RD SHARMA

Solutions

Class 10 Maths

Chapter 9

Ex 9.5

1. Find the sum of the following arithmetic progressions:

(i) 50, 46, 42, ... to 10 terms

(ii) 1, 3, 5, 7, ... to 12 terms

(iii) 3, 9/2, 6, 15/2, ... to 25 terms

(iv) 41, 36, 31, ... to 12 terms

(v) a + b, a – b, a – 3b, ... to 22 terms

(vi) $(x - y)^2, (x^2 + y^2), (x + y)^2, \ldots$, to n terms

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots to\ n\ terms$

(viii) $-26, -24, -22, \ldots to\ 36\ terms$

Sol:

In an A.P let first term = a, common difference = d, and there are n terms. Then, sum of n terms is,

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

(i) Given progression is,

50, 46, 42, ……to 10 term.

First term (a) = 50

Common difference (d) = 46 – 50 = –4

n^{th} term = 10

Then $S_{10} = \frac{10}{2}\{2.50 + (10-1) - 4\}$

$= 5\{100 - 9.4\}$

$= 5\{100 - 36\}$

$= 5 \times 64$

$\therefore S_{10} = 320$

(ii) Given progression is,

1, 3, 5, 7, …..to 12 terms

First term difference (d) = 3 – 1 = 2

n^{th} term = 12

Sum of n^{th} terms $S_{12} = \frac{12}{2} \times \{2.1 + (12 - 1).2\}$

$= 6 \times \{2 + 22\} = 6.24$

$\therefore S_{12} = 144.$

(iii) Given expression is

$3, \frac{9}{2}, 6, \frac{15}{2}, \dots \dots .to\ 25\ terms$

First term (a) = 3

Common difference (d) $= \frac{9}{2} - 3 = \frac{3}{2}$

Sum of n^{th} terms $S_n, given\ n = 25$

$S_{25} = \frac{n}{2}(2a + (n-1).d)$

$S_{25} = \frac{25}{2}\left(2.3 + (25-1).\frac{3}{2}\right)$

$= \frac{25}{2}\left(6 + 24.\frac{3}{2}\right)$

$= \frac{25}{2}(6 + 36)$

$= \frac{25}{2}(42)$

$\therefore S_{25} = 525$

(iv) Given expression is,

41, 36, 31, …… to 12 terms.

First term (a) = 41

Common difference (d) = 36 – 41 = – 5

Sum of n^{th} terms $S_n, given\ n = 12$

$S_{12} = \frac{n}{2}(2a(n-1).d)$

$= \frac{12}{6}(2.41 + (12-1).-5)$

$= 6(82 + 11.(-5))$

$= 6(27)$

$= 162$

$\therefore S_{12} = 162.$

(v) a + b, a – b, a – 3b, …… to 22 terms

First term (a) = a + b

Common difference (d) = a – b – a – b = –2b

Sum of n[th] terms $S_n = \frac{n}{2}\{2a(n-1).d\}$

Here n = 22

$S_{22} = \frac{22}{2}\{2.(a+b) + (22-1).-2b\}$

$= 11\{2(a+b) - 22b\}$

= 11 {2a – 20b}

= 22a – 440b

∴ S_{22} = 22a – 440b

(vi) $(x-y)^2, (x^2+y^2), (x+y)^2, \ldots\ldots to\ n\ terms$

First term (a) = $(x-y)^2$

Common difference (d) = $x^2 + y^2 - (x-y)^2$

$= x^2 + y^2 - (x^2 + y^2 - 2xy)$

$= x^2 + y^2 - x^2 + y^2 + 2xy$

= 2xy

Sum of n[th] terms $S_n = \frac{n}{2}\{2a(n-1).d\}$

$= \frac{n}{2}\{2(x-y)^2 + (n-1).2xy\}$

$= n\{(x-y)^2 + (n-1)xy\}$

$\therefore S_n = n\{(x-y)^2 + (n-1).xy\}$

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots\ldots to\ n\ terms$

$First\ term\ (a) = \frac{x-y}{x+y}$

Common difference (d) $= \frac{3x-2y}{x+y} - \frac{x-y}{x+y}$

$= \frac{3x-2y-x+y}{x+y}$

$= \frac{2x-y}{x+y}$

Sum of n terms $S_n = \frac{n}{2}\{2a + (n-1).d\}$

$= \frac{n}{2}\left\{2.\frac{x-y}{x+y} + (n-1).\frac{2x-y}{x+y}\right\}$

$= \frac{n}{2(x+y)}\{2(x-y) + (n-1)(2x-y)\}$

$= \frac{n}{2(x+y)}\{2x - 2y + 2nx - ny - 2x + y\}$

$= \frac{n}{2(x+y)}\{n(2x-y) - y\}$

$\therefore S_n = \frac{n}{2(x+y)}\{n(2x-y) - y\}$

(viii) Given expression $-26, -24, -22, \ldots\ldots.$ To 36 terms

First term (a) = -26

Common difference (d) = $-24-(-26) = -24+26 = 2$

Sum of n terms $S_n = \frac{n}{2}\{2a+(n-1)d\}$

Sum of n terms $S_n = \frac{36}{2}\{2.-26+(36-1)2\}$

$= 18[-52+70]$

$= 18.18$

$= 324$

$\therefore S_n = 324$

2. Find the sum to n term of the A.P. 5, 2, — 1, —4, —7, ...

Sol:

Given AP is 5, 2, -1, -4, -7, …..

