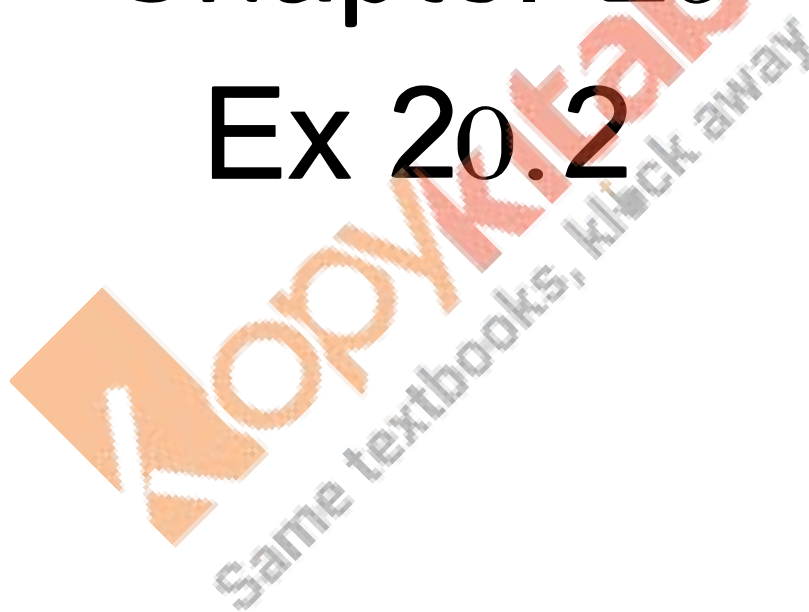


RD Sharma
Solutions
Class 11 Maths
Chapter 20
Ex 20.2



Geometric Progressions Ex 20.2 Q 1

Let the three number in G.P be $\frac{a}{r}, a, ar$

$$\text{Sum of these numbers} = \frac{a}{r} + a + ar = 65$$

3375 = Product of these numbers

$$3375 = \left(\frac{a}{r}\right)(a)(ar) = a^3$$

$$a^3 = (5)^3 \times (3)^3 = (15)^3$$

$$\Rightarrow a = 15$$

$$a\left(\frac{1}{r} + 1 + r\right) = 65$$

$$15\left(\frac{1}{r} + 1 + r\right) = \frac{65}{15} = \frac{13}{3}$$

$$\frac{1+r+r^2}{r} = \frac{13}{3}$$

$$3 + 3r + 3r^2 = 13r$$

$$3r^2 - 10r + 3 = 0$$

$$3r^2 - r - 9r + 3 = 0$$

$$r(3r - 1) - 3(3r - 1) = 0$$

$$r = 3, \frac{1}{3} \quad r = \frac{1}{3} \text{ or } r = 3$$

$\therefore$ G.P. is a, ar, ar^2

$\therefore$ G.P. is 45, 15, 5 or 5, 15, 45

Geometric Progressions Ex 20.2 Q 2

Let the three numbers be a, ar, ar^2 in G.P., where a is first term and r is the common ratio.

Then,

$$a + ar + ar^2 = 38$$

$$a(1 + r + r^2) = 38 \quad \text{---(i)}$$

and

$$(a)(ar)(ar)^2 = 1728$$

$$a^3 r^3 = 1728 = 4^3 3^3 = (12)^3$$

$$a^3 = \frac{12^3}{r^3} \Rightarrow \frac{12}{r} = a$$

Putting $a = \frac{12}{r}$ in (i)

$$\frac{12}{r}(1 + r + r^2) = 38$$

$$12 + 12r + 12r^2 = 38r$$

$$12r^2 - 26r + 12 = 0$$

$$6r^2 - 13r + 6 = 0$$

$$6r^2 - 9r - 4r + 6 = 0$$

$$3r(3r - 3) - 2(3r - 3) = 0$$

$$r = \frac{3}{2}, \frac{2}{3}$$

$$a = \frac{12}{\frac{3}{2}} = 8 \text{ or } \frac{12}{\frac{2}{3}} = 18$$

$\therefore$ G.P. is 8, 12, 18.

