RD SHARMA

Solutions

Class 10 Maths

Chapter 15

Ex15.3

Q1. AB is a chord of a circle with center O and radius 4 cm. AB is of length 4 cm and divides the circle into two segments. Find the area of the minor segment.

Soln:

Given data:

Radius of the circle with center‘O’, r = 4 cm = OA = OB

Length of the chord AB = 4 cm

OAB is an equilateral triangle and angle AOB = 60° + θ

Angle subtended at centre θ = 60°

Area of the segment (shaded region) = (area of sector) – (area of triangleAOB)

= θ360 $\times\prod r^2 - \sqrt{3}4\,(\text{side})^2$ $\frac{\theta}{360} \times \prod r^2 - \frac{\sqrt{3}}{4}(\text{side})^2$

= 60360 $\times\prod 4^2 - \sqrt{3}4\,(4)^2$ $\frac{60}{360} \times \prod 4^2 - \frac{\sqrt{3}}{4}(4)^2$

On solving the above equation, we get,

= 58.67 – 6.92 = 51.75 cm2

Therefore, the required area of the segment is 51.75 cm2

Q2. A chord PQ of length 12 cm subtends an angle 120 at the center of a circle. Find the area of the minor segment cut off by the chord PQ.

Soln:

We know that,

Area of the segment = θ360 $\times\prod r^2 - \sqrt{3}4\,(\text{side})^2$ $\frac{\theta}{360} \times \prod r^2 - \frac{\sqrt{3}}{4}(\text{side})^2$

We have,

$\angle POQ = 120 \text{and} PQ = 12\text{cm}$ $\angle POQ = 120\text{and}PQ = 12\text{cm}$

PL = PQ x (0.5)

= 12 x 0.5 = 6 cm

Since, $\angle POQ \angle POQ$ = 120

$\angle POL \angle POL$ = $\angle QOL \angle QOL$ = 60

In triangle OPQ, we have

sinθ=PLOA $\sin\theta = \frac{PL}{OA}$,

sin 60° = 6OA $\frac{6}{OA}$,

OA = 12√3 $\frac{12}{\sqrt{3}}$

Thus ,OA = 12√3 $\frac{12}{\sqrt{3}}$

Now using the value of r and angle θ we will find the area of minor segment.

A=4{4π–3√3}cm² $A = 4\,\{4\pi - 3\sqrt{3}\}\ cm^2$.

Q 3. A chord of circle of radius 14 cm makes a right angle at the centre. Find the areas of minor and major segments of the circle.

Soln:

Given data:

Radius (r) = 14 cm

Angle subtended by the chord with the centre of the circle, θ = 90°

Area of minor segment (ANB) = (area of ANB sector) – (area of the triangle AOB)

= θ360 ×∏r² $\frac{\theta}{360} \times \prod r^2$ – 0.5 x OA x OB

= 90360 ×∏14² $\frac{90}{360} \times \prod 14^2$ – 0.5 x 14 x 14 = 154 – 98 = 56 cm2

Therefore the area of the minor segment (ANB) = 56 cm2

Area of the major segment (other than shaded) = area of circle – area of segment ANB

= ∏r²–56cm² $\prod r^2 - 56cm^2$

= 3.14 x 14 x 14 – 56 = 616 – 56 = 560 cm2

Therefore, the area of the major segment = 560 cm2.

Q 4. A chord 10 cm long is drawn in a circle whose radius is 5√2cm $5\sqrt{2}cm$. Find the area of both segments.

Soln:

Given data: Radius of the circle , r = $5\sqrt{2}cm 5\sqrt{2}cm$ = OA = OB

Length of the chord AB = 10cm

In triangle OAB , OA2 +OB2 = $(5\sqrt{2})^2+(5\sqrt{2})^2 (5\sqrt{2})^2+(5\sqrt{2})^2$ = 50 + 50 = 100 =(AB)2

Hence, pythogoras theorem is satisfied.

Therefore OAB is a right angle triangle.

Angle subtended by the chord with the centre of the circle, θ = 90°

Area of segment (minor) = shaded region = area of sector – area of triangle OAB

= $\theta 360 \times \prod r^2 \frac{\theta}{360} \times \prod r^2$ – 0.5 x OA x OB

= $90360 \times \prod (5\sqrt{2})^2 - 0.5x(5\sqrt{2})^2 x(5\sqrt{2})^2 \frac{90}{360} \times \prod (5\sqrt{2})^2 - 0.5x(5\sqrt{2})^2 x(5\sqrt{2})^2$

= $11007 - 1007 = 10007 cm^2 \frac{1100}{7} - \frac{100}{7} = \frac{1000}{7} cm^2$

Therefore, Area of segment (minor) = $10007 cm^2 \frac{1000}{7} cm^2$.

Q5. A chord AB of circle of radius 14 cm makes an angle of 60° at the centre. Find the area of the minor segment of the circle.

Soln:

Given data: radius of the circle (r) = 14 cm = OA = OB

Angle subtended by the chord with the centre of the circle, θ = 60°

In triangle AOB, angle A = angle B [angle opposite to equal sides OA and OB] = x

By angle sum property, $\angle A+\angle B+\angle O=180 \angle A+\angle B+\angle O=180$

X + X + 60° =180°

2X = 120°, X = 60°

All angles are 60°, triangle OAB is equilateral OA = OB = AB

= area of the segment (shaded region in the figure) = area of sector– area of triangle OAB

= $\theta 360 \times \prod r^2 - \sqrt{3}4(-AB)^2 \frac{\theta}{360} \times \prod r^2 - \frac{\sqrt{3}}{4}(-AB)^2$

On solving the above equation we get,

= 308–147√33 cm2 $\frac{308-147\sqrt{3}}{3}\text{cm}^2$

Therefore, area of the segment (shaded region in the figure) = 308–147√33 cm2 $\frac{308-147\sqrt{3}}{3}\text{cm}^2$.

