Geometric Progressions Ex 20.3 Q 5

Here,

$$3,\ \frac{3}{2},\ \frac{3}{4},\ \ldots \text{ is a G.P.}$$

and $S_n = \frac{3069}{512},\ a = 3, r = \frac{1}{2}$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r}$$

$$\frac{3069}{512} = \frac{3\left(1 - \left(\frac{1}{2}\right)^n\right)}{1 - \frac{1}{2}}$$

$$\frac{3069}{512} = \frac{3\left(2^n - 1\right)}{2^n \times \frac{1}{2}}$$

$$\frac{1023}{512} = \frac{2\left(2^n - 1\right)}{2^n}$$

$$10232^n = 1024.2^n - 1024$$

$$1024 = 2^n$$

$\Rightarrow \quad 2^{10} = 2^n$

$\Rightarrow \quad n = 10$

Geometric Progressions Ex 20.3 Q 6

$2 + 6 + 18 + \ldots$

$$S_n = 728$$

Now,

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$a = 2,\ r = \frac{6}{2} = 3$$

$$728 = \frac{2\left(3^n - 1\right)}{3 - 1}$$

$$728 = \frac{2\left(3^n - 1\right)}{2} = \left(3^n - 1\right)$$

$$728 + 1 = 3^n$$

$$729 = 3^n$$

$$(3)^6 = 3^n$$

$\Rightarrow \quad n = 6$

Geometric Progressions Ex 20.3 Q 7

$\sqrt{3}, 3, 3\sqrt{3}, \ldots$

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$a = \sqrt{3},\ r = \frac{3}{\sqrt{3}} = \sqrt{3},\ S_n = 39 + 13\sqrt{3}$$

Putting into formula

$$39 + 13\sqrt{3} = \frac{\sqrt{3}\left(\left(\sqrt{3}\right)^n - 1\right)}{\sqrt{3} - 1}$$

$$39 + 13\sqrt{3} = \frac{\left(\sqrt{3}\right)^{n+1} - \sqrt{3}}{\sqrt{3} - 1}$$

$$\left(39 + 13\sqrt{3}\right)\left(\sqrt{3} - 1\right) = \left(\sqrt{3}\right)^{n+1} - \sqrt{3}$$

$$39\sqrt{3} - 39 + 39 - 13\sqrt{3} = \left(\sqrt{3}\right)^{n+1} - \sqrt{3}$$

$$26\sqrt{3} + \sqrt{3} = \left(\sqrt{3}\right)^{n+1}$$

$$\left(27\sqrt{3}\right)^1 = \left(\sqrt{3}\right)^{n+1}$$

$$\left(\sqrt{3}\right)^6\left(\sqrt{3}\right)^1 = \left(\sqrt{3}\right)^{n+1}$$

$$7 = n + 1$$

$$\Rightarrow \quad n = 6$$

Geometric Progressions Ex 20.3 Q 8

$3, 6, 12, \ldots n\ 381$

$$a = 3,\ r = \frac{6}{3} = 2,\ n = ?\ S_n = 381$$

We know that

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$381 = \frac{3(2)^n - 1}{2 - 1}$$

$$\frac{381}{3} = 2^n - 1$$

$$127 = 2^n - 1$$

$$128 = 2^n$$

$$2^7 = 2^n$$

$$n = 7$$

Geometric Progressions Ex 20.3 Q 9

$r = 3$, last term is 486

Sum of terms $= S_n = 728$, $a = ?$

We know that

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$728 = \frac{a\left(3^n - 1\right)}{3 - 1}$$

Also, $t_n = ar^{n-1}$

$t_n = 486$

$\therefore \quad 486 = a(3)^{n-1}$

$a\left(3^{n-1}\right) = 3^5 \times 2$

$3^{n-1} = 3^5$

$n = 6$

and $\quad a = 2$

Geometric Progressions Ex 20.3 Q 10

Let Sum of first three terms $= a + ar + ar^2$

$$\text{The ratio} = \frac{a + ar + ar^2}{a + ar + ar^2 + ar^3 + ar^4 + ar^5}$$

$$= \frac{1 + r + r^2}{1 + r + r^2 + r^3 + r^4 + r^5}$$

$$= \frac{1 + r + r^2}{1 + r + r^2 + r^3(1 + r + r^2)} \quad \text{.......................(1)}$$

