2a-1}{3} = \frac{3}{1} \text{ and } \frac{-3}{b-2} = \frac{3}{1} \\ \Rightarrow & \frac{3(2a-1)}{3} = 3 \text{ and } -9 = 3(b-2) \\ \Rightarrow & 2a-1 = 1 \text{ and } -9 = 3b-10 \\ \Rightarrow & 2a = 2 \text{ and } -9+10 = 3b \\ \Rightarrow & a = \frac{1}{2} \text{ and } 1 = 3b \\ \Rightarrow & a = \frac{1}{2} \text{ and } b = \frac{1}{3} \end{aligned}$$

Hence, the given system of equations will have infinitely many solutions,

if $a = \frac{1}{2}$ and $b = \frac{1}{3}$.

Q37

Find the value of a and b for which the following system of equations has infinitely many solutions.

$$2x - (2a+5)y = 5$$

$$(2b+1)x - 5y = 15$$

Solution

The given system of equations is

$$2x - (2a+5)y - 5 = 0$$

$$(2b+1)x - 9y - 15 = 0$$

It is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 2$, $b_1 = -(2a+5)$, $c_1 = -5$
and, $a_2 = (2b+1)$, $b_2 = -9$, $c_2 = -15$

The given system of equations will have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\textcircled{1} \quad \frac{2}{2b+1} = \frac{-(2a+5)}{-9} = \frac{5}{15}$$

$$\textcircled{2} \quad \frac{2}{2b+1} = \frac{2a+5}{9} = \frac{1}{3}$$

$$\textcircled{3} \quad \frac{2}{2b+1} = \frac{1}{3} \text{ and } \frac{2a+5}{9} = \frac{1}{3}$$

$$\textcircled{4} \quad 6 = 2b+1 \text{ and } \frac{3(2a+5)}{9} = 1$$

$$\textcircled{5} \quad 0 = 1 - 2b \text{ and } 2a+5 = 3$$

$$\textcircled{6} \quad 6 = 2b \text{ and } 2a = -2$$

$$\textcircled{7} \quad \frac{5}{2} = b \text{ and } a = \frac{-2}{2} = -1$$

Hence, the given system of equations will have infinitely many solutions,

(if $a = -1$ and $b = \frac{5}{2}$)

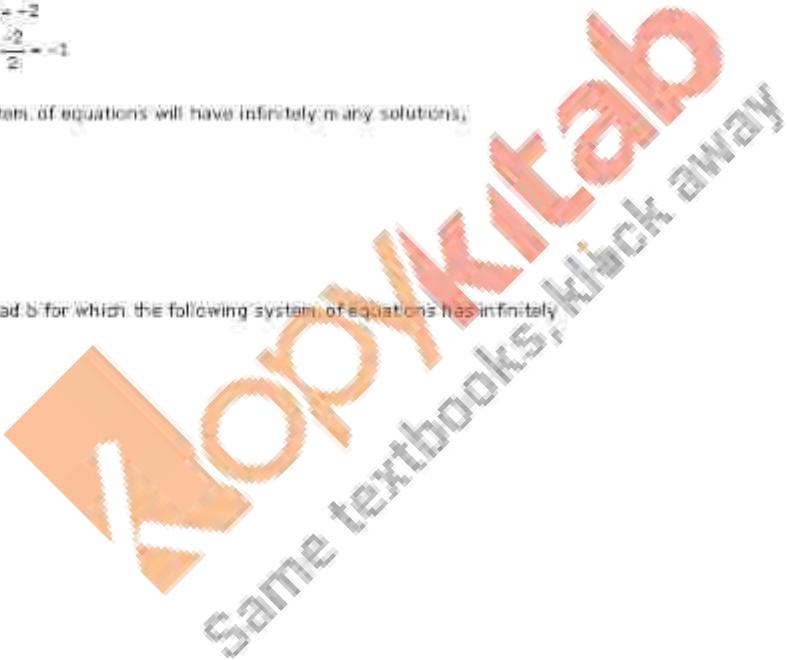
Q38

Find the value of a and b for which the following system of equations has infinitely many solutions:

$$(a-1)x + 3y = 2$$

$$6x + (1-2b)y = 6$$

Solution



The given system of equations is

$$\begin{cases} (a-1)x + 3y - 2 = 0 \\ 5x + (1-2b)y - 6 = 0 \end{cases}$$

It is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = a-1$, $b_1 = 3$, $c_1 = -2$

and, $a_2 = 5$, $b_2 = 1-2b$, $c_2 = -6$

The given system of equations will have infinite number of solutions, if

$$\begin{aligned} \frac{a_1}{a_2} &= \frac{b_1}{b_2} = \frac{c_1}{c_2} \\ \Rightarrow \frac{a-1}{5} &= \frac{3}{1-2b} = \frac{-2}{-6} \\ \Rightarrow \frac{a-1}{5} &= \frac{3}{1-2b} = \frac{1}{3} \\ \Rightarrow \frac{a-1}{5} &= \frac{1}{3} \text{ and } \frac{3}{1-2b} = \frac{1}{3} \\ \Rightarrow 3(a-1) &= 5 \text{ and } 3 \times 3 = 1-2b \\ \Rightarrow 3a-3 &= 5 \text{ and } 9 = 1-2b \\ \Rightarrow a = 2+1 &\text{ and } 2b = 1-9 \\ \Rightarrow a = 3 &\text{ and } 2b = -8 \\ \Rightarrow a = 3 \text{ and } b &= \frac{-8}{2} = -4 \end{aligned}$$

Hence, the given system of equations will have infinitely many solutions, if $a = 3$ and $b = -4$.

Q39

Find the value of a and b for which the following system of equations has infinitely many solutions.

$$\begin{cases} 3x + 4y = 12 \\ (a+b)x + 2(a-b)y = 5a-1 \end{cases}$$

Solution

The given system of equations is:

$$3x + 4y - 12 = 0$$

$$(a+b)x + 2(a-b)y - (5a-1) = 0$$

It is of the form:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 3$, $b_1 = 4$, $c_1 = -12$,

and, $a_2 = a+b$, $b_2 = 2(a-b)$, $c_2 = -(5a-1)$.

The given system of equations will have infinite number of solutions, if

$$\begin{aligned} & \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \\ \Rightarrow & \frac{3}{a+b} = \frac{4}{2(a-b)} = \frac{-12}{-(5a-1)} \\ \Rightarrow & \frac{3}{a+b} = \frac{2}{a-b} = \frac{12}{5a-1} \\ \Rightarrow & \frac{3}{a+b} = \frac{2}{a-b} \text{ and } \frac{2}{a-b} = \frac{12}{5a-1} \\ \Rightarrow & 3(a-b) = 2(a+b) \text{ and } 2(5a-1) = 12(a-b) \\ \Rightarrow & 3a - 3b = 2a + 2b \text{ and } 10a - 2 = 12a - 12b \\ \Rightarrow & 3a - 2a = 2b + 3b \text{ and } 10a - 12a = -12b + 2 \\ \Rightarrow & a = 5b \text{ and } -2a = -12b + 2 \end{aligned}$$

Substituting $a = 5b$ in $-2a = -12b + 2$, we get:

$$-2(5b) = -12b + 2$$

$$-10b = -12b + 2$$

$$12b - 10b = 2$$

$$2b = 2$$

$$b = 1$$

Putting $b = 1$ in $a = 5b$, we get:

$$a = 5 \times 1 = 5$$

Hence, the given system of equations will have infinitely many solutions, if $a = 5$ and $b = 1$.

Q40

Find the values of a and b for which the following system of equations has infinitely many solutions:

$$2x + 3y = 7$$

$$(a-b)x + (a+b)y = 3a + b - 2$$

Solution

Q41

Find the value of a and b for which the following system of equations has infinitely many solutions:

$$2x + 3y - 7 = 0$$

$$(a-1)x + (a+1)y = (3a-1)$$

Solution

The given system of equations is:

$$2x + 3y - 7 = 0$$

$$(a-1)x + (a+1)y - (3a-2) = 0$$

It is of the form:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 2$, $b_1 = 3$, $c_1 = -7$

and, $a_2 = a-1$, $b_2 = a+1$, $c_2 = -(3a-2)$

The given system of equations will have infinite number of solutions, if

$$\begin{aligned} \frac{a_1}{a_2} &= \frac{b_1}{b_2} = \frac{c_1}{c_2} \\ \Rightarrow \frac{2}{a-1} &= \frac{3}{a+1} = \frac{-7}{-(3a-2)} \\ \Rightarrow \frac{2}{a-1} &= \frac{3}{a+1} = \frac{7}{3a-1} \\ \Rightarrow \frac{2}{a-1} &= \frac{3}{a+1} \text{ and } \frac{3}{a+1} = \frac{7}{3a-1} \\ \Rightarrow 2(a+1) &= 3(a-1) \text{ and } 3(3a-1) = 7(a+1) \\ \Rightarrow 2a+2 &= 3a-3 \text{ and } 9a-3 = 7a+7 \\ \Rightarrow 2a-3a &= -3-2 \text{ and } 9a-7a = 7+3 \\ \Rightarrow -a &= -5 \text{ and } 2a = 10 \\ \Rightarrow a &= 5 \text{ and } a = \frac{10}{2} = 5 \\ \Rightarrow a &= 5 \end{aligned}$$

Hence, the given system of equations will have infinitely many solutions, if $a = 5$.

Q42

Find the value of a and b for which the following system of equations has infinitely many solutions:

$$2x + 3y = 7$$

$$(a-1)x + (a+2)y = 3a$$

Solution

The given system of equations is:

$$2x + 3y - 7 = 0$$

$$(a-1)x + (a+2)y - 3a = 0$$

It is of the form:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 2$, $b_1 = 3$, $c_1 = -7$

and, $a_2 = a-1$, $b_2 = a+2$, $c_2 = -3a$

The given system of equations will have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{2}{a-1} = \frac{3}{a+2} = \frac{-7}{-3a}$$

$$\Rightarrow \frac{2}{a-1} = \frac{3}{a+2} = \frac{7}{3a}$$

$$\Rightarrow \frac{2}{a-1} = \frac{3}{a+2} \text{ and } \frac{7}{a+2} = \frac{-7}{3a}$$

$$\Rightarrow 2(a+2) = 3(a-1) \text{ and } 7 \cdot 3a = -7(a+2)$$

$$\Rightarrow 2a + 4 = 3a - 3 \text{ and } 21a = -7a - 14$$

$$\Rightarrow 2a - 3a = -3 - 4 \text{ and } 28a = -14$$

$$\Rightarrow -a = -7 \text{ and } 2a = -14$$

$$\Rightarrow a = 7 \text{ and } a = \frac{14}{2} = -7$$

$$\Rightarrow a = 7$$

Hence, the given system of equations will have infinitely many solutions, if $a = 7$.

Q43

Find the values of a and b for which the following system of equations has infinitely many solutions:

$$x + 2y = 1$$

$$(a - b)x + (a + b)y = a + b - 2$$

Solution

The given system of equations may be written as:

$$x + 2y - 1 = 0$$

$$(a-b)x + (a+b)y - (a+b-2) = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = 1$, $b_1 = 2$, $c_1 = -1$

And, $a_2 = a-b$, $b_2 = a+b$, $c_2 = -(a+b-2)$

For the system of equations to have infinite solutions,

$$\begin{aligned} \frac{a_1}{a_2} &= \frac{b_1}{b_2} = \frac{c_1}{c_2} \\ \frac{1}{a-b} &= \frac{2}{a+b} = \frac{-1}{-(a+b-2)} \\ \frac{1}{a-b} &= \frac{2}{a+b} = \frac{1}{a+b-2} \end{aligned}$$

$$\text{Now, } \frac{1}{a-b} = \frac{2}{a+b}$$

$$\Rightarrow a+b = 2a-2b$$

$$\Rightarrow a-3b = 0 \quad \dots(1)$$

$$\text{And, } \frac{1}{a-b} = \frac{1}{a+b-2}$$

$$\Rightarrow a+b-2 = a-b$$

$$\Rightarrow 2b = 2$$

$$\Rightarrow b = 1$$

Substituting $b = 1$ in (1), we have:

$$a-3x+1=0$$

$$\Rightarrow a=3$$

Hence, the given system of equations will have infinite number of solutions for $a=3$ and $b=1$.

Q44

Find the values of a and b for which the following system of equations has infinitely many solutions:

$$2x + 3y = 7$$

$$2ax + ay = 28 - by$$

Solution

The given system of equations may be written as

$$2x + 3y - 7 = 0$$

$$2ax + ay + by - 28 = 0 \Rightarrow 2ax + (a+b)y - 28 = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = 2$, $b_1 = 3$, $c_1 = -7$

And, $a_2 = 2a$, $b_2 = a+b$, $c_2 = -28$

For the system of equations to have infinite solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{-28}$$

$$\Rightarrow \frac{1}{a} = \frac{3}{a+b} = \frac{1}{4}$$

$$\text{Now, } \frac{1}{a} = \frac{1}{4}$$

$$\Rightarrow a = 4$$

$$\text{And, } \frac{1}{a} = \frac{3}{a+b}$$

$$\Rightarrow a+b = 3a$$

$$\Rightarrow 2a = b$$

$$\Rightarrow 2 \times 4 = b$$

$$\Rightarrow b = 8$$

Hence, the given system of equations will have infinite number of solutions for $a = 4$ and $b = 8$.

Q45

For which value(s) of λ , do the pair of linear equations $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have no solution?

Solution

The given system of equations may be written as

$$\lambda x + y - \lambda^2 = 0$$

$$x + \lambda y - 1 = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = \lambda$, $b_1 = 1$, $c_1 = -\lambda^2$

And, $a_2 = 1$, $b_2 = \lambda$, $c_2 = -1$

For the system of equations to have no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{\lambda}{1} = \frac{1}{\lambda} \neq \frac{-\lambda^2}{-1}$$

$$\Rightarrow \frac{\lambda}{1} = \frac{1}{\lambda} \neq \frac{\lambda^2}{1}$$

$$\text{Now, } \frac{\lambda}{1} = \frac{1}{\lambda}$$

$$\Rightarrow \lambda^2 = 1$$

$$\Rightarrow \lambda = \pm 1$$

$$\text{And, } \frac{1}{\lambda} = \frac{\lambda^2}{1}$$

$$\Rightarrow \lambda^2 \neq 1 \Rightarrow \lambda \text{ cannot be } (+1)$$

$$\text{Hence, } \lambda = -1$$

Thus, the given system of equations will have no solution if $\lambda = -1$.

Q46

For which value(s) of λ , do the pair of linear equations $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have infinitely many solutions?

Solution

The given system of equations may be written as:

$$\lambda x + y - \lambda^2 = 0$$

$$x + \lambda y - 1 = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = \lambda$, $b_1 = 1$, $c_1 = -\lambda^2$

And, $a_2 = 1$, $b_2 = \lambda$, $c_2 = -1$

For the system of equations to infinite solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{\lambda}{1} = \frac{1}{\lambda} = \frac{-\lambda^2}{-1}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{1}{\lambda} = \frac{\lambda^2}{1}$$

$$\text{Now, } \frac{\lambda}{1} = \frac{1}{\lambda}$$

$$\Rightarrow \lambda^2 = 1$$

$$\Rightarrow \lambda = \pm 1$$

$$\text{And, } \frac{1}{\lambda} = \frac{\lambda^2}{1}$$

$\Rightarrow \lambda^2 = 1 \Rightarrow \lambda$ cannot be (-1) .

Hence, $\lambda = 1$

Thus, the given system of equations will have infinite number of solution if $\lambda = 1$.

Q47

For which value(s) of λ , do the pair of linear equations $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have a unique solution?

Solution

The given system of equations may be written as:

$$\lambda x + y - \lambda^2 = 0$$

$$x + \lambda y - 1 = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = \lambda$, $b_1 = 1$, $c_1 = -\lambda^2$

And, $a_2 = 1$, $b_2 = \lambda$, $c_2 = -1$

For the system of equations to have unique,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \frac{\lambda}{1} \neq \frac{1}{\lambda}$$

$$\Rightarrow \lambda^2 \neq 1$$

$$\Rightarrow \lambda \neq \pm 1$$

Thus, the given system of equations will have unique solution for all real values of λ except ± 1 .



Exercise 3.6**Q1**

5 pens and 6 pencils together cost Rs 9 and 3 pens and 2 pencils cost Rs 5. Find the cost of 1 pen and 1 pencil.

Solution

Let the cost of a pen be Rs x and that of a pencil be Rs y . Then,

$$5x + 6y = 9 \quad \text{---(i)}$$

$$\text{and, } 3x + 2y = 5 \quad \text{---(ii)}$$

Multiplying equation (i) by 2 and equation (ii) by 6, we get

$$10x + 12y = 18 \quad \text{---(iii)}$$

$$18x + 12y = 30 \quad \text{---(iv)}$$

Subtracting equation (iii) by equation (iv), we get

$$18x - 10x + 12y - 12y = 30 - 18$$

$$\Rightarrow 8x = 12$$

$$\Rightarrow x = \frac{12}{8} = \frac{3}{2} = 1.5$$

Substituting $x = 1.5$ in equation (i), we get

$$5 \times 1.5 + 6y = 9$$

$$\Rightarrow 7.5 + 6y = 9$$

$$\Rightarrow 6y = 9 - 7.5$$

$$\Rightarrow 6y = 1.5$$

$$\Rightarrow y = \frac{1.5}{6} = \frac{1}{4} = 0.25$$

Hence, cost of one pen = Rs 1.50 and cost of one pencil = Rs 0.25.

Q2

7 audio cassettes and 3 video cassettes cost Rs 1110, while 5 audio cassettes and 4 video cassettes cost Rs 1350. Find the cost of an audio cassette and a videocassette.

Solution

Let the cost of a audio cassette be Rs x and that of a video cassette be Rs y . Then;

$$7x + 3y = 1110 \quad \text{---(i)}$$

$$\text{and, } 5x + 4y = 1350 \quad \text{---(ii)}$$

Multiplying equation (i) by 4 and equation (ii) by 3, we get

$$28x + 12y = 4440 \quad \text{---(iii)}$$

$$15x + 12y = 4050 \quad \text{---(iv)}$$

Subtracting equation (iv) from equation (iii), we get

$$28x - 15x + 12y - 12y = 4440 - 4050$$

$$\Rightarrow 13x = 390$$

$$\Rightarrow x = \frac{390}{13} = 30$$

Substituting $x = 30$ in equation (i), we get:

$$7 \times 30 + 3y = 1110$$

$$\Rightarrow 210 + 3y = 1110$$

$$\Rightarrow 3y = 1110 - 210$$

$$\Rightarrow 3y = 900$$

$$\Rightarrow y = \frac{900}{3} = 300$$

Hence, cost of one audio cassette = Rs 30 and cost of one video cassette = Rs 300.

Q3

Reena has pens and pencils which together are 40 in number. If she has 5 more pencil and 5 less pens, then number of pencils would become 4 times the number of pens. Find the original number of pens and pencils.

Solution

Let the number of pens be x , and that of pencils be y . Then,

$$\begin{aligned}x + y &= 40 \quad \text{---(i)} \\ \text{and, } (y + 5) &= 4(x - 5) \\ \Rightarrow y + 5 &= 4x - 20 \\ \Rightarrow 5 + 20 &= 4x - y \\ \Rightarrow 4x - y &= 25 \quad \text{---(ii)}\end{aligned}$$

Adding equation (i) and equation (ii), we get

$$\begin{aligned}x + 4x &= 40 + 25 \\ \Rightarrow 5x &= 65 \\ \Rightarrow x &= \frac{65}{5} = 13\end{aligned}$$

Putting $x = 13$ in equation (i), we get

$$\begin{aligned}13 + y &= 40 \\ \Rightarrow y &= 40 - 13 = 27\end{aligned}$$

Hence, Reena has 13 pens and 27 pencils.

Q4

4 tables and 3 chairs, together, cost Rs 2,250 and 3 tables and 4 chairs cost of Rs 1450. Find the cost of 2 chairs and 1 table.

Solution

Let the cost of a table be Rs x and that of a chair be Rs y . Then,

$$\begin{aligned}4x + 3y &= 2,250 \quad \text{---(i)} \\ \text{and, } 3x + 4y &= 1450 \quad \text{---(ii)}\end{aligned}$$

Multiplying equation (i) by 4 and equation (ii) by 3, we get

$$\begin{aligned}16x + 12y &= 9000 \quad \text{---(iii)} \\ 9x + 12y &= 5550 \quad \text{---(iv)}\end{aligned}$$

Subtracting equation (iv) by equation (iii), we get

$$\begin{aligned}16x - 9x &= 9000 - 5550 \\ \Rightarrow 7x &= 3150 \\ \Rightarrow x &= \frac{3150}{7} = 450\end{aligned}$$

Putting $x = 450$ in equation (i), we get

$$\begin{aligned}4 \times 450 + 3y &= 2,250 \\ \Rightarrow 1800 + 3y &= 2250 \\ \Rightarrow 3y &= 2250 - 1800 \\ \Rightarrow 3y &= 450 \\ \Rightarrow y &= \frac{450}{3} = 150\end{aligned}$$

$$\Rightarrow 2y = 2 \times 150 = 300$$

Cost of 2 chairs = Rs 300

and Cost of 1 table = Rs 450

The cost of 2 chairs and 1 table = 300 + 450 = Rs 750.

Q5

3 bags and 4 pens together cost Rs 257 whereas 4 bags and 3 pens together cost Rs 324. Find the total cost of 1 bag and 10 pens.

Solution

Let the cost of a bag be Rs x and that of a pen be Rs y . Then,

$$\begin{aligned} 3x + 4y &= 257 \quad \text{---(i)} \\ \text{and, } 4x + 3y &= 324 \quad \text{---(ii)} \end{aligned}$$

Multiplying equation (i) by 3 and equation (ii) by 4, we get:

$$\begin{aligned} 9x + 12y &= 771 \quad \text{---(iii)} \\ 16x + 12y &= 1296 \quad \text{---(iv)} \end{aligned}$$

Subtracting equation (iv) by equation (iii), we get:

$$\begin{aligned} 16x - 9x &= 1296 - 771 \\ \Rightarrow 7x &= 525 \\ \Rightarrow x &= \frac{525}{7} = 75 \end{aligned}$$

Cost of a bag = Rs 75

Putting $x = 75$ in equation (i), we get:

$$\begin{aligned} 3 \times 75 + 4y &= 257 \\ \Rightarrow 225 + 4y &= 257 \\ \Rightarrow 4y &= 257 - 225 \\ \Rightarrow 4y &= 32 \\ \Rightarrow y &= \frac{32}{4} = 8 \\ \text{Cost of a pen} &= \text{Rs } 8 \\ \text{Cost of 10 pens} &= 8 \times 10 = \text{Rs } 80 \end{aligned}$$

Hence, the total cost of 1 bag and 10 pens = $75 + 80 = \text{Rs } 155$.

Q6

5 books and 7 pens together cost Rs 70 whereas 7 books and 5 pens together cost Rs 77. Find the total cost of 1 book and 2 pens.

Solution

Let the cost of a book be Rs x and that of a pen be Rs y . Then,

$$5x + 7y = 79 \quad \dots(i)$$

$$\text{and, } 7x + 5y = 97 \quad \dots(ii)$$

Multiplying equation (i) by 5 and equation (ii) by 7, we get:

$$25x + 35y = 395 \quad \dots(iii)$$

$$49x + 35y = 679 \quad \dots(iv)$$

Subtracting equation (iii) by equation (iv), we get:

$$40x - 25y = 395 - 679$$

$$\therefore 24x = 144$$

$$\therefore x = \frac{144}{24} = 6$$

Cost of a book = Rs 6

Putting $x = 6$ in equation (i), we get:

$$5 \times 6 + 7y = 79$$

$$\therefore 30 + 7y = 79$$

$$\therefore 7y = 79 - 30$$

$$\therefore 7y = 49$$

$$\therefore y = \frac{49}{7} = 7$$

Cost of a pen = Rs 7

Cost of 2 pens = $2 \times 7 = \text{Rs } 14$

Hence, the total cost of 1 book and 2 pens = $6 + 14 = \text{Rs } 20$.

