RD Sharma Solutions Class 11 Maths Chapter 20 Ex 20.6

Geometric Progressions Ex 20.6 Q 1

6 Geometric means between 27 and $\frac{1}{81}$

Let $G_1, G_2, G_3, G_4, G_5, G_6$ be 6 geometric means between $a = 27$ and $b = \frac{1}{81}$.

Then, $27, G_1, G_2, G_3, G_4, G_5, G_6, \frac{1}{81}$ is a G.P. with common ratio r given by

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$= \left(\frac{\frac{1}{81}}{27}\right)^{\frac{1}{6+1}} = \left(\frac{1}{81 \times 27}\right)^{\frac{1}{7}} = \left(\frac{1}{3^7}\right)^{\frac{1}{7}}$$

$$\therefore \quad G_1 = ar = 27\left(\frac{1}{3}\right) = 9$$

$$G_2 = ar^2 = 27 \times \frac{1}{9} = 3$$

$$G_3 = ar^3 = 27 \times \frac{1}{27} = 1$$

$$G_4 = ar^4 = 27 \times \frac{1}{27 \times 3} = \frac{1}{3}$$

$$G_5 = ar^5 = 27 \times \frac{1}{3^5} = \frac{1}{9}$$

$$G_6 = ar^6 = 27 \times \frac{1}{36} = \frac{1}{27}$$

Hence, $9, 3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$ are 6 geometric means between 27 and $\frac{1}{81}$.

Geometric Progression Ex 20.6 Q 2

5 Geometric means between 16 and $\frac{1}{4}$

Let $G_1, G_2, G_3, G_4, G_5,$ be five geometric means between 16 and $\frac{1}{4}$.

$$16, G_1, G_2, G_3, G_4, G_5, \frac{1}{4} \text{ is a G.P. with } a = 16, b = \frac{1}{4}.$$

Then,

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$= \left(\frac{\frac{1}{4}}{16}\right)^{\frac{1}{5+1}} = \left(\frac{1}{2^6}\right)^{\frac{1}{6}} = \frac{1}{2}$$

$\therefore$
$$G_1 = ar = 16 \times \frac{1}{2} = 8$$

$$G_2 = ar^2 = 16 \times \frac{1}{4} = 4$$

$$G_3 = ar^3 = 16 \times \frac{1}{8} = 2$$

$$G_4 = ar^4 = 16 \times \frac{1}{16} = 1$$

$$G_5 = ar^5 = 16 \times \frac{1}{2^5} = \frac{1}{2}$$

Hence, $8, 4, 2, 1, \frac{1}{2}$ are five geometric means between 16 and $\frac{1}{4}$.

Geometric Progression Ex 20.6 Q 3

5 Geometric means between $\frac{32}{9}$ and $\frac{81}{2}$

Let $G_1, G_2, G_3, G_4, G_5,$ be five geometric means between $\frac{32}{9}$ and $\frac{81}{2}$.

Then, $\frac{32}{9}, G_1, G_2, G_3, G_4, G_5, \frac{81}{2}$ is a G.P. with $a = \frac{32}{9}, b = \frac{81}{2}$.

Then,

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$= \left(\frac{\frac{81}{2}}{\frac{32}{9}}\right)^{\frac{1}{5+1}} = \left(\frac{81}{2} \times \frac{9}{32}\right)^{\frac{1}{6}} = \left(\frac{3^6}{2^6}\right) = \frac{3}{2}$$

Thus, $G_1 = ar = \frac{32}{9} \times \frac{3}{2} = \frac{16}{3}$

$$G_2 = ar^2 = \frac{32}{9} \times \frac{9}{4} = 8$$

$$G_3 = ar^3 = \frac{32}{9} \times \frac{27}{8} = 12$$

$$G_4 = ar^4 = \frac{32}{9} \times \frac{3^4}{2^4} = 2 \times 9 = 18$$

$$G_5 = ar^5 = \frac{32}{9} \times \frac{3^5}{2^5} = 27$$

Hence, $\frac{16}{3}, 8, 12, 18, 27$ are five geometric means between $\frac{32}{9}$ and $\frac{81}{2}$.

Geometric Progressions Ex 20.6 Q 4

(i) 2 and 8

Geometric means between a and $b = \sqrt{ab}$ ---(i)

Here, $a = 2, b = 8$

$\therefore$ Geometric means $= \sqrt{2 \times 8} = \sqrt{16} = 4$

(ii) a^3b and ab^3

Using (i)

$a = a^3b, b = ab^3$

Geometric means $= \sqrt{a^3b \times ab^3} = \sqrt{a^4b^4} = a^2b^2$

(iii) -8 and -2

Using (ii)

$a = -8, b = -2$

Geometric means $= \sqrt{-8 \times -2} = \sqrt{16} = 4, -4$

Geometric Progressions Ex 20.6 Q 5

a is geometric means between 2 and $\frac{1}{4}$.

