RD SHARMA

Solutions

Class 10 Maths

Chapter 9

Ex 9.4

Question 1. If 12^{th} of an A.P is 82 and 18^{th} term is 124. Then find out the 24^{th} term.

Solution: Given: $a_{12} = 82$ and $a_{18} = 124$

we know : $a_n = a + (n-1)$ c.d

=> $a_{12} = a + (12-1)$ c.d

=> 82 = a + 11c.d —(1)

=> 124 = a + (18-1) c.d

=> 124 = a + 17c.d —(2)

Subtracting (2) from (1)

=> (a + 17c.d) – (a + 11c.d) = 124 – 82

=> a +17c.d – a – 11c.d = 42

=> 6c.d = 42

=> c.d = 7

Here we have, Common Difference (c.d) = 7

putting c.d = 7 in equation (1), we get

=> a + 11 X 7 = 82

=> a = 82 – 77

=> a = 5

Now, we have First Term (a) = 5

we have to find 24^{th} term

a_{24} = a + (24 -1) c.d

= 5 + 23 X 7

= 5 + 161

= **166**

Question 2. In an A.P. the 24^{th} term is twice the 10^{th} term. Prove that the 72^{nd} term is twice the 34^{th} term.

Solution: Given

24^{th} term is twice the 10^{th} term

$a_{24} = 2 \times a_{10}$ . . . . . . (1)

let, first term be a

and common difference be d

we know, n^{th} term is $a_n = a + (n - 1)d$

from equation (1), we have

$a + (24-1)d = 2(a + (10-1)d)$

=> $a + 23d = 2 (a + 9d)$

=> $a + 23 d = 2a + 18d$

=> $(23 - 18)d = a$

=> $a = 5d$

we have to prove that,

72^{nd} term is twice the 34^{th} term

=> $a_{72} = 2 \times a_{34}$

=> $a + (72-1)d = 2 [a + (34-1)d]$

=> $a + 71d = 2(a + 33d)$

=> $a + 71d = 2a + 66d$

putting the value of a = 5 in the above equation,

=> $5d + 71d = 2 (5d) + 66d$

=> $76d = 76d$

Hence it is proved…

Question 3. If the $(m+1)^{th}$ term of an A.P. is twice the $(n+1)^{th}$term of the A.P. Then prove that: $(3m+1)^{th}$is twice the $(m+n+1)^{th}$term.

Solution: From the question, we have

$a_{(m+1)} = 2\ a_{(n+1)}$

Let, First term = a

and Common Difference = d

=> a + (m + 1 – 1)d = 2[a+(n + 1 -1)d]

=> a +md = 2a + 2nd

=> a = md – 2nd

=> a = (m-2n)d —(2)

We have to prove, $a_{(3m+1)} = 2\ a_{(m+n+1)}$

=> a + (3m + 1 – 1)d = 2 [a + (m + n + 1 -1)d]

=> a + 3md = 2a + 2(m + n)d

putting the value of a = (m – 2n)d, from equation (1)

=> (m – 2n)d + 3md = 2[(m – 2n)d] + 2(m + n)d

=> m -2n + 3m = 2m -4n + 2m + 2n

=> 4m – 2n = 4m – 2n

Hence it is proved…

Question 4. If the n^{th} term of the A.P. 9,7,5, … is same as the n^{th} term of the A.P. 15, 12 , 9, … find n.

Solution: we have here,

First sequence is 9,7,5, …

First term (a) = 9,

Common Difference (c.d) = 9 – 7 = -2

n^{th} term = a + (n-1)c.d

=> a_n = 9 + (n-1)(-2)

= 9 – 2n + 2

= 11 – 2n

Second sequence is 15, 12, 9, …

here, First term (a) = 15

Common Difference (c.d) = 12 -15 = -3

n^{th} term = a + (n-1)d

=> a'_n = 15 + (n-1)(-3)

= 15 -3n +3

= 18 -3n

We are given in the question that the n^{th} term of both the A.P.s are same,

So, we can write it as

$a_n = a'_n$

=> 11 – 2n = 18 – 3n

=> **n = 7**

So, the 7^{th} term of both the A.P.s will be equal.

Question 5. Find the 13^{th} term from the end in the following A.P.

(i). 4, 9, 14, … , 254.