Q7

Jamila sold a table and a chair for Rs. 1050, thereby making a profit of 10% on a table and 25% on the chair. If she had taken profit of 25% on the table and 10% on the chair she would have got Rs. 1065. Find the cost price of each.

Solution

Let the cost price of table be Rs. x and that of chair be Rs. y .

Then, selling price of table when sold at 10% profit = Rs. $\left(x + \frac{10x}{100}\right) = \text{Rs. } \left(\frac{110x}{100}\right)$

And, selling price of chair when sold at 25% profit = Rs. $\left(y + \frac{25y}{100}\right) = \text{Rs. } \left(\frac{125y}{100}\right)$

According to question,

$$\frac{110x}{100} + \frac{125y}{100} = 1050$$

$$\Rightarrow 110x + 125y = 105000 \quad \dots(i)$$

Now, selling price of table when sold at 25% profit = Rs. $\left(x + \frac{25x}{100}\right) = \text{Rs. } \left(\frac{125x}{100}\right)$

And, selling price of chair when sold at 10% profit = Rs. $\left(y + \frac{10y}{100}\right) = \text{Rs. } \left(\frac{110y}{100}\right)$

According to question,

$$\frac{125x}{100} + \frac{110y}{100} = 1065$$

$$\Rightarrow 125x + 110y = 106500 \quad \dots(ii)$$

Adding equations (i) and (ii), we get:

$$235x + 235y = 211500$$

$$\Rightarrow x + y = 900 \quad \dots(iii)$$

Subtracting equation (i) from (ii), we get:

$$15x - 15y = 1500$$

$$\Rightarrow x - y = 100 \quad \dots(iv)$$

Adding equations (iii) and (iv), we get:

$$2x = 1000$$

$$\Rightarrow x = 500$$

$$\Rightarrow 500 + y = 900 \Rightarrow y = 400$$

Hence, the cost price of table is Rs. 500 and that of chair is Rs. 400.

Q8

Susan invested certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. She received Rs.1860 as annual interest. However, had she interchanged the amount of investment in the two schemes, she would have received Rs.20 more as annual interest. How much money did she invest in each scheme?

Solution

Let Susan invested Rs. x in scheme A and Rs. y in scheme B.

$$\text{Then, S.I. on Rs. } x \text{ at 8\% p.a. for 1 year} = \text{Rs. } \frac{x \times 8 \times 1}{100} = \text{Rs. } \frac{8x}{100}$$

$$\text{And, S.I. on Rs. } y \text{ at 9\% p.a. for 1 year} = \text{Rs. } \frac{y \times 9 \times 1}{100} = \text{Rs. } \frac{9y}{100}$$

According to question,

$$\frac{8x}{100} + \frac{9y}{100} = 1860$$

$$\Rightarrow 8x + 9y = 186000 \quad \dots (i)$$

$$\text{Now, S.I. on Rs. } x \text{ at 9\% p.a. for 1 year} = \text{Rs. } \frac{x \times 9 \times 1}{100} = \text{Rs. } \frac{9x}{100}$$

$$\text{And, S.I. on Rs. } y \text{ at 8\% p.a. for 1 year} = \text{Rs. } \frac{y \times 8 \times 1}{100} = \text{Rs. } \frac{8y}{100}$$

According to question,

$$\frac{9x}{100} + \frac{8y}{100} = 1860 + 20$$

$$\Rightarrow 9x + 8y = 188000 \quad \dots (ii)$$

Adding equations (i) and (ii), we get:

$$17x + 17y = 374000$$

$$\Rightarrow x + y = 22000 \quad \dots (iii)$$

Subtracting equation (i) from (ii), we get:

$$x - y = 374000$$

$$\Rightarrow x - y = 2000 \quad \dots (iv)$$

Adding equations (iii) and (iv), we get:

$$2x = 24000$$

$$\Rightarrow x = 12000$$

$$\Rightarrow 12000 + y = 22000 \Rightarrow y = 10000$$

Hence, the money invested in scheme A is Rs. 12000

and in scheme B is Rs. 10000.

Q9

The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, he buys 3 bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.

Solution

Let the cost of a bat be x and y , respectively.

According to the given information,

$$7x + 6y = 3800 \dots\dots\dots(1)$$

$$3x + 5y = 1750 \dots\dots\dots(2)$$

From (1), we obtain,

$$y = \frac{3800 - 7x}{6} \dots\dots\dots(3)$$

Substituting this value in equation (2), we obtain

$$3x + 5\left(\frac{3800 - 7x}{6}\right) = 1750$$

$$3x + \frac{19000 - 35x}{6} = 1750$$

$$3x - \frac{35x}{6} = 1750 - \frac{19000}{6}$$

$$\frac{18x - 35x}{6} = \frac{10500 - 19000}{6}$$

$$\frac{17x}{6} = \frac{8500}{6}$$

$$x = 500 \dots\dots\dots(4)$$

Substituting this equation (3), we obtain,

$$y = \frac{3800 - 7 \times 500}{6}$$

$$= \frac{300}{6} = 50$$

Hence, the cost of a bat is Rs 500 and that of a ball is Rs 50.

Concept Insight: Cost of bats and balls need to be found as the cost of a bat and bat will be taken as the variables. Apply the conditions of total cost of bats and balls algebraic equations will be obtained. The pair of equations can then be solved by suitable substitution.

Q10

A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs. 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Solution

Let the fixed charge for first three days and each day charge thereafter be Rs x and Rs y , respectively.

According to the question,

$$x + 4y = 27 \dots\dots\dots(1)$$

$$x + 2y = 21 \dots\dots\dots(2)$$

Subtracting equation (2) from equation (1), we obtain:

$$2y = 6$$

$$y = 3$$

Substituting the value of y in equation (1), we obtain:

$$x + 12 = 27$$

$$x = 15$$

Hence, the fixed charge is Rs 15 and the charge per day is Rs 3.

Q11

The cost of 4 pens and 4 pencil boxes is Rs. 100. Three times the cost of a pen is Rs. 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.

Solution

Let the cost of a pen be Rs. x and that of pencil be Rs. y .

According to question, we have

$$4x + 4y = 100$$

$$\Rightarrow x + y = 25 \quad \dots (i)$$

$$\text{And, } 3x - y = 15 \quad \dots (ii)$$

Adding equations (i) and (ii), we get

$$4x = 40$$

$$\Rightarrow x = 10$$

Substituting $x = 10$ in (i), we have

$$10 + y = 25$$

$$\Rightarrow y = 15$$

Hence, the cost of a pen is Rs. 10 and that of a pencil is Rs. 15.

Q12

One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital?

Solution

Let the money with the first person and second person be Rs x and Rs y respectively.

According to the question,

$$x + 100 = 2(y - 100)$$

$$x + 100 = 2y - 200$$

$$x - 2y = -300 \quad \dots (1)$$

$$6(x - 10) = (y + 10)$$

$$6x - 60 = y + 10$$

$$6x - y = 70 \quad \dots (2)$$

Multiplying equation (2) by 2, we obtain:

$$12x - 2y = 140 \quad \dots (3)$$

Subtracting equation (1) from equation (3), we obtain:

$$11x = 140 + 300$$

$$11x = 440$$

$$x = 40$$

Putting the value of x in equation (1), we obtain:

$$40 - 2y = -300$$

$$40 + 300 = 2y$$

$$2y = 340$$

$$y = 170$$

Thus, the two friends had Rs. 40 and Rs. 170 with them.

Q13

A and B each have a certain number of mangoes. A says to B, "if you give 30 of your mangoes, I will have twice as many as left with you." B replies, "if you give me 10, I will have thrice as many as left with you." How many mangoes does each have?

Solution

Suppose A has x mangoes and B has y mangoes.

According to the given conditions, we have

$$\begin{aligned}x + 30 &= 2(y - 30) \\ \Rightarrow x + 30 &= 2y - 60 \\ \Rightarrow x - 2y &= -60 - 30 \\ \Rightarrow x - 2y &= -90\end{aligned}\quad \text{---(i)}$$

$$\begin{aligned}\text{and, } y + 10 &= 3(x - 10) \\ \Rightarrow y + 10 &= 3x - 30 \\ \Rightarrow 10 + 30 &= 3x - y \\ \Rightarrow 3x - y &= 40\end{aligned}\quad \text{---(ii)}$$

Multiplying equation (i) by 3 and equation (ii) by 1, we get

$$\begin{aligned}3x - 6y &= -270 \quad \text{---(iii)} \\ 3x - y &= 40 \quad \text{---(iv)}\end{aligned}$$

Subtracting equation (iv) by equation (iii), we get

$$-6y - (-y) = -270 - 40$$

$$\begin{aligned}\Rightarrow -6y + y &= -310 \\ \Rightarrow -5y &= -310 \\ \Rightarrow y &= \frac{-310}{-5} = 62\end{aligned}$$

Putting $y = 62$ in equation (ii), we get

$$\begin{aligned}x - 2 \times 62 &= -90 \\ \Rightarrow x - 124 &= -90 \\ \Rightarrow x &= -90 + 124 \\ \Rightarrow x &= 34\end{aligned}$$

Hence, A has 34 mangoes and B has 62 mangoes.

Q14

Vijay had some bananas, and he divided them into two lots A and B. He sold first lot at the rate of Rs.2 for 3 bananas and the second lot at the rate of Rs.1 per banana and got a total of Rs.400. If he had sold the first lot at the rate of Rs.1 per banana and the second lot at the rate of Rs.4 per five bananas, his total collection would have been Rs.480. Find the total number of bananas he had.

Solution

Let the number of bananas in lot A be x and that in lot B be y .

Then, total number of bananas = $x + y$.

According to question,

$$\begin{aligned}2 \times \frac{x}{3} + 1 \times \frac{y}{1} &= 400 \\ \Rightarrow 2x + 3y &= 1200 \quad \dots(i)\end{aligned}$$

$$\text{And, } 1 \times \frac{x}{1} + 4 \times \frac{y}{5} = 480$$

$$\Rightarrow 5x + 4y = 2300 \quad \dots(ii)$$

Adding equations (i) and (ii), we get

$$7x + 7y = 3500$$

$$\Rightarrow x + y = 500$$

Hence, Vijay had 500 bananas.

Q15

On selling a T.V. at 5% gain and a fridge at 10% gain, a shopkeeper gains Rs.2000. But if he sells the T.V. at 10% gain and the fridge at 5% loss, he gains Rs.1500 on the transaction. Find the actual prices of T.V. and fridge.

Solution

Let the price of a TV be Rs x and that of a fridge be Rs y . Then, we have:

$$\frac{5x}{100} + \frac{10y}{100} = 2500$$

$$\Rightarrow 5x + 10y = 250000$$

$$\Rightarrow 5(x + 2y) = 250000$$

$$\Rightarrow x + 2y = 50000 \quad \text{---(i)}$$

$$\text{and, } \frac{10x}{100} - \frac{5y}{100} = 1500$$

$$\Rightarrow 10x - 5y = 150000$$

$$\Rightarrow 5(2x - y) = 150000$$

$$\Rightarrow 2x - y = 30000 \quad \text{---(ii)}$$

Multiplying equation (i) by 2, we get

$$4x - 2y = 60000 \quad \text{---(iii)}$$

Adding equation (ii) and equation (iii), we get

$$x + 4x = 40000 + 60000$$

$$\Rightarrow 5x = 100000$$

$$\Rightarrow x = 20000$$

Putting $x = 20000$ in equation (ii), we get:

$$20000 + 2y = 40000$$

$$\Rightarrow 2y = 40000 - 20000$$

$$\Rightarrow y = \frac{20000}{2} = 10000$$

Hence, the actual price of TV = Rs 20,000

and, the actual price of fridge = Rs 10,000.



Exercise 3.7**Q1**

The sum of two numbers is 8. If their sum is four times their difference, find the numbers.

Solution

Let the numbers be x and y . Then, we have

$$\begin{aligned}x + y &= 8 \quad \text{---(i)} \\ \text{and, } x + y &= 4(x - y) \\ \Rightarrow x + y &= 4x - 4y \\ \Rightarrow 0 &= 4x - 4y - x + y \\ \Rightarrow 3x - 5y &= 0 \quad \text{---(ii)}\end{aligned}$$

Multiplying equation (i) by 5 and, we get

$$5x + 5y = 40 \quad \text{---(iii)}$$

Adding equation (i) and equation (ii), we get

$$\begin{aligned}3x + 5x &= 40 \\ \Rightarrow 8x &= 40 \\ \Rightarrow x &= \frac{40}{8} = 5\end{aligned}$$

Putting $x = 5$ in equation (i), we get

$$\begin{aligned}5 + y &= 8 \\ \Rightarrow y &= 8 - 5 = 3\end{aligned}$$

Hence, the required numbers are 5 and 3.

Q2

The sum of digits of a two digit numbers is 13. If the number is subtracted from the one obtained by interchanging the digits, the result is 45. What is the number?

Solution

Let the digit in the units place be x and digit in the tens place be y . Then,

$$x + y = 13 \quad [\text{given}] \quad \text{---(i)}$$

and, Number = $10y + x$

Number obtained by reversing the digits = $10x + y$

It is given that the number is subtracted from the one obtained by interchanging the digits, the result is 45.

i.e., {Number obtained by interchanging the digits} - Number = 45

$$\begin{aligned}10x + y - (10y + x) &= 45 \\ \Rightarrow 9x - 9y &= 45 \\ \Rightarrow 9(x - y) &= 45 \\ \Rightarrow x - y &= 5 \quad \text{---(ii)}\end{aligned}$$

Adding equation (i) and equation (ii), we get

$$\begin{aligned}2x &= 13 + 5 \\ \Rightarrow 2x &= 18 \\ \Rightarrow x &= \frac{18}{2} = 9\end{aligned}$$

Putting $x = 9$ in equation (i), we get

$$\begin{aligned}9 + y &= 13 \\ \Rightarrow y &= 13 - 9 = 4\end{aligned}$$

Hence, the number is $10y + x = 10 \times 4 + 9 = 49$.

Q3

A number consists of two digits whose sum is five. When the digits are reversed, the number becomes greater by nine. Find the number.

Solution

Let the digit in the units place be x and digit in the tens place be y . Then,

$$\begin{aligned}x + y &= 5 \quad [\text{given}] \quad \text{---(i)} \\ \text{and, } \text{Number} &= 10y + x\end{aligned}$$

Number obtained by reversing the digits = $10x + y$

It is given that if the digits are reversed, the number becomes greater by nine.

i.e., Number obtained by interchanging the digits = Number + 9.

$$\begin{aligned}10x + y &= 10y + x + 9 \\ \Rightarrow 10x - x + y - 10y &= 9 \\ \Rightarrow 9x - 9y &= 9 \\ \Rightarrow 9(x - y) &= 9 \\ \Rightarrow x - y &= 1 \quad \text{---(ii)}\end{aligned}$$

Adding equation (i) and equation (ii), we get

$$\begin{aligned}2x &= 5 + 1 \\ \Rightarrow 2x &= 6 \\ \Rightarrow x &= \frac{6}{2} = 3\end{aligned}$$

Putting $x = 3$ in equation (i), we get

$$\begin{aligned}x + y &= 5 \\ \Rightarrow 3 + y &= 5 \\ \Rightarrow y &= 5 - 3 = 2\end{aligned}$$

Hence, the number is $10y + x = 10 \times 2 + 3 = 23$.

Q4

The sum of digits of a two-digit number is 15. The number obtained by reversing the order of digits of the given number exceeds the given number by 9. Find the given number.

Solution

Let the digit in the units place be x and digit in the ten's place be y . Then,

$$\text{Number} = 10y + x$$

Number formed by reversing the digits = $10x + y$

According to the given conditions, we have

$$x + y = 15 \quad \dots(1)$$

$$\text{and, } 10x + y + 10y + x = 9$$

$$\Rightarrow 10x + x + y - 10y = 9$$

$$\Rightarrow 9x - 9y = 9$$

$$\Rightarrow 9(x - y) = 9$$

$$\Rightarrow x - y = 1 \quad \dots(2)$$

Adding equation (1) and equation (2), we get

$$2x = 15 + 1$$

$$\Rightarrow 2x = 16$$

$$\Rightarrow x = \frac{16}{2} = 8$$

Putting $x = 8$ in equation (1), we get

$$8 + y = 15$$

$$\Rightarrow y = 15 - 8 = 7$$

Hence, the required number is $10y + x = 10 \times 7 + 8 = 78$.

Q5

The sum of two-digit number and the number formed by reversing the order of digits is 66. If the two digits differ by 2, find the number. How many such numbers are there?

Solution

Let the ten's and the unit's digits in the first number be x and y , respectively. So, the first number may be written as $10x + y$ in the expanded form (for example, 56 = $10(5) + 6$)

When the digits are reversed, x becomes the unit's digit and y becomes the ten's digit. This number, in the expanded notation is $10y + x$. (For example, when 56 is reversed, we get 65 = $10(6) + 5$).

According to the given condition,

$$(10x + y) + (10y + x) = 66$$

$$\text{i.e., } 11(x + y) = 66$$

$$\text{i.e., } x + y = 6 \quad \dots(1)$$

We are also given that the digits differ by 2, therefore, either

$$x - y = 2 \quad \dots(2)$$

$$\text{Or, } y - x = 2 \quad \dots(3)$$

If $x - y = 2$, then solving (1) and (2) by elimination, we get $x = 4$ and $y = 2$.

In this case, we get the number 42.

If $y - x = 2$, then solving (1) and (3) by elimination, we get $x = 2$ and $y = 4$. In this case, we get the number 24.

Thus, there are two such numbers 42 and 24.

Q6

The sum of two space numbers is 1000 and the difference between their space squares is 256000. Find the space numbers.

Solution

Let the larger number be x and the smaller number be y . Then,

$$x+y = 1000 \quad \text{---(i)}$$

$$\text{and, } x^2 - y^2 = 256000$$

$$\text{Now, } x^2 - y^2 = 256000$$

$$\Rightarrow (x+y)(x-y) = 256000$$

$$\Rightarrow 1000(x-y) = 256000 \quad [x+y=1000]$$

$$\Rightarrow x-y = 256 \quad \text{---(ii)}$$

Adding equation (i) and equation (ii), we get

$$2x = 1000 + 256$$

$$\Rightarrow x = \frac{1256}{2} = 628$$

Putting $x = 628$ in equation (i), we get

$$628+y = 1000$$

$$\Rightarrow y = 1000 - 628 = 372$$

Hence, the required numbers are 628 and 372.

Q7

The sum of a two digit number and the number obtained by reversing the order of its digits is 99. If the digits differ by 3, find the number.

Solution

Let the digit in the unit's place be x and the digit at the ten's place be y . Then,

$$\text{Number} = 10y+x$$

The number obtained by reversing the order of the digits is $10x+y$.

According to the given conditions, we have

$$(10y+x)+(10x+y) = 99$$

$$\Rightarrow 11x+11y = 99$$

$$\Rightarrow 11(x+y) = 99$$

$$\Rightarrow x+y = 9$$

$$\text{and, } x-y = 3 \quad [\text{Difference of digits is 3}]$$

Thus, we have the following sets of simultaneous equations:

$$x+y = 9 \quad \text{---(i)}$$

$$\text{and, } x-y = 3 \quad \text{---(ii)}$$

$$\text{or}$$

$$x+y = 9 \quad \text{---(iii)}$$

$$x-y = -3 \quad \text{---(iv)}$$

Adding equation (i) and (iii), we get

$$2y = 9+3$$

$$\Rightarrow y = \frac{12}{2} = 6$$

Putting $x = 6$ in equation (i), we get:

$$6+y = 9$$

$$\Rightarrow y = 9-6 = 3$$

The required number is $10y+x = 10 \times 3 + 6 = 36$

Adding equation (ii) and equation (iv), we get

$$2x = 9-3$$

$$\Rightarrow x = \frac{6}{2} = 3$$

Putting $x = 3$ in equation (iii), we get:

$$3+y = 9$$

$$\Rightarrow y = 9-3 = 6$$

The required number is $10y+x = 10 \times 6 + 3 = 63$

Hence, the required number is 36 or, 63.

Q8

A two-digit number is 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

Solution

Let the digit in the unit's place be x and the digit at the ten's place be y . Then,

$$\text{Number} = 10y + x$$

The number obtained by reversing the order of the digits is $10x + y$.

According to the given conditions, we have

$$10y + x = 4(x + y)$$

$$\Rightarrow 10y + x = 4x + 4y$$

$$\Rightarrow 0 = 4x - y + 4y - 10y$$

$$\Rightarrow 0 = 3x - 6y$$

$$\Rightarrow 3x - 6y = 0$$

$$\Rightarrow 3\{x - 2y\} = 0$$

$$\Rightarrow x - 2y = 0 \quad \text{---(i)}$$

$$\text{and, } 10y + x + 18 = 10x + y$$

$$\Rightarrow 18 = 10x - x + y - 10y$$

$$\Rightarrow 18 = 9x - 9y$$

$$\Rightarrow 9x - 9y = 18$$

$$\Rightarrow 9(x - y) = 18$$

$$\Rightarrow x - y = 2 \quad \text{---(ii)}$$

Subtracting equation (ii) from equation (i), we get

$$-2y - (-y) = 0 - 2$$

$$\Rightarrow -2y + y = -2$$

$$\Rightarrow -y = -2$$

$$\Rightarrow y = 2$$

Putting $y = 2$ in equation (ii), we get

$$x = 2 + 2$$

$$\Rightarrow x = 4$$

Hence, the required number is $10y + x = 10 \times 2 + 4 = 24$.

Q9

A two-digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

Solution

Let the digit in the unit's place be x and the digit at the ten's place be y . Then,
 Number = $10y + x$

The number obtained by reversing the order of the digits is $10x + y$.

According to the given conditions, we have

$$\begin{aligned} & 10y + x = 4(x + y) - 3 \\ \Rightarrow & 10y + x = 4x + 4y + 3 \\ \Rightarrow & 10y - 4y + x - 4x = 3 \\ \Rightarrow & 6y - 3x = 3 \\ \Rightarrow & 3(2y - x) = 3 \\ \Rightarrow & 2y - x = 1 \quad \text{---(i)} \\ \text{and, } & 10y + x + 18 = 10x + y \\ \Rightarrow & 18 = 10x - x + y - 10y \\ \Rightarrow & 18 = 9x - 9y \\ \Rightarrow & 9x - 9y = 18 \\ \Rightarrow & 9(x - y) = 18 \\ \Rightarrow & x - y = \frac{18}{9} = 2 \quad \text{---(ii)} \end{aligned}$$

Adding equation (i) and equation (ii), we get

$$\begin{aligned} & 2y - y = 1 + 2 \\ \Rightarrow & y = 3 \\ \text{Putting } y = 3 \text{ in equation (ii), we get} \\ \Rightarrow & x - 3 = 2 \\ \Rightarrow & x = 2 + 3 = 5 \end{aligned}$$

Hence, the required number is $10y + x = 10 \times 3 + 5 = 35$.

Q10

A two-digit number is 4 more than 6 times the sum of its digits. If 18 is subtracted from the number, the digits are reversed. Find the number.

Solution

Let the digit in the unit's place be x and the digit at the ten's place be y . Then,

$$\text{Number} = 10y + x$$

The number obtained by reversing the order of the digits is $10x + y$.

