RD Sharma Solutions Class 11 Maths Chapter 19 Ex 19.5

Arithematic Progressions Ex 19.5 Q1(i)

$\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ will be in A.P if $\frac{c+a}{b} - \frac{b+c}{a} = \frac{a+b}{c} - \frac{c+a}{b}$

if $\quad \frac{ca + a^2 - b^2 - cb}{ab} = \frac{ab + b^2 - c^2 - ac}{bc}$

LHS $\quad \Rightarrow \frac{ca + a^2 - b^2 - cb}{ab}$

$\Rightarrow \quad \frac{c^2a + a^2c - b^2c - c^2b}{abc}$

$\Rightarrow \quad \frac{c(a-b)[a+b+c]}{abc}$ ---(i)

RHS $\quad \Rightarrow \frac{ab + b^2 - c^2 - ac}{bc}$

$\Rightarrow \quad \frac{a^2b + ab^2 - ac^2 - a^2c}{abc}$

$\Rightarrow \quad \frac{a(b-c)[a+b+c]}{abc}$ ---(ii)

and since $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$

$c(b-a) = a(b-c)$ ---(iii)

$\therefore \quad$ LHS = RHS and the given terms are in A.P.

Arithematic Progressions Ex 19.5 Q1(ii)

$a(b+c), b(c+a), c(a+b)$ are in A.P if $b(c+a) - a(b+c) = c(a+b) - b(c+a)$

LHS $\quad = b(c+a) - a(b+c)$

$\quad = bc + ab - ab - ac$

$= c(b-a)$ ---(i)

RHS $= c(a+b) - b(c+a)$

$= ca + cb - bc - ba$

$= a(c-b)$ ---(ii)

and $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$\therefore$ $\frac{1}{a} - \frac{1}{b} = \frac{1}{b} - \frac{1}{c}$

or $c(b-a) = a(c-b)$ ---(iii)

From (i), (ii) and (iii)

$a(b+c), b(c+a), c(a+b)$ are in A.P

Arithematic Progressions Ex 19.5 Q2

$\frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b}$ are in A.P if $\frac{b}{a+c} - \frac{a}{b+c} = \frac{c}{a+b} - \frac{b}{a+c}$

LHS $= \frac{b}{a+c} - \frac{a}{b+c}$

$\Rightarrow$ $\frac{b^2 + bc - a^2 - ac}{(a+c)(b+c)}$

$\Rightarrow$ $\frac{(b-a)(a+b+c)}{(a+c)(b+c)}$ ---(i)

RHS $= \frac{a}{a+b} - \frac{b}{a+c}$

$\Rightarrow$ $\frac{ca + c^2 - b^2 - ab}{(a+b)(b+c)}$

$\Rightarrow \qquad \frac{(c-b)(a+b+c)}{(a+b)(b+c)}$ ---(ii)

and $\quad a^2, b^2, c^2$ are in A.P

$\therefore \quad b^2 - a^2 = c^2 - b^2$ ---(iii)

Substituting $b^2 - a^2$ with $c^2 - b^2$

(i) = (ii)

$\therefore \quad \frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b}$ are in A.P

Arithematic Progressions Ex 19.5 Q3(i)

$a^2(b+c), b^2(c+a), c^2(a+b)$ are in A.P.

If $b^2(c+a) - a^2(b+c) = c^2(a+b) - b^2(a+c)$

$\Rightarrow \quad b^2c + b^2a - a^2b - a^2c = c^2a + c^2b - b^2a - b^2c$

Given, $b - a = c - b$ [a, b, c are in A.P]

$c(b^2 - a^2) + ab(b-a) = a(c^2 - b^2) + bc(c-b)$

$(b-a)(ab+bc+ca) = (c-b)(ab+bc+ca)$

Cancelling $ab+bc+ca$ from both sides

$b - a = c - b$

$2b = c + a$ which is true

Hence, $a^2(b+c), (c+a)b^2$ and $c^2(a+b)$ are also in A.P.

Arithematic Progressions Ex 19.5 Q3(ii)

(ii) T.P $b+c-a, c+a-b, a+b-c$ are in A.P.

$b+c-a, c+a-b, a+b-c$ are in A.P only if $(c+a-b)-(b+c-a)=(a+b-c)-(c+a-b)$

LHS $\Rightarrow (c+a-b)-(b+c-a)$

$\Rightarrow \quad 2a-2b \quad$ ---(i)

RHS $\Rightarrow (a+b-c)-(c+a-b)$

$\Rightarrow \quad 2b-2c \quad$ ---(ii)

Since,

a, b, c are in A.P

$\therefore \quad b-a=c-b$

or $\quad a-b=b-c \quad$ ---(iii)

From (i), (ii) and (iii)

LHS = RHS

Thus, given numbers

$b+c-a, c+a-b, a+b-c$ are in A.P

Arithematic Progressions Ex 19.5 Q3(iii)

To prove $bc-a^2, ca-b^2, ab-c^2$ are in A.P

$$\left(ca-b^2\right)-\left(bc-a^2\right)=\left(ab-c^2\right)-\left(ca-b^2\right)$$