Solution: we have,

First term (a) = 4 and common difference (c.d) = 9 -4 = 5

last term here (l) = 254

n^{th} term from the end is : l – (n-1)d

we have to find 13^{th} term from end then : l – 12d

= 254 – 12 X 5

= 254 – 60

= **194**

(ii). 3, 5, 7, 9, …, 201.

Solution: we have,

First term (a) = 3 and common difference (c.d) = 5 – 3 = 2 last term here (l) = 201

n^{th} term from the end is : l – (n-1)d

we have to find 13^{th} term from end then : l – 12d

= 201 – 12 X 2

= 201 – 24

= **177**

(iii). 1, 4, 7, 10, … , 88.

Solution: we have,

First term (a) = 1 and common difference (c.d) = 4 -1 = 3

last term here (l) = 88

n^{th} term from the end is : l – (n-1)d

we have to find 13^{th} term from end then : l – 12d

= 88 – 12 X 3 = 88 – 36 = **52**

Question 6. The 4^{th} term of an A.P. is three times the first term and the 7^{th} term exceeds the twice the

third term by 1. Find the A.P.

Solution: Given, 4^{th} term of the A.P = thrice the first term

=> a_4 = 3 first term

Assuming first term to be ‘a’ and the common difference be ‘d’

we have, a+ (4 -1)d = 3 a

=> a + 3d = 3 a

=> a = 32d $\frac{3}{2}d$ —(1)

and also it is given that,

the 7^{th} term exceeds the twice of the 3^{rd} term by 1

=> a_7 + 1 = 2 X a_3

=> a+ (7-1)d +1 = 2[a + (3-1)d]

=> a + 6d +1 = 2a + 4d

=> a = 2d + 1 —(2)

putting the value of a = $32d\frac{3}{2}d$ from equation (1) in equation (2)

$32d\frac{3}{2}d$ = 2d + 1

=> $32d\frac{3}{2}d$ – 2d = 1

=> $3d–4d2=1\frac{3d-4d}{2} = 1$

=> -d = 2

=> d = -2

put d = -2 in $a=32da = \frac{3}{2}d$

=> a = $32(-2)\frac{3}{2}(-2)$

=> a = -3

Now, we have a = -3 and d = -2, so the A.P. is -2, -5, -8, -11 , …

Question 7. Calculate the third term and the n^{th} term of an A.P. whose 8^{th} term and 13^{th} term are 48

and 78 respectively.

Solution: Given, a_8 = 48 and a_{13} = 78

n^{th} term of an A.P. is: a_n = a + (n-1)d

so,

a_8 = a + (8 – 1)d = a + 7d —(1)

a_{13} = a + (13 – 1)d = a + 12d —(2)

Equating (1) and (2), we get.

=> a + 12d – (a + 7d) = 78 – 48

=> a +12d – a -7d = 30

=> 5d = 30

=> **d = 6**

Putting the value of d = 6 in equation (1),

a + 7 X 6 = 48

=> a + 42 = 48

=> **a = 4**

Now, we have the first term (a) and the common difference (d) with us,

So, n^{th} term will be: $a_n = a + (n-1)d$

= 4 + (n-1)6

= 4 + 6n – 6

$a_n = 6n – 2$

and the 3^{rd} term will be

$a_3 = 6 X 3 – 2$

$a_n = 16$

Question 8. How many three digit numbers are divisible with 3?

Solution: We know the first three digit number which is divisible by 3 is 102 and the last three digit number which is divisible by 3 is 999

So, here we have

First term (a) = 102

Common Difference (c.d) = 3

last term or n^{th} term (l) = 999

=> $a_n = 999$

=> a + (n – 1)c.d = 999

=> 102 + (n – 1)3 = 999

=> 102 + 3n – 3 = 999

=> 99 + 3n = 999

=> 3n = 900

=> n = 300

Therefore, there are 300 terms in the sequence.

Question 9. An A.P. has 50 terms and the first term is 8 and the last term is 155. Find the 41st term from the A.P.

Solution: Given,

First term (a) = 8

Number of terms (n) = 50

Last term (a_n) = 148

=> $a_n = a + (n – 1)d$

=> 155 = 8 + (50 – 1)d

=> 49 d = 147

=> d = 3

now, 41st term will be: a + (41-1)d

=> 8 + 40 X 3

=> **128**

Question 10. The sum of 4th and 8th term of an A.P. is 24 and the sum of the 6th and 10th terms is 34. Find the first term and the common difference of the A.P.

