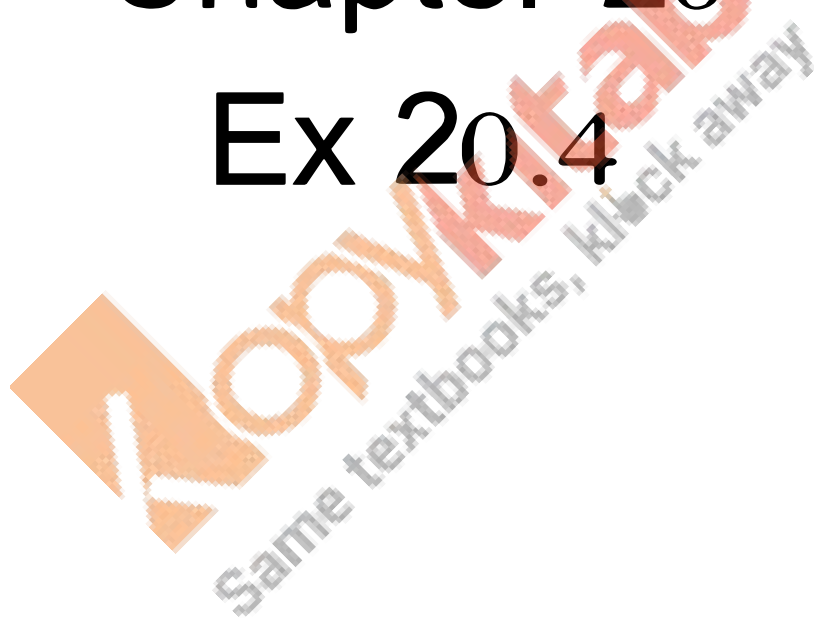


RD Sharma
Solutions
Class 11 Maths
Chapter 20
Ex 20.4



Geometric Progressions Ex 20.4 Q 1

$$S_{\infty} = 1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots$$

$$\Rightarrow a = 1, r = -\frac{1}{3}$$

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{1}{1 + \frac{1}{3}}$$

$$S_{\infty} = \frac{3}{4}$$

$$S_{\infty} = 8 + 4\sqrt{2} + 4 + \dots$$

$$\Rightarrow a = 8, r = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{8}{1 - \frac{1}{\sqrt{2}}}$$

$$= \frac{8\sqrt{2}}{\sqrt{2}-1} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}$$

$$= \frac{8(2+\sqrt{2})}{2-1}$$

$$S_{\infty} = 8(2+\sqrt{2})$$

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$$\begin{aligned}
 S_{\infty} &= \frac{2}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \dots \\
 &= \left(\frac{2}{5} + \frac{2}{5^3} + \dots \right) + \left(\frac{3}{5^2} + \frac{3}{5^4} + \dots \right) \\
 S_{\infty} &= S'_{\infty} + S''_{\infty}
 \end{aligned}$$

For

$$\begin{aligned}
 S'_{\infty} &= \frac{a}{1-r} \\
 &= \frac{\frac{2}{5}}{1 - \frac{1}{25}} \\
 &= \frac{2}{5} \times \frac{25}{24}
 \end{aligned}$$

$$S'_{\infty} = \frac{5}{12}$$

$$\begin{aligned}
 S''_{\infty} &= \frac{\frac{3}{25}}{1 - \frac{1}{25}} \\
 &= \frac{3}{25} \times \frac{25}{24} \\
 &= \frac{3}{24}
 \end{aligned}$$

$$\begin{aligned}
 S_{\infty} &= S'_{\infty} + S''_{\infty} \\
 &= \frac{5}{12} + \frac{3}{24} \\
 &= \frac{13}{24} \\
 S_{\infty} &= \frac{13}{24}
 \end{aligned}$$

This infinite G.P has first term $a=10$ and common ratio $r = -\frac{9}{10} = -0.9$

Thus the sum of the infinite G.P will be:

$$\begin{aligned}
 10 - 9 + 8.9 - 7.29 + \dots \infty &= \frac{a}{1-r} \quad [\text{Since } |r| < 1] \\
 &= \frac{10}{1 - (-0.9)} \\
 &= \frac{10}{1.9} \\
 &= \frac{100}{19}
 \end{aligned}$$