Then, $a = \sqrt{2 \times \frac{1}{4}}$

$a = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

Geometric Progressions Ex 20.6 Q 6

Let the first term of a GP is a and common ratio of the series is r.

The $(n+2)$th term is ar^{n+1}.

The GM of a and ar^{n+1} will be:

$$G_1 = \sqrt{a \cdot ar^{n+1}} = \left(a^2 r^{n+1}\right)^{\frac{1}{2}}$$

Now the n GM in between a and ar^{n+1} are:

$$ar, ar^2, \cdots, ar^n$$

Therefore the product of n GM will be:

$$\begin{aligned} ar \times ar^2 \times \cdots \times ar^n &= a^n r^{1+2+3+\cdots+n} \\ &= a^n r^{\frac{n(n+1)}{2}} \\ &= \left(a^2 r^{n+1}\right)^{\frac{n}{2}} \\ &= G_1^n \end{aligned}$$

Hence it is proved.

Geometric Progressions Ex 20.6 Q 7

Given,

$$A.M = 25$$
$$G.M = 20$$

Now, $A.M = \frac{a+b}{2} = 25$

and, $G.M = \sqrt{ab} = 20$

$$a + b = 50, ab = 400$$

$$\begin{aligned} (a-b) &= \sqrt{(a+b)^2 - 4ab} \\ &= \sqrt{(50)^2 - 1600} \\ &= \sqrt{2500 - 1600} \\ &= \pm 30 \end{aligned}$$

$$a - b = \pm 30$$
$$\underline{a + b = 50}$$
$$2a = 80$$
$$a = 40$$

Also, $-2b = -20$

$b = 10$

$\therefore$ The numbers are 40,10.

Geometric Progressions Ex 20.6 Q 8

A.M. between two numbers a and b $(a > b)$ is $\frac{a+b}{2}$

Also, geometric mean between 2 numbers is $\sqrt{ab}$

Given,

$$A.M = 2G.M$$

$$\Rightarrow \quad \frac{a+b}{2} = 2\sqrt{ab}$$

$$a+b = 4\sqrt{ab}$$

$$\frac{a+b}{2\sqrt{ab}} = \frac{2}{1}$$

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{2+1}{2-1} = \frac{3}{1} \quad \text{[By componendo and dividendo]}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2} = \frac{\left(\sqrt{3}\right)^2}{(1)^2}$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{3}}{1}$$

By componendo and dividendo, we get

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)+\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)-\left(\sqrt{a}-\sqrt{b}\right)} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\frac{a}{b} = \frac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}-1\right)^2} = \frac{3+1+2\sqrt{3}}{3+1-2\sqrt{3}}$$

$$= \frac{4+2\sqrt{3}}{4-2\sqrt{3}}$$

$$\frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

Thus, $a : b = \left(2+\sqrt{3}\right) : \left(2-\sqrt{3}\right)$.

Geometric Progressions Ex 20.6 Q 9

Let A.M = A between a and b

G.M = G_1 and G_2 between a and b

$$\Rightarrow \quad A = \frac{a+b}{2}$$

$a, G_1 G_2, b$ is G.P. with common ratio $r = \left(\frac{b}{a}\right)^{\frac{1}{3}}$

$$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{3}}$$

$$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{3}} = a^{\frac{1}{3}}b^{\frac{2}{3}}$$

Now,

$$G_1^{\,2} = a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}$$

$$G_2^{\,2} = a^{\frac{2}{3}}b^{\frac{4}{3}}$$

Then,

$$\frac{G^2{}_1}{G_2} + \frac{G^2{}_2}{G_1} = \frac{a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}{a^{\frac{1}{3}}b^{\frac{2}{3}}} + \frac{a^{\frac{2}{3}}b^{\frac{4}{3}}}{a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

$$= a^{2-\frac{2}{3}-\frac{1}{3}}b^{\frac{2}{3}-\frac{2}{3}} + a^{\frac{2}{3}-2+\frac{2}{3}}b^{\frac{4}{3}-\frac{2}{3}}$$

$$= a^{\frac{3}{3}}b^0 + a^0 b$$

$$= a + b$$

$$= 2a$$

$$= \text{RHS}$$