Let $A = 1 + r + r^2$ ....(2)

$$Ratio = \frac{A}{A + r^3 A} = \frac{125}{152}$$

$$\frac{1}{1 + r^3} = \frac{125}{152}$$

$$152 = 125 + 125\, r^3$$

$$r^3 = \frac{27}{125}$$

$$r = \frac{3}{5}$$

Geometric Progressions Ex 20.3 Q 11

$t_4 = \frac{1}{27}$, $t_7 = \frac{1}{729}$, $t_n = ar^{n-1}$

Where $t_n = n^{th}$ term, r = common difference, n = number of terms.

$$t_4 = ar^3 = \frac{1}{27} \qquad \text{---(i)}$$

$$t_7 = ar^6 = \frac{1}{729} \qquad \text{---(ii)}$$

Dividing(ii) by (i), we get

$$\frac{t_7}{t_4} = \frac{ar^6}{ar^3} = r^3 = \frac{27}{729} = \frac{1}{27}, \quad r = \frac{1}{3}$$

Sum of n term $= S_n = \frac{a\left(1 - r^n\right)}{1 - r}$ ---(i)

When, $r = 3$, $t_4 = ar^3 = \frac{1}{27}$

$$a\left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

$$a = 1$$

Substituting $a = 1$, $r = \frac{1}{3}$ in(i)

$$S_n = \frac{1\left(1 - \left(\frac{1}{3}\right)^n\right)}{1 - \frac{1}{3}}$$

$$= \frac{1 - \left(\frac{1}{3}\right)^n}{\frac{2}{3}}$$

$$= \frac{3}{2}\left(1 - \left(\frac{1}{3}\right)^n\right)$$

Geometric Progressions Ex 20.3 Q 12

$$\sum_{n=1}^{10}\left\{\left(\frac{1}{2}\right)^{n-1} + \left(\frac{1}{5}\right)^{n+1}\right\}$$

$$= \sum_{n=1}^{10}\left(\frac{1}{2}\right)^{n-1} + \sum_{n=1}^{10}\left(\frac{1}{5}\right)^{n+1}$$

$$= 1 + \frac{1}{2} + \frac{1}{2^2} + \ldots\ldots + \frac{1}{5^2} + \frac{1}{5^3} + \frac{1}{5^4} + \ldots\ldots$$

$$= \frac{(1 - \frac{1}{2^{10}})}{1 - \frac{1}{2}} + \frac{\frac{1}{5}(1 - \frac{1}{5^{10}})}{1 - \frac{1}{5}}$$

$$= \frac{2^{10} - 1}{2^9} + \frac{5^{10} - 1}{5^{11}}$$

Geometric Progressions Ex 20.3 Q 13

Fifth term of series is

$ar^{5-1} = 81 \ldots\ldots\ldots\ldots\ldots(1)$

Second term of series is

$ar = 24 \quad \ldots\ldots\ldots\ldots\ldots(2)$

Dividing (2) by (1) we get,

$$\frac{ar}{ar^4} = \frac{24}{81} = \frac{8}{27}$$

$$r^3 = \frac{27}{8}$$

$$r = \frac{3}{2}$$

Substituting r in (2), we get,

$$a = \frac{24 \times 2}{3} = 16$$

$$Sum = \frac{16\left[\left(\frac{3}{2}\right)^8 - 1\right]}{\frac{3}{2} - 1}$$

$$= \frac{16\left[3^8 - 2^8\right]}{2^7}$$

$$= \frac{6305}{8}$$

Geometric Progressions Ex 20.3 Q14

S_1 = sum of n terms,
S_1 = sum of $2n$ terms,
S_1 = sum of $3n$ terms.