According to the given conditions, we have:

$$10y + x = 6(x + y) + 4$$

$$\Rightarrow 10y + x = 6x + 6y + 4$$

$$\Rightarrow 10y - 6y + x - 6x = 4$$

$$\Rightarrow 4y - 5x = 4 \quad \dots(i)$$

$$\text{and, } 10y + x - 18 = 10x + y$$

$$\Rightarrow -18 = 10x - x + y - 10y$$

$$\Rightarrow -18 = 9x - 9y$$

$$\Rightarrow 9x - 9y = -18$$

$$\Rightarrow 3(x - y) = -18$$

$$\Rightarrow x - y = -2 \quad \dots(ii)$$

Multiplying equation (i) by 3, we get

$$4x - 4y = -12 \quad \dots(iii)$$

Adding equation (ii) and equation (iii), we get

$$-5x + 4y = -4 - 8$$

$$\Rightarrow -x = -4$$

$$\Rightarrow x = 4$$

Putting $x = 4$ in equation (ii), we get:

$$4 - y = -2$$

$$\Rightarrow -y = -2 - 4$$

$$\Rightarrow -y = -6$$

$$\Rightarrow y = 6$$

Hence, the required number is $10y + x = 10 \times 6 + 4 = 64$.

Q11

A two-digit number is 4 times the sum of its digits and twice the product of the digits. Find the number.

Solution

Let the digit in the unit's place be x and the digit at the ten's place be y . Then,
 Number = $10y + x$

The number obtained by reversing the order of the digits is $10x + y$.

According to the given conditions, we have:

$$\begin{aligned} & 10y + x = 4(x + y) \\ \Rightarrow & 10y + x = 4x + 4y \\ \Rightarrow & 10y - 4y + x - 4x = 0 \\ \Rightarrow & 6y - 3x = 0 \\ \Rightarrow & -3x + 6y = 0 \\ \Rightarrow & 3x - 6y = 0 \\ \Rightarrow & 3(x - 2y) = 0 \\ \Rightarrow & x - 2y = 0 \\ \Rightarrow & x = 2y \quad \text{--- (i)} \\ \text{and, } & 10y + x = 2(x + y) \\ \Rightarrow & 10y + x = 2xy \quad \text{--- (ii)} \end{aligned}$$

Substituting $x = 2y$ in equation (i), we get

$$\begin{aligned} & 10y + 2y = 2 + (2y) \times y \\ \Rightarrow & 12y = 4y^2 \\ \Rightarrow & 3y = y^2 \\ \Rightarrow & y^2 - 3y = 0 \\ \Rightarrow & y(y - 3) = 0 \\ \Rightarrow & y = 0 \quad \text{or} \quad y = 3 \quad [\because \text{Ten's digit can not be } 0] \\ \Rightarrow & y = 3 \end{aligned}$$

Putting $y = 3$ in equation (i), we get:

$$x = 2 \times 3 = 6$$

Hence, the required number is $10y + x = 10 \times 3 + 6 = 36$.

Q12

A two-digit number is such that the product of its digits is 20. If 9 is added to the number, the digits interchange their places. Find the number.

Solution

Let the digit in the unit's place be x and the digit at the ten's place be y . Then,
 Number = $10y + x$

The number obtained by reversing the order of the digits is $10x + y$.
 According to the given conditions, we have

$$x + y = 20 \quad \text{---(i)}$$

$$\therefore y = \frac{20}{x} \quad \text{---(ii)}$$

$$\text{and, } 10y + x + 9 = 10x + y$$

$$\Rightarrow 9 = 10x - x + y - 10y$$

$$\Rightarrow 9 = 9x - 9y$$

$$\Rightarrow 9x - 9y = 9$$

$$\Rightarrow 9(x - y) = 9$$

$$\Rightarrow x - y = 1 \quad \text{---(iii)}$$

Substituting $y = \frac{20}{x}$ in equation (ii), we get

$$x - \frac{20}{x} = 1$$

$$\Rightarrow x^2 - 20 = x$$

$$\Rightarrow x^2 - x - 20 = 0$$

$$\Rightarrow x^2 - 5x + 4x - 20 = 0$$

$$\Rightarrow x(x - 5) + 4(x - 5) = 0$$

$$\Rightarrow (x + 4)(x - 5) = 0$$

$$\Rightarrow x - 5 = 0 \quad \text{or} \quad x + 4 = 0$$

$$\Rightarrow x = 5$$

Putting $x = 5$ in $y = \frac{20}{x}$, we get

$$y = \frac{20}{5} = 4$$

Hence, the required number is $10y + x = 10 \times 4 + 5 = 45$.

[A digit can not be negative]

Q13

The difference between two numbers is 26, and one number is three times the other.
 Find them.

Solution

Let the larger number be x and the smaller number be y . Then,

$$x - y = 26 \quad \text{---(i)}$$

$$\text{and, } x = 3y \quad \text{---(ii)}$$

Substituting $x = 3y$ in equation (i), we get

$$3y - y = 26$$

$$\Rightarrow 2y = 26$$

$$\Rightarrow y = \frac{26}{2} = 13$$

Putting $y = 13$ in equation (ii), we get

$$x = 3 \times 13 = 39$$

Hence, the required numbers are 39 and 13.

Q14

The sum of digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Solution

Let the units digit and tens digit of the number be x and y respectively.

$$\text{Number} = 10y + x$$

Number after reversing the digits = $10x + y$

According to the question,

$$x + y = 9 \quad \dots (1)$$

$$9(10y + x) = 2(10x + y)$$

$$90y + 9x = 20x + 2y$$

$$-x + 8y = 0 \quad \dots (2)$$

Adding equations (1) and (2), we obtain:

$$9y = 9$$

$$y = 1$$

Substituting the value of y in equation (1), we obtain:

$$x = 8$$

Thus, the number is $10y + x = 10 \times 1 + 8 = 18$.

Concept insight: This problem talks about a two-digit number. Here, remember that a two-digit number xy can be expanded as $10x + y$. Then, using the two given conditions, a pair of linear equations can be formed which can be solved by eliminating one of the variables.

Q15

Seven times a two-digit number is equal to four times the number obtained by reversing the digits. If the difference between the digits is 3. Find the number.

Solution

Let the digit in the unit's place be x and the digit at the ten's place be y . Then,

$$\text{Number} = 10y + x$$

The number obtained by reversing the order of the digits is $10x + y$.

According to the given conditions, we have

$$7(10y + x) = 4(10x + y)$$

$$\Rightarrow 70y + 7x = 40x + 4y$$

$$\Rightarrow 70y - 4y = 40x - 7x$$

$$\Rightarrow 66y = 33x \Rightarrow 2y = x$$

$$\Rightarrow 33(x - 2y) = 0$$

$$\Rightarrow x - 2y = 0$$

$$\Rightarrow x = 2y$$

$$\text{and, } x - y = 3$$

$$\Rightarrow x - y = 3 \quad \text{or} \quad x - y = -3$$

$$\Rightarrow 2y - y = 3 \quad \text{or} \quad 2y - y = -3$$

$$\Rightarrow y = 3 \quad \text{or} \quad y = -3$$

$$\Rightarrow y = 3$$

Putting $y = 3$ in $x = 2y$, we get

$$x = 2 \times 3 = 6$$

Hence, the required number is $10y + x = 10 \times 3 + 6 = 36$.

Q16

Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.

Solution

Let the two numbers be x and y .

According to question,

$$\begin{aligned}x &= 5 \\y &= 6\end{aligned}$$

$$\Rightarrow 6x = 5y$$

$$\Rightarrow 6x - 5y = 0 \quad \dots(i)$$

$$\text{And, } \frac{x-8}{y-8} = \frac{4}{5}$$

$$\Rightarrow 5x - 40 = 4y - 32$$

$$\Rightarrow 5x - 4y = 8 \quad \dots(ii)$$

Multiplying equation (i) by 4 and (ii) by 5, we get

$$24x - 20y = 0 \quad \dots(iii)$$

$$25x - 20y = 40 \quad \dots(iv)$$

Subtracting equation (iii) from (iv), we have

$$x = 40$$

Substituting $x = 40$ in (i), we have

$$6x - 5y = 0$$

$$\Rightarrow 5y = 240$$

$$\Rightarrow y = 48$$

Hence, the two numbers are 40 and 48.

Q17

A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.

Solution

Let the digit at unit's place be x and the digit at tens place be y .

Then, number = $10y + x$

According to given conditions, we have

$$10y + x = 8(x + y) - 5$$

$$\Rightarrow 10y + x = 8x + 8y - 5$$

$$\Rightarrow 7x - 2y - 5 = 0$$

$$\text{And, } 10y + x = 16(y - x) + 3$$

$$\Rightarrow 10y + x = 16y - 16x + 3$$

$$\Rightarrow 17x - 6y - 3 = 0$$

By cross-multiplication, we have

$$\begin{array}{rcccl}x & & -y & & 1 \\ -2x(-3) - (-6)x(-5) & = & 7x(-3) - 17x(-5) & = & 7x(-6) - 17x(-2) \\ \hline -6 - 30 & = & -21 + 85 & = & -42 + 34 \\ \hline -36 & = & 64 & = & -8 \\ \hline \Rightarrow \frac{x}{-24} & = & \frac{-y}{64} & = & \frac{1}{-8} \\ \hline \end{array}$$

$$\text{Now, } \frac{x}{-24} = \frac{1}{-8} \Rightarrow -8x = -24 \Rightarrow x = 3$$

$$\text{And, } \frac{-y}{64} = \frac{1}{-8} \Rightarrow 8y = 64 \Rightarrow y = 8$$

Hence, the number = $10 \times 8 + 3 = 80 + 3 = 83$.

Exercise 3.8**Q1**

The numerator of a fraction is 4 less than the denominator. If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is eight times the numerator. Find the fraction.

Solution

Let the numerator and denominator of the fraction be x and y respectively. Then

$$\text{Fraction} = \frac{x}{y}$$

It is given that

$$\text{Numerator} = \text{Denominator} - 4$$

$$\Rightarrow x = y - 4$$

$$\Rightarrow x - y = -4 \quad \text{---(i)}$$

According to the given condition, we have

$$8(x - 2) = (y + 1)$$

$$\Rightarrow 8x - 16 = y + 1$$

$$\Rightarrow 8x - y = 1 + 16$$

$$\Rightarrow 8x - y = 17 \quad \text{---(ii)}$$

Subtracting equation (i) by equation (ii), we get

$$8x - x - y - (-y) = 17 - (-4)$$

$$\Rightarrow 7x + y = 17 + 4$$

$$\Rightarrow 7x = 21$$

$$\Rightarrow x = \frac{21}{7} = 3$$

Putting $x = 3$ in equation (i), we get

$$3 - y = -4$$

$$\Rightarrow -y = -4 - 3$$

$$\Rightarrow -y = -7$$

$$\Rightarrow y = 7$$

Hence, the fraction is $\frac{3}{7}$.

Q2

A fraction becomes $\frac{9}{11}$ if 2 is added to both numerator and the denominator. If 3 is added to both the numerator and the denominator it becomes $\frac{5}{8}$. Find the fraction.

Solution

Let the fraction be $\frac{x}{y}$

According to the given information,

$$\frac{+2}{+2} = \frac{9}{11}$$

$$11x + 22 = 9y + 18$$

$$11x - 9y = -4 \quad (1)$$

$$\frac{x+3}{y+3} = \frac{5}{6}$$

$$6x + 18 = 5y + 15$$

$$6x - 5y = -3 \quad (2)$$

$$\text{From equation (1), we obtain } x = \frac{-4+9y}{11} \quad (3)$$

Substituting this in equation (2), we obtain

$$6\left(\frac{-4+9y}{11}\right) - 5y = -3$$

$$-24 + 54y - 55y = -33$$

$$-y = -9$$

$$y = 9 \quad (4)$$

Substituting this in equation (3), we obtain

$$x = \frac{-4+81}{11} = 7$$

Hence, the fraction is $\frac{7}{9}$.

Q3

A fraction becomes $\frac{1}{3}$ if 1 is subtracted from both its numerator and denominator. If 1 is added to both the numerator and denominator, it becomes $\frac{1}{2}$. Find the fraction.

Solution

Let the fraction be $\frac{x}{y}$.

Then, according to the given conditions, we have

$$\begin{aligned} \text{i) } & \frac{x-1}{y+1} = \frac{1}{2} \\ & y+1 = 2(x-1) \\ & 2x-2 = y+1 \\ & 2x-y = 1+2 \\ \Rightarrow & 2x-y = 3 \quad \dots (i) \end{aligned}$$

$$\begin{aligned} \text{And, } & \frac{x+1}{y+1} = \frac{1}{2} \\ & y+1 = 2(x+1) \\ & 2x+2 = y+1 \\ & 2x-y = 1-2 \\ \Rightarrow & 2x-y = -1 \quad \dots (ii) \end{aligned}$$

Subtracting equation (ii) by equation (i), we get

$$\begin{aligned} & 3x-2x = 2-(-1) \\ \Rightarrow & x = 2+1=3 \end{aligned}$$

Putting $x=3$ in equation (i), we get

$$\begin{aligned} & 3x3-y=2 \\ \text{i) } & 9-y=2 \\ & -y=2-9 \\ & -y=-7 \\ \Rightarrow & y=7 \end{aligned}$$

Hence, the fraction is $\frac{3}{7}$.

Q4

If we add 1 to the numerator and subtract 1 from the denominator, a fraction becomes 1. It also becomes $1/2$ if we only add 1 to the denominator. What is the fraction?

Solution

Let the fraction be $\frac{x}{y}$.

According to the question,

$$\begin{aligned} \frac{x+1}{y-1} &= 1 \\ \Rightarrow x-y &= -2 \quad \dots (1) \end{aligned}$$

$$\begin{aligned} \frac{x}{y+1} &= \frac{1}{2} \\ \Rightarrow 2x &= y+1 \quad \dots (2) \end{aligned}$$

Subtracting equation (1) from equation (2), we obtain:

$$x = 3$$

Substituting this value of x in equation (1), we obtain:

$$3-y=-2$$

$$-y=-5$$

$$y=5$$

Hence, the fraction is $\frac{3}{5}$.

Concept insight: This problem talks about a fraction. The numerator and denominator of the fraction are not known so we represent these as variables x and y respectively where variable y must be non zero. Then, a pair of linear equations can be formed from the given conditions. The pair of equations can then be solved by eliminating a suitable variable.

Q5

The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$. Find the fraction.

Solution

Let the fraction be $\frac{x}{y}$.

Then, according to the given conditions, we have

$$x + y = 12 \quad \text{---(i)}$$

$$\text{and, } \frac{x}{y+3} = \frac{1}{2}$$

$$\Rightarrow 2x = y + 3$$

$$\Rightarrow 2x - y = 3 \quad \text{---(ii)}$$

Adding equation (i) and equation (ii), we get

$$x + 2x = 12 + 3$$

$$\Rightarrow 3x = 15$$

$$\Rightarrow x = \frac{15}{3} = 5$$

Putting $x = 5$ in equation (i), we get:

$$5 + y = 12$$

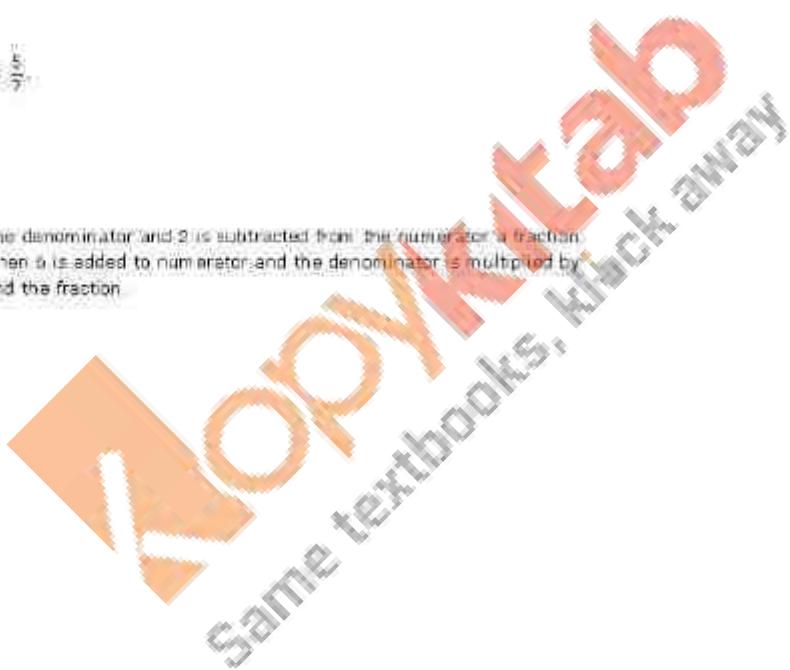
$$\Rightarrow y = 12 - 5 = 7$$

Hence, the fraction is $\frac{5}{7}$.

Q6

When 3 is added to the denominator and 2 is subtracted from the numerator a fraction becomes $\frac{1}{4}$. And, when 6 is added to numerator and the denominator is multiplied by 3, it becomes $\frac{2}{3}$. Find the fraction.

Solution



Let the fraction be $\frac{x}{y}$.

Then, according to the given conditions, we have

$$\begin{aligned} \frac{x-2}{y} &= \frac{1}{4} \\ y+3 &= 4 \\ \Rightarrow 4(x-2) &= y+3 \\ \Rightarrow 4x-8 &= y+3 \\ \Rightarrow 4x-y &= 3+8 \\ \Rightarrow 4x-y &= 11 \quad \text{---(i)} \end{aligned}$$

$$\begin{aligned} \text{And, } \frac{x+5}{3y} &= \frac{2}{3} \\ \Rightarrow \frac{3(x+5)}{3y} &= 2 \\ \Rightarrow x+5 &= 2y \\ \Rightarrow x-2y &= -5 \quad \text{---(ii)} \end{aligned}$$

Multiplying equation (i) by 2, we get

$$8x-2y = 22 \quad \text{---(iii)}$$

Subtracting equation (ii), from equation (iii), we get

$$\begin{aligned} 8x-y &= 22 - (-5) \\ \Rightarrow 8x-x &= 22+5 \\ \Rightarrow 7x &= 27 \\ \Rightarrow x &= \frac{27}{7} = 4 \end{aligned}$$

Putting $x = 4$ in equation (ii), we get

$$\begin{aligned} 4-2y &= -5 \\ \Rightarrow -2y &= -5-4 \\ \Rightarrow -2y &= -9 \\ \Rightarrow y &= \frac{-9}{-2} = \frac{9}{2} \end{aligned}$$

Hence, the fraction is $\frac{4}{\frac{9}{2}} = \frac{8}{9}$.

Q7

The sum of a numerator and denominator of a fraction is 8. If the denominator is increased by 2, the fraction reduces to $\frac{1}{3}$. Find the fraction.

Solution

Let the fraction be $\frac{x}{y}$.

Then, according to the given conditions, we have

$$x+y=18 \quad \text{---(i)}$$

$$\begin{aligned} \text{And, } \frac{x}{y+2} &= \frac{1}{3} \\ \Rightarrow 3x &= y+2 \\ \Rightarrow 3x-y &= 2 \end{aligned} \quad \text{---(ii)}$$

Adding equation(i) and equation(ii), we get

$$\begin{aligned} x+3x &= 18+2 \\ \Rightarrow 4x &= 20 \\ \Rightarrow x &= \frac{20}{4} = 5 \end{aligned}$$

Putting $x=5$ in equation(ii), we get

$$\begin{aligned} 5+y &= 18 \\ \Rightarrow y &= 18-5 \\ \Rightarrow y &= 13 \end{aligned}$$

Hence, the fraction is $\frac{5}{13}$.

Q8

If 2 is added to the numerator of a fraction, it reduces to $\frac{1}{2}$ and if 1 is subtracted from the denominator, it reduce to $\frac{1}{3}$. Find the fraction.

Solution

Let the fraction be $\frac{x}{y}$.

Then, according to the given conditions, we have

$$\begin{aligned} \frac{x+2}{y} &= \frac{1}{2} \\ \Rightarrow 2(x+2) &= y \\ \Rightarrow 2x+4 &= y \\ \Rightarrow 2x-y &= -4 \end{aligned} \quad \text{---(i)}$$

$$\begin{aligned} \text{And, } \frac{x}{y-1} &= \frac{1}{3} \\ \Rightarrow 3x &= y-1 \\ \Rightarrow 3x-y &= -1 \end{aligned} \quad \text{---(ii)}$$

Subtracting equation(i) by equation(ii), we get

$$\begin{aligned} 3x-2x &= -1+4 \\ \Rightarrow x &= 3 \end{aligned}$$

Putting $x=3$ in equation(i), we get

$$\begin{aligned} 2\times 3-y &= -4 \\ \Rightarrow 6-y &= -4 \\ \Rightarrow -y &= -4-6 \\ \Rightarrow -y &= -10 \\ \Rightarrow y &= 10 \end{aligned}$$

Hence, the fraction is $\frac{3}{10}$.

Q9

The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2:3. Determine the fraction.

Solution

Let the fraction be $\frac{x}{y}$.

Then, according to the given conditions, we have

$$\begin{aligned} \text{i) } & x + y = 2x + 4 \\ \text{ii) } & x + y - 2x = 4 \\ \text{iii) } & -x + y = 4 \quad \text{---(i)} \end{aligned}$$

$$\begin{aligned} \text{And, } & \frac{x+3}{y+3} = \frac{2}{3} \\ \Rightarrow & 3(x+3) = 2(y+3) \\ \text{iv) } & 3x + 9 = 2y + 6 \\ \text{v) } & 3x - 2y = 6 - 9 \\ \text{vi) } & 3x - 2y = -3 \quad \text{---(ii)} \end{aligned}$$

Multiplying equation (i) by 3, we get

$$-3x + 3y = 12 \quad \text{---(iii)}$$

Adding equation (i) and equation (iii), we get

$$\begin{aligned} -2y + 3y &= -3 + 12 \\ y &= 9 \end{aligned}$$

Putting $y = 9$ in equation (i), we get

$$\begin{aligned} -x + 9 &= 4 \\ \text{vii) } & -x = 4 - 9 \\ \text{viii) } & -x = -5 \\ \text{ix) } & x = 5 \end{aligned}$$

Hence, the fraction is $\frac{5}{9}$.

Q10

If the numerator of a fraction is multiplied by 2 and the denominator is reduced by 5, the fraction becomes $\frac{6}{5}$. And, if the denominator is doubled and the numerator is increased by 8, the fraction becomes $\frac{2}{5}$. Find the fraction.

Solution

Let the fraction be $\frac{x}{y}$.

Then, according to the given conditions, we have

$$\begin{aligned} \text{Q11. } & \frac{2x}{y-5} = \frac{6}{5} \\ \Rightarrow & 5 \times 2x = 6(y-5) \\ \Rightarrow & 10x = 6y - 30 \\ \Rightarrow & 10x - 6y = -30 \\ \Rightarrow & 2(5x - 3y) = -30 \\ \Rightarrow & 5x - 3y = -15 \quad \text{---(i)} \end{aligned}$$

$$\begin{aligned} \text{And, } & \frac{x+8}{2y} = \frac{2}{5} \\ \Rightarrow & 5(x+8) = 2 \times 2y \\ \Rightarrow & 5x + 40 = 4y \\ \Rightarrow & 5x - 4y = -40 \quad \text{---(ii)} \end{aligned}$$

Subtracting equation (ii) by equation (i), we get

$$\begin{aligned} & -3y + 4y = -15 + 40 \\ \Rightarrow & y = 25 \end{aligned}$$

Putting $y = 25$ in equation (i), we get

$$\begin{aligned} & 5x - 3 \times 25 = -15 \\ \Rightarrow & 5x - 75 = -15 \\ \Rightarrow & 5x = -15 + 75 \\ \Rightarrow & 5x = 60 \\ \Rightarrow & x = \frac{60}{5} = 12 \end{aligned}$$

Hence, the fraction is $\frac{12}{25}$.

Q11

The sum of the numerator and denominator of a fraction is 3 less than twice the denominator. If the numerator and denominator are decreased by 1, the numerator becomes half the denominator. Find the fraction.

Solution

Let the fraction be $\frac{x}{y}$.