The G.P can be written as follows:

$$\begin{aligned}\frac{1}{3} + \frac{1}{5^2} + \frac{1}{3^3} + \frac{1}{5^4} + \frac{1}{3^5} + \frac{1}{5^6} + \dots \infty &= \left(\frac{1}{3} + \frac{1}{3^3} + \frac{1}{3^5} + \dots \infty \right) + \left(\frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6} + \dots \infty \right) \\ &= \frac{\frac{1}{3}}{1 - \frac{1}{3^2}} + \frac{\frac{1}{5^2}}{1 - \frac{1}{5^2}} \\ &= \frac{3}{8} + \frac{1}{24} \\ &= \frac{10}{24} \\ &= \frac{5}{12}\end{aligned}$$

Geometric Progressions Ex 20.4 Q 2

$$g^{\frac{1}{3}} \times g^{\frac{1}{9}} \times g^{\frac{1}{27}} \dots \infty$$

$$= g^{\left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \infty \right)}$$

$$= g^{\left(\frac{\frac{1}{3}}{1 - \frac{1}{3}} \right)}$$

$$= g^{\left(\frac{1}{3} \times \frac{3}{2} \right)}$$

$$= g^{\frac{1}{2}}$$

$$= 3$$

So,

$$g^{\frac{1}{3}} \times g^{\frac{1}{9}} \times g^{\frac{1}{27}} \dots \infty = 3$$

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$$\left[\text{Using } S_{\infty} = \frac{a}{1-r} \right]$$

Geometric Progressions Ex 20.4 Q 3

$$2^{\frac{1}{4}}, 4^{\frac{1}{8}}, 8^{\frac{1}{16}}, 16^{\frac{1}{32}}, \dots, \infty$$

$$= 2^{\frac{1}{4}}, 2^{\frac{2}{8}}, 2^{\frac{3}{16}}, 2^{\frac{4}{32}}, \dots, \infty$$

$$= \left(\frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \dots, \infty \right)$$

$$= 2$$

$$= 2^5 \text{----- (1)}$$

$$S = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \dots, \infty$$

$$S = \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots, \infty \right) 2$$

$$\frac{S}{2} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots, \infty$$

$$= \frac{1}{4}$$

$$1 - \frac{1}{2}$$

$$= \frac{1}{4} \times \frac{2}{1}$$

$$S = \frac{1}{2}$$

$$S = 1$$

Thus $2^{\frac{1}{4}}, 4^{\frac{1}{8}}, 8^{\frac{1}{16}}, 16^{\frac{1}{32}}, \dots, \infty = 2^1 = 2$

Geometric Progressions Ex 20.4 Q4

$$S_p = 1 + r^p + r^{2p} + \dots + \infty$$

$$S_p = \frac{1}{1 - r^p}$$

$$s_p = 1 - r^p + r^{2p} + \dots + \infty$$

$$s_p = \frac{1}{1 + r^p}$$

Now,

$$S_p + s_p = \frac{1}{1 - r^p} + \frac{1}{1 + r^p}$$

$$= \frac{2}{1 - r^{2p}}$$

$$S_p + s_p = 2 \times S_{2p}$$

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Geometric Progressions Ex 20.4 Q5.

Here,

$$a = 4$$

$$a_3 - a_5 = \frac{31}{81}$$

$$ar^2 - ar^4 = \frac{32}{81}$$

$$r^2 4(1 - r^2) = \frac{32}{81}$$

$$r^2(1 - r^2) = \frac{8}{81}$$

Let $r^2 = A$

$$A(1 - A) = \frac{8}{81}$$

$$A - A^2 = \frac{8}{81}$$

$$81A - 81A^2 = 8$$

$$81A^2 - 81A + 8 = 0$$

$$A = \frac{81 \pm \sqrt{(81)^2 - 4 \times 81 \times 8}}{81 \times 2}$$

$$= \frac{81 \pm \sqrt{6561 - 2592}}{162}$$

$$= \frac{81 \pm \sqrt{3969}}{162}$$

$$= \frac{81 \pm 63}{162}$$

$$= \frac{81 + 63}{162} \text{ or } \frac{81 - 63}{162}$$

$$= \frac{144}{162} \text{ or } \frac{18}{162}$$

$$r^2 = \frac{8}{9} \text{ or } \frac{1}{9}$$

$$r = \pm \frac{2\sqrt{2}}{3} \text{ or } \pm \frac{1}{3}$$

Since it is a decreasing G.P.

$$r = \frac{2\sqrt{2}}{3}, \frac{1}{3}$$

$$S_{\infty} = \frac{4}{1 - \frac{2\sqrt{2}}{3}} \text{ and } S_{\infty} = \frac{4}{1 - \frac{1}{3}}$$

$$S_{\infty} = \frac{12}{3 - 2\sqrt{2}}, 6$$

Geometric Progressions Ex 20.4 Q6.

