RD SHARMA
Solutions
Class 10 Maths
Chapter 14
Ex *14.5*

1. Find the area of a triangle whose vertices are

(i) $(6,3),(-3,5)$ and $(4,-2)$

(ii) $\left[\left(at_1^2, 2at_1\right),\left(at_2^2, 2at_2\right)\left(at_3^2, 2at_3\right)\right]$

(iii) $(a, c+a),(a, c)$ *and* $(-a, c-a)$

Sol:

(i) Area of a triangle is given by

$$\frac{1}{2}\left[x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1+y_2\right)\right]$$

Here, $x_1=6, y_1=3, x_2=-3, y_2=5, x_3=4, y_3=-2$

Let $A(6,3), B(-3,5)$ and $C(4,-2)$ be the given points

Area of $\Delta ABC=\frac{1}{2}\left[6(5+2)+(-3)(-2-3)+4(3-5)\right]$

$=\frac{1}{2}\left[6\times 7-3\times(-5)+4(-2)\right]$

$=\frac{1}{2}[42+15-8]$

$=\frac{49}{2}\, sq.units$

(ii) Let $A=(x_1,y_1)=\left(at_1^2,2at_1\right)$

$B=(x_2,y_2)=\left(at_2^2,2at_2\right)$

$=(x_3,y_3)=\left(at_3^2,2at_3\right)$ be the given points.

The area of ΔABC

$$=\frac{1}{2}\left[at_1^2\left(2at_2-2at_3\right)+at_2^2\left(2at_3-2at_1\right)+at_3^2\left(2at_1-2at_2\right)\right]$$

$$=\frac{1}{2}\left[2a^2t_1^2t_2-2a^2t_1^2t_3+2a^2t_2^2t_3-2a^2t_2^2t_1+2a^2t_3^2t_1-2a^2t_3^2t_2\right]$$

$$=\frac{1}{2}\times 2\left[a^2t_1^2\left(t_2-t_3\right)+a^2t_2^2\left(t_3-t_1\right)+a^2+t_3^2\left(t_1-t_2\right)\right]$$

$$=a^2\left[t_1^2\left(t_2-t_3\right)+t_2^2\left(t_3-t_1\right)+t_3^2\left(t_1-t_2\right)\right]$$

(iii) Let $A=(x_1,y_1)=(a,c+a)$

$B=(x_2,y_2)=(a,c)$

$C=(x_3,y_3)=(-a,c-a)$ be the given points

The area of ΔABC

$$=\frac{1}{2}\left[a\left(c-\{c-a\}\right)+a\left(c-a-(c+a)\right)+(-a)(c+a-a)\right]$$

$$=\frac{1}{2}\left[a\left(c-c+a\right)+a\left(c-a-c-a\right)-a\left(c+a-c\right)\right]$$

$$=\frac{1}{2}\left[a\times a+ax(-2a)-a\times a\right]$$

$$=\frac{1}{2}\left[a^2-2a^2-a^2\right]$$

$$=\frac{1}{2}\times(-2a)^2$$

$$=-a^2$$

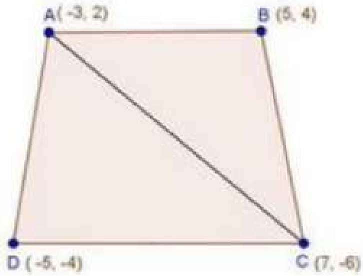
2. Find the area of the quadrilaterals, the coordinates of whose vertices are

(i) (—3, 2), (5, 4), (7, — 6) and (—5, — 4)

(ii) (1, 2), (6, 2), (5, 3) and (3, 4)

(iii) (—4, — 2), (—3, — 5), (3, — 2), (2, 3)

Sol:



Let $A(-3,2), B(5,4), C(7,-6)$ and $D(-5,-4)$ be the given points.