Then, according to the given conditions, we have

$$\begin{aligned} & x+y = 3y-3 \\ \Rightarrow & x-2y = y-3 \\ \Rightarrow & x-y = -3 \quad \text{---(i)} \end{aligned}$$

And, $(x-1) = \frac{1}{2}(y-1)$

$$\begin{aligned} \Rightarrow & 2(x-1) = y-1 \\ \Rightarrow & 2x-2 = y-1 \\ \Rightarrow & 2x-y = -1+2 \\ \Rightarrow & 2x-y = 1 \quad \text{---(ii)} \end{aligned}$$

Subtracting equation (i) and equation (ii), we get

$$\begin{aligned} & 2y-x = 1+3 \\ \Rightarrow & x = 4 \end{aligned}$$

Putting $x = 4$ in equation (i), we get

$$\begin{aligned} & 4-y = -3 \\ \Rightarrow & -y = -3-4 \\ \Rightarrow & -y = -7 \\ \Rightarrow & y = 7 \end{aligned}$$

Hence, the fraction is $\frac{4}{7}$.



Exercise 3.9**Q1**

A father is three times as old as his son. After twelve years, his age will be twice as that of his son then. Find their present ages.

Solution

Suppose father's age be x and that of son's be y . Then,

$$x = 3y \quad \text{---(i)}$$

Twelve years later, father's age will be $(x + 12)$ years and son's age will be $(y + 12)$ years.

$$x + 12 = 2(y + 12)$$

$$x + 12 = 2y + 24$$

$$x - 2y = 24 - 12$$

$$x - 2y = 12 \quad \text{---(ii)}$$

Substituting $x = 3y$ in equation (ii), we get

$$3y - 2y = 12$$

$$\therefore y = 12$$

Putting $y = 12$ in equation (i), we get

$$x = 3 \times 12 = 36$$

Hence, father's age is 36 years and son's age is 12 years.

Q2

Ten years later, A will be twice as old as B and five years ago, A was three times as old as B. What are the present ages of A and B?

Solution

Let the age of A and B be x and y years respectively. Then,

Ten years later, the age of A will be $(x+10)$ years and, the age of B will be $(y+10)$ years.

$$\begin{aligned} \Rightarrow & x+10 = 2(y+10) \\ \Rightarrow & x+10 = 2y+20 \\ \Rightarrow & x-2y = 20-10 \\ \Rightarrow & x-2y = 10 \quad \text{---(i)} \end{aligned}$$

Five years ago, A's age = $(x-5)$ years

B's age = $(y-5)$ years

Using the given information, we get

$$\begin{aligned} \Rightarrow & x-5 = 3(y-5) \\ \Rightarrow & x-5 = 3y-15 \\ \Rightarrow & x-3y = -15+5 \\ \Rightarrow & x-3y = -10 \quad \text{---(ii)} \end{aligned}$$

Subtracting equation (ii) from equation (i), we get

$$\begin{aligned} \Rightarrow & -2y + 3y = 10 + 10 \\ \Rightarrow & y = 20 \end{aligned}$$

Putting $y = 20$ in equation (i), we get

$$\begin{aligned} \Rightarrow & x-2 \times 20 = 10 \\ \Rightarrow & x-40 = 10 \\ \Rightarrow & x = 10+40 = 50 \end{aligned}$$

Hence, A's age = 50 years

B's age = 20 years.

Q3

Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

Solution

List present age of Nuri and Sonu by x and y respectively.

According to the question,

$$\begin{aligned} (x-5) &= 3(y-5) \\ x-3y &= -10 \quad \text{---(1)} \\ (x+10) &= 2(y+10) \\ x-2y &= 10 \quad \text{---(2)} \end{aligned}$$

Subtracting equation (1) from equation (2), we obtain:

$$y = 20$$

Substituting the value of y in equation (1), we obtain:

$$x-60 = -10$$

$$x = 50$$

Thus, the age of Nuri and Sonu are 50 years and 20 years respectively.

Q4

Six years hence a man's age will be three times the age of his son and three years ago he was nine times as old as his son. Find their present ages.

Solution

Let the present age of Man's be x years and the present age of his son's be y years.

Six years hence, Man's age = $(x + 6)$ years.

$$\text{Son's age} = (y + 6) \text{ years}$$

Using the given information, we get:

$$x + 6 = 3(y + 6)$$

$$\Rightarrow x + 6 = 3y + 18$$

$$\Rightarrow x - 3y = 18 - 6$$

$$\Rightarrow x - 3y = 12 \quad \text{---(i)}$$

Three years ago, Man's age = $(x - 3)$ years.

$$\text{Son's age} = (y - 3) \text{ years}$$

Using the given information, we get:

$$x - 3 = 9(y - 3)$$

$$\Rightarrow x - 3 = 9y - 27$$

$$\Rightarrow x - 9y = -27 + 3$$

$$\Rightarrow x - 9y = -24 \quad \text{---(ii)}$$

Subtracting equation (ii) from equation (i), we get:

$$-3y + 9y = 12 + 24$$

$$\Rightarrow 6y = 36$$

$$\Rightarrow y = \frac{36}{6} = 6$$

Putting $y = 6$ in equation (i), we get:

$$x - 3 \times 6 = 12$$

$$\Rightarrow x - 18 = 12$$

$$\Rightarrow x = 12 + 18 = 30$$

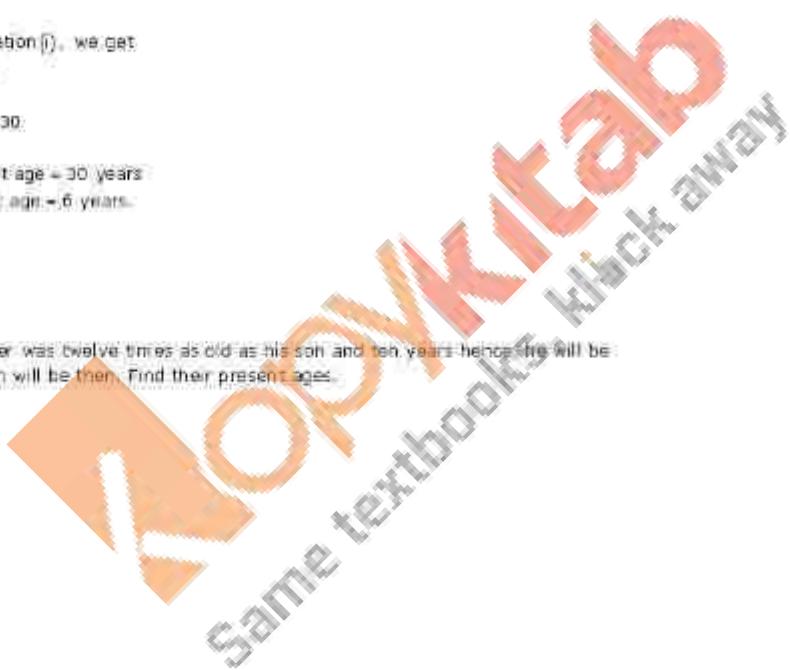
Hence, Man's present age = 30 years.

Son's present age = 6 years.

Q5

Ten years ago, a father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be then. Find their present ages.

Solution



Let the present age of father be x years and the present age of son be y years.

$$\begin{aligned}\text{Ten years ago, Father's age} &= (x-10) \text{ years} \\ \text{Son's age} &= (y-10) \text{ years}\end{aligned}$$

Using the given information, we get

$$\begin{aligned}x-10 &= 12(y-10) \\ \Rightarrow x-10 &= 12y-120 \\ \Rightarrow x-12y &= -120+10 \\ \Rightarrow x-12y &= -110 \quad \text{---(i)}\end{aligned}$$

$$\text{Ten years hence, Father's age} = (x+10) \text{ years}$$

$$\text{Son's age} = (y+10) \text{ years}$$

Using the given information, we get

$$\begin{aligned}x+10 &= 2(y+10) \\ \Rightarrow x+10 &= 2y+20 \\ \Rightarrow x-2y &= 20-10 \\ \Rightarrow x-2y &= 10 \quad \text{---(ii)}\end{aligned}$$

Subtracting equation (i) from equation (ii), we get

$$\begin{aligned}-2y+12y &= 10+110 \\ \Rightarrow 10y &= 120 \\ \Rightarrow y &= \frac{120}{10} = 12\end{aligned}$$

Putting $y = 12$ in equation (i), we get

$$\begin{aligned}x-2 \times 12 &= 10 \\ \Rightarrow x-24 &= 10 \\ \Rightarrow x &= 10+24 = 34\end{aligned}$$

Hence, present age of father is 34 years and present age of son is 12 years.

Q6

The present age of a father is three more than three times the age of the son. Three years hence father's age will be 10 years more than twice the age of the son. Determine their present ages.

Solution

Let the present age of father be x years and the present age of son be y years. Then,

$$\begin{aligned} x &= 3y + 3 && \text{[Given]} \\ \Rightarrow x - 3y &= 3 && \text{---(i)} \end{aligned}$$

Three years hence, Father's age = $(x + 3)$ years.

Son's age = $(y + 3)$ years

Using the given information, we get:

$$\begin{aligned} x + 3 &= 2(y + 3) + 10 \\ \Rightarrow x + 3 &= 2y + 5 + 10 \\ \Rightarrow x - 2y &= 10 - 3 \\ \Rightarrow x - 2y &= 13 && \text{---(ii)} \end{aligned}$$

Subtracting equation (i) by equation (ii), we get:

$$\begin{aligned} -2y + 3y &= 13 - 3 \\ \Rightarrow y &= 10 \end{aligned}$$

Putting $y = 10$ in equation (i), we get:

$$\begin{aligned} x - 3 \times 10 &= 3 \\ \Rightarrow x - 30 &= 3 \\ \Rightarrow x &= 3 + 30 = 33 \end{aligned}$$

Hence, present age of father is 33 years and present age of son is 10 years.

Q7

A father is three times as old as his son. In 12 years time, he will be twice as old as his son. Find the present ages of father and the son.

Solution

Let the present age of father be x years and the present age of son be y years. Then,

$$x = 3y \quad \text{---(i)}$$

Twelve years hence, Father's age = $(x + 12)$ years.

Son's age = $(y + 12)$ years.

Using the given information, we get:

$$\begin{aligned} x + 12 &= 2(y + 12) \\ \Rightarrow x + 12 &= 2y + 24 \\ \Rightarrow x - 2y &= 24 - 12 \\ \Rightarrow x - 2y &= 12 && \text{---(ii)} \end{aligned}$$

Substituting $x = 3y$ in equation (ii), we get:

$$\begin{aligned} 3y - 2y &= 12 \\ \Rightarrow y &= 12 \end{aligned}$$

Putting $y = 12$ in equation (i), we get:

$$x = 3y = 3 \times 12 = 36$$

Hence, present age of father is 36 years and present age of son is 12 years.

Q8

Father's age is three times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of father.

Solution

Let the present age of father be x years and the sum of the present ages of two children be y years. Then,

$$x = 3y \quad \text{---(i)}$$

Five years hence, Father's age = $(x + 5)$ years.

$$\text{Sum of children's age} = (y + 5 + 5) = (y + 10) \text{ years}$$

Using the given information, we get

$$\begin{aligned} x + 5 &= 2(y + 10) \\ \Rightarrow x + 5 &= 2y + 20 \\ \Rightarrow x - 2y &= 20 - 5 \\ \Rightarrow x - 2y &= 15 \end{aligned} \quad \text{---(ii)}$$

Substituting $x = 3y$ in equation (ii), we get

$$\begin{aligned} 3y - 2y &= 15 \\ \Rightarrow y &= 15 \end{aligned}$$

Putting $y = 15$ in equation (i), we get

$$x = 3 \times 15 = 45$$

Hence, present age of father is 45 years.

Q9

Two years ago, a father was five times as old as his son. Two years later, his age will be nine times the age of the son. Find the present ages of father and son.

Solution

Let the present age of father be x years and the present age of son be y years.

$$\begin{aligned} \text{Two years ago, Father's age} &= (x - 2) \text{ years} \\ \text{Son's age} &= (y - 2) \text{ years} \end{aligned}$$

Using the given information, we get

$$\begin{aligned} x - 2 &= 5(y - 2) \\ \Rightarrow x - 2 &= 5y - 10 \\ \Rightarrow x - 5y &= -10 + 2 \\ \Rightarrow x - 5y &= -8 \end{aligned} \quad \text{---(i)}$$

$$\begin{aligned} \text{Two years later, Father's age} &= (x + 2) \text{ years} \\ \text{Son's age} &= (y + 2) \text{ years} \end{aligned}$$

Using the given information, we get:

$$\begin{aligned} x + 2 &= 9(y + 2) + 8 \\ \Rightarrow x + 2 &= 9y + 18 + 8 \\ \Rightarrow x - 9y &= 14 - 2 \\ \Rightarrow x - 9y &= 12 \end{aligned} \quad \text{---(ii)}$$

Subtracting equation (i) by equation (ii), we get

$$\begin{aligned} -5y + 5y &= 12 + 8 \\ \Rightarrow 2y &= 20 \\ \Rightarrow y &= \frac{20}{2} = 10 \end{aligned}$$

Putting $y = 10$ in equation (i), we get

$$\begin{aligned} x - 2 &= 10 \times 5 = 50 \\ \Rightarrow x - 2 &= 50 \\ \Rightarrow x &= 50 + 2 = 52 \end{aligned}$$

Hence, present age of father is 52 years and present age of son is 10 years.

Q10

A is elder to **B** by 2 years. **A**'s father **C** is twice as old as **A**, and **A** is twice as old as his sister **S**. If the ages of the father and sister differ by 40 years, find the age of **A**.

Solution

Suppose father's age be x years and that of sister's age be y years. Then,

$$x - y = 40 \quad \text{---(i)}$$

Now, **B**'s is twice as old as his sister,

$$B\text{'s age} = 2y \quad \text{---(ii)}$$

and, father's is twice as old as **A**,

$$x = 2(A\text{'s age})$$

$$\Rightarrow A\text{'s age} = \frac{x}{2} \quad \text{---(iii)}$$

It is also given that, **A** is elder to **B** by 2 years:

$$\frac{x}{2} = 2y + 2 \quad [\text{Using (i) and (iii)}]$$

$$\Rightarrow x = 4y + 4$$

$$\Rightarrow x - 4y = 4 \quad \text{---(iv)}$$

Subtracting equation (iv) from equation (i), we get

$$-y + 4y = 40 - 4$$

$$\Rightarrow 3y = 36$$

$$\Rightarrow y = \frac{36}{3} = 12$$

Putting $y = 12$ in equation (i), we get:

$$x - 12 = 40$$

$$\Rightarrow x = 40 + 12 = 52$$

Put $x = 52$ in equation (iii), we get

$$A\text{'s age} = \frac{52}{2} = 26 \text{ years.}$$

Q11

The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differs by 30 years. Find the ages of Ani and Biju.

Solution

The difference between the ages of Ani and Biju is given as 3 years. So, either Biju is 3 years older than Ani or Ani is 3 years older than Biju.

Let the age of Ani and Biju be x years and y years respectively.
Age of Dharam = $2x - z = 2x$ years.

$$\text{Age of Cathy} = \frac{y}{2} \text{ years}$$

Case I: Ani is older than Biju by 3 years
 $x - y = 3 \quad \dots (1)$

$$\begin{aligned} &\text{begin mathsize 12px style 2 straight } x \text{ minus straight } y \text{ over 2 equals 30 end style} \\ &4x - y = 60 \quad \dots (2) \end{aligned}$$

Subtracting (1) from (2), we obtain:

$$3x = 60 - 3 = 57$$

$$x = \frac{57}{3} = 19$$

$$\text{Age of Ani} = 19 \text{ years}$$

$$\text{Age of Biju} = 19 - 3 = 16 \text{ years}$$

Case II: Biju is older than Ani by 3 years
 $y - x = 3 \quad \dots (3)$

$$\begin{aligned} &\text{begin mathsize 12px style 2 straight } x \text{ minus straight } y \text{ over 2 equals 30 end style} \\ &4x - y = 60 \quad \dots (4) \end{aligned}$$

Adding (3) and (4), we obtain:

$$3x = 63$$

$$x = 21$$

$$\text{Age of Ani} = 21 \text{ years}$$

$$\text{Age of Biju} = 21 + 3 = 24 \text{ years}$$

Concept Insight: In this problem, ages of Ani and Biju are the unknown quantities. So, we represent them by variables x and y . Now, note that here it is given that the ages of Ani and Biju differ by 3 years. So, it is not mentioned that which one is older. So, the most important point in this question is to consider both cases. Ani is older than Biju and, Biju is older than Ani. For second condition the relation on the ages of Dharam and Cathy can be implemented. Pair of linear equations can be solved using a suitable algebraic method.

Q12

Two years ago, Salim was thrice as old as his daughter and six years later, he will be four years older than twice her age. How old are they now?

Solution

Let Salim's present age be x years and his daughter's age be y years.

Two years ago,

$$\text{Salim's age} = (x - 2) \text{ years}$$

$$\text{Daughter's age} = (y - 2) \text{ years}$$

Using the given information, we have:

$$x - 2 = 3(y - 2)$$

$$\Rightarrow x - 3y + 4 = 0 \quad \dots (i)$$

Six years hence,

$$\text{Salim's age} = (x + 6) \text{ years}$$

$$\text{Daughter's age} = (y + 6) \text{ years}$$

Using the given information, we have:

$$x + 6 = 2(y + 6) + 4$$

$$\Rightarrow x - 2y = 10 \quad \dots (ii)$$

Subtracting (i) from (ii), we get

$$y = 14$$

$$\Rightarrow x - 3(14) = -4$$

$$\Rightarrow x = 38$$

Thus, present age of Salim is 38 years and that of his daughter is 14 years.

Q13

The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Solution

Let father's age be x years and
the sum of the ages of his two children be y years.

Then,

$$x = 2y$$

$$\Rightarrow x - 2y = 0 \quad \dots \text{(i)}$$

20 years hence,

Father's age = $(x + 20)$ years

Sum of the ages of two children = $y + 20 + 20 = (y + 40)$ years

Then, we have

$$x + 20 = y + 40$$

$$\Rightarrow x - y = 20 \quad \dots \text{(ii)}$$

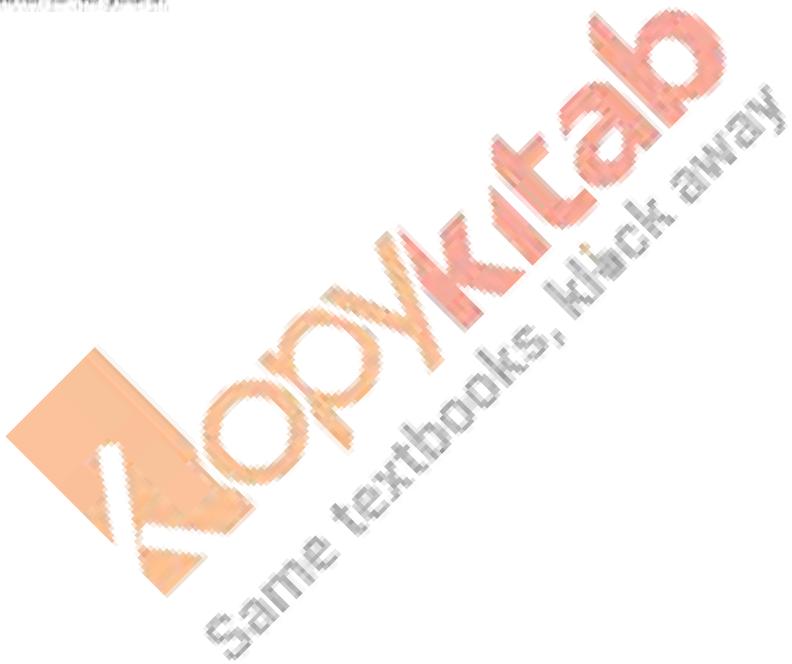
Multiplying (ii) by 2, we get

$$2x - 2y = 40 \quad \dots \text{(iii)}$$

Subtracting (i) from (iii), we have

$$x = 40$$

Thus, the age of father is 40 years.



Exercise 3.10**Q1**

Points A and B are 70 km apart on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 7 hours; but if they travel towards each other, they meet in one hour. Find the speed of the two cars.

Solution

Let X and Y be two cars starting from points A and B respectively.

Let the speed of car X be x km/hr and that of car Y be y km/hr.

Case I: When two cars move in the same directions:

Suppose two cars meet at point Q. Then,

Distance travelled by car X = AQ

Distance travelled by car Y = BQ

It is given that two cars meet in 7 hours.

Distance travelled by car X in 7 hours = $7x$ km

$$\Rightarrow AQ = 7x$$

Distance travelled by car Y in 7 hours = $7y$ km

$$\Rightarrow BQ = 7y$$

Clearly, $AQ + BQ = AB$

$$\Rightarrow 7x + 7y = 70 \quad [\because AB = 70 \text{ km}]$$

$$\Rightarrow 7(x + y) = 70$$

$$\Rightarrow x + y = 10 \quad \text{---(i)}$$

Case II: When two cars move in opposite directions:

Suppose two cars meet at point P. Then,

Distance travelled by car X = AP ,

Distance travelled by car Y = BP ,

In this case, two cars meet in 1 hour.

Distance travelled by car X in 1 hour = x km

$$\Rightarrow AP = x$$

Distance travelled by car Y in 1 hour = y km

$$\Rightarrow BP = y$$

Clearly, $AP + BP = AB$

$$\Rightarrow x + y = 70 \quad \text{---(ii)}$$

Adding equation (i) and equation (ii), we get

$$2x = 10 + 70$$

$$\Rightarrow 2x = 80$$

$$\Rightarrow x = \frac{80}{2} = 40$$

Putting $x = 40$ in equation (ii), we get

$$40 + y = 70$$

$$\Rightarrow y = 70 - 40 = 30$$

Hence, Speed of car X is 40 km/hr and speed of car Y is 30 km/hr.

Q2

A sailor goes 8 km downstream in 40 minutes and returns in 1 hours. Determine the speed of the sailor in still water and the speed of the current.

Solution

Let the speed of the sailor in still water be x km/hr and the speed of the current be y km/hr.

Then,

$$\text{Speed downstream} = (x + y) \text{ km/hr}$$

$$\text{Speed in return journey} = (x - y) \text{ km/hr}$$

$$\text{Now, Time taken to cover 8 km downstream} = \frac{8}{x+y} \text{ hrs}$$

$$\text{But, Time taken to cover 6 km downstream is 40 minutes}$$

$$\Rightarrow \frac{8}{x+y} = \frac{40}{60} \quad \left[40 \text{ minutes} = \frac{40}{60} \text{ hrs} \right]$$

$$\Rightarrow \frac{8}{x+y} = \frac{2}{3}$$

$$\Rightarrow \frac{8 \times 3}{2} = x+y$$

$$\Rightarrow 4 \times 3 = x+y$$

$$\Rightarrow x+y = 12 \quad \text{---(i)}$$

$$\text{and, Time taken in return journey} = \frac{8}{x-y} \text{ km/hr}$$

$$\text{But, Time taken in return journey is 1 hour.}$$

$$\Rightarrow \frac{8}{x-y} = 1$$

$$\Rightarrow x-y = 8 \quad \text{---(ii)}$$

Adding equation (i) and equation (ii), we get

$$2x = 12+8$$

$$\Rightarrow 2x = 20$$

$$\Rightarrow x = \frac{20}{2} = 10$$

Putting $x = 10$ in equation (i), we get

$$10+y = 12$$

$$\Rightarrow y = 12-10$$

$$\Rightarrow y = 2$$

Hence, Speed of the sailor in still water = 10 km/hr

Speed of the current = 2 km/hr

Q3

The boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of stream and that of the boat in still water.

Solution

Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr.