Solution:

Let's assume first term be a and common difference be d

Given 4th term + 8th term = 24

=> $a_4 + a_8 = 24$

=> (a + (4 – 1)d) + (a + (8 -1)d) = 24

=> a + 3d + a + 7d = 24

=> 2a + 10d = 24 —(1)

And 6th term + 10th term = 34

=> $a_6 + a_{10} = 34$

=> (a + 5d) + (a + 9d) = 34

=> 2a + 14d = 34 —(2)

Subtracting equation (1) from (2),we get

=> (2a + 14d) – (2a + 10d) = 34 -24

=> 2a + 14d – 2a – 10d = 10

=> 4d = 10

=> d = 52 $\frac{5}{2}$

Put d = 52 $\frac{5}{2}$ in equation (1)

=> 2a + 10 X 52 $\frac{5}{2}$ = 24

=> 2a + 25 = 24

=> 2a = -1

=> a = − 12 $-\frac{1}{2}$

Therefore, we have a = − 12 $-\frac{1}{2}$ and d = 52 $\frac{5}{2}$

Question 11. The first term of an A.P. is 7 and its 100^{th} term is -488, Find the 50^{th} term of the same A.P.

Solution: Given,

First term (a) = 7

100^{th} term (a_{100}) = -488

we know, $a_n = a + (n – 1)d$

=> (a_{100}) = a + (100 -1)d

=> 7 + 99d = -488

=> 99d = -495

=> **d = -5**

Now, we have the common difference (d) = 5

We have to find out the 50^{th} term of the A.P.

Then, a_{50} = a + 49 d

= 7 + 49 X (-5)

= 7 – 245

= **– 238**

So, the 50^{th} term of the A.P. is -238

Question 12. Find $a_{40} – a_{30}$ of the following A.P.

(i). 3, 5, 7, 9, . . .

Solution: Provided A.P. is 3, 5, 7, 9, . . .

So, we have first term (a) = 3 and the common difference (d) is 5 – 3 = 2

we have to find $a_{40} – a_{30}$ = (a + 39d) – (a + 29d)

= a + 39 d – a -29 d

= 10 d

= 10 X 2

= **20**

(ii). 4, 9, 14, 19, . . .

Solution: Given A.P. is 4, 9, 14, 19, . . .

Common difference (d) = 9 – 4 = 5

we have to find $a_{40} – a_{30}$ = 10 d

= 10 X 5 = 50

Question 13. Write the expression $a_m – a_n$ for the A.P. a, a + d, a + 2d, . . .

Solution: General Arithmetic Progression

a, a + d, a + 2d, . . .

$a_m – a_n$ = (a + (m – 1)d) – (a + (n – 1)d)

=> a + md – d – a -nd +d

=> md – kd

=> (m – n) d —(1)

Hence find the common difference of the A.P. for which

(i). 11th term is 5 and 13th term is 79

Solution: Given,

11th term (a_{11}) = 5

and 13th term (a_{13}) = 79

from equation (1),

taking m = 11 and n = 13

=> $a_m - a_n$ = (13 -11) d

=> 79 – 5 = 2d

=> 74 = 2d

=> **d = 37**

(ii). $a_{10} - a_5$ = 200

Solution: Given,

here we have the difference between the 10th term and 5th term

Putting the value of m and n in equation (1) as 10 and 5, we have

=> $a_{10} - a_5$ = (10 – 5)d

=> 200 = 5 d

=> **d = 40**

(iii). 20th term is 10 more than the 18th term

Solution: Given,

a_{20} + 10 = a_{18}

=> $a_{20} - a_{18}$ = 10

from equation (1), we have

$a_m - a_n = (m - n) d$

=> $a_{20} - a_{18} = (20 - 18) d$

=> 10 = 2d

=> **d = 5**

Question 15. Find n if the given value of x is the n term if the given A.P.

(i) 1,2111,3111,4111,....:x=14111 $1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \ldots : x = \frac{141}{11}$

(ii) 512,11,1612,22,...:x=550 $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots : x = 550$

(iii) -1 , -3, -5, -7, . . . : x = -151

(iv) 25, 50, 70, 100, . . . : x = 1000

Solution:

(i) Given sequence is

1,2111,3111,4111,....:x=14111 $1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \ldots : x = \frac{141}{11}$

first term (a) = 1

Common difference (d) = 2111−1 $\frac{21}{11} - 1$

= 21−1111 $\frac{21-11}{11}$

= 1011 $\frac{10}{11}$

n^{th} term $a_n = a + (n-1) \times d$

=> 17111=1+(n−1).1011 $\frac{171}{11} = 1 + (n-1).\frac{10}{11}$

=> 17111−1=(n−1).1011 $\frac{171}{11} - 1 = (n-1).\frac{10}{11}$

=> 171−1111=(n−1).1011 $\frac{171-11}{11} = (n-1).\frac{10}{11}$

=> 16011=(n−1).1011 $\frac{160}{11} = (n-1).\frac{10}{11}$

=> (n−1)=16011X1110 $(n-1) = \frac{160}{11} X \frac{11}{10}$

=> n = 17

(ii) Given sequence is

5 12 ,11,16 12 ,22,...:x=550 $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots : x = 550$

first term (a) = 5 12 = 112 $5\frac{1}{2} = \frac{11}{2}$

Common Difference (d) = 11– 112 = 112 $11 - \frac{11}{2} = \frac{11}{2}$

n^{th} term a_n = a + (n-1) X d

=> 550= 112 +(n–1). 112 $550 = \frac{11}{2} + (n-1).\ \frac{11}{2}$

=> 550= 112 [1+n–1] $550 = \frac{11}{2}[1 + n - 1]$

=> n = 550 X 211 $\frac{2}{11}$

=> 100

(iii) Given sequence is,

-1 , -3, -5, -7, . . . : x = -151

first term (a) = -1

Common Difference (d) = -3 – (-1)

= -3 + 1

= -2

n^{th} term a_n = a + (n-1) X d

=> -151 = -1 + (n – 1) X -2

=> -151 = -1 – 2n + 2

=> – 151 = 1 – 2n

=> 2n = 152

=> n = 76

(iv) Given sequence is,

25, 50, 70, 100, . . . : x = 1000

First term (a) = 25

Common Difference (d) = 50 – 25 = 25

n^{th} term $a_n = a + (n-1) X d$

we have $a_n = 1000$

=> $1000 = 25 + (n – 1) 25$

=> $975 = (n – 1)25$

=> $n – 1 = 39$

=> $n = 40$

Question 16. If an A.P. consists of n terms with the first term a and n^{th} term 1. Show that the sum of the m^{th} term from the beginning and the m^{th} term from the end is (a + 1).

Solution: First term of the sequence is a

Last term (l) = a + (n – 1) d

Total no. of terms = n

Common Difference = d

m^{th} term from the beginning $a_m = a + (n – 1)d$

m^{th} term from the end = $l + (n – 1)(-d)$

=> $a_{(n – m + 1)} = l – (n – 1)d$

=> $a_m + a_{(n – m + 1)} = a + (n – 1)d + (l – (n – 1)d)$

= $a + (n-1)d + l – (n-1)d$

= $a + l$

Question 17. Find the A.P. whose third term is 16 and seventh term exceeds its fifth term by 12.

Solution: Given, $a_3 = 16$

=> $a + (3 – 1)d = 16$

=> $a + 2d = 16$ … (i)

and $a_7 – 12 = a_5$

=> a + 6d -12 = a + 4d

=> 2d = 12

=> d = 6

Put d = 6 in equation (1)

a + 2 X 6 = 16

=> a + 12 = 16

=> a = 4. So, the sequence is 4, 10, 16, . . .

Question 18. The 7th term of an A.P is 32 and its 13th term is 62. Find the A.P.

Solution: Given

a_7 = 32

=> a + (7 – 1)d = 32

=> a + 6d = 32 … (i)

and a_{13} = 62

=> a + (13 – 1)d = 62

=> a + 12d = 62 …(ii)

equation (ii) – (i), we have

(a + 12d) – (a + 6d) = 62 – 32

=> 6d = 30

=> d = 5

Putting d = 5 in equation (i)

a + 6 X 5 = 32

=> a = 32 – 30

=> a = 2

So, the obtained A.P. is

2, 7, 12, 17, . . .

Question 19. Which term of the A.P. 3, 10, 17, . . . will be 84 more than its 13th term?

Solution:

Given A.P. is 3, 10 , 17, . . .

First term (a) = 3

Common Difference (d) = 10 -3 = 7

Let n^{th} term of the A.P. will be 84 more than its 13^{th} term, then

$a_n = 84 + a_{13}$

=> a + (n – 1)d = a + (13 – 1)d + 84

=> (n – 1) X 7 = 12 X 7 + 84

=> n – 1 = 24

=> n = 25

Hence, 25^{th} ter, of the given A.P. is 84 more than the 13^{th} term.