Area of ΔABC

$= \frac{1}{2}\left[-3(4+6)+5(-6-2)+7(2-4)\right]$

$= \frac{1}{2}\left[-3\times 10+5\times(-8)+7(-2)\right]$

$= \frac{1}{2}[-30-40-14]$

$= -42$

But area cannot be negative

$\therefore$ Area of $\Delta ADC = 42$ square units

Area of ΔADC

$$=\frac{1}{2}\left[-3(-6+4)+7(-4-2)+(-5)(2+6)\right]$$

$$=\frac{1}{2}\left[-3(-2)+7(-6)-5\times 8\right]$$

$$=\frac{1}{2}[6-42-40]$$

$$=\frac{1}{2}\times -76$$

$$=-38$$

But area cannot be negative

$\therefore$ Area of $\Delta ADC=38$ square units

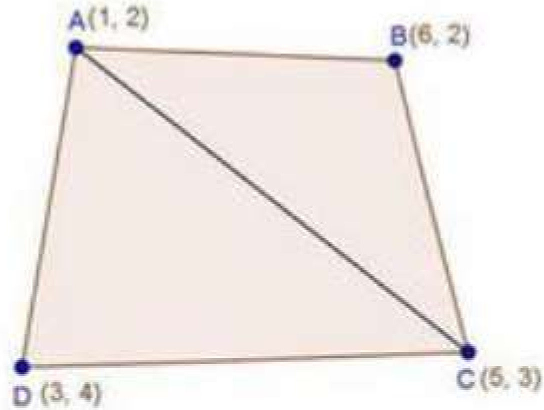
Now, area of quadrilateral $ABCD$

$= Ar.\ of\ ABC + Ar\ of\ ADC$

$=(42+38)$

$=80$ square. units

(i)



Let $A(1,2), B(6,2), C(5,3)$ and $(3,4)$ be the given points

Area of ΔABC

$$=\frac{1}{2}\left[1(2-3)+6(3-2)+5(2-2)\right]$$

$$=\frac{1}{2}\left[-1+6\times(1)+0\right]$$

$$=\frac{1}{2}[-1+6]$$

$$=\frac{5}{2}$$

Area of ΔADC

$$=\frac{1}{2}\left[1(3-4)+5(4-2)+3(2-3)\right]$$

$$=\frac{1}{2}\left[-1\times 5\times 2+3(-1)\right]$$

$=\frac{1}{2}[-1+10-3]$

$=\frac{1}{2}[6]$

$=3$

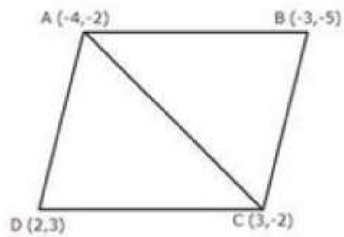
Now, Area of quadrilateral $ABCD$

$=$ Area of $ABC+$ Area of ADC

$=\left(\frac{5}{2}+3\right) sq.\ units$

$=\frac{11}{2}\ sq.\ units$

(ii)



Let $A(-4,-2), B(-3,-5), C(3,-2)$ and $D(2,3)$ be the given points

Area of $\Delta ABC=\frac{1}{2}|(-4)(-5+2)-3(-2+2)+3(-2+5)|$

$=\frac{1}{2}|(-4)(-3)-3(0)+3(3)|$

$=\frac{21}{2}$

Area of $\Delta ACD=\frac{1}{2}|(-4)(3+2)+2(-2+2)+3(-2-3)|$

$=\frac{1}{2}|-4(5)+2(0)+3(-5)|=\frac{-35}{2}$

But area can't be negative, hence area of $\Delta ADC=\frac{35}{2}$

Now, area of quadrilateral $(ABCD)=ar(\Delta ABC)+ar(\Delta ADC)$

Area (quadrilateral $ABCD$) $=\frac{21}{2}+\frac{35}{2}$

Area (quadrilateral $ABCD$) $=\frac{56}{2}$

Area (quadrilateral $ABCD$) $=28$ square. Units