LHS $=\left(a-b^2-bc+a^2\right)$

$=(a-b)[a+b+c] \quad$ ---(i)

$$\text{RHS} = ab - c^2 - ca + b^2$$
$$= (b-c)[a+b+c] \qquad \text{---(ii)}$$

and since a, b, c are in ab

$$b - c = a - b$$

$\therefore$ LHS = RHS

and

Thus, $bc - a^2, ca - b^2, ab - c^2$ are in A.P

Arithematic Progressions Ex 19.5 Q4

(i) If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\text{LHS} = \frac{1}{b} - \frac{1}{a}$$
$$= \frac{a-b}{ab} = \frac{c(a-b)}{abc} \qquad \text{---(i)}$$

$$\text{RHS} = \frac{1}{c} - \frac{1}{b}$$
$$= \frac{a(b-c)}{abc} \qquad \text{---(ii)}$$

Since, $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P

$$\frac{b+c}{a} - \frac{c+a}{b} = \frac{c+a}{b} - \frac{a+b}{c}$$

$$\frac{b^2 + cb - ac - a^2}{ab} = \frac{c^2 + ac - ab - b^2}{bc}$$

$\Rightarrow \quad \frac{(b-a)(a+b+c)}{ab} = \frac{(c-b)(a+b+c)}{bc}$

or $\quad \frac{a(b-c)}{abc} = \frac{c(a-b)}{abc}$ ---(iii)

From (i), (ii) and (iii)

LHS = RHS

Hence, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

(ii) If bc, ca, ab are in A.P

Then,

$ca - bc = ab - ca$

$c(a-b) = a(b-c)$ ---(i)

If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$

$\Rightarrow \quad c(a-b) = a(b-c)$ ---(ii)

Thus, the condition necessary to prove bc, ca, ab in A.P is fullfilled.

Thus, bc, ca, ab, are in A.P.

Arithematic Progressions Ex 19.5 Q5

(i) If $(a-c)^2 = 4(a-b)(b-c)$

Then,

$a^2 + c^2 - 2ac = 4(ab - b^2 - ac + bc)$

$\Rightarrow \quad a^2 + c^2 4b^2 + 2ac - 4ab - 4bc = 0$

$\Rightarrow \quad (a + c - 2b)^2 = 0$ $\quad \left[\text{Using } (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc\right]$

$\therefore \quad a + c - 2b = 0$

or $\quad a + c = 2b$

and since,

a, b, c are in A.P [Given]

$a + c = 2b$

Hence proved.

$(a-c)^2 = 4(a-b)(b-c)$

(ii) If $a^2 + c^2 + 4ac = 2(ab + bc + ca)$

Then,

$a^2 + c^2 + 2ac - 2ab - 2bc = 0$

or $\quad (a + c - b)^2 - b^2 = 0$ $\quad \left[\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc\right]$

or $\quad b = a + c - b$

or $\quad 2b = a + c$

$b = \frac{a+c}{2}$

and since,

a, b, c are in A.P

$b = \frac{a+c}{2}$

Thus, $a^2 + c^2 + 4ac = 2(ab + bc + ca)$Hence proved.

(iii) If $a^3 + c^3 + 6abc = 8b^3$

or $\quad a^3 + c^3 - (2b)^3 + 6abc = 0$

or $\quad a^3 + (-2b)^3 + c^3 + 3 \times a \times (-2b) \times c = 0$

$\therefore \quad (a - 2b + c) = 0$ $\quad \left[\begin{array}{l}\because x^3 + y^3 + z^3 + 3xyz = 0 \\ \text{or if } x + y + z = 0\end{array}\right]$

or $\qquad a+c=2b$

$$a-b=c-b$$

and since, a, b, c are in A.P

Thus, $a-b=c-b$

Hence proved. $a^3+c^3+6abc=8b^3$

Arithematic Progressions Ex 19.5 Q6

Here,

$$a\left(\frac{1}{b}+\frac{1}{c}\right),\ b\left(\frac{1}{c}+\frac{1}{a}\right),\ c\left(\frac{1}{a}+\frac{1}{b}\right) \text{ are in A.P.}$$

$$\Rightarrow \quad a\left(\frac{1}{b}+\frac{1}{c}\right)+1,\ b\left(\frac{1}{c}+\frac{1}{a}\right)+1,\ c\left(\frac{1}{a}+\frac{1}{b}\right)+1 \text{ are in A.P.}$$

$$\Rightarrow \quad \left(\frac{ac+ab+bc}{bc}\right),\ \left(\frac{ab+bc+ac}{ac}\right),\ \left(\frac{cb+ac+ab}{ab}\right) \text{ are in A.P.}$$

$$\Rightarrow \quad \frac{1}{bc},\ \frac{1}{ac},\ \frac{1}{ab} \text{ are in A.P.}$$

$$\Rightarrow \quad \frac{abc}{bc},\ \frac{abc}{ac},\ \frac{abc}{ab} \text{ are in A.P.}$$

$$\Rightarrow \quad a,\ b,\ c \text{ are in A.P.}$$