Question 20. Two arithmetic progressions have the same common difference. The difference between their 100^{th} terms is 100. What is the difference between their 1000^{th} terms?

Solution:

Let the two A.P. be a_1, a_2 , a_3 , . . . and b_1, b_2 , b_3, . . .

a_n = a1 + (n – 1)d and b_n = a1 + (n – 1)d

Since common difference of two equation is same and given difference between 100^{th}

terms is 100

=> $a_{100} – b_{100}$ = 100

=> a + (100 – 1)d – [b + (100 – 1)d] = 100

=> a + 99d -b – 99d = 100

=> a + b = 100 … (1)

Difference between 100^{th} term is

=> $a_{1000} – b_{1000}$

= a + (1000 – 1)d – [b + (1000 – 1)d]

= a + 999d – b – 999d = a – b = 100 (from equation 1)

Therefore, Difference between 1000^{th} terms of two A.P. is 100.

Question 21. For what value of n, the n^{th} terms of the Arithmetic Progression 63, 65, 67, . . . and , 3, 10, 17, . . . are equal?

Solution:

Given two A.P.s are:

63, 65, 67, . . . and 3, 10, 17, . . .

First term for first A.P. is (a) = 63

Common difference (d) is 65 – 63 = 2

n^{th} term (a_n) = a + (n – 1)d

= 63 + (n – 1) 2

First term for second A.P. is (a') = 3

Common Difference (d') = 10 – 3

= 7

n^{th} term (a'_n) = a' + (n – 1)d

= 3 + (n – 1) 7

Let n^{th} term of the two sequence be equal then,

=> 63 + (n – 1)2 = 3 + (n – 1)7

=> 60 = (n – 1).7 – (n – 1).2

=> 60 = 5(n – 1)

=> n – 1 = 12

=> n = 13

Hence, the 13^{th} term of both the A.P.s are same.

Question 22. How many multiple of 4 lie between 10 and 250?

Solution: Multiple of 4 after 10 is 12 and multiple of 4 before 250 is 120/4, remainder is 2, so, 250 – 2 = 248

248 is the last multiple of 4 before 250

the sequence is

12,. . . . . , 248

with first term (a) = 12

Last term (l) = 258

Common Difference (d) = 4

n^{th} term (a_n) = a + (n – 1)d

Here n^{th} term a_n = 248

=> 248 = a + (n – 1)d

=> 12 + (n – 1)4 = 248

=> (n – 1)4 = 236

=> n – 1 = 59

=> n = 59 + 1

=> N = 60

Therefore, there are 60 terms between 10 and 250 which are multiples of 4

Question 23. How many three digit numbers are divisible by 7 ?

Solution: The three digit numbers are 100, ... , 999

105 us the first 3 digit number which is divisible by 7

and when we divide 999 with 7 remainder is 5, so, 999 – 5 = 994

994 Is the last three digit number which is divisible by 7 .

The sequence here is

105, ... , 994

First term (a) = 105

Last term (l) = 994

Common Difference (d) = 7

Let there are n numbers in the sequence then,

=> $a_n = 994$

=> $a + (n – 1)d = 994$

=> $105 + (n – 1)7 = 994$

=> $(n – 1) X 7 = 889$

=> $n – 1 = 127$

=> $n = 128$

Therefore, there are 128 three digit numbers which are divisible by 7.

Question 24. Which term of the A.P. 8, 14, 20, 26, . . . will be 72 more than its 41st term?

Solution: Given sequence

8, 14, 20, 26, . . .

Let its n term be 72 more than its 41st term

=> $a_n = a_{41} + 72$ ... (1)

For the given sequence,

first term (a) = 8,

Common Difference (d) = 14 – 8 = 6

from equation (1), we have

$a_n = a_{41} + 72$

=> $a + (n – 1)d = a + (41 – 1)d + 72$

=> $8 + (n – 1)6 = 8 + 40 X 6 + 72$

=> $(n – 1)6 = 312$

=> $n – 1 = 52$

=> $n = 53$

Therefore, 53rd term is 72 more than its 41st term.

Question 25. Find the term of the Arithmetic Progression 9, 12, 15, 18, . . . which is 39 more than its 36th term.

Solution: Given A.P. is

9, 12, 15, 18 , . . .