$$a = 1$$

$$a_n = a_{n+1} + a_{n+2} + a_{n+3} + \dots$$

$$ar^{n-1} = ar^n + ar^{n+1} + ar^{n+2} + \dots$$

$$ar^{n-1} = ar^n (1 + r + r^2 + \dots)$$

$$1 = r \left(\frac{1}{1-r} \right)$$

$$1 - r = r$$

$$1 = 2r$$

$$r = \frac{1}{2}$$

G.P. is $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$

Geometric Progressions Ex 20.4 Q 7

$$a + ar = 5$$

$$a(1+r) = 5 \text{ ----- (1)}$$

$$a_n = 3(a_{n+1} + a_{n+2} + a_{n+3} + \dots)$$

$$ar^{n-1} = 3(ar^n + ar^{n+1} + ar^{n+2} + \dots)$$

$$ar^{n-1} = 3ar^n (1+r+r^2 + \dots)$$

$$1 = 3r \left(\frac{1}{1-r} \right)$$

$$1-r = 3r$$

$$1 = 4r$$

$$r = \frac{1}{4}$$

$$a(1+r) = 5$$

$$a \left(\frac{5}{4} \right) = 5$$

$$a = 4$$

G.P. is $4, 1, \frac{1}{4}, \frac{1}{16}, \dots$

Geometric Progressions Ex 20.4 Q8

$$0.125125125\dots = 0.\overline{125}$$

$$= 0.125 + 0.000125 + 0.000000125 + \dots$$

$$= \frac{125}{10^3} + \frac{125}{10^6} + \frac{125}{10^9} + \dots$$

$$= \frac{125}{10^3} \left(1 + \frac{1}{10^3} + \frac{1}{10^6} + \dots \right)$$

$$= \frac{125}{10^3} \left(\frac{1}{1 - \frac{1}{1000}} \right)$$

$$= \frac{125}{1000} \left(\frac{1000}{999} \right)$$

$$0.125125125\dots = \frac{125}{999}$$

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Geometric Progressions Ex 20.4 Q 9

$$\begin{aligned}0.4\overline{23} &= 0.4 + 0.0232323\dots\dots\dots \\&= 0.4 + 0.023 + 0.00023 + \dots\dots\dots \\&= 0.4 + \frac{23}{10^3} + \frac{23}{10^5} + \dots\dots\dots \\&= 0.4 + \frac{23}{10^3} \left(1 + \frac{1}{10^2} + \frac{1}{10^4} + \dots\dots\dots \right) \\&= 0.4 + \frac{23}{1000} \left(\frac{1}{1 - \frac{1}{100}} \right) \\&= 0.4 + \frac{23}{1000} \left(\frac{100}{99} \right) \\&= \frac{4}{10} + \frac{23}{990} \\&= \frac{396 + 23}{990} \\0.4\overline{23} &= \frac{419}{990}\end{aligned}$$

Geometric Progressions Ex 20.4 Q 10

Let a be first term and r be common ratio of G.P. Here,

$$\begin{aligned}\frac{a_n}{(a_{n+1} + a_{n+2} + \dots\infty)} &= \frac{ar^{n-1}}{ar^n + ar^{n+1} + \dots} \\&= \frac{ar^{n-1}}{ar^n (1 + r + r^2 + \dots\infty)} \\&= \frac{ar^{n-1}}{ar^n \left(\frac{1}{1-r} \right)} \\&= \left(\frac{1-r}{r} \right)\end{aligned}$$

Since r is a constant, so

$$\left(\frac{a_n}{a_{n+1} + a_{n+2} + \dots\infty} \right) = k \text{ (constant)}$$

Such that $k = \left(\frac{1-r}{r} \right)$

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Geometric Progressions Ex 20.4 Q 11

$$\begin{aligned}0.\overline{3} &= 0.3333\dots \\ &= 0.3 + 0.03 + 0.003 + \dots \\ &= \frac{3}{10} + \frac{3}{10^2} + \frac{3}{10^3} + \dots \\ &= \frac{3}{10} \left(1 + \frac{1}{10} + \frac{1}{10^2} + \dots \right) \\ &= \frac{3}{10} \left(\frac{1}{1 - \frac{1}{10}} \right) \\ &= \frac{3}{10} \times \frac{10}{9} \\ &= \frac{3}{9}\end{aligned}$$

$$0.\overline{3} = \frac{1}{3}$$

$$\begin{aligned}0.\overline{231} &= 0.231231231\dots \\ &= 0.231 + 0.000231 + 0.000000231 + \dots \\ &= \frac{231}{10^3} + \frac{231}{10^6} + \frac{231}{10^9} + \dots \\ &= \frac{231}{10^3} \left(1 + \frac{1}{10^3} + \frac{1}{10^6} + \dots \right) \\ &= \frac{231}{1000} \left(\frac{1}{1 - \frac{1}{1000}} \right)\end{aligned}$$