Then,

$$\text{Speed upstream} = \{x - y\} \text{ km/hr}$$

$$\text{Speed downstream} = \{x + y\} \text{ km/hr}$$

$$\text{Now, Time taken to cover 30 km upstream} = \frac{30}{x-y} \text{ hrs}$$

$$\text{Time taken to cover 44 km downstream} = \frac{44}{x+y} \text{ hrs}$$

But, Total time of journey is 10 hours

$$\frac{30}{x-y} + \frac{44}{x+y} = 10 \quad \dots(1)$$

$$\text{Time taken to cover 40 km upstream} = \frac{40}{x-y} \text{ hrs}$$

$$\text{Time taken to cover 55 km downstream} = \frac{55}{x+y} \text{ hrs}$$

But, Total time of journey is 13 hours

$$\frac{40}{x-y} + \frac{55}{x+y} = 13 \quad \dots(2)$$

Putting $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$, in equation (1) and (2), we get

$$30u + 44v = 10 \quad \dots(3)$$

$$40u + 55v = 13 \quad \dots(4)$$

By cross-multiplication, we get

$$\frac{u}{44 \times (-13) - (-10) \times 55} = \frac{-v}{30 \times (-13) - (-10) \times 40} = \frac{1}{30 \times 55 - 44 \times 40}$$

$$\frac{u}{-572 + 550} = \frac{-v}{-390 + 400} = \frac{1}{1650 - 1760}$$

$$\frac{u}{-22} = \frac{-v}{10} = \frac{1}{-110}$$

$$u = \frac{-22}{-110} \text{ and } -v = \frac{10}{-110}$$

$$u = \frac{1}{5} \text{ and } v = \frac{1}{11}$$

$$\text{Now, } u = \frac{1}{5}$$

$$\Rightarrow \frac{1}{x-y} = \frac{1}{5}$$

$$\Rightarrow x-y=5$$

$$\text{And, } v = \frac{1}{11}$$

$$\Rightarrow \frac{1}{x+y} = \frac{1}{11}$$

$$\Rightarrow x+y=11 \quad \dots(5)$$

Adding equation (5) and equation (6), we get:

$$2y = 5+11$$

$$\Rightarrow x = \frac{16}{2} = 8$$

Putting $x=8$ in equation (5), we get:

$$8+y=11$$

$$\Rightarrow y=11-8$$

$$\Rightarrow y=3$$

Hence, Speed of the stream = 3 km/hr

Speed of the boat in still water = 8 km/hr

Q4

A boat goes 24 km upstream and 28 km downstream in 6 hrs. It goes 30 km upstream and 21 km downstream in $6\frac{1}{2}$ hrs. Find the speed of the boat in still water and also speed of the stream.

Solution

Let

Speed of the boat be x and speed of the stream be y .

From the given data

we get

$$\begin{aligned}\frac{24}{x-y} + \frac{28}{x+y} &= 6 \\ \frac{30}{x-y} + \frac{21}{x+y} &= 6.5\end{aligned}$$

Let

$$\begin{aligned}\frac{1}{x-y} &= X \\ \frac{1}{x+y} &= Y\end{aligned}$$

Then the equation becomes

$$24X + 28Y = 6 \quad \dots \dots (i)$$

$$30X + 21Y = 6.5 \quad \dots \dots (ii)$$

Solving (i) and (ii) we get

$$X = \frac{1}{6} \text{ and } Y = \frac{1}{14}$$

$$\text{So } x-y=6 \text{ and } x+y=14$$

Hence

$$x = 10 \text{ kmph and } y = 4 \text{ kmph}$$

Speed of the boat is 10 kmph

Speed of the stream is 4 kmph

Q5

A man walks a certain distance with certain speed. If he walks $1/2$ km an hour faster, he takes 1 hour less. But, if he walks 1 km an hour slower, he takes 3 more hours. Find the distance covered by the man and his original rate of walking.

Solution

Let the original speed of man be x km/hr and the actual time taken by y hours. Then,
 Distance covered = $\{xy\}$ km $\rightarrow (i)$ [As $D = S \times T$]

If the speed is increased by $\frac{1}{2}$ km/hr then time of journey is reduced by 1 hour i.e., when

speed is $\left(x + \frac{1}{2}\right)$ km/hr, time of journey is $\{y - 1\}$ hours.

$$\text{Distance covered} = \left(x + \frac{1}{2}\right)(y - 1)$$

$$\Rightarrow xy = xy + x + \frac{1}{2}y - \frac{1}{2} \quad [\text{using (i)}]$$

$$\Rightarrow x + \frac{1}{2}y + \frac{1}{2} = 0$$

$$\Rightarrow 2x + y + 1 = 0 \quad \rightarrow (ii)$$

When the speed is reduced by 1 km/hr, then the time of journey is increased by 3 hours i.e.,
 When speed is $\{x - 1\}$ km/hr time of journey is $\{y + 3\}$ hours.

$$\text{Distance covered} = \{x - 1\}(y + 3)$$

$$\Rightarrow xy - \{x - 1\}(y + 3) \quad [\text{using (i)}]$$

$$\Rightarrow xy = xy + 3x - y - 3$$

$$\Rightarrow 0 = 3x - y - 3$$

$$\Rightarrow 3x - y - 3 = 0 \quad \rightarrow (iii)$$

Thus, we obtain the following system of equations:

$$2x + y + 1 = 0$$

$$3x - y - 3 = 0$$

By using cross-multiplication, we have

$$\begin{aligned} & \frac{x}{(-1) \times (-3) - \{1\}} = \frac{-y}{(2) \times (-3) - \{1\} \times \{3\}} = \frac{1}{(2) \times (-1) - \{1\} \times \{2\}} \\ \Rightarrow & \frac{x}{3+1} = \frac{-y}{-6-3} = \frac{1}{-2+3} \\ \Rightarrow & \frac{x}{4} = \frac{-y}{-9} = 1 \end{aligned}$$

$$\Rightarrow \frac{x}{4} = 1 \text{ and } \frac{y}{-9} = 1$$

$$\Rightarrow x = 4 \text{ and } y = -9$$

Putting the value of x and y in equation (i), we obtain

$$\text{Distance} = \{4 \times 9\} \text{ km} = 36 \text{ km}$$

Hence, distance covered by man = 36 km.

Original rate of walking = 4 km/hr

Q6

A person rowing at the rate of 5 km/h in still water, takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of stream.

Solution

Given, speed of boat in still water = 5 km/hr

Let the speed of the stream be x km/hr.

Speed of the boat upstream = $(5-x)$ km/hr

Speed of the boat downstream = $(5+x)$ km/hr

It is given that

Time to cover 40 km upstream = 3 × time to cover 40 km downstream

$$\Rightarrow \frac{40}{5-x} = 3 \times \frac{40}{5+x}$$

$$\Rightarrow \frac{40}{5-x} = \frac{120}{5+x}$$

$$\Rightarrow \frac{1}{5-x} = \frac{3}{5+x}$$

$$\Rightarrow 5+x = 15-3x$$

$$\Rightarrow 4x = 10$$

$$\Rightarrow x = \frac{10}{4}$$

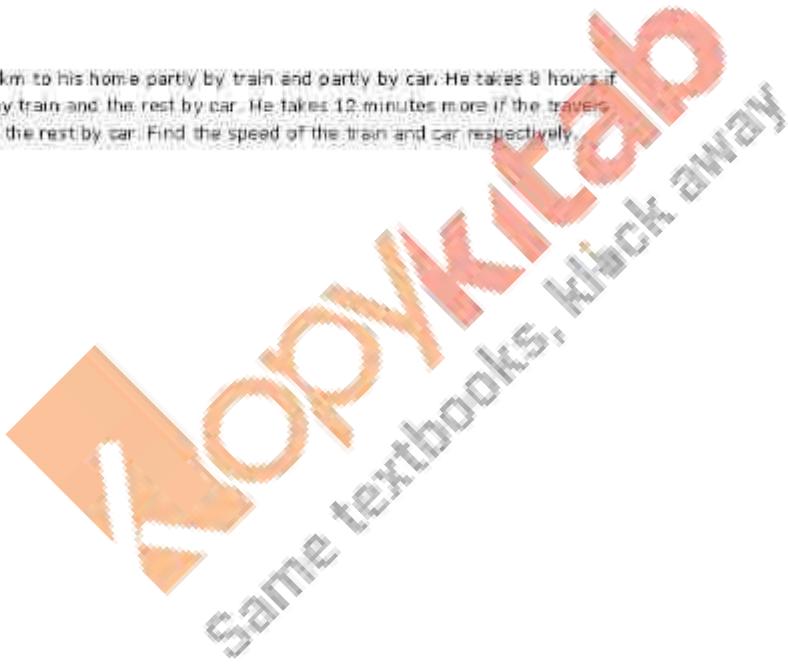
$$\Rightarrow x = 2.5$$

Thus, the speed of the stream is 2.5 km/hr.

Q7

Ramkish travels 760 km to his home partly by train and partly by car. He takes 8 hours if he travels 150 km by train and the rest by car. He takes 12 minutes more if the travels 240 km by train and the rest by car. Find the speed of the train and car respectively.

Solution



Let the speed of the train be x km/hr and that of the car be y km/hr. We have following cases:

Case I When Ramesh travels 160 km by train and the rest by car:

In this case, we have,

$$\text{Time taken by Ramesh to travel 160 km by train} = \frac{160}{x} \text{ hrs}$$

$$\text{Time taken by Ramesh to travel } (760 - 160) + 600 \text{ km by car} = \frac{600}{y} \text{ hrs}$$

$$\therefore \text{Total time taken by Ramesh to cover 760 km} = \frac{160}{x} + \frac{600}{y}$$

It is given that the total time taken is 8 hours.

$$\therefore \frac{160}{x} + \frac{600}{y} = 8$$

$$\Rightarrow 8 \left[\frac{20}{x} + \frac{75}{y} \right] = 8$$

$$\Rightarrow \frac{20}{x} + \frac{75}{y} = 1 \quad \text{---(i)}$$

Case II When Ramesh travels 240 km by train and the rest by car:

In this case, we have

$$\text{Time taken by Ramesh to travel 240 km by train} = \frac{240}{x} \text{ hrs}$$

$$\text{Time taken by Ramesh to travel } (760 - 240) + 520 \text{ km by car} = \frac{520}{y} \text{ hrs}$$

In this case, total time of the journey is 8 hrs 12 minutes

$$\therefore \frac{240}{x} + \frac{520}{y} = 8 \text{ hrs 12 minutes}$$

$$\Rightarrow \frac{240}{x} + \frac{520}{y} = 8 \frac{12}{60}$$

$$\Rightarrow \frac{240}{x} + \frac{520}{y} = \frac{41}{5} \quad \text{---(ii)}$$

Thus, we obtain the following system of equations:

$$\frac{20}{x} + \frac{75}{y} = 1$$

$$\frac{240}{x} + \frac{520}{y} = \frac{41}{5}$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the above system reduces to

$$20u + 75v = 1 \quad \text{---(iii)}$$

$$240u + 520v = \frac{41}{5} \quad \text{---(iv)}$$

Multiplying equation (iii) by 12, we get

$$240u + 900v = 12 \quad \text{---(v)}$$

Subtracting equation (iv) by equation (v), we get

$$900v - 520v = 12 - \frac{41}{5}$$

$$\Rightarrow 380v = \frac{60 - 41}{5}$$

$$\Rightarrow 380v = \frac{19}{5}$$

$$\Rightarrow v = \frac{19}{5} \times \frac{1}{380}$$

$$\Rightarrow v = \frac{1}{5} \times \frac{1}{20}$$

$$\Rightarrow v = \frac{1}{100}$$

Putting $v = \frac{1}{100}$ in equation(v), we get

$$240u + 900 \times \frac{1}{100} = 12$$

$$\Rightarrow 240u + 9 = 12$$

$$\Rightarrow 240u = 12 - 9 = 3$$

$$\Rightarrow u = \frac{3}{240} = \frac{1}{80}$$

$$\text{Now, } u = \frac{1}{80}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{80}$$

$$\Rightarrow x = 80$$

$$\text{And, } v = \frac{1}{100}$$

$$\Rightarrow \frac{1}{y} = \frac{1}{100}$$

$$\Rightarrow y = 100$$

Hence, speed of the train = 80 km/hr

Speed of the car = 100 km/hr

Q8

A man travels 600 km partly by train and partly by car. If he travels 400 km by train and the rest by car, it takes him 6 hours and 30 minutes. But, if he travels 200 km by train and the rest by car, he takes half an hour longer. Find the speed of the train and that of the car.

Solution

Let the speed of the train be x km/hr and that of the car be y km/hr. We have following cases:

Case I When Ramesh travels 400 km by train and the rest by car:

In this case, we have

$$\text{Time taken by the man to travel } 400 \text{ km by train} = \frac{400}{x}$$

$$\text{Time taken by the man to travel } (600 - 400) = 200 \text{ km by car} = \frac{200}{y}$$

In this case, total time of the journey is 6 hrs 30 minutes.

$$\Rightarrow \frac{400}{x} + \frac{200}{y} = 6 \text{ hrs 30 minutes}$$

$$\Rightarrow \frac{400}{x} + \frac{200}{y} = 6\frac{1}{2}$$

$$\Rightarrow \frac{400}{x} + \frac{200}{y} = \frac{13}{2} \quad \text{---(i)}$$

Case II When he travels 200 km by train and the rest by car:

In this case, we have

$$\text{Time taken by the man to travel } 200 \text{ km by train} = \frac{200}{x} \text{ hrs}$$

$$\text{Time taken by the man to travel } (600 - 200) = 400 \text{ km by car} = \frac{400}{y} \text{ hrs}$$

In this case, total time of journey is $\left(\frac{13}{2} + \frac{1}{2}\right) = 7$ hrs.

$$\Rightarrow \frac{200}{x} + \frac{400}{y} = 7 \quad \text{---(ii)}$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, in equation (i) and (ii), we get

$$400u + 200v = \frac{13}{2} \quad \text{---(iii)}$$

$$200u + 400v = 7 \quad \text{---(iv)}$$

Multiplying equation (ii) by 2, we get

$$800u + 400v = 13 \quad \text{---(v)}$$

Subtracting equation (iv) by equation (v), we get

$$800u - 200u = 13 - 7$$

$$\Rightarrow 600u = 6$$

$$\Rightarrow u = \frac{6}{600} = \frac{1}{100}$$

Putting $u = \frac{1}{100}$ in equation (iv), we get

$$200 \times \frac{1}{100} + 400v = 7$$

$$\Rightarrow 2 + 400v = 7$$

$$\Rightarrow 400v = 7 - 2$$

$$\Rightarrow 400v = 5$$

$$\Rightarrow v = \frac{5}{400} = \frac{1}{80}$$

$$\begin{aligned} \text{Now, } v &= \frac{1}{100} \\ \Rightarrow \frac{1}{x} &= \frac{1}{100} \\ \Rightarrow x &= 100 \end{aligned}$$

$$\begin{aligned} \text{And, } v &= \frac{1}{60} \\ \Rightarrow \frac{1}{y} &= \frac{1}{60} \\ \Rightarrow y &= 60 \end{aligned}$$

Hence, speed of the train = 100 km/hr
Speed of the car = 60 km/hr.

Q9

Places A and B are 90 km apart from each other on a highway. A car starts from A and other from B at the same time. If they move in the same direction, they meet in 9 hours and if they move in opposite directions, they meet in 1 hour and 20 minutes. Find the speeds of the cars.

Solution



Let X and Y be two cars starting from points A and B , respectively.
Let the speed of car X be x km/hr and that of car Y be y km/hr.

Case I, When two cars move in the same direction:

Suppose two cars meet at point Q . Then,

Distance travelled by car $X = AQ$

Distance travelled by car $Y = BQ$

It is given that two cars meet in 8 hours.

Distance travelled by car X in 8 hours = $8x$ km

$$\Rightarrow AQ = 8x$$

Distance travelled by car Y in 8 hours = $8y$ km

$$\Rightarrow BQ = 8y$$

Clearly, $AQ - BQ = AB$

$$\Rightarrow 8x - 8y = 80 \quad [AB = 80 \text{ km}]$$

$$\Rightarrow 8(x - y) = 80$$

$$\Rightarrow x - y = 10 \quad \text{---(i)}$$

Case II, When two cars move in opposite directions:

Suppose two cars meet at point P . Then,

Distance travelled by car $X = AP$,

Distance travelled by car $Y = BP$

In this case, two cars meet in 1 hour 20 minutes = $1\frac{1}{3} = \frac{4}{3}$ hrs

Distance travelled by car X in $\frac{4}{3}$ hours = $\frac{4}{3}x$ km

$$\Rightarrow AP = \frac{4}{3}x$$

Distance travelled by car Y in $\frac{4}{3}$ hours = $\frac{4}{3}y$ km

$$\Rightarrow BP = \frac{4}{3}y$$

Clearly, $AP + BP = AB$

$$\Rightarrow \frac{4}{3}x + \frac{4}{3}y = 80$$

$$\Rightarrow \frac{4}{3}(x + y) = 80$$

$$\Rightarrow x + y = \frac{80 \times 3}{4}$$

$$\Rightarrow x + y = 60 \quad \text{---(ii)}$$

Adding equation (i) and equation (ii), we get

$$2x = 10 + 60$$

$$\Rightarrow 2x = 70$$

$$\Rightarrow x = \frac{70}{2} = 35$$

Putting $x = 35$ in equation (ii), we get,

$$35 + y = 60$$

$$\Rightarrow y = 60 - 35 = 25$$

Hence, speed of car X is 35 km/hr and speed of car Y is 25 km/hr.

Q10

A boat goes 12 km upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream in the same time. Find the speed of the boat in still water and the speed of the stream.

Solution

Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr.

Then,

$$\text{Speed upstream} = \{x - y\} \text{ km/hr}$$

$$\text{Speed downstream} = \{x + y\} \text{ km/hr}$$

$$\text{Now, Time taken to cover 12 km upstream} = \frac{12}{x-y} \text{ hrs}$$

$$\text{Time taken to cover 40 km downstream} = \frac{40}{x+y} \text{ hrs}$$

But, Total time of journey is 8 hours

$$\frac{12}{x-y} + \frac{40}{x+y} = 8 \quad \text{---(i)}$$

$$\text{Time taken to cover 16 km upstream} = \frac{16}{x-y} \text{ hrs.}$$

$$\text{Time taken to cover 32 km downstream} = \frac{32}{x+y} \text{ hrs.}$$

But, Total time of journey is 8 hours.

$$\frac{16}{x-y} + \frac{32}{x+y} = 8 \quad \text{---(ii)}$$

Putting $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$, in equation (i) and (ii), we get

$$12u + 40v = 8$$

$$\Rightarrow 4(3u + 10v) = 8$$

$$\Rightarrow 3u + 10v = 2$$

$$\Rightarrow 3u + 10v - 2 = 0 \quad \text{---(iii)}$$

$$\text{And, } 16u + 32v = 8$$

$$\Rightarrow 8(2u + 4v) = 8$$

$$\Rightarrow 2u + 4v = 1$$

$$\Rightarrow 2u + 4v - 1 = 0 \quad \text{---(iv)}$$

By cross-multiplication, we get

$$\frac{u}{10 \times (-1) - (-2) \times 4} = \frac{-v}{(-2) \times 2 - 3 \times 4} = \frac{1}{3 \times 4 - 2 \times 10}$$

$$\Rightarrow \frac{u}{-10 + 8} = \frac{-v}{-3 + 4} = \frac{1}{12 - 20}$$

$$\Rightarrow \frac{u}{-2} = \frac{-v}{1} = \frac{1}{-8}$$

$$\Rightarrow \frac{u}{-2} = \frac{1}{-8} \text{ and } \frac{-v}{1} = \frac{1}{-8}$$

$$\Rightarrow u = \frac{2}{8} \text{ and } v = \frac{1}{8}$$

$$\Rightarrow u = \frac{1}{4} \text{ and } v = \frac{1}{8}$$

$$\text{Now, } u = \frac{1}{4}$$

$$\Rightarrow \frac{1}{x-y} = \frac{1}{4}$$

$$\Rightarrow x - y = 4 \quad \text{---(v)}$$

CopyKitab
Same textbooks, klick away

$$\begin{aligned} \text{And, } y &= \frac{1}{8} \\ \Rightarrow \frac{1}{x+y} &= \frac{1}{3} \\ \Rightarrow x+y &= 3 \end{aligned}$$

$\quad \quad \quad = \{v\}$

Adding equation $\{v\}$ and equation $\{vi\}$, we get:

$$\begin{aligned} 2x &= 4+8 \\ \Rightarrow x &= \frac{12}{2} = 6 \end{aligned}$$

Putting $x = 10$ in equation $\{vi\}$, we get:

$$\begin{aligned} 6+y &= 8 \\ \Rightarrow y &= 8-6 = 2 \end{aligned}$$

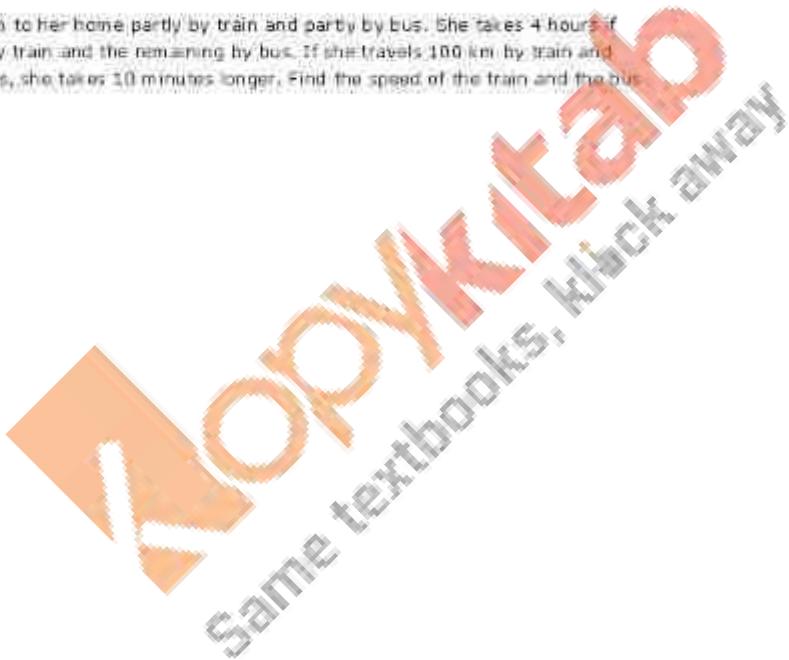
Hence, speed of the boat in still water = 6 km/hr

Speed of the stream = 2 km/hr

Q11

Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 50 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Solution



Let the speed of the train be x km/hr and that of the bus be y km/hr. We have the following cases:

Case I: When Rochi travels 60 km by train and the rest by bus.
In this case, we have

$$\text{Time taken by Rochi to travel } 60 \text{ km by train} = \frac{60}{x} \text{ hrs}$$

$$\text{Time taken by Rochi to travel } (300 - 60) = 240 \text{ km by bus} = \frac{240}{y} \text{ hrs}$$

$$\text{Total time taken by Rochi to cover } 300 \text{ km} = \frac{60}{x} + \frac{240}{y}$$

It is given that the total time taken is 4 hours.

$$\frac{60}{x} + \frac{240}{y} = 4$$

$$\Rightarrow 4 \left[\frac{10}{x} + \frac{60}{y} \right] = 4$$

$$\Rightarrow \frac{15}{x} + \frac{60}{y} = 1 \quad \text{---(i)}$$

Case II: When Rochi travels 100 km by train and the rest by bus.