Q 6. Ab is the diameter of a circle with centre 'O' . C is a point on the circumference such that ∠COB $\angle COB$ = θ. The area of the minor segmentcut off by AC is equal to twice the area of sector BOC. Prove that sinθ2.cosθ2 $\sin\frac{\theta}{2}.\cos\frac{\theta}{2}$ = ∏(12−θ120) $\prod(\frac{1}{2}-\frac{\theta}{120})$.

Soln:

Given data: AB is a diameter of circle with centre O,

Also, ∠COB $\angle COB$ = θ = Angle subtended

Area of sector BOC = θ360 ×∏r2 $\frac{\theta}{360}\times\prod r^2$

Area of segment cut off by AC = (area of sector) – (area of triangle AOC)

∠AOC $\angle AOC$ = 180 – θ ∠AOCand∠BOC $\angle AOC \text{and} \angle BOC$ from linear pair]

Area of sector = (180−θ)360 ×π×r2=π×r22−πθr2360 $\frac{(180-\theta)}{360}\times\pi\times r^2=\frac{\pi\times r^2}{2}-\frac{\pi\theta r^2}{360}$

In triangle AOC , drop a perpendicular AM , this bisects ∠AOC $\angle AOC$ and side AC.

Now, In triangle AMO, sin∠AOM=AMOA=sin(180−θ2)=AMr

$$\sin\angle AOM=\frac{AM}{OA}=\sin(\frac{180-\theta}{2})=\frac{AM}{r}$$

AM=rsin(90−θ2)=rcosθ2

$$AM=r\sin(90-\frac{\theta}{2})=r\cos\frac{\theta}{2}$$

cos∠AOM=OMOA=cos(90−θ2)=OMr⇒OM=rsinθ2

$$\cos\angle AOM=\frac{OM}{OA}=\cos(90-\frac{\theta}{2})=\frac{OM}{r}\Rightarrow OM=r\sin\frac{\theta}{2}$$

Area of segment= πr22−πθr2360−12(AC×OM)[AC=2AM]

$$\frac{\pi r^2}{2}-\frac{\pi\theta r^2}{360}-\frac{1}{2}(AC\times OM)[AC=2AM]$$

= πr22−πθr2360−12(2rcosθ2rsinθ2)=r2[π2−πθ360−cosθ2sinθ2]

$$\frac{\pi r^2}{2}-\frac{\pi\theta r^2}{360}-\frac{1}{2}(2r\cos\frac{\theta}{2}r\sin\frac{\theta}{2})=r^2[\frac{\pi}{2}-\frac{\pi\theta}{360}-\cos\frac{\theta}{2}\sin\frac{\theta}{2}]$$

Area of segment by AC = 2 (Area of sector BOC)

r2[π2−πθ360−cosθ2.sinθ2]=2r2[πθ360] $r^2[\frac{\pi}{2}-\frac{\pi\theta}{360}-\cos\frac{\theta}{2}.\sin\frac{\theta}{2}]=2r^2[\frac{\pi\theta}{360}]$

On solving the above equation we get,

cosθ2×sinθ2=π(12−θ120) $\cos\frac{\theta}{2} \times \sin\frac{\theta}{2} = \pi(\frac{1}{2} - \frac{\theta}{120})$

Hence proved that, cosθ2.sinθ2=π(12−θ120) $\cos\frac{\theta}{2}.\sin\frac{\theta}{2} = \pi(\frac{1}{2} - \frac{\theta}{120})$.

Q 7. A chord a circle subtends an angle θ at the center of the circle. The area of the minor segment cut off by the chord is one-eighth of the area of the circle. Prove that 8sinθ2.cosθ2+π=πθ45 $8\sin\frac{\theta}{2}.\cos\frac{\theta}{2} + \pi = \frac{\pi\theta}{45}$ **.**

Soln:

Let the area of the given circle be = r

We know that, area of a circle = π r2

AB is a chord, OA and OB are joined. Drop a OM such that it is perpendicular to AB, this OM bisects AB as well as ∠AOM $\angle AOM$

∠AOM=∠MOB=12(0)=θ2,AB=2AM $\angle AOM = \angle MOB = \frac{1}{2}(0) = \frac{\theta}{2}, AB = 2AM$

Area of segment cut off by AB = (area of sector) – (area of the triangle formed)

θ360×πr2−12×AB×OM=r2[πθ360]−12.2rsinθ2.cosθ2
$\frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times AB \times OM = r^2[\frac{\pi\theta}{360}] - \frac{1}{2}.2r\sin\frac{\theta}{2}.\cos\frac{\theta}{2}$

Area of segment = 18 $\frac{1}{8}$ (area of circle)

r2[πθ360−sinθ2.cosθ2]=18πr2 $r^2[\frac{\pi\theta}{360} - \sin\frac{\theta}{2}.\cos\frac{\theta}{2}] = \frac{1}{8}\pi r^2$

On solving the above equation we get,

8sinθ2.cosθ2+π=πθ45 $8\sin\frac{\theta}{2}.\cos\frac{\theta}{2} + \pi = \frac{\pi\theta}{45}$

Hence proved, 8sinθ2.cosθ2+π=πθ45 $8\sin\frac{\theta}{2}.\cos\frac{\theta}{2} + \pi = \frac{\pi\theta}{45}$.