Here we have,

First term (a) = 9

Common Difference (d) = 12 – 9 = 3

Let its nth term is 39 more than its 36th term

So, a_n = 39 + a_{36}

=> a + (n – 1)d = 39 + a + (36 – 1)d

=> (n -1)3 = 39 + 35 X 3

=> (n -1)3 = (13 + 35) 3

=> n – 1 = 48

=> n = 49

Therefore, 49^{th} term of the A.P. 39 more than its 36^{th} term.

Question 26. Find the 8^{th} term from the end of the A.P. 7, 10, 13, . . . , 184.

Solution:

Given A.P. is 7, 10, 13, . . . , 184

First term (a) = 7

Common Difference (d) = 10 – 7 = 3

last term (l) = 184

n^{th} term from end = l – (n – 1)d

8^{th} term from end = 184 – (8 – 1)3

= 184 – 7 X 3

= 184 – 21

= 183

Therefore, 8^{th} term from the end is 183

Question 27. Find the 10^{th} term from the end of the A.P. 8, 10, 12, . . . , 126

Solution: Given A.P. is 8, 10, 12, . . . , 126

First term (a) = 8

Common Difference (d) = 10 – 8 = 2

Last term (l) = 126

n^{th} term from end is : l – (n -1)d

So, 10^{th} term from end is : l – (10 – 1)d

= 126 – 9 X 2

= 126 – 18

= 108

Therefore, 109 is the 10^{th} term from the last in the A.P. 8, 10 ,12 , . . 126.

Question 28. The sum of 4^{th} and 8^{th} term of an A.P. is 24 and the sum of 6^{th} and 10^{th} term is 44. Find the Arithmetic Progression.

Solution: Given $a_4 + a_8 = 24$

=> a + (4 – 1)d + a + (8 – 1)d = 24

=> 2a + 3d + 7d = 24

=> 2a + 10d = 24 …(1)

and $a_6 + a_{10} = 44$

=> a + (6 – 1)d + a + (10 – 1)d = 44

=> 2a + 5d + 9 d = 44

=> 2a + 14d = 44 …(2)

equation (2) – equation (1), we get

2a + 14d – (2a + 10d) = 44 -24

=> 4d = 20

=> d = 5

Put d = 5 in equation (1), we get

2a + 10X5 = 24

=> 2a = 24 – 50

=> 2a = -26

=> a = -13

The A.P is -13, – 7, -2, . . .

Question 29: Which term of the A.P. is 3, 15, 27, 39, . . . will be 120 more than its 21st term?

Solution: Given A.P. is 3, 15, 27, 39, . . .

First term (a) = 3

Common Difference (d) = 15 – 3 = 12

Let nth term is 120 more than 21st term

=> $a_n = 120 + a_{21}$

=> a + (n – 1)d = 120 + a + (21 – 1)d

=> (n -1)d = 120 + 20d

=> (n – 1)12 = 120 + 20 X 12

=> n – 1 = 10 + 20

=> n = 31

Therefore, 31st term of the A.P. is 120 more than the 21st term.

Question 30. The 17th term of an A.P. is 5 more than twice its 8th term. If the 11th term of the A.P. is 43. Find the nth term.

Solution: Given

17th term of an A.P is 5 more than twice its 8th term

=> $a_{17} = 5 + 2a_8$

=> a + (17 – 1)d = 5 + 2 [a + (8 – 1)d]

=> a + 16d = 5 + 2a + 14d

=> a + 5 = 2d … (1)

and 11th term of the A>P. is 43

$a_{11} = 43$

=> $a + (11 - 1)d = 43$

=> $a + 10d = 43$

=> $a + 5 \times 2d = 43$

from equation (1)

=> $a + 5 \times (a + 5) = 43$

=> $a + 5a + 25 = 43$

=> $6a = 18$

=> $a = 3$

Putting the value of a = 3, in equation (1), we get

$3 + 5 = 2d$

=> $2d = 8$

=> $d = 4$

We have to find the n^{th} term $(a_n) = a + (n - 1)d$

$= 3 + (n - 1)4$

$= 3 + 4n - 4$

$= 4n - 1$

Therefore, n^{th} term is $4n - 1$

Question 31. Find the number of all three digit natural number which are divisible by 9.

Solution: First three-digit number that is divisible by 9 is 108.

Next number is 108 + 9 = 117.

And the last three-digit number that is divisible by 9 is 999.

Thus, the progression will be 108, 117, …. , 999.