$$0.\overline{231} = \frac{231}{999}$$

$$\begin{aligned}
3.5\bar{2} &= 3 + 0.52222\dots \\
&= 3 + 0.5 + 0.02 + 0.002 + 0.0002 + \dots \\
&= 3.5 + \frac{2}{10^2} + \frac{2}{10^3} + \frac{2}{10^4} + \dots \\
&= 3.5 + \frac{2}{10^2} \left(1 + \frac{1}{10} + \frac{1}{10^2} + \dots \right) \\
&= \frac{35}{10} + \frac{2}{100} \left(\frac{1}{1 - \frac{1}{10}} \right) \\
&= \frac{35}{10} + \frac{2}{100} \times \left(\frac{10}{9} \right) \\
&= \frac{35}{10} + \frac{2}{90} \\
&= \frac{315 + 2}{90} \\
3.5\bar{2} &= \frac{317}{90}
\end{aligned}$$

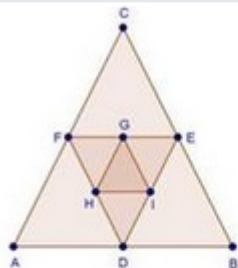
The rational number can be written as:

$$\begin{aligned}
0.6\bar{8} &= 0.6 + 0.08 + 0.008 + 0.0008 + \dots \infty \\
&= \frac{3}{5} + 8[0.01 + 0.001 + 0.0001 + \dots \infty] \\
&= \frac{3}{5} + 8 \left[\frac{1}{100} + \frac{1}{1000} + \dots \infty \right]
\end{aligned}$$

This is an infinite GP with first term $\frac{1}{100}$ and common ratio $\frac{1}{10}$

$$\begin{aligned}
&= \frac{3}{5} + 8 \cdot \frac{1}{100} \cdot \frac{1}{1 - \frac{1}{10}} \\
&= \frac{3}{5} + \frac{4}{45} \\
&= \frac{31}{45}
\end{aligned}$$

Geometric Progressions Ex 20.4 Q12



Side of triangle = 18 cm.

$$AD = BD = 9 \text{ cm.}$$

$$DE = BE = 9 \text{ cm.}$$

$$GI = IF = \frac{9}{2} \text{ cm.}$$

Sides of the triangles are 18, 9, $\frac{9}{2}$,

(i) sum of perimeters of the equilateral triangle = $\left(54 + 27 + \frac{27}{2} + \dots\right)$

$$= \frac{54}{1 - \frac{1}{2}}$$

$$= 54 \times 2$$

Perimeter = 108 cm.

(ii) sum of area of equilateral triangle

$$= \left[\frac{\sqrt{3}}{4} (18)^2 + \frac{\sqrt{3}}{4} (9)^2 + \frac{\sqrt{3}}{4} \left(\frac{9}{2}\right)^2 + \dots \right]$$

$$= \frac{\sqrt{3}}{4} \left[324 + 81 + \frac{81}{4} + \dots \right]$$

$$= \frac{\sqrt{3}}{4} \left[\frac{324}{1 - \frac{1}{4}} \right]$$

$$= \frac{\sqrt{3}}{4} \left[\frac{324 \times 4}{3} \right]$$

$$= \sqrt{3} (108)$$

Geometric Progressions Ex 20.4 Q13

$$S = a + ar + ar^2 + ar^3 + \dots$$

$$S = \frac{a}{1-r} \quad \text{--- (1)}$$

$$S_1 = a^2 + a^2r^2 + a^2r^4 + a^2r^6 + \dots$$

$$S_1 = \frac{a^2}{1-r^2} \quad \text{--- (2)}$$

$$S^2 = \frac{a^2}{(1-r)^2}$$

$$S^2 = \frac{S_1(1-r^2)}{(1-r^2)}$$

$$(1-r)S^2 = S_1(1+r)$$

$$S^2 - S^2r = S_1 + S_1r$$

$$S_1r + S^2r = S^2 - S_1$$

$$r = \frac{S^2 - S_1}{S_1 + S^2}$$

Put r in equation (1)

$$S(1-r) = a$$

$$a = S \left[1 - \frac{S^2 - S_1}{S^2 + S_1} \right]$$

$$a = S \left[\frac{S^2 + S_1 - S^2 + S_1}{S^2 + S_1} \right]$$

$$a = \frac{2SS_1}{S^2 + S_1}$$

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