Geometric Progressions Ex 20.2 Q 3

Let the first three terms of G.P. are $\frac{a}{r}$, a , ar

Here,

$$\frac{a}{r} + a + ar = \frac{13}{12} \quad \text{---(i)}$$

$$\text{and } \frac{a}{r} \times a \times ar = -1$$

$$\Rightarrow a^3 = -1$$

$$\Rightarrow a = -1$$

Put $a = -1$ in equation (i),

$$\frac{-1}{r} + (-1) - r = \frac{13}{12}$$

$$\Rightarrow -1 - r - r^2 = \frac{13}{12}r$$

$$\Rightarrow -12 - 12r - 12r^2 = 13r$$

$$\Rightarrow 12r^2 + 12r + 13r + 12 = 0$$

$$\Rightarrow 12r^2 + 25r + 12 = 0$$

$$\Rightarrow 12r^2 + 16r + 9r + 12 = 0$$

$$\Rightarrow 4r(3r + 4) + 3(3r + 4) = 0$$

$$\Rightarrow (4r + 3)(3r + 4) = 0$$

$$r = \frac{-3}{4}, \frac{-4}{3}$$

So,

$$\text{Required G.P. is, } \frac{4}{3}, -1, \frac{3}{4}, \dots$$

$$\text{or } \frac{3}{4}, -1, \frac{4}{3}, \dots$$

Geometric Progressions Ex 20.2 Q4

Let the three numbers in G.P. be $\frac{a}{r}, a, ar$ then product of these numbers $\left(\frac{a}{r}\right)(a)(ar)$

$$\Rightarrow a^3 = 125 = 5^3$$
$$a = 5$$

Also, sum of these products in pair

$$\left(\frac{a}{r}\right)(a) + (a)(ar) + \left(\frac{a}{r}\right)(ar) = 87 \frac{1}{2} = \frac{195}{2}$$

$$\frac{a^2}{r} + a^2r + a^2 = a^2 \left(\frac{1}{r} + r + 1 \right)$$

$$= (5)^2 \left(\frac{1+r^2+r}{r} \right) = \frac{195}{2}$$

$$1+r^2+r = \left(\frac{195}{2 \times 25} \right)^r$$

$$2(1+r^2+r) = \frac{39}{5}r$$

$$10 + 10r^2 + 10r = 39r$$

$$10r^2 - 29r + 10 = 0$$

$$10r^2 - 25r - 4r + 10 = 0$$

$$5r(2r-5) - 2(2r-5) = 0$$

$$r = \frac{5}{2}, \frac{2}{5}$$

$\therefore$ G.P. is $\frac{a}{r}, a, ar$

$$10, 5, \frac{5}{2}, \dots \text{ or } \frac{5}{2}, 5, 10, \dots$$

Geometric Progressions Ex 20.2 Q5

Let the three numbers in G.P. be $\frac{a}{r}, a, ar$

then product of them is $\left(\frac{a}{r}\right)(a)(ar) = 21$ ---(i)

$$= \frac{a}{r}(1+r+r^2) = 21$$

and sum of their squares

$$\frac{a^2}{r^2} + a^2 + a^2r^2 = a^2 \frac{(1+r^2+r^4)}{r^2} = 189$$
 ---(ii)

Now,

$$a(1+r+r^2) = 21r$$
 ---(iii)

Then, $a^2(1+r+r^2)^2 = 441r^2$ [squaring]

$$a^2(1+r^2+r^4) + 2a^2r(1+r+r^2) = 441r$$

$$189r^2 + 2ar \times 21r = 441r^2$$

Dividing both sides by $21r^2$

$$9 + 2a = 21$$

$$2a = 21 - 9 = 12$$

$$a = 6 \Rightarrow a = 6$$

Putting in (iii)

$$6(1+r+r^2) = 21r$$

$$6 + 6r + 6r^2 - 21r = 0$$

$$6r^2 - 15r + 6 = 0$$

$$6r^2 - 12r - 3r + 6 = 0$$

$$\Rightarrow 6r(r-2) - 3(r-2) = 0$$

$$r = 2, \frac{1}{2}$$

$\therefore$ G.P. is 3, 6, 12 or 12, 6, 3.