In this case, we have

$$\text{Time taken by Rochi to travel } 100 \text{ km by train} = \frac{100}{x} \text{ hrs}$$

$$\text{Time taken by Rochi to travel } (300 - 100) = 200 \text{ km by bus} = \frac{200}{y} \text{ hrs}$$

In this case, total time of the journey is 4 hrs 10 minutes.

$$\frac{100}{x} + \frac{200}{y} = 4 \text{ hrs 10 minutes}$$

$$\Rightarrow \frac{100}{x} + \frac{200}{y} = 4\frac{1}{6}$$

$$\Rightarrow \frac{100}{x} + \frac{200}{y} = \frac{25}{6}$$

$$\Rightarrow 25 \left(\frac{4}{x} + \frac{8}{y} \right) = \frac{25}{6}$$

$$\Rightarrow \frac{4}{x} + \frac{8}{y} = \frac{1}{6}$$

$$\Rightarrow 6 \left(\frac{4}{x} + \frac{8}{y} \right) = 1$$

$$\Rightarrow \frac{24}{x} + \frac{48}{y} = 1 \quad \text{---(ii)}$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, in equation (i) and (ii), we get

$$15u + 60v = 1 \quad \text{---(i)}$$

$$24u + 48v = 1 \quad \text{---(ii)}$$

By cross-multiplication, we have

$$\frac{u}{60 \times (-1) - 48 \times (-1)} = \frac{-v}{15 \times (-1) - 24 \times (-1)} = \frac{1}{15 \times 48 - 60 \times 24}$$

$$\Rightarrow \frac{u}{-60 + 48} = \frac{-v}{-15 + 24} = \frac{1}{720 - 1440}$$

$$\begin{aligned} \Rightarrow & \frac{u}{-12} = \frac{-v}{9} = \frac{1}{-720} \\ \Rightarrow & \frac{u}{-12} = \frac{1}{-720} \text{ and } \frac{-v}{9} = \frac{1}{-720} \\ \Rightarrow & u = \frac{-12}{-720} \text{ and } v = \frac{-9}{-720} \\ \Rightarrow & u = \frac{1}{60} \text{ and } v = \frac{1}{80} \end{aligned}$$

Now, $u = \frac{1}{60}$

$$\begin{aligned} \Rightarrow & \frac{1}{x} = \frac{1}{60} \\ \Rightarrow & x = 60 \text{ km/hr} \end{aligned}$$

and, $v = \frac{1}{80}$

$$\begin{aligned} \Rightarrow & \frac{1}{y} = \frac{1}{80} \\ \Rightarrow & y = 80 \text{ km/hr} \end{aligned}$$

Hence, speed of the train = 60 km/hr
Speed of the car = 80 km/hr.

Q12

Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

Solution

Let the speed of Ritu in still water and the speed of stream be x km/h and y km/h respectively.

Speed of Ritu while rowing upstream = $(x-y)$ km/h
Speed of Ritu while rowing downstream = $(x+y)$ km/h

According to the question,

$$2(x+y) = 20$$

$$\Rightarrow x+y=10 \quad \dots (1)$$

$$2(x-y)=4$$

$$\Rightarrow x-y=2 \quad \dots (2)$$

Adding equations (1) and (2), we obtain:

$$2x=12$$

$$\Rightarrow x=6$$

Putting the value of x in equation (2), we obtain:

$$y=4$$

Thus, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h.

Q13

A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.

Solution

Let the speed of boat in still water be x km/hr and the speed of the stream be y km/hr.

Speed of the boat upstream = $(x - y)$ km/hr

Speed of the boat downstream = $(x + y)$ km/hr

Now, time taken by boat to travel 30 km upstream = $\frac{30}{x-y}$

Time taken by boat to travel 28 km downstream = $\frac{28}{x+y}$

Then, we have $\frac{30}{x-y} + \frac{28}{x+y} = 7$ (i)

Also, time taken by boat to travel 21 km upstream = $\frac{21}{x-y}$

Time taken by boat to travel 21 km downstream = $\frac{21}{x+y}$

Then, we have $\frac{21}{x-y} + \frac{21}{x+y} = 5$ (ii)

Putting $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$ in equations (i) and (ii), we get

$$30u + 28v = 7$$

$$\Rightarrow 30u + 28v - 7 = 0 \quad \dots \text{(iii)}$$

$$21u + 21v = 5$$

$$\Rightarrow 21u + 21v - 5 = 0 \quad \dots \text{(iv)}$$

By cross multiplication, we have

$$\frac{u}{28(-5) - 21(-7)} = \frac{-v}{30(-5) - 21(-7)} = \frac{1}{30 \times 21 - 21 \times 28}$$

$$\Rightarrow \frac{u}{-140 + 147} = \frac{-v}{-150 + 147} = \frac{1}{560 - 588}$$

$$\Rightarrow \frac{u}{7} = \frac{-v}{3} = \frac{1}{-28}$$

$$\text{Now, } \Rightarrow \frac{u}{7} = \frac{1}{-28} \Rightarrow 42u = 7 \Rightarrow u = \frac{7}{42} = \frac{1}{6} = \frac{1}{x-y} = \frac{1}{6}$$

$$\Rightarrow x - y = 6 \quad \dots \text{(iii)}$$

$$\text{And, } \frac{v}{3} = \frac{1}{-28} \Rightarrow 42v = -3 \Rightarrow v = \frac{-3}{42} = \frac{1}{-14} = \frac{1}{x+y} = \frac{1}{-14}$$

$$\Rightarrow x + y = 14 \quad \dots \text{(iv)}$$

Adding (iii) and (iv), we get $2x = 20 \Rightarrow x = 10$

$$\Rightarrow 10 + y = 14 \Rightarrow y = 4$$

Thus, the speed of the boat in still water is 10 km/hr and the speed of the stream is 4 km/hr.

Q14

Abdul travelled 300 km by train and 200 km by taxi. It took him 5 hours 30 minutes. But if he travels 260 km by train and 240 km by taxi he takes 6 minutes longer. Find the speed of the train and that of the taxi.

Solution

Let the speed of the train be x km/hr and that of the taxi be y km/hr. We have the following cases:

Case I: When Abdul travels 300 km by train and 200 km by taxi.

In this case, we have

$$\text{Time taken by Abdul to travel 300 km by train} = \frac{300}{x} \text{ hrs}$$

$$\text{Time taken by Abdul to travel 200 km by taxi} = \frac{200}{y} \text{ hrs}$$

$$\text{Total time taken by Abdul} = \frac{300}{x} + \frac{200}{y}$$

It is given that the total time taken is 5 hours 30 minutes.

$$\frac{300}{x} + \frac{200}{y} = 5 \text{ hours } 30 \text{ minutes}$$

$$\Rightarrow \frac{300}{x} + \frac{200}{y} = 5\frac{1}{2}$$

$$\Rightarrow \frac{300}{x} + \frac{200}{y} = \frac{11}{2}$$

$$\Rightarrow \frac{600}{x} + \frac{400}{y} = 11$$

---(i)

Case II: When Abdul travels 260 km by train and 240 km by taxi.

In this case, we have

$$\text{Time taken by Abdul to travel 260 km by train} = \frac{260}{x} \text{ hrs}$$

$$\text{Time taken by Abdul to travel 240 km by taxi} = \frac{240}{y} \text{ hrs}$$

In this case, total time of the journey is {5 hours 30 minutes + 5 minutes}

$$= 5\frac{1}{2} + \frac{1}{10}$$

$$= \frac{11}{2} + \frac{1}{10}$$

$$= \frac{55+1}{10}$$

$$= \frac{56}{10}$$

$$= \frac{28}{5}$$

$$= \frac{28}{5} \text{ hrs}$$

$$\Rightarrow \frac{260}{x} + \frac{240}{y} = \frac{28}{5}$$

$$\Rightarrow \left(\frac{65}{x} + \frac{60}{y} \right) \times \frac{28}{5}$$

$$\Rightarrow \frac{65}{x} + \frac{60}{y} = \frac{7}{5}$$

$$\Rightarrow \frac{65 \times 5}{x} + \frac{60 \times 5}{y} = 7$$

$$\Rightarrow \frac{325}{x} + \frac{300}{y} = 7$$

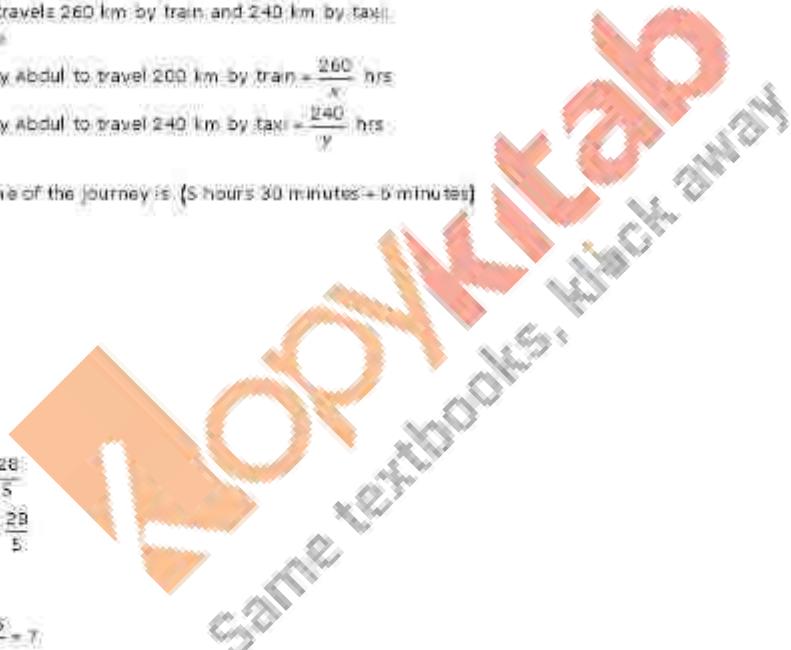
---(ii)

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, in equation (i) and (ii), we get

$$600u + 400v = 11$$

$$\Rightarrow 600u + 400v - 11 = 0$$

---(iii)



$$\text{And, } \begin{aligned} & 325u + 300v = 7 \\ \Rightarrow & 325u + 300v - 7 = 0 \end{aligned} \quad \text{---(iv)}$$

By cross-multiplication, we have

$$\begin{aligned} & \frac{u}{400 \times \{-7\} - \{-11\} \times 300} = \frac{-v}{600 \times \{-7\} - \{-11\} \times 325} = \frac{1}{600 \times 300 - 400 \times 325} \\ \Rightarrow & \frac{u}{-2800 + 3300} = \frac{-v}{-4200 + 3575} = \frac{1}{180000 - 130000} \\ \Rightarrow & \frac{u}{500} = \frac{-v}{-625} = \frac{1}{50000} \\ \Rightarrow & \frac{u}{500} = \frac{v}{625} = \frac{1}{50000} \\ \Rightarrow & u = \frac{1}{50000} \text{ and } v = \frac{625}{50000} \\ \Rightarrow & u = \frac{500}{50000} \text{ and } v = \frac{625}{50000} \\ \Rightarrow & u = \frac{1}{100} \text{ and } v = \frac{1}{80} \end{aligned}$$

$$\text{Now, } u = \frac{1}{100}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{100}$$

$$\Rightarrow x = 100$$

$$\text{And, } v = \frac{1}{80}$$

$$\Rightarrow \frac{1}{y} = \frac{1}{80}$$

$$\Rightarrow y = 80$$

Hence, speed of the train = 100 km/hr
Speed of the taxi = 80 km/hr.

Q15

A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And if the train were slower by 10 km/h, it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Solution

Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel be d km.

Now, Speed = $\frac{\text{Distance travelled}}{\text{Time taken to travel that distance}}$

$$x = \frac{d}{t}$$

$$\text{Or, } d = xt \quad \dots (1)$$

According to the question,

$$(x+10) = \frac{d}{(t-2)}$$

$$(x+10)(t-2) = d$$

$$xt + 10t - 2x - 20 = d$$

By using equation (1), we obtain:
 $-2x + 10t = 20 \quad \dots (2)$

$$(x-10) = \frac{d}{(t+3)}$$

$$(x-10)(t+3) = d$$

$$xt - 10t + 3x - 30 = d$$

By using equation (1), we obtain:
 $3x - 10t = 30 \quad \dots (3)$

Adding equations (2) and (3), we obtain:

$$x = 50$$

Substituting the value of x in equation (2), we obtain:

$$(-2) \times (50) + 10t = 20$$

$$-100 + 10t = 20$$

$$10t = 120$$

$$t = 12$$

From equation (1), we obtain:

$$d = xt = 50 \times 12 = 600$$

Thus, the distance covered by the train is 600 km.

Concept Insight: To solve this problem, it is very important to remember the relation $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$. Now, all these three quantities are unknown. So, we will represent these

by three different variables.

By using the given conditions, a pair of equations will be obtained. Mind one thing that the equations obtained will not be linear. But they can be reduced to linear form by using the fact that $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$. Then two linear equations can be formed which can be solved easily by elimination method.

Q16

Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hours. What are the speeds of two cars?

Solution

Let the speed of first car and second car be u km/h and v km/h respectively.

According to the question,

$$5(u - v) = 100$$

$$\Rightarrow u - v = 20 \quad \dots (1)$$

$$1(u + v) = 100$$

$$\Rightarrow u + v = 100 \quad \dots (2)$$

Adding equations (1) and (2), we obtain:

$$2u = 120$$

$$u = 60$$

Substituting the value of u in equation (2), we obtain:

$$v = 40$$

Hence, speed of the first car is 60 km/h and speed of the second car is 40 km/h.

Q17

While covering a distance of 30 km, Ajeeet takes 2 hours more than Amit. If Ajeeet doubles his speed, he would take 1 hour less than Amit. Find their speeds of walking.

Solution

Let the speed of Ajeeet and Amit be x km/hr and y km/hr respectively. Then,

$$\text{Time taken by Ajeeet to cover 30 km} = \frac{30}{x} \text{ hrs}$$

$$\text{And, Time taken by Amit to cover 30 km} = \frac{30}{y} \text{ hrs}$$

By the given conditions, we have

$$\begin{aligned} \frac{30}{x} - \frac{30}{y} &= 2 \\ \Rightarrow \frac{15}{x} - \frac{15}{y} &= 1 \quad \text{--- (i)} \end{aligned}$$

If Ajeeet doubles his pace, then speed of Ajeeet is $2x$ km/hr.

$$\text{Time taken by Ajeeet to cover 30 km} = \frac{30}{2x} \text{ hrs}$$

$$\text{Time taken by Amit to cover 30 km} = \frac{30}{y} \text{ hrs}$$

According to the given conditions, we have

$$\begin{aligned} \frac{30}{x} - \frac{30}{2x} &= 1 \\ \Rightarrow \frac{30}{y} - \frac{15}{x} &= 1 \quad \text{--- (ii)} \end{aligned}$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, in equations (i) and (ii), we get

$$15u - 15v = 1 \quad \text{--- (iii)}$$

$$30v - 15u = 1 \quad \text{--- (iv)}$$

Adding equation (iii) and equation (iv), we get

$$30v - 15v = 1 + 1$$

$$\Rightarrow 15v = 2$$

$$\Rightarrow v = \frac{2}{15}$$

Putting $v = \frac{2}{15}$ in equation (ii), we get

$$15u - 15 \times \frac{2}{15} = 1$$

$$\Rightarrow 15u - 2 = 1$$

$$\Rightarrow 15u = 1 + 2$$

$$\Rightarrow 15u = 3$$

$$\Rightarrow u = \frac{3}{15} = \frac{1}{5}$$

$$\text{Now, } u = \frac{1}{5}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{5}$$

$$\Rightarrow x = 5$$

$$\text{And, } v = \frac{2}{15}$$

$$\Rightarrow \frac{1}{y} = \frac{2}{15}$$

$$\Rightarrow y = \frac{15}{2} = 7.5$$

Hence, Ajeeet's speed = 5 km/hr and Amit's speed = 7.5 km/hr.

Q18

A takes 3 hours more than B to walk a distance of 30 km. But, if A doubles his pace (speed), he is ahead of B by $1\frac{1}{2}$ hours. Find the speeds of A and B.

Solution

Let the speed of A and B be x km/hr and y km/hr respectively. Then,

$$\text{Time taken by A to cover 30 km} = \frac{30}{x} \text{ hrs}$$

$$\text{And, Time taken by B to cover 30 km} = \frac{30}{y} \text{ hrs.}$$

By the given conditions, we have:

$$\begin{aligned} \frac{30}{x} - \frac{30}{y} &= 2 \\ \Rightarrow \frac{30}{x} - \frac{30}{y} &= 2 \quad \text{--- (i)} \end{aligned}$$

If A doubles his pace, then speed of A is $2x$ km/hr

$$\text{Time taken by A to cover 30 km} = \frac{30}{2x} \text{ hrs.}$$

$$\text{Time taken by B to cover 30 km} = \frac{30}{y} \text{ hrs.}$$

According to the given conditions, we have

$$\begin{aligned} \frac{30}{x} - \frac{30}{2x} &= 1 \\ \Rightarrow \frac{30}{x} - \frac{30}{2x} &= 1 \\ \Rightarrow \frac{10}{x} - \frac{10}{2x} &= 1 \\ \Rightarrow \frac{10}{x} - \frac{5}{x} &= 1 \\ \Rightarrow \frac{5}{x} &= 1 \\ \Rightarrow \frac{-5}{x} + \frac{10}{x} &= 1 \\ \Rightarrow \frac{-10 + 20}{x} &= 1 \quad \text{--- (ii)} \end{aligned}$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, in equation (i) and (ii), we get:

$$\begin{aligned} \Rightarrow 10u - 10v &= 1 \quad \text{--- (iii)} \\ \Rightarrow -10u + 20v &= 1 \quad \text{--- (iv)} \end{aligned}$$

Adding equations (iii) and (iv), we get

$$-10v + 20v = 1 - 1$$

$$\Rightarrow 10v = 2$$

$$\Rightarrow v = \frac{2}{10} = \frac{1}{5}$$

Putting $v = \frac{1}{5}$ in equation (iii), we get

$$10u - 10 \times \frac{1}{5} = 1$$

$$\Rightarrow 10u - 2 = 1$$

$$\Rightarrow 10u = 1 + 2$$

$$\Rightarrow 10u = 3$$

$$\Rightarrow u = \frac{3}{10}$$

$$\text{Now, } u = \frac{3}{10}$$

$$\Rightarrow \frac{1}{x} = \frac{3}{10}$$

$$\Rightarrow x = \frac{10}{3}$$

$$\text{And, } v = \frac{1}{5}$$

$$\Rightarrow \frac{1}{y} = \frac{1}{5}$$

$$\Rightarrow y = 5$$

Hence, A's speed = $\frac{10}{3}$ km/hr and B's speed = 5 km/hr.



Exercise 3.11**Q1**

If in a rectangle, the length is increased and breadth reduced each by 2 units, the area is reduced by 28 square units. If, however, the length is reduced by 1 unit and the breadth increased by 2 units, the area increases by 33 square units. Find the area of the rectangle.

Solution

Let the length and breadth of the rectangle be x and y units respectively. Then,

$$\text{Area} = xy \text{ sq. units}$$

If length is increased by 2 units and the breadth is reduced by 2 units, then area is reduced by 28 square units.

$$\begin{aligned} & xy - 28 = (x+2)(y-2) \\ \Rightarrow & xy - 28 = xy + 2x - 2y - 4 \\ \Rightarrow & -28 + 4 = -2x + 2y \\ \Rightarrow & -24 = -2(x - y) \\ \Rightarrow & 2(x - y) = 24 \\ \Rightarrow & x - y = 12 \quad \text{---(i)} \end{aligned}$$

When length is reduced by 1 unit and the breadth is increased by 2 units, then area is increased by 33 square units.

$$\begin{aligned} & xy + 33 = (x-1)(y+2) \\ \Rightarrow & xy + 33 = xy + 2x - y - 2 \\ \Rightarrow & 33 + 2 = 2x - y \\ \Rightarrow & 35 = 2x - y \\ \Rightarrow & 2x - y = 35 \quad \text{---(ii)} \end{aligned}$$

Subtracting equation (i) by equation (ii), we get:

$$\begin{aligned} & 2x - x = 35 - 12 \\ \Rightarrow & x = 23 \end{aligned}$$

Putting $x = 23$ in equation (i), we get:

$$\begin{aligned} & 23 - y = 12 \\ \Rightarrow & -y = 12 - 23 \\ \Rightarrow & -y = -11 \\ \Rightarrow & y = 11 \\ \therefore & \text{Area} = x \times y \\ & = 23 \times 11 \\ & = 253 \text{ sq. units} \quad [x = 23, y = 11] \end{aligned}$$

Hence, area of rectangle is 253 sq. units.

Q2

The area of a rectangle remains the same if the length is increased by 2 metres and the breadth is decreased by 3 metres. The area remains unaffected if the length is decreased by 7 metres and breadth is increased by 5 metres. Find the dimensions of the rectangle.

Solution

Let the length and breadth of the rectangle be x m and y m respectively. Then,

$$\text{Area} = xy \text{ m}^2$$

If length is increased by 7m and the breadth is decreased by 3m, the area remains same:

$$\begin{aligned} & xy = (x+7)(y-3) \\ \Rightarrow & xy = xy + 7y - 3x - 21 \\ \Rightarrow & 3x - 7y = -21 \quad \text{---(i)} \end{aligned}$$

When length is decreased by 7m and breadth is increased by 5m, then area remains unaffected

$$\begin{aligned} & xy = (x-7)(y+5) \\ \Rightarrow & xy = xy + 5x - 7y - 35 \\ \Rightarrow & 5x - 7y = 35 \quad \text{---(ii)} \\ \Rightarrow & 5x - 7y = 35 \end{aligned}$$

Subtracting equation (i) from (ii), we get

$$\begin{aligned} & 5x - 3x = 35 - (-21) \\ \Rightarrow & 2x = 56 \\ \Rightarrow & x = \frac{56}{2} = 28 \end{aligned}$$

Putting $x = 28$ in equation (i), we get

$$\begin{aligned} & 5 \times 28 - 7y = 35 \\ \Rightarrow & 140 - 7y = 35 \\ \Rightarrow & -7y = 35 - 140 \\ \Rightarrow & -7y = -105 \\ \Rightarrow & y = \frac{105}{7} = 15 \end{aligned}$$

Hence, length and breadth of the rectangle are 28m and 15m respectively.

Q3

In a rectangle, if the length is increased by 3 metres and breadth is decreased by 4 metres, the area of the rectangle is reduced by 67 square metres. If length is reduced by 1 metre and breadth is increased by 4 metres, the area is increased by 89 square metres. Find the dimensions of the rectangle.

Solution

Let the length and breadth of the rectangle be x m and y m respectively. Then,

$$\text{Area} = xy \text{ m}^2$$

If length is increased by 3m and breadth is decreased by 4m, then area is reduced by 57m^2 ,

$$xy - 57 = (x+3)(y-4)$$

$$\Rightarrow xy - 57 = xy + 3y - 4x - 12$$

$$\Rightarrow 4x - 3y = -12 + 57$$

$$\Rightarrow 4x - 3y = 45 \quad \text{---(i)}$$

When length is decreased by 1m and breadth is increased by 4m, then area is increased by 89m^2

$$xy + 89 = (x-1)(y+4)$$

$$\Rightarrow xy + 89 = xy + 4x - y - 4$$

$$\Rightarrow 89 + 4 = 4x - y$$

$$\Rightarrow 93 = 4x - y$$

$$\Rightarrow 4x - y = 93 \quad \text{---(ii)}$$

Subtracting equation (i) by equation (ii), we get

$$-y + 3y = 93 - 45$$

$$\Rightarrow 2y = 48$$

$$\Rightarrow y = \frac{48}{2} = 24$$

Putting $y = 24$ in equation (ii), we get

$$4x - 24 = 93$$

$$\Rightarrow 4x = 93 + 24$$

$$\Rightarrow 4x = 117$$

$$\Rightarrow x = \frac{117}{4} = 28.75$$

Hence, length and breadth of the rectangle are 28.75m and 24m respectively.

Q4

The incomes of X and Y are in the ratio of $8:7$ and their expenditures are in the ratio $19:16$. If each saves Rs 1250, find their incomes.

Solution

Let the income of X be Rs $8x$ and the income of Y be Rs $7y$.