All are three digit numbers which are divisible by 9, and thus forms an A.P.

having first term (a) : 108

last term (l) = 999

and the common difference (d) as 9

We know that, n^{th} term (a_n) = a + (n – 1)d

According to the question,

999 = 108 + (n – 1)9

=> 999 = 108 + 9n – 9

=> 999 = 99 + 9n

=> 999 = 9n

=> 999 – 99

=> 9n = 900

=>n = 100

Therefore, There are 100 three digit terms which are divisible by 9.

Question 32. The 19^{th} term of an A.P. is equal to three times its 6^{th} term. if its 9^{th} term is 19, find the A.P.

Solution: Let a be the first term

and d be the common difference.

We know that, nth term = an= a + (n – 1)d

According to the question,

$a_{19} = 3a_6$

=> a + (19 – 1)d= 3(a + (6 – 1)d)

=> a + 18d= 3a + 15d

=> 18d- 15d= 3a – a

=>3d= 2a

=> a = 32d …. (1)

Also, a9 = 19

=> a+(9 – 1)d= 19

=> a+ 8d= 19 ….(2)

On substituting the values of (1) in (2), we get

=> 32d + 8d= 19

=> 3d+ 16d= 19 x 2

=> 19d= 38

=>d = 2

Now,   a = 32x2   [From (1)]

a= 3

Therefore, The A.P. is : 3, 5, 7, 9, . . .

Question 33. The 9^{th} term of an A.P. is equal to 6 times its second term. If its 5^{th} term is 22, find the A.P.

Solution: Let a be the first term

and d be the common difference.

We know that, nth term (a_n) = a + (n – 1)d

According to the question,

a9 = 6a2

=> a + (9 – 1)d = 6(a + (2 – 1)d)

=> a+8d= 6a+6d

=> 8d-6d= 6a-a

=> 2d = 5a

=> a = $25\frac{2}{5}$ …. (1)

Also, a_5 = 22

=> a+(6 – 1)d= 22

=> a + 4d = 22   ….(2)

On substituting the values of (1) in (2), we get

$25\frac{2}{5}$ d + 4d = 22

=> 2d + 20d = 22 X 5

=> 22d = 110

=> d = 5

Now, a = $25\frac{2}{5}$ X 5   [From (1)]

=> a = 2

Thus, the A.P. is : 2, 7, 12, 17, . . .

Question 34. The 24^{th} term of an A.P. is twice its 10^{th} term. Show that its 72^{nd} term is 4 times its 15^{th} term.

Solution: Let a be the first term

and d be the common difference.

We know that,

n^{th} term (a_n) = a + (n – 1)d

According to the question,

$a_{24} = 2 a_{10}$

=> a + (24 – 1)d = 2(a + (10 – 1)d)

=> a + 23d = 2a + 18d

=>23d – 18d = 2a – a

=> 5d = a

=> a = 5d …. (1)

Also,

a_{72} = a + (72 – 1) d

= 5d + 71d [From (1)]

= 76d …. (2)

and

a_{15} = a + (15 – 1) d

= 5d + 14d [From (1)]

= 19d …. (3)

On comparing (2) and (3), we get

=> 76 d = 4 X 19 d

=> a_{72} = 4 X a_{15}

Thus, 72^{nd} term of the given A.P. is 4 times its 15^{th} term.

Question 35. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Solution: Since, the number is divisible by both 2 and 5, means it must be divisible by 10.

In the given numbers, first number that is divisible by 10 is 110.

Next number is 110 + 10 = 120.

The last number that is divisible by 10 is 990.

Thus, the progression will be 110, 120, …, 990.

All the terms are divisible by 10,

and thus forms an A.P. having first term as 110

and the common difference as 10.

We know that,

nth term = an= a + (n – 1)d

According to the question,

990 = 110 + (n – 1)10

=> 990 = 110 + 10n – 10

=> 10n = 990 – 100

=> n = 89

Thus, the number of natural numbers between 101 and 999 which are divisible by both 2 and 5 is 89.

Question 36. If the seventh term of an A.P. is 1/9 and its 9thterm is 1/7, find the 63rd term.

Solution: Let a be the first term and d be the common difference.