Geometric Progressions Ex 20.2 Q6

Let the numbers are: $\frac{a}{r}$, a and ar .

Then

$$\frac{a}{r} + a + ar = 14$$

Again the numbers $a+1$, $ar+1$ and ar^2-1 are in A.P, therefore

$$2(a+1) = (ar-1) + \left(\frac{a}{r}+1\right)$$

$$2(a+1) = ar + \frac{a}{r}$$

$$2(a+1) = 14 - a$$

$$3a = 12$$

$$a = 4$$

Now we have

$$\frac{4}{r} + 4 + 4r = 14$$

$$2 - 5r + 2r^2 = 0$$

$$2r^2 - 4r - r + 2 = 0$$

$$2r(r-2) - 1(r-2) = 0$$

$$(r-2)(2r-1) = 0$$

$$r = 2, \frac{1}{2}$$

Thus the numbers are: 2, 4, 8 or 8, 4, 2.

Geometric Progressions Ex 20.2 Q7

Let the number in G.P. are $\frac{a}{r}$, a , ar

So,

$$\frac{a}{r} \times a \times ar = 216$$

$$\Rightarrow a^3 = 216$$

$$\Rightarrow a = 6$$

And also given,

$$\frac{a}{r} + 2, a + 8, ar + 6 \text{ are in A.P.}$$

$$2(a + 8) = \left(\frac{a}{r} + 2\right) + (ar + 6)$$

$$\Rightarrow 2(6 + 8) = \left(\frac{6 + 2r}{r}\right) + 6r + 6$$

$$\Rightarrow 28r = 6 + 2r + 6r^2 + 6r$$

$$\Rightarrow 6r^2 - 20r + 6 = 0$$

$$\Rightarrow 6r^2 - 18r - 2r + 6 = 0$$

$$\Rightarrow 6r(r - 3) - 2(r - 3) = 0$$

$$\Rightarrow (r - 3)(6r - 2) = 0$$

$$r = 3, r = \frac{1}{3}$$

So,

Required G.P. is 18, 6, 2, ...

or, 2, 6, 18, ...

Geometric Progressions Ex 20.2 Q8

Let three numbers in G.P. are $\frac{a}{r}$, a , ar

Here,

$$\frac{a}{r} \times a \times ar = 729$$

$$\Rightarrow a^3 = 729$$

$$\Rightarrow a = 9$$

And,

$$\left(\frac{a}{r} \times a\right) + (a \times ar) + \left(\frac{a}{r} \times ar\right) = 819$$

$$\Rightarrow \frac{81}{r} + 81r + 81 = 819$$

$$\Rightarrow \frac{9}{r} + 9r + 9 = 91$$

$$\Rightarrow 9 + 9r^2 + 9r = 91r$$

$$\Rightarrow 9r^2 - 82r + 9 = 0$$

$$\Rightarrow 9r^2 - 81r - r + 9 = 0$$

$$\Rightarrow 9r(r - 9) - 1(r - 9) = 0$$

$$r = 9, \frac{1}{9}$$

So, required G.P. are

81, 9, 1, ...

or, 1, 9, 81, ...

Geometric Progressions Ex 20.2 Q9

Let the numbers are $\frac{a}{r}$, a and ar . Then we have

$$\frac{a}{r} + a + ar = \frac{39}{10}$$

And

$$\frac{a}{r} \cdot a \cdot ar = 1$$

$$a^3 = 1$$

$$a = 1$$

Now we have

$$\frac{1}{r} + 1 + r = \frac{39}{10}$$

$$1 + r + r^2 = \frac{39}{10}r$$

$$r^2 - \frac{29}{10}r + 1 = 0$$

$$10r^2 - 29r + 10 = 0$$

$$(2r - 5)(5r - 2) = 0$$

$$r = \frac{5}{2}, \frac{2}{5}$$

Thus the numbers are: either $\frac{2}{5}, 1, \frac{5}{2}$ or $\frac{5}{2}, 1, \frac{2}{5}$.