Further, let the expenditures of X and Y be $19y$ and $16y$ respectively. Then,

$$\text{Saving of } X = 8x - 19y$$

$$\text{Saving of } Y = 7y - 16y$$

$$8x - 19y = 1250 \quad \text{---(i)}$$

$$\text{and, } 7y - 16y = 1250 \quad \text{---(ii)}$$

Multiplying equation (i) by 7, and equation (ii) by 8, we get

$$56x - 133y = 8750 \quad \text{---(iii)}$$

$$56y - 128y = 10,000 \quad \text{---(iv)}$$

Subtracting equation (iv) from equation (iii), we get

$$-133y + 128y = 8750 - 10,000$$

$$\Rightarrow -5y = -1250$$

$$\Rightarrow y = \frac{-1250}{-5} = 250$$

Putting $y = 250$ in equation (i), we get

$$8x - 19 \times 250 = 1250$$

$$\Rightarrow 8x - 4750 = 1250$$

$$\Rightarrow 8x = 1250 + 4750$$

$$\Rightarrow x = \frac{6000}{8} = 750$$

Thus, X's income $= 8x = 8 \times 750 = \text{Rs } 6000$

Y's income $= 7y = 7 \times 750 = \text{Rs } 5250$

Q5

A and B each has some money. If A gives Rs 30 to B, then B will have twice the money left with A. But, if B gives Rs 10 to A, then A will have thrice as much as is left with B. How much money does each have?

Solution

Let A and B each has Rs x and Rs y respectively. Then,

$$\begin{aligned} & 2(x - 30) = y + 30 \quad [\text{given}] \\ \Rightarrow & 2x - 60 = y + 30 \\ \Rightarrow & 2x - y = 30 + 60 \\ \Rightarrow & 2x - y = 90 \quad \rightarrow (i) \end{aligned}$$

And, $x + 10 = 3(y - 10)$

$$\begin{aligned} \Rightarrow & x + 10 = 3y - 30 \\ \Rightarrow & x - 3y = -30 - 10 \\ \Rightarrow & x - 3y = -40 \quad \rightarrow (ii) \end{aligned}$$

Multiplying equation (ii) by 2, we get:

$$2x - 6y = -80 \quad \rightarrow (iii)$$

Subtracting equation (iii) from equation (i), we get

$$\begin{aligned} & -y + 6y = 90 + 80 \\ \Rightarrow & 5y = 170 \\ \Rightarrow & y = \frac{170}{5} = 34 \end{aligned}$$

Putting $y = 34$ in equation (i), we get

$$\begin{aligned} & 2x - 34 = 90 \\ \Rightarrow & 2x = 90 + 34 \\ \Rightarrow & 2x = 124 \\ \Rightarrow & x = \frac{124}{2} = 62 \end{aligned}$$

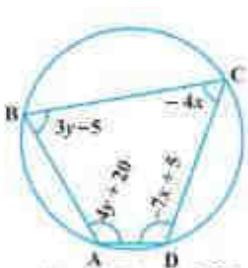
Hence, A's money = Rs 62

B's money = Rs 34

Q6

ABCD is a cyclic quadrilateral such that $\angle A = (4y + 20)^\circ$, $\angle B = (3y - 5)^\circ$, $\angle C = (2x + 10)^\circ$ and $\angle D = (7x + 5)^\circ$. Find the four angles.

Solution



We know that the sum of the measures of opposite angles in a cyclic quadrilateral is 180° .

$$\angle A + \angle C = 180$$

$$4y + 20 - 4x = 180$$

$$-4x + 4y = 180$$

$$x - y = -40 \quad \dots (1)$$

Also, $\angle B + \angle D = 180$

$$3y - 5 - 7x + 20 = 180$$

$$-7x + 3y = 180 \quad \dots (2)$$

Multiplying equation (1) by 3, we obtain:

$$3x - 3y = -120 \quad \dots (3)$$

Adding equations (2) and (3), we obtain:

$$-4x = 0$$

$$x = -15$$

Substituting the value of x in equation (1), we obtain:

$$-15 - y = -40$$

$$y = -15 + 40 = 25$$

$$\angle A = 4y + 20 = 4(25) + 20 = 120^\circ$$

$$\angle B = 3y - 5 = 3(25) - 5 = 70^\circ$$

$$\angle C = -4x = -4(-15) = 60^\circ$$

$$\angle D = -7x + 20 = -7(-15) + 20 = 110^\circ$$

Concept Insight: The most important idea to solve this problem is by using the fact that the sum of the measures of opposite angles in a cyclic quadrilateral is 180° . By using this relation, two linear equations can be obtained which can be solved easily by eliminating a suitable variable.

Q7

2 men and 7 boys can do a piece of work in 6 days. The same work is done in 3 days by 4 men and 4 boys. How long would it take one man and one boy to do it?

Solution

Suppose that one man alone can finish the work in x days and one boy alone can finish it in y days. Then,

$$\text{One man's one day's work} = \frac{1}{x}$$

$$\text{One boy's one day's work} = \frac{1}{y}$$

$$2 \text{ men's one day's work} = \frac{2}{x}$$

$$7 \text{ boy's one day's work} = \frac{7}{y}$$

Since 2 men and 7 boys can finish the work in 4 days:

$$\begin{aligned} & \frac{2}{x} + \frac{7}{y} = \frac{1}{4} \\ \Rightarrow & 4 \left(\frac{2}{x} + \frac{7}{y} \right) = 1 \\ \Rightarrow & \frac{8}{x} + \frac{28}{y} = 1 \quad \text{--- (i)} \end{aligned}$$

Again, 4 men and 4 boys can finish the same work in 3 days:

$$\begin{aligned} & \frac{4}{x} + \frac{4}{y} = \frac{1}{3} \\ \Rightarrow & 3 \left(\frac{4}{x} + \frac{4}{y} \right) = 1 \\ \Rightarrow & \frac{12}{x} + \frac{12}{y} = 1 \quad \text{--- (ii)} \end{aligned}$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in equations (i) and (ii), we get

$$\begin{aligned} 8u + 28v &= 1 \quad \text{--- (iii)} \\ 12u + 12v &= 1 \quad \text{--- (iv)} \end{aligned}$$

Multiplying equation (iii) by 3 and equation (iv) by 2, we get

$$\begin{aligned} 24u + 84v &= 3 \quad \text{--- (v)} \\ 24u + 24v &= 2 \quad \text{--- (vi)} \end{aligned}$$

Subtracting equation (v) from equation (vi), we get

$$\begin{aligned} 24v - 84v &= 2 - 3 \\ -60v &= -1 \\ v &= \frac{1}{60} \end{aligned}$$

Putting $v = \frac{1}{60}$ in equation (iv), we get

$$\begin{aligned} 12u + 12 \times \frac{1}{60} &= 1 \\ 12u + \frac{1}{5} &= 1 \\ 12u &= 1 - \frac{1}{5} \\ 12u &= \frac{5-1}{5} = \frac{4}{5} \\ u &= \frac{4}{5 \times 12} \\ &= \frac{1}{5 \times 3} \\ &= \frac{1}{15} \end{aligned}$$

$$\text{Now, } \frac{1}{x} = \frac{1}{15}$$

$$\Rightarrow \frac{1}{y} = \frac{1}{60}$$

$$\Rightarrow x = 15$$

$$\text{And, } \frac{1}{y} = \frac{1}{60}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{60}$$

$$\Rightarrow y = 60$$

Hence, one man alone can finish the work in 15 days
one boy alone can finish the work in 60 days.

Q8

In $\triangle ABC$, $\angle A = x^\circ$, $\angle B = (3x - 2)^\circ$, $\angle C = y^\circ$. Also, $\angle C - \angle B = 9^\circ$. Find the three angles.

Solution

It is given that,

$$\angle A = x^\circ \quad \text{---(i)}$$

$$\angle B = (3x - 2)^\circ \quad \text{---(ii)}$$

$$\angle C = y^\circ \quad \text{---(iii)}$$

$$\text{And, } \angle C - \angle B = 9^\circ \quad \text{---(iv)}$$

Putting $\angle C = y^\circ$ and $\angle B = (3x - 2)^\circ$ in equation (iv), we get

$$y - (3x - 2) = 9$$

$$\Rightarrow y - 3x + 2 = 9$$

$$\Rightarrow y - 3x = 9 - 2$$

$$\Rightarrow -3x + y = 7 \quad \text{---(v)}$$

We know that, the sum of angles of a triangle is 180°

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow x + 3x - 2 + y = 180^\circ$$

$$\Rightarrow 4x + y = 180 + 2$$

$$\Rightarrow 4x + y = 182 \quad \text{---(vi)}$$

Subtracting equation (v) from equation (vi), we get

$$4x + 3x - 182 = 7$$

$$\Rightarrow 7x = 175$$

$$\Rightarrow x = \frac{175}{7} = 25$$

Putting $x = 25$ in equation (v), we get

$$-3 \times 25 + y = 7$$

$$\Rightarrow -75 + y = 7$$

$$\Rightarrow y = 7 + 75$$

$$\Rightarrow y = 82$$

$$\therefore \angle A = x^\circ = 25^\circ$$

$$\angle B = (3x - 2)^\circ = (3 \times 25 - 2)^\circ = (75 - 2)^\circ = 73^\circ$$

$$\text{And, } \angle C = y^\circ = 82^\circ$$

Q9

In a cyclic quadrilateral $ABCD$, $\angle A = (2x + 4)^\circ$, $\angle B = (y + 3)^\circ$, $\angle C = (2y + 10)^\circ$, $\angle D = (4x - 5)^\circ$. Find the four angles.

Solution

We know that the sum of the opposite angles of a cyclic quadrilateral is 180° .

In the cyclic quadrilateral $ABCD$, angles A and C and angles B and D form pairs of opposite angles.

$$\angle A + \angle C = 180^\circ \text{ and } \angle B + \angle D = 180^\circ$$

$$\text{Now, } \angle A + \angle C = 180^\circ$$

$$\Rightarrow 2x + 4 + 2y + 10 = 180$$

[$\because \angle A = (2x + 4)^\circ$ and $\angle C = (2y + 10)^\circ$]

$$\Rightarrow 2x + 2y + 14 = 180$$

$$\Rightarrow 2x + 2y = 180 - 14$$

$$\Rightarrow 2x + 2y = 166$$

$$\Rightarrow 2(x + y) = 166$$

$$\Rightarrow x + y = \frac{166}{2} = 83$$

$$\Rightarrow x + y = 83 \quad \text{---(i)}$$

$$\text{Now, } \angle B + \angle D = 180^\circ$$

$$\Rightarrow x + 3 + 4x - 5 = 180$$

[$\angle B = (x + 3)^\circ$ and $\angle D = (4x - 5)^\circ$]

$$\Rightarrow 5x - 2 = 180$$

$$\Rightarrow 5x = 182 \quad \text{---(ii)}$$

Subtracting equation (i) from equation (ii), we get

$$4x - x = 182 - 83$$

$$\Rightarrow 3x = 99$$

$$\Rightarrow x = 33$$

Putting $x = 33$ in equation (i), we get

$$33 + y = 83$$

$$\Rightarrow y = 83 - 33$$

$$\Rightarrow y = 50$$

$$\text{Hence, } \angle A = (2x + 4)^\circ$$

$$= (2 \times 33 + 4)^\circ$$

$$\Rightarrow \angle A = 70^\circ$$

$$\angle B = (x + 3)^\circ$$

$$= (33 + 3)^\circ$$

$$\Rightarrow \angle B = 36^\circ$$

$$\angle C = (2y + 10)^\circ$$

$$= (2 \times 50 + 10)^\circ$$

$$\Rightarrow \angle C = 110^\circ$$

And,

$$\angle D = (4x - 5)^\circ$$

$$= (4 \times 33 - 5)^\circ$$

$$= (132 - 5)^\circ$$

$$\Rightarrow \angle D = 127^\circ$$

Q10

Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many question were there in the test?

Solution

Let the number of right answers and wrong answers be x and y respectively.

According to the question,

$$3x - y = 40 \dots\dots\dots(1)$$

$$4x - 3y = 50$$

$$\Rightarrow 2x - y = 25 \dots\dots\dots(2)$$

Subtracting equation (2) from equation (1), we obtain:

$$x = 15$$

Substituting the value of x in equation (2), we obtain:

$$30 - y = 25$$

$$y = 5$$

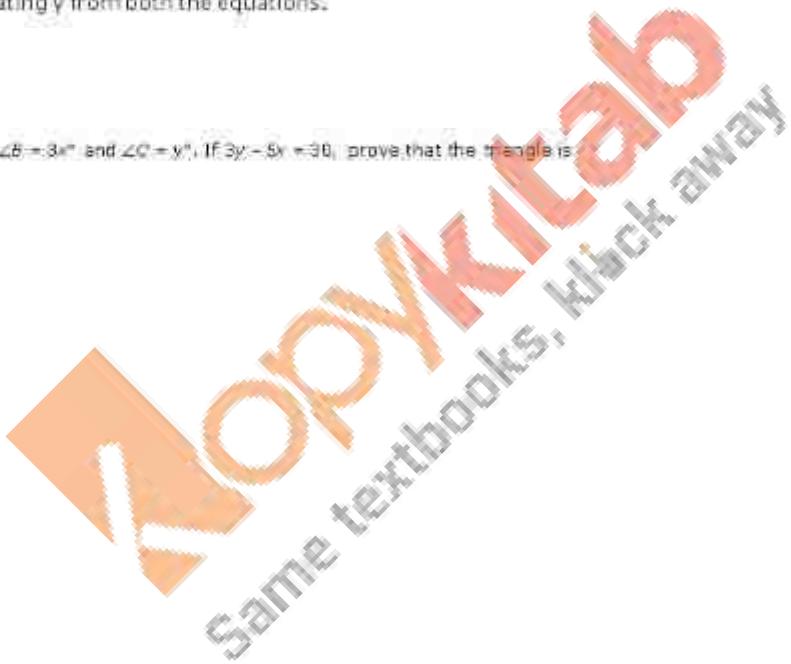
Thus, the number of right answers and the number of wrong answers is 15 and 5 respectively. Therefore the total number of questions is 20.

Concept insight: In this problem, the number of write answers and the number of wrong answers answered by Yash are the unknown variable y . It has the same coefficient in both the equations, so it will be easier to find the solution by eliminating y from both the equations.

Q11

In a $\triangle ABC$, $\angle A = x^\circ$, $\angle B = 3x^\circ$ and $\angle C = y^\circ$. If $3y - 5x = 30$, prove that the triangle is right angled.

Solution



We have,

$$\angle A = x^\circ \quad \text{---(i)}$$

$$\angle B = 3x^\circ \quad \text{---(ii)}$$

$$\text{And, } \angle C = y^\circ \quad \text{---(iii)}$$

We know that, the sum of angles of a triangle is 180° .

$$\angle A + \angle B + \angle C = 180^\circ$$

$$x + 3x + y = 180$$

$$\therefore 4x + y = 180 \quad \text{---(iv)}$$

[Using (i), (ii) and (iii)]

Now,

$$2y - 5x = 30 \quad \text{---(v)} \quad [\text{given}]$$

Multiplying equation (v) by 3, we get

$$12x + 3y = 540 \quad \text{---(vi)}$$

Subtracting equation (v) from equation (vi), we get:

$$12x + 3y = 540 - 30$$

$$\therefore 17x = 510$$

$$\therefore x = \frac{510}{17} = 30$$

Putting $x = 30$ in equation (iv), we get

$$4 \times 30 + y = 180$$

$$\therefore 120 + y = 180$$

$$\therefore y = 180 - 120 = 60$$

Now, $\angle B = 3x^\circ$

$$\therefore \angle B = 3 \times 30^\circ = 90^\circ$$

$\triangle ABC$ is the right angle triangle.

Hence proved.

Q12

The car hire charges in a city comprise of fixed charges together with the charge for the distance covered. For a journey of 12 km, the charge paid is Rs 89 and for a journey of 20 km, the charge paid is Rs. 145. What will a person have to pay for travelling a distance of 30 km?

Solution

Let the fixed charges of the car be Rs x and the running charges be Rs y/km .

According to the given condition, we have:

$$\begin{aligned}x + 12y &= 89 \quad (1) \\x + 20y &= 145 \quad (2)\end{aligned}$$

Subtracting equation (1) from equation (2), we get:

$$\begin{aligned}20y - 12y &= 145 - 89 \\8y &= 56 \\y &= \frac{56}{8} = 7\end{aligned}$$

Putting $y = 7$ in equation (1), we get:

$$\begin{aligned}x + 12 \times 7 &= 89 \\x + 84 &= 89 \\x &= 89 - 84 = 5\end{aligned}$$

Total charges from travelling a distance of 30 km:

$$\begin{aligned}&x + 30y \\&5 + 30 \times 7 \\&5 + 210 \\&= \text{Rs } 215\end{aligned}$$

Hence, total charges from travelling a distance of 30 km is Rs 215.

Q13

A part of monthly hostel charges in a college are fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days, he has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 28 days, pays Rs 1180 as hostel charges. Find the fixed charge and the cost of food per day.

Solution

Let the fixed charge of the food and the charge for food per day be x and y respectively.

According to the question,

$$x + 20y = 1000 \quad (1)$$

$$x + 28y = 1180 \quad (2)$$

Substituting this value of y in equation (1) from equation (2), we obtain:

$$6y = 180$$

$$y = 30$$

Substituting this value of y in equation (1), we obtain:

$$x + 20 \times 30 = 1000$$

$$x = 1000 - 600$$

$$x = 400$$

Thus, the fixed charge of the food and the charge per day are Rs 400 and Rs 30 respectively.

Concept insight: Here, the fixed charge of the food and charge for food per day are the unknown quantities. So they are taken as variables x and y . The two equations can then be obtained by using the given conditions. You will observe that the variable x has the same coefficient in both the equations, so it will be easier to find the solution by eliminating x from both the equations. Also, one can solve the system by other methods.

Q14

Half the perimeter of a garden, whose length is 4 m more than its width is 36 m. Find the dimensions of the garden.

Solution

Let the length and breadth of the garden be x m and y m respectively. Then,

$$x = y + 4 \quad \text{---(i)}$$

$$\text{And, } \frac{1}{2} [\text{perimeter of a garden}] = 36$$

$$\Rightarrow \frac{1}{2} [2(x+y)] = 36$$

[perimeter of rectangle = $2(L+B)$]

$$\Rightarrow x + y = 36$$

Substituting $x = y + 4$ in equation (i), we get

$$y + 4 + y = 36$$

$$\Rightarrow 2y = 36 - 4$$

$$\Rightarrow 2y = 32$$

$$\Rightarrow y = \frac{32}{2} = 16$$

Putting $y = 16$ in equation (i), we get

$$x = 16 + 4 = 20$$

Hence, the length and breadth of the garden are 20m and 16m respectively.

Q15

The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Solution

Let the larger angle be x and smaller angle be y .

We know that the sum of the measures of angles of a supplementary pair is always 180° .

According to the given information,

$$x + y = 180^\circ \quad (1)$$

$$x - y = 18^\circ \quad (2)$$

From (1), we obtain

$$x = 180^\circ - y \quad (3)$$

Substituting this in equation (2), we obtain

$$180^\circ - y - y = 18^\circ$$

$$162^\circ = 2y$$

$$81^\circ = y \quad (4)$$

Putting this in equation (3), we obtain

$$x = 180^\circ - 81^\circ$$

$$= 99^\circ$$

Hence, the angles are 99° and 81° .

Concept insight: This problem talks about the measure of two supplementary angles. So, the two angles will be written as variables. The pair of equations can be formed using the fact that the sum of two supplementary angles is 180° and using the condition given in the problem. The pair of equations can then be solved by suitable substitution.

Q16

2 Women and 5 men can together finish a piece of embroidery in 4 days, while 3 women and 8 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the embroidery, and that taken by 1 man alone.

Solution

Let the number of days taken by a woman and a man to finish the work be x and y respectively.

$$\text{Work done by a woman in 1 day} = \frac{1}{x}$$

$$\text{Work done by a man in 1 day} = \frac{1}{y}$$

According to the question,

$$4\left(\frac{2}{x} + \frac{3}{y}\right) = 1$$

$$\Rightarrow \frac{2}{x} + \frac{3}{y} = \frac{1}{4}$$

$$3\left(\frac{2}{x} + \frac{6}{y}\right) = 1$$

$$\Rightarrow \frac{2}{x} + \frac{6}{y} = \frac{1}{3}$$

$$\text{Let } \frac{1}{x} = p \text{ and } \frac{1}{y} = q$$

The given equations reduce to:

$$2p + 5q = \frac{1}{4}$$

$$\Rightarrow 8p + 20q = 1$$

$$3p + 6q = \frac{1}{3}$$

$$\Rightarrow 9p + 18q = 1$$

Using cross-multiplication, we obtain:

$$\frac{P}{-20 - (-18)} = \frac{q}{-9 - (-8)} = \frac{1}{144 - 100}$$

$$\frac{P}{-2} = \frac{q}{-1} = \frac{1}{36}$$

$$\frac{P}{-2} = \frac{1}{-36}, \frac{q}{-1} = \frac{1}{-36}$$

$$P = \frac{1}{12}, q = \frac{1}{36}$$

$$P = \frac{1}{12}, q = \frac{1}{36}$$

$$x = 12, y = 36$$

Thus, the number of days taken by a woman and a man to finish the work is 12 and 36.

Q17

Meena went to a bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all. Find how many notes Rs 50 and Rs 100 she received.

Solution

Let the number of Rs 50 notes and Rs 100 notes be x and y respectively.

According to the question,

$$x + y = 25 \quad \dots (1)$$

$$50x + 100y = 2000 \quad \dots (2)$$

Multiplying equation (1) by 50, we obtain:

$$50x + 50y = 1250$$

Subtracting equation (3) from equation (2), we obtain:

$$50y = 750$$

$$y = 15$$

Substituting the value of y in equation (1), we obtain:

$$x = 10$$

Hence, Meena received 10 notes of Rs 50 and 15 notes of Rs 100.

Concept Insight: This problem talks about two types of notes, Rs 50 notes and Rs 100 notes. And the number of both these notes with Meena is not known. So, we denote the number of Rs 50 notes and Rs 100 notes by variables x and y respectively. Now two linear equations can be formed by the given conditions which can be solved by eliminating one of the variables.

Q18

There are two examination rooms A and B. If 10 candidates are sent from A to B, the number of students in each room is same. If 20 candidates are sent from B to A, the number of students in A is double the number of students in B. Find the number of students in each room.

Solution

Let the number of candidates in rooms A and B be x and y respectively. Then,

$$\begin{aligned} & x - 10 = y + 10 \quad [\text{given}] \\ \Rightarrow & x - y = 10 + 10 \\ \Rightarrow & x - y = 20 \quad \text{---(i)} \end{aligned}$$

$$\begin{aligned} \text{And, } & 2(y - 20) = x + 20 \quad [\text{given}] \\ \Rightarrow & 2y - 40 = x + 20 \\ \Rightarrow & 2y - x = 20 + 40 \\ \Rightarrow & -x + 2y = 60 \quad \text{---(ii)} \end{aligned}$$

Adding equations (i) and (ii), we get:

$$\begin{aligned} -y + 2y &= 20 + 60 \\ y &= 80 \end{aligned}$$

Putting $y = 80$ in equation (i), we get:

$$\begin{aligned} x - 80 &= 20 \\ x &= 20 + 80 \\ x &= 100 \end{aligned}$$

Hence, number of students in room A = 100 and number of students in room B = 80.

Q19

A railway half ticket costs half the full fare and the reservation charge is the same on half ticket as on full ticket. One reserved first class ticket from Mumbai to Ahmedabad costs Rs. 216 and one full and one half reserved first class tickets cost Rs. 327. What is the basic first class full fare and what is the reservation charge?

Solution

Let the cost of the full fare be Rs x and that of the reservation charge be Rs y . Then;

$$x + y = 216 \quad \text{---(i)} \quad [\text{given}]$$

And,

$$(x + y) + \left(\frac{1}{2}x + y\right) = 327 \quad [\text{given}]$$

$$x + y + \frac{1}{2}x + y = 327$$

$$\therefore x + \frac{1}{2}x + 2y = 327$$

$$\therefore \frac{3x}{2} + 2y = 327$$

$$\therefore 3x + 4y = 654 \quad \text{---(ii)}$$

Multiplying equation (i) by 4, we get

$$4x + 4y = 864 \quad \text{---(iii)}$$

Subtracting equation (ii) from equation (iii), we get

$$4x - 3x = 864 - 654$$

$$\therefore x = 210$$

Putting $x = 210$ in equation (i), we get

$$210 + y = 216$$

$$\therefore y = 216 - 210 = 6$$

Hence, the cost of the full fare = Rs 210 and, the cost of the reservation charge = Rs 6.