We know that, nth term = an= a + (n – 1)d

According to the question,

$a_7 = 19\frac{1}{9}$

=> a+(7 – 1)d = $19\frac{1}{9}$

=> a+ 6d = 19 $\frac{1}{9}$ ….(1)

Also, a_9 = 17 $\frac{1}{7}$

=> a + (9 – 1)d = 17 $\frac{1}{7}$

=> a + 8d = 17 $\frac{1}{7}$ ….(2)

On Subtracting (1) from (2), we get

=> 8d – 6d = 17 $\frac{1}{7}$ – 19 $\frac{1}{9}$

=> 2d = 9–763 $\frac{9-7}{63}$

=> 2d = 263 $\frac{2}{63}$

=> d= 163 $\frac{1}{63}$

Put value of d = 163 $\frac{1}{63}$ in equation (1), we get

=> a + 6 X 163 $\frac{1}{63}$ = 19 $\frac{1}{9}$

=> a = 19 $\frac{1}{9}$ – 663 $\frac{6}{63}$

=> a = 7–663 $\frac{7-6}{63}$

=> a = 163 $\frac{1}{63}$

Therefore, a_{63} = a + (63 – 1)d

= 163 $\frac{1}{63}$ + 6263 $\frac{62}{63}$

= 6363 $\frac{63}{63}$

= 1

Thus, 63^{rd} term of the given A.P. is 1.

Question 37. The sum of 5^{th} and 9^{th} terms of an A.P. is 30. If its 25^{th} term is three times its 8^{th} term, Find the A.P.

Solution: Let a be the first term and d be the common difference.

We know that, nth term (a_n) = a + (n – 1)d

According to the question,

$a_5 + a_9 = 30$

=> a + (5 – 1)d + a + (9 – 1)d = 30

=> a + 4d + a + 8d = 30

=> 2a + 12d = 30

=> a + 6d = 15 …. (1)

Also, $a_{25} = 3(a_8)$

=> a + (25 – 1)d = 3[a + (8 – 1)d]

=> a + 24d = 3a + 21d

=> 3a – a = 24d – 21d

=> 2a = 3d

=> a = $32d\frac{3}{2}d$ ….(2)

Substituting the value of (2) in (1), we get 32d+6d= 15

=> $32d\frac{3}{2}d$ + 6d = 15

=> 3d + 12d = 15 x 2

=> 15d = 30

=> d = 2

now, a = $32d\frac{3}{2}d$ X 2 [From (1)]

=> a = 3

Therefore, the A.P. is 3, 5, 7, 9, . . .

Question 38. Find whether 0 (zero) is a term of the A.P. 40, 37, 34, 31, . . .

Solution: Let a be the first term and d be the common difference.

We know that, nth term = an = a + (n – 1)d

It is given that a = 40, d = -3

and a_n= 0

According to the question,

=> 0 = 40 + (n – 1)(-3)

=> 0 = 40 – 3n + 3

=> 3n = 43

=> n = 433 $\frac{43}{3}$ …. (1)

Here, n is the number of terms, so must be an integer.

Thus, there is no term where 0 (zero) is a term of the A.P. 40, 37, 34, 31,. . .

Question 39. Find the middle term of the A.P. 213, 205, 197, . . . 37.

Solution: Let a be the first term and d be the common difference.

We know that, nth term (a_n) = a + (n – 1)d

It is given that a = 213,

d = -8

and a_n = 37

According to the question,

=> 37 = 213 + (n – 1)(-8)

=> 37 =213 – 8n+ 8

=> 8n = 221 – 37

=> an = 184

=> n=23 ….(1)

Therefore, total number of terms is 23.

Since, there is odd number of terms.

So, Middle term will be 23 + 12th term, i.e., the 12th term.

a_{12} =213 + (12 – 1)(-8)

a_{12} = 213 – 88

= 125

Thus, the middle term of the A.P. 213, 205, 197, . . . , 37 is 125.

Question 40. If the 5th term of the A.P. is 31 and 25th term is 140 more than the 5th term, find the A.P.

Solution: Let a be the first term and d be the common difference.

We know that, nth term $(a_n) = a + (n - 1)d$

According to question,

$a_6 = 31$

=> $a + (5 - 1) = 31$

=> $a + 4d = 31$

=> $a = 31 - 4d$ . . . .(1)Also, $a_{25} = 140 + a_5$

=> $a + (25 - 1) = 140 + 31$

=> $a + 24d = 171$ . . . . (3)

On substituting the values of (1) in (2), we get

$31 - 4d + 24d = 171$

=> $20d = 171 - 31$

=> $20d = 140$

=> $d = 7$

=> $a = 31 - 4 X 7$ [From (1)]

=> $a = 3$

Thus, the A.P. obtained is 3, 10 , 17, 24, . .