Then, $S_1^2 + S_2^2$

$$= (S_n)^2 + (S_{2n})^2$$
$$= \left(\frac{a(1-r^n)}{1-r}\right)^2 + \left(\frac{a(1-r^{2n})}{1-r}\right)^2$$
$$= \frac{a^2}{(1-r)^2}\left[\left(1-(r)^n\right)^2 + \left(1-r^{2n}\right)^2\right]$$
$$= \frac{a^2}{(1-r)^2}\left[1 + r^{2n} - 2r^n + 1 + r^{4n} - 2r^{2n}\right]$$
$$= \frac{a^2}{(1-r)^2}\left[2 - r^{2n} - 2r^n + r^{4n}\right] \quad \text{---(i)}$$

Also, $S_1(S_2 + S_3)$

$$= \frac{a(1-r^n)}{1-r}\left(\frac{a(1-r^{2n})}{1-r} + \frac{a(1-r^{3n})}{1-r}\right)$$
$$= \frac{a^2}{(1-r)^2}\left[(1-r)^n\left(1-r^{2n}\right) + \left(1-r^n\right)\left(1-r^{3n}\right)\right]$$
$$= \frac{a^2}{(1-r)^2}\left[1 - r^{2n} - r^n + r^{3n} - r^{3n} - r^n + 1 + r^{4n}\right]$$
$$= \frac{a^2}{(1-r)^2}\left[2 - r^{2n} - 2r^n + r^{4n}\right] \quad \text{---(ii)}$$

(i) = (ii) Hence, $S_1^2 + S_2^2 = S_1(S_2 + S_3)$

Geometric Progressions Ex 20.3 Q15

$S_1, S_2, \ldots, S_n$ are the sums of n terms of G.P. $a = 1$, $r = 1, 2, 3, \ldots, n$

Then, $S_1 + S_2 + 2S_3 + 3S_4 + \ldots + (n-1)S_n$

$$\frac{1(1^n - 1)}{1-1} + \frac{1(2^n - 1)}{2-1} + \frac{2(3^n - 1)}{3-1} + \ldots + (n-1)1\left(\frac{1^n - 1}{1-1}\right)$$
$$= 2^n - 1 + 23^n - 1 + 3.4^n - 1 + \ldots$$
$$= 2^n + 3^n + 4^n + \ldots + n^n$$

Geometric Progressions Ex 20.3 Q16.

Let the G.P. be $2n, 2, 2n+4, \dots$

Then, $S_n = \dfrac{a\left(r^n - 1\right)}{r-1}$, $a = 2n$, $r = 2$

$\therefore \quad S_n = \dfrac{2n\left(2^n - 1\right)}{2-1} = 2n^{n+1} - 2n$

Then the G.P. of odd term

$a_1 + a_3 + a_5 + \dots a_{2n-1}$

According to the question

Sum of all terms = 5 (sum of terms occupying the odd places)

$a_1 + a_2 + a_3 + \dots + a_{2n} = 5\left(a_1 + a_3 + a_5 + \dots + a_{2n-1}\right)$

$a + ar + ar^2 + \dots + ar^{2n-1} = 5\left(a + ar^2 + ar^4 + \dots + ar^{2n-2}\right)$

$\dfrac{a\left(1 - r^{2n}\right)}{1-r} = 5\left(\dfrac{a\left(1 - (r)^2\right)^n}{1-r^2}\right)$

$\dfrac{a}{1-r}$ is cancelled on both side

$1 - r^{2n} = \dfrac{5\left(1 - r^{2n}\right)}{1+r}$

$1 + r - r^{2n} - r^{2n+1} = 5 - 5r^{2n}$

$r^{2n+1} - 4r^{2n} - r + 4 = 0$

$r^{2n}(r-4) - 1(r-4) = 0$

$r^{2n} = 1, \; r = 4$

$\Rightarrow \quad r = 4$

Geometric Progressions Ex 20.3 Q17

Given $\sum_{n=1}^{100} a_{2n} = \alpha$

$\Rightarrow \quad a_2 + a_4 + a_6 + \dots + a_{200} = \alpha$ ---(i)

also, $\sum_{n=1}^{100} a_{2n-1} = \beta$

$\Rightarrow \quad a_1 + a_3 + a_5 + \dots + a_{199} = \beta$ ---(ii)

Sum of G.P,

$S_n = \dfrac{a\left(1 - r^n\right)}{1-n}$

$= a = a_2, r = r^2, n = 100$

$ar + ar^3 + ar^5 + \dots + ar^{199} = \alpha$

$ar\dfrac{\left(1 - \left(r^2\right)^{100}\right)}{1-r^2} = \alpha$ ---(iii)