Q20

A wizard having powers of mystic incantations and magical medicines seeing a cock-fight going on, spoke privately to both the owners of cocks. To one he said; if your bird wins, then you give me your stake-money, but if you do not win, I shall give you two-third of that. Going to the other, he promised in the same way to give three-fourths. From both of them his gain would be only 12 gold coins. Find the stake of money each of the cock-owners have.

Solution

Let the stake money of first and second cock-owners be Rs. x and Rs. y respectively. Then,

$$\begin{aligned} y - \frac{2}{3}x &= 12 \\ \Rightarrow 3y - 2x &= 36 \quad \text{---(i)} \end{aligned}$$

$$\begin{aligned} \text{And, } x - \frac{3}{4}y &= 12 \\ \Rightarrow 4x - 3y &= 48 \quad \text{---(ii)} \end{aligned}$$

Multiplying equation (i) by 2, we get

$$6y - 4x = 72 \quad \text{---(iii)}$$

Adding equations (i) and (iii), we get:

$$\begin{aligned} -3y + 5y &= 48 + 72 \\ \Rightarrow 2y &= 120 \\ \Rightarrow y &= \frac{120}{2} = 40 \end{aligned}$$

Putting $y = 40$ in equation (i), we get:

$$\begin{aligned} 4x - 3 \times 40 &= 48 \\ \Rightarrow 4x - 120 &= 48 \\ \Rightarrow 4x &= 48 + 120 \\ \Rightarrow 4x &= 168 \\ \Rightarrow x &= \frac{168}{4} = 42 \end{aligned}$$

Hence, the stake of money of 1st cock-owner = 42 gold coins and, the stake of money of 2nd cock-owner = 40 gold coins.

Q21

The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row there would be 2 rows more. Find number of students in the class.

Solution

Let the number of students be x and the number of rows be y . Then,

$$\text{Number of students in each row} = \frac{x}{y}$$

When 3 students are extra in each row, there is 1 row less i.e., when each row has

$$\left(\frac{x}{y} + 3\right) \text{ students, the number of rows is } (y-1)$$

$$\begin{aligned} & \text{Total number of students} = \text{Number of rows} \times \text{Number of students in each row} \\ \Rightarrow & x = \left(\frac{x}{y} + 3\right)(y-1) \\ \Rightarrow & x = x - \frac{x}{y} + 3y - 3 \\ \Rightarrow & \frac{x}{y} - 3y + 3 = 0 \quad \text{---(i)} \end{aligned}$$

If 3 students are less in each row, then there are 2 rows more i.e., when each row

$$\left(\frac{x}{y} - 3\right) \text{ students, the number of rows is } (y+2)$$

$$\begin{aligned} & \text{Total number of students} = \text{Number of rows} \times \text{Number of students in each row} \\ \Rightarrow & x = \left(\frac{x}{y} - 3\right)(y+2) \\ \Rightarrow & x = x + \frac{2x}{y} - 3y - 6 \\ \Rightarrow & \frac{-2x}{y} + 3y + 6 = 0 \quad \text{---(ii)} \end{aligned}$$

Multiplying equation (i) by 2, we get

$$\frac{2x}{y} - 6y + 6 = 0 \quad \text{---(iii)}$$

Adding equation (ii) and equation (iii), we get

$$\begin{aligned} & 3y - 6y + 6 + 6 = 0 \\ \Rightarrow & -3y = -12 \\ \Rightarrow & y = \frac{-12}{-3} = 4 \end{aligned}$$

Putting $y = 4$ in equation (i), we get

$$\begin{aligned} & \frac{x}{4} - 3 \times 4 + 3 = 0 \\ \Rightarrow & \frac{x}{4} - 12 + 3 = 0 \\ \Rightarrow & \frac{x}{4} = 9 \\ \Rightarrow & x = 9 \times 4 \\ \Rightarrow & x = 36 \end{aligned}$$

Hence, the number of students in the class is 36.

Q22

One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital?

Solution

Let the money with the first person and second person be Rs x and Rs y respectively.

According to the question,

$$x + 100 = 2(y - 100)$$

$$x + 100 = 2y - 200$$

$$x - 2y = -300 \quad \dots (1)$$

$$6(x - 10) = (y + 10)$$

$$6x - 60 = y + 10$$

$$6x - y = 70 \quad \dots (2)$$

Multiplying equation (2) by 2, we obtain:

$$12x - 2y = 140 \quad \dots (3)$$

Subtracting equation (1) from equation (3), we obtain:

$$11x = 140 + 300$$

$$11x = 440$$

$$x = 40$$

Putting the value of x in equation (1), we obtain:

$$40 - 2y = -300$$

$$40 + 300 = 2y$$

$$2y = 340$$

$$y = 170$$

Thus, the two friends had Rs 40 and Rs 170 with them.

Concept insight: This problem talks about the amount of capital with two friends. So, we will represent them by variables x and y respectively. Now, using the given conditions, a pair of linear equations can be formed which can then be solved easily using elimination method.

Q23

A shopkeeper sells a saree at 8% profit and a sweater at 10% discount, thereby getting a sum of Rs. 1008. If she had sold the saree at 10% profit and sweater at 8% discount, she would have got Rs. 1028. Find the cost price of the saree and the list price (price before discount) of the sweater.

Solution

Let the cost price of a saree be Rs. x ,

and the list price of the sweater be Rs. y .

$$\text{S.P. of saree at 8\% profit} = \text{Rs.} \left(x + \frac{8x}{100} \right) = \text{Rs.} \left(\frac{108x}{100} \right)$$

$$\text{S.P. of sweater at 10\% discount} = \text{Rs.} \left(y - \frac{10y}{100} \right) = \text{Rs.} \left(\frac{90y}{100} \right)$$

According to question,

$$\frac{108x}{100} + \frac{90y}{100} = 1008$$

$$\Rightarrow 108x + 90y = 100800 \quad \dots (i)$$

$$\text{Now, S.P. of saree at 10\% profit} = \text{Rs.} \left(x + \frac{10x}{100} \right) = \text{Rs.} \left(\frac{110x}{100} \right)$$

$$\text{S.P. of sweater at 8\% discount} = \text{Rs.} \left(y - \frac{8y}{100} \right) = \text{Rs.} \left(\frac{92y}{100} \right)$$

According to question,

$$\frac{110x}{100} + \frac{92y}{100} = 1028$$

$$\Rightarrow 110x + 92y = 102800 \quad \dots (ii)$$

Subtracting (i) from (ii), we get:

$$2x + 2y = 2000$$

$$\Rightarrow x + y = 1000$$

$$\Rightarrow x = 1000 - y \quad \dots (iii)$$

Substituting (iii) in (i), we get:

$$108(1000 - y) + 90y = 100800$$

$$\Rightarrow 108000 - 108y + 90y = 100800$$

$$\Rightarrow -18y = -7200 \Rightarrow y = 400$$

$$\Rightarrow x = 1000 - 400 = 600$$

Thus, the cost price of saree is Rs. 600 and the list price of sweater is Rs. 400.

Q24

In a competitive examination, one mark is awarded for each correct answer while $\frac{1}{2}$ mark is deducted for every wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly?

Solution

Let Jayanti answered x questions correctly and y questions wrongly.

According to information given in question,

$$x+y=120 \quad \dots (i)$$

$$1x + -\frac{1}{2}y = 90$$

$$\Rightarrow 2x-y=180 \quad \dots (ii)$$

Adding equations (i) and (ii), we get:

$$3x=300$$

$$\Rightarrow x=100$$

Thus, Jayanti answered 100 questions correctly.

Q25

A shopkeeper gives book on rent for reading. She takes a fixed charge for the first two days, and an additional charge for each day thereafter. Latika paid Rs.22 for a book kept for 6 days, while Rs.16 for the book kept for four days. Find the fixed charges and charge for each extraday.

Solution

Let the fixed charge for first three days be Rs. x ,

and the additional charge for each day thereafter be Rs. y .

Then, according to question, we have

$$x+4y=22 \quad \dots (i)$$

$$x+2y=16 \quad \dots (ii)$$

Subtracting equation (ii) from (i), we have

$$2y=6$$

$$\Rightarrow y=3$$

Substituting $y=3$ in (ii), we have

$$x+2(3)=16$$

$$\Rightarrow x+6=16$$

$$\Rightarrow x=10$$

Thus, the fixed charge for first three days is Rs. 10

and the additional charge for each day thereafter is Rs. 3.

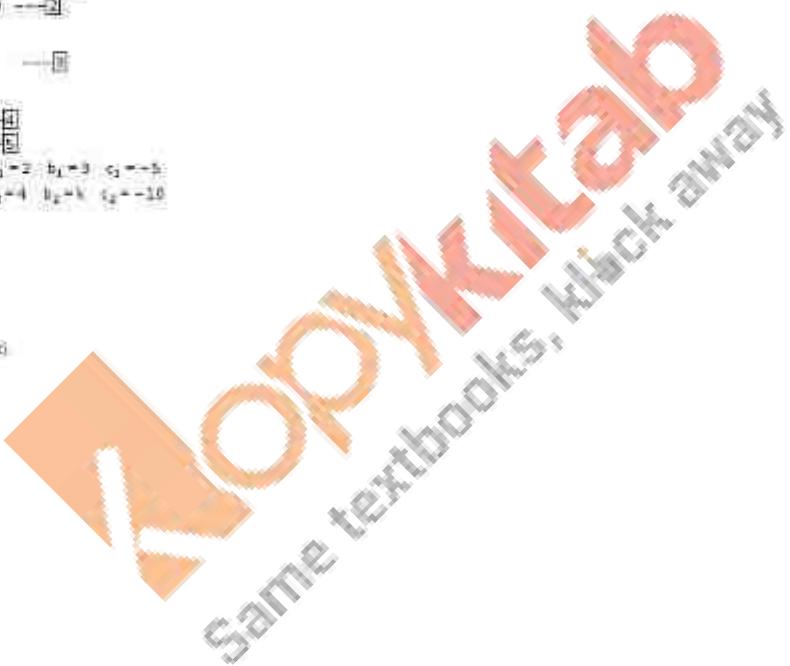
Exercise 3.114

Q1**Solution****Q2****Solution**

we know, if $a_1x + b_1y + c_1 = 0 \quad \text{--- } [1]$
 $a_2x + b_2y + c_2 = 0 \quad \text{--- } [2]$
 for infinite solution:
 $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \text{--- } [3]$
 Given equation are:
 $2x + 3y = 5 \quad \text{--- } [4]$
 $4x + 6y = 10 \quad \text{--- } [5]$
 from equations [4] & [5] $a_1 = 2, b_1 = 3, c_1 = -5$
 from equations [4] & [5] $a_2 = 4, b_2 = 6, c_2 = -10$
 from equation (3)
 $\frac{2}{4} = \frac{3}{6} = \frac{-5}{-10}$
 $= \frac{1}{2} = \frac{1}{2}$
 $= \boxed{1 = 1}$
 So, the correct option is (c).

Q3**Solution****Q4**

The value of k for which the system of equations
 $3x + 5y = 11$ and $(k+1)3y = 11$, has a non-zero solution,
 is
 (a) 0 (b) 2 (c) 5 (d) 6

Solution

If the equations are $a_1x + b_1y + c_1 = 0 \quad \text{--- [1]}$
 $a_2x + b_2y + c_2 = 0 \quad \text{--- [2]}$

for infinite solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \quad \text{--- [3]}$$

Given equations are:

$$3x + 5y = 0 \quad \text{--- [4]}$$

$$kx + 10y = 0 \quad \text{--- [5]}$$

on comparing [1] [2] [4] [5]

$$\begin{aligned} \therefore a_1 &= 3 & b_1 &= 5 \\ a_2 &= k & b_2 &= 10 \end{aligned}$$

from [3] $\frac{3}{k} = \frac{5}{10}$

$$\rightarrow k = 6$$

So, the correct option is (c).



Exercise 3.115

Q1**Solution**

If $a_1x + b_1y + c_1 = 0 \quad \dots \text{[1]}$
 $a_2x + b_2y + c_2 = 0 \quad \dots \text{[2]}$

For inconsistent solution:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \quad \dots \text{[3]}$$

Given equations are:

$$3x + y = 3 \quad \dots \text{[4]}$$

$$(2k-1)x + (k+1)y = 2k+1 \quad \dots \text{[5]}$$

From [3] & [4]

$$-a_1 = 3 \quad b_1 = 1 \quad c_1 = -3$$

From [2] & [5]

$$a_2 = 2k-1 \quad b_2 = k+1 \quad c_2 = 2k+1$$

From [3]

$$\frac{3}{2k-1} = \frac{1}{k+1}$$

$$= 3k - 3 = 2k - 1$$

$$= \boxed{k = 2}$$

So, the correct option is (d).

Q2

If $am = bl$, then the system of equations
 $ax + by = c$
 $lx + my = n$

- (a) has a unique solution
- (b) has no solution
- (c) has infinite many solution
- (d) may or may not have a solution

Solution

For unique solution:

$$\frac{a}{l} = \frac{b}{m}$$

$$= am = bl$$

which is the given condition.

Hence the given equation

$$am = bl$$

is the condition to a unique solution.

So, the correct option is (a).

Q3

If the system of equations

$$2x + 3y = 7$$

$$2ax + (a+b)y = 2b$$

has infinite many solutions, then

- (a) $a = 2b$ (b) $b = 2a$ (c) $a+2b = 0$ (d) $2a+b=0$

Solution**Q4**

Solution**Q5**

If $2x-3y=7$ and $(a+b)x - (a+b-3)y = 4a+b$ represent coincident lines than a and b satisfy the equation
 (a) $a+5b=0$ (b) $51+b=0$ (c) $a-5b=0$ (d) $5a-b=0$

Solution

For coincident lines

$$\begin{aligned} \frac{2}{a+b} &= \frac{-3}{-(a+b-3)} = \frac{7}{4a+b} \\ \frac{2}{a+b} &= \frac{7}{4a+b} \\ 8a+2b &= 7a+7b \\ a-5b &= 0 \\ a-5b &= 0 \end{aligned}$$

So, the correct option is (a).

Q6

If a pair of linear equations in two variables is consistent, then the lines represented by two equations are
 (a) Intersecting (b) parallel
 (c) always coincident (d) intersecting or coincident

Solution

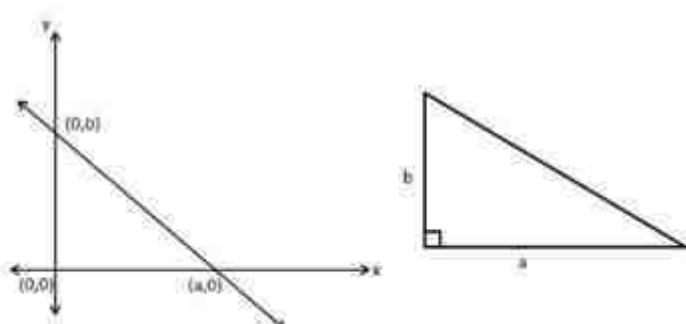
Consistent solution means either linear equations have unique solutions or infinite solutions.
 ⇒ In case of unique solution; lines are intersecting

⇒ If solutions are infinite, lines are coincident.

So, lines are either intersecting or coincident

So, the correct option is (d).

Q7**Solution**



Interception x-axis = a
Interception y-axis = b

$$\text{Area of triangle} = \frac{1}{2} \times a \times b$$

$$= \frac{1}{2} ab$$

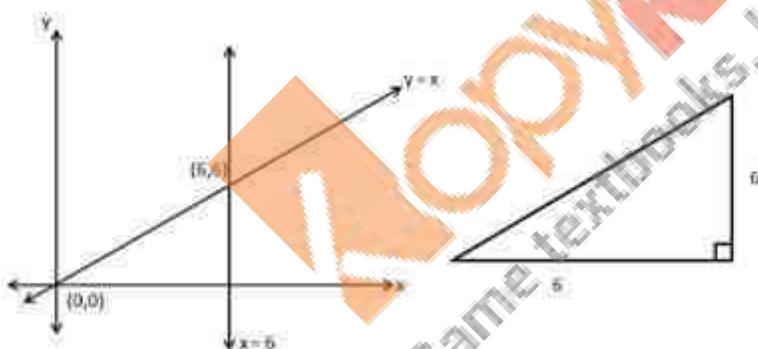
So, the correct option is (c)

Q8

The area of the triangle formed by the lines $y=x$, $x=6$, and $y=0$ is

- (a) 36 sq. units
- (b) 18 sq. units
- (c) 9 sq. units
- (d) 72 sq. units

Solution



Q9

If the system of equations $2x + 3y = 5$, $4x + ky = 10$ has infinitely many solutions, then $k =$

- (a) 1
- (b) $\frac{1}{2}$
- (c) 3
- (d) 6

Solution

Q10

If the system of equations $kx - 5y = 2$, $6x + 2y = 7$ has no solution, then $k =$

- (a) -10
- (b) -5
- (c) -6
- (d) -15

Solution

Equations $a_1x + b_1y + c_1 = 0 \quad \text{--- [1]}$
 and $a_2x + b_2y + c_2 = 0 \quad \text{--- [2]}$

have no solution, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \text{--- [3]}$$

On comparing with given equation to [1] & [2] we get:

$$\begin{aligned} a_1 &= k & b_1 &= -5 & c_1 &= -2 \\ a_2 &= 6 & b_2 &= 2 & c_2 &= 7 \end{aligned}$$

From equation [3]

$$\begin{aligned} \frac{k}{6} &= \frac{-5}{2} = \frac{2}{7} \\ \Rightarrow k &= -15 \end{aligned}$$

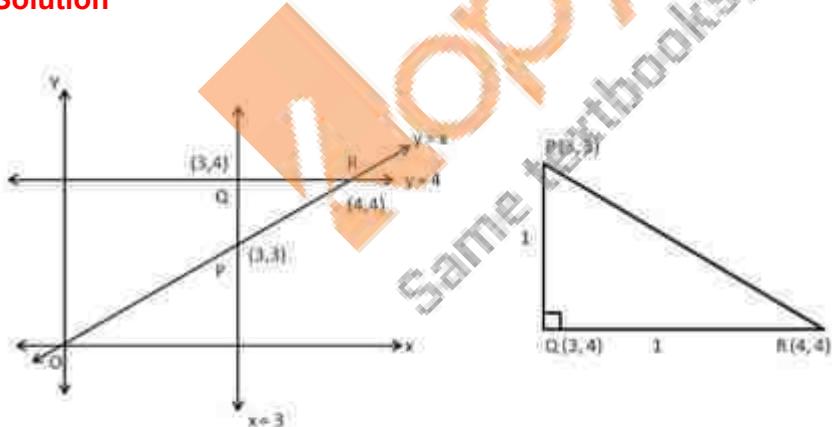
So, the correct option is (d).

Q11

The area of the triangle formed by the lines $x = 3$, $y = 4$ and $x = y$ is

- (a) $\frac{1}{2}$ sq. unit
- (b) 1 sq. unit
- (c) 2 sq. unit
- (d) None of these

Solution

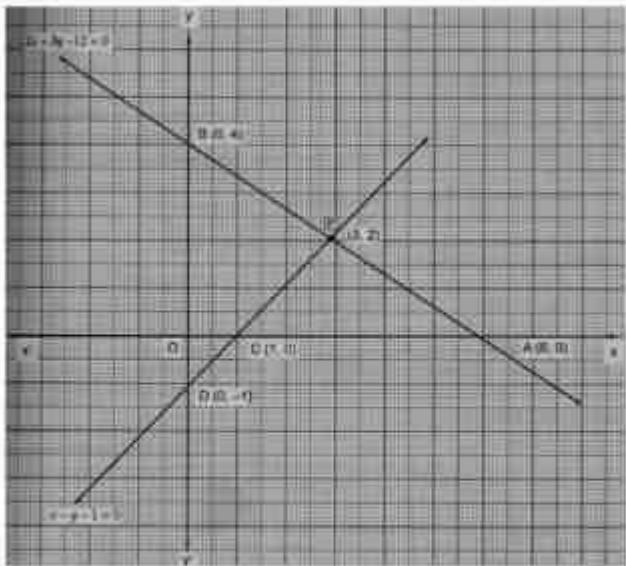


$$\text{area of triangle} = \frac{1}{2} \times 1 \times 1$$

$$= \frac{1}{2} \text{ sq. unit}$$

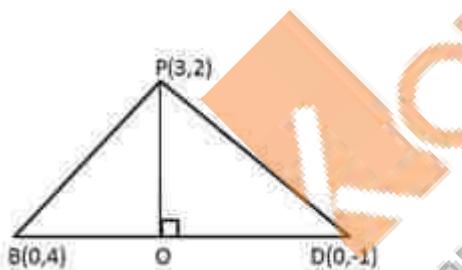
So, the correct option is (a).

Exercise 3.116

Q1

The area of the triangle formed by the lines $2x + 3y = 12$, $x - y - 1 = 0$ and $x = 0$

- (a) 7 sq. units
- (b) 7.5 sq. units
- (c) 6.5 sq. units
- (d) 6 sq. units

Solution**Q2**

The sum of the digits of a two digit number is 9. If 27 is added to it, the digits of the number get reversed. The number is

- 25
- 72
- 63
- 36

Solution

Let the digit at unit's place be x and the digit at ten's place be y .

Then, number = $10y + x$.

According to given conditions, we have

$$x + y = 9 \dots (i)$$

$$\text{And, } 10y + x + 27 = 10x + y$$

$$\Rightarrow 9x - 9y = 27$$

$$\Rightarrow x - y = 3 \dots (ii)$$

Adding equations (i) and (ii), we have

$$2x = 12 \Rightarrow x = 6$$

$$\Rightarrow y = 9 - 6 = 3$$

Hence, the number = 36.

Hence, correct option is (d).

Q3

If $x = a$, $y = b$ is the solution of the system of equations $x - y = 2$ and $x + y = 4$, then the values of a and b are, respectively

3 and 1

3 and 5

5 and 3

-1 and -3

Solution

Since $x = a$ and $y = b$ is the solution of given system of equations $x - y = 2$ and $x + y = 4$, we have

$$a - b = 2 \dots (i)$$

$$a + b = 4 \dots (ii)$$

Adding (i) and (ii), we have

$$2a = 6 \Rightarrow a = 3$$

$$\Rightarrow b = 4 - 3 = 1$$

Hence, correct option is (a).

Q4

For what value k , do the equations $3x - y + 2 = 0$ and $8x - ky + 16 = 0$ represent coincident lines?

$\frac{1}{2}$
 $\frac{2}{3}$
 $\frac{3}{2}$
 $\frac{4}{3}$

Solution

The given system of equations are

$$3x - y + 8 = 0$$

$$6x - ky + 16 = 0$$

This system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = 3, b_1 = -1, c_1 = 8$

And, $a_2 = 6, b_2 = -k, c_2 = 16$

For the lines to be coincident, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$$

$$\text{Now, } \frac{3}{6} = \frac{-1}{-k} \Rightarrow 3k = 6 \Rightarrow k = 2$$

Hence, correct option is (c).

Q5

Aruna has only Rs. 1 and Rs. 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is Rs. 75, then the number of Rs. 1 and Rs. 2 coins are, respectively

35 and 15

35 and 20

15 and 35

25 and 25

Solution

Let there be x coins of Rs. 1 and y coins of Rs. 2.

Then, we have

$$x + y = 50 \quad \dots(i)$$

$$x + 2y = 75 \quad \dots(ii)$$

Subtracting (i) from (ii), we have

$$y = 25$$

$$\Rightarrow x = 50 - 25 = 25$$

Thus, number of Re. 1 coins is 25 and the number of Rs. 2 coins is 25.

Hence, correct option is (d).