$a = 5, d = 2-5 = -3$

$S_n = \frac{n}{2}\{2a+(n-1)d\}$

$= \frac{n}{2}\{2.5+(n-1)-3\}$

$= \frac{n}{2}\{10-3(n-1)\}$

$= \frac{n}{2}\{13-3n\}$

$\therefore S_n = \frac{n}{2}(13-3n)$

3. Find the sum of n terms of an A.P. whose th terms is given by $a_n = 5 - 6n$.

Sol:

Given nth term $a_n = 5-6n$

Put n = 1, $a_1 = 5-6.1 = -1$

We know, first term $(a_1) = -1$

Last term $(a_n) = 5-6n = 1$

Then $S_n = \frac{n}{2}(-1+5-6n)$

$= \frac{n}{2}(4-6n) = \frac{n}{2}(2-3n)$

4. If the sum of a certain number of terms starting from first term of an A.P. is 25, 22, 19, ... is 116. Find the last term.

Sol:

Given AP is 25, 22, 19, ……

First term (a) = 25, d = 22 – 25 = −3.

Given, $S_n = \frac{n}{2}(2a + (n-1)d)$

$116 = \frac{n}{2}(2 \times 25 + (n-1) - 3)$

$232 = n(50 - 3(n-1))$

$232 = n(53 - 3n)$

$232 = 53n - 3n^2$

$3n^2 - 53n + 232 = 0$

$(3n - 29)(n - 8) = 0$

$\therefore n = 8$

$\Rightarrow a_8 = 25 + (8-1) - 3$

$\therefore n = 8, a_8 = 4$

$= 25 - 21 = 4$

5. (i) How many terms of the sequence 18, 16, 14, ... should be taken so that their

(ii) How many terms are there in the A.P. whose first and fifth terms are −14 and 2 respectively and the sum of the terms is 40?

(iii) How many terms of the A.P. 9, 17, 25, . . . must be taken so that their sum is 636?

(iv) How many terms of the A.P. 63, 60, 57, . . . must be taken so that their sum is 693?

Sol:

(i) Given sequence, 18, 16, 14, …

$a = 18, d = 16 - 18 = -2.$

Let, sum of n terms in the sequence is zero

$S_n = 0$

$\frac{n}{2}(2a + (n-1)d) = 0$

$\frac{n}{2}(2.18 + (n-1) - 2) = 0$

$n(18 - (n-1)) = 0$

$n(19 - n) = 0$

n = 0 or n = 19

(ii) $\because n = 0$ is not possible. Therefore, sum of 19 numbers in the sequence is zero.

Given, a = -14, a 5 = 2

$a + (5-1)d = 2$

$-14 + 4d = 2$

$4d = 16 \Rightarrow d = 4$

Sequence is −14, −10, -6, −2, 2, ……

Given $S_n = 40$

$40 = \frac{n}{2}\{2(-14) + (n-1)4\}$

$80 = n(-28 + 4n - 4)$

$80 = n(-32 + 4n)$

$4(20) = 4n(-8 + n)$

$n^2 - 8n - 20 = 0$

$(n - 10)(n + 2) = 0$

$n = 10 \text{ or } n = -2$

∴ Sum of 10 numbers is 40 (Since -2 is not a natural number)

(iii) Given AP 9, 17, 25, …….

a = 9, d = 17 – 9 = 8, and $S_n = 636$

$636 = \frac{n}{2}(2.9 + (n-1)8)$

$1272 = n(18 - 8 + 8n)$

1272 = n(10 + 8n)

2 × 636 = 2n(5 + 4n)

$636 = 5n + 4n^2$

$4n^2 + 5n - 636 = 0$

(4n + 53) (n – 12) = 0

∴ n = 12 (Since n $\frac{-53}{4}$ is not a natural number)

Therefore, value of n is 12.

(iv) Given AP, 63, 60, 57, ……

a =63, d = 60 – 63 = -3 $S_n = 693$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$693 = \frac{n}{2}(2.63 + (n-1) - 3)$

1386 = n(126 – 3n + 3)

1386 = (129 – 3n)n

$3n^2 - 129n + 1386 = 0$

$n^2 - 43n + 462 = 0$

n = 21, 22

∴ Sum of 21 or 22 term is 693