$a + ar^2 + ar^4 + \dots + ar^{198} = \beta$

$\dfrac{a\left(1 - \left(r^2\right)^{100}\right)}{1-r^2} = \beta$ ---(iv)

$r(\beta) = \alpha$

$r = \dfrac{\alpha}{\beta}$ [From (iv) and (v)]

Geometric Progressions Ex 20.3 18

Let the seried be $a_1 + a_2 + a_3 + \ldots + a_{2n}$

It is given that $a_1 = 1, a_2 = a, a_3 = ac, a_4 = a^2c, a_5 = a^2c^2, \ldots$

$\therefore$ Sum of $2n$ term

$a_1 + a_2 + a_3 + \ldots + a_{2n}$

$= 1 + a + ac + a^2c + a^2c^2 + \ldots + 2n$ term

$= (1 + a) + ac(1 + a) + a^2c^2(1 + a) + \ldots + n$ term

$$= (1 + a)\frac{\left(1 - (ac)^n\right)}{1 - ac}$$

$$= (a + 1)\frac{\left((ac)^n - 1\right)}{ac - 1}.$$

Geometric Progressions Ex 20.3 Q19.

Sum of first n term of G.P.

$= a + a_2 + a_3 + \ldots + a_n$

$= a + ar + ar^2 + \ldots + ar^{n-1}$ $\quad \left[\because t_n = ar^{n-1}\right]$ ---(i)

Also sum of term from

$(n+1)^{th}$ to $(2n)^{th}$ term is

$= a_{n+1} + a_{n+2} + \ldots + a_{2n}$

$= ar^n + ar^{n-1} + \ldots + ar^{2n-1}$ ---(ii)

Ratio of (i) and (ii) is

$$= \frac{a + ar + ar^2 + \ldots ar^{n-1}}{ar^n + ar^{n-1} + \ldots + ar^{2n-1}} \quad \left[\because S_n = \frac{a\left(1 - r^n\right)}{1 - r}\right]$$

$$= \frac{\frac{a\left(1 - r^n\right)}{1 - r}}{\frac{ar^n\left(1 - r^n\right)}{1 - r}}$$

$$= \frac{1}{r^n}$$

Geometric Progressions Ex 20.3 Q20

Given,

$a,\ b$ are roots of the equation $x^2 - 3x + p = 0$

$\Rightarrow \quad a + b = 3,\ ab = p$

and $\quad c,\ d$ are roots of the equation $x^2 - 12x + q = 0$

$\Rightarrow \quad c + d = 12,\ cd = q$

Let $b = ar,\ c = ar^2$ and $d = ar^3$, then $a + b = 3$ and $c + d = 12$

$a(1+r) = 3$ and $ar^2(1+r) = 12$

$$\Rightarrow \quad \frac{ar^2(1+r)}{a(1+r)} = \frac{12}{3}$$

$\Rightarrow \quad r = 2$

and $a(r+1) = 3$

$\Rightarrow \quad a = 1$

$p = ab$

$= a \times ar$

$p = 2$

$q = cd$

$= ar^2 \times ar^3$

$= 2^5$

$a = 32$

$$\frac{q+p}{q-p} = \frac{32+2}{32-2}$$

$$= \frac{34}{30}$$

$(q+p) : (q-p) = 17 : 15$

Geometric Progressions Ex 20.3 Q21.

$$\text{Sum} = \frac{3069}{512} = \frac{3(1-\frac{1}{2^n})}{\frac{1}{2}}$$

$$1 - \frac{1}{2^n} = \frac{3069}{512\times 6} = \frac{1023}{512\times 2}$$

$$1 - \frac{1023}{1024} = \frac{1}{2^n}$$

$$\frac{1}{2^n} = \frac{1}{1024}$$

$n = 10$

Geometric Progressions Ex 20.3 Q22.

To find number of ancestors, we will find the sum of $2, 2^2, 2^3, \ldots$

$$\text{Number of ancestors} = \frac{2(2^{10}-1)}{2-1}$$

$= 2(1024 - 1)$

$= 2 \times 1